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PRESENTATION CONTENTS

ABSTRACT ORAL PRESENTATIONS

Statistical Evaluation of Relationships Between Water Quality Parameters of Güzelsu (Engil) Stream In Lake Van Basin (965)

Aysenur Gümüş[1], Adnan Aldemir[1,2*]

[1]Van Yüzüncü Yıl University, Faculty of Engineering, Department of Chemical Engineering, 65080, Van, Turkey

[2]Van Yüzüncü Yıl University, Faculty of Engineering, Department of Industrial Engineering, 65080, Van, Turkey

*Corresponding author e-mail: adnanaldemir@yyu.edu.tr

Abstract

A surface water quality (SWQ) that describes the physical, chemical, and biological transport and degradation processes of pollutants and their impact on aquatic ecosystems can support policy decision-making for environmental management. Lake Van which is the Turkey's largest lake is the system encompasses lakes, ponds, rivers and streams in a closed basin with water from many different sources. The obtained data in this study were used to perform water quality analysis based on the evaluated parameters from Güzelsu (Engil) Stream. A total of 22 water quality parameters were used to determine water quality at the discharge into Lake Van. Monitoring sampling and analysis of Güzelsu (Engil) Stream were carried out by the Regional Directorate of State Hydraulic Works (Turkey). These variables were evaluated with Turkish Surface Water Quality Regulation (TSWQR, 2015). As a result, Güzelsu (Engil) Stream has Class I water quality in terms of seasonal conditions and water parameters. Among the parametric analysis methods, trend distribution, normality, correlation, matrix table, regression and normal distribution of the data set were examined and the relationships between parameters were interpreted statistically. According to the results, most of the parameters were within the normal range, solid matter, alkalinity and color effects were correlated, and matrix relations with regression equations were related to the other parameters.

Keywords: *Van Lake basin, Güzelsu (Engil) stream, Water quality, Regression, Correlation*

Role of Alternative energy in the management of Energy crisis in Pakistan: A Statistical Analysis (966)

Jalib Ali Baig[1], Aqsa Zafar[1], Aleesha Malik[1], Farooq Ali Awan[1], Muzzamil Hussain[1], Danish Hassan[1*]

[1]Department of Textile & Clothing, National Textile University Karachi Campus

*Corresponding author e-mail: danish10ansari@gmail.com

Abstract

This study focuses on the analysis of solar energy data collected from five meteorological stations across Pakistan: Islamabad, Karachi, Lahore, Bahawalpur, and Hyderabad. The dataset comprises 96,933 observations recorded at 10-minute intervals from October 16, 2014, to December 22, 2016. The variables under investigation include Barometric Pressure (BP), Relative Humidity (RH), Integrated Circuit Maximum (IC Max), Error Maximum, Logger Voltage, and Logger Temperature. Descriptive statistical techniques, such as histograms, box plots, and normal probability plots, were employed to analyze the data. Additionally, advanced statistical methods, including measures of central tendency, coefficient of variation, variance, range, kurtosis, skewness, quartiles, and Multiple Linear Regression (MLR), were applied to derive meaningful insights. The findings reveal that the MLR model exhibits significant predictive accuracy for Karachi, with its effectiveness diminishing as the distance from Karachi to other cities increases. This suggests a spatial dependency in the applicability of the model. The primary objective of this research is to highlight the potential of solar energy as a viable and cost-effective alternative to conventional electricity sources. By raising awareness of its benefits, this study aims to encourage households to adopt solar energy solutions, particularly in regions experiencing persistent electricity shortages. The results underscore the feasibility of solar energy as a sustainable and economical option for residential energy needs.

Keywords: *IC Max, Error Max, Logger voltage, Logger temperature, Relative Humidity, Alternative energy, Regression analysis.*

Moment-Based Approximation for Variance of Residual Waiting Time Process (1014)

Tahir Khaniyev[1*], Tulay Yazır[2,3], Aynur Coban[1]

[1]TOBB University of Economics and Technology, Faculty of Engineering, Department of Industrial Engineering, Turkey
[2]Karadeniz Technical University, Faculty of Science, Department of Mathematics, Turkey
[3]Ankara Yıldırım Beyazıt University, Faculty of Engineering and Natural Sciences, Department of Mathematics, Turkey

*Corresponding author e-mail: tahirkhaniyev@etu.edu.tr

Abstract

The residual waiting time process plays an important role in solving many problems that arise in reliability, queuing and inventory control theory. Let $W(t)$ *be a residual waiting time process generated by a sequence of independent, positively distributed random variables. In various applications, especially in service systems, transportation and communication networks, the calculation of the expected value and variance of the residual waiting time process plays a critical role. In the literature, one-term asymptotic expansions for the expected value and variance of the residual waiting time process are available. However, these expansions may not yield effective results for values of the parameter* t *that are not large enough. For this purpose, in this study, the following approximated formulas were obtained for the expected value and variance of residual waiting time process by using Kambo's moment-based approximation method:* $E\big(W(t)\big) \approx A + B\exp(-kt)$; $Var\big(W(t)\big) \approx C + (Dt + E)\exp(-kt)$. *Here; A, B, C, D, E and* k *are constants expressed by the first three moments of the random variable* X*.Unlike similar expansions in the literature, the proposed approximated formulas for expected value and variance of residual waiting time process provide sufficiently accurate approximations even small values of the parameter* t*. Additionally, the precision of the obtained results was tested on specific examples.*

Keywords: *Residual Waiting Time Process, Moment-Based Approximation, Renewal Process*

Comprehensive Advances in Stability Analysis for Crop Breeding: Univariate, Multivariate, and Multi-Trait Innovations (1015)

Hossein Zeinalzadeh-Tabrizi[1*]

[1]Kyrgyz-Turkish Manas University, Faculty of Agriculture, Department of Horticulture and Agronomy, Kyrgyzstan

*Corresponding author e-mail: h.zeynelzade@manas.edu.kg

Abstract

Stability analysis is essential in crop breeding to assess genotypic performance and adaptability across diverse environments, ensuring resilience amid climate variability. This review synthesizes traditional and cutting-edge stability analysis methods, integrating univariate, multivariate, and novel multi-trait approaches into a cohesive framework. Traditional univariate methods, such as Eberhart and Russell's regression, Wricke's ecovalence, Shukla's stability variance, and the coefficient of variation, offer straightforward evaluations of genotype-by-environment (G×E) interactions for single traits but falter in capturing multi-trait complexity. Multivariate techniques, including Additive Main Effects and Multiplicative Interaction (AMMI), Genotype plus Genotype-by-Environment (GGE) biplot, and Principal Component Analysis (PCA), provide robust tools to dissect G×E interactions across multiple traits, with AMMI revealing stability patterns and GGE biplot visualizing genotypic-environmental dynamics. Emerging methods further enhance this landscape: the Multi-Trait Stability Index (MTSI) integrates stability and multi-trait performance using factor analysis to prioritize ideotypes, while the Weighted Average of Stability and Biological Index (WASBI) combines stability with biological yield. Additionally, indices are categorized into five key approaches: univariate performance selection index (UPSI, e.g., mean performance), univariate stability selection indices (USSIs, e.g., environmental variance, AMMI stability value [ASV], modified AMMI stability value [MASV], modified AMMI stability index [MASI], and weighted average of absolute scores [WAASB]), multivariate performance selection indices (MPSIs, e.g., genotype by trait [GT], genotype by yield × trait [GYT], and ideotype-design index [FAI-BLUP]), univariate performance and stability selection indices (UPSSIs, e.g., harmonic mean of genotypic value [HMGV], relative performance of genotypic value [RPGV], and WAASBY), and multivariate performance and stability selection index (MPSSI, e.g., MTSI). Supported by high-throughput phenotyping and computational advances, these methods—including WAASB and its derivative WAASBY, which fuse AMMI and BLUP—enable precise selection of stable, high-performing genotypes. This review elucidates their strengths, limitations, and synergies, emphasizing their role in addressing food security. Future prospects, such as genomic integration and machine learning, promise to further refine stability analysis in crop breeding.

Keywords: *Stability Analysis, Crop Breeding, Genotype-by-Environment Interaction (G×E), Multi-Trait Stability Index (MTSI), Weighted Average of Absolute Scores (WAASB)*

Psychological Approaches in Chemistry Education (1019)

Gulnara Duruskari[1], Gulbadam Akbarova[1*], Khalil Nagiyev[1]

[1]Baku State University, Azerbaijan

*Corresponding author e-mail: gduruskari@mail.ru

Abstract

Emphasizing the significance of the subject, teaching and fostering an interest in chemistry, and developing students' analytical thinking and problem-solving skills related to real-life issues are key priorities in education. Discussing issues such as the environmental problems created by the field of chemistry and the effects of pharmaceutical products on the human body fosters the formation of values such as patriotism, responsibility, humanism, and goal-orientedness in students. To achieve this, the initial phase involves familiarizing students with the problem, and developing skills in proper research and presentation is essential in the teaching process. Real-life events, such as chemical accidents and environmental disasters, provide students with an opportunity to think more deeply about the practical application of chemistry. The effectiveness of presentations, alongside theoretical knowledge, enhances students' thinking abilities and their capacity to solve real-life problems. This approach creates an environment where students can develop the skills to analyze and solve contemporary problems.

Keywords: *Teaching and fostering interest in chemistry, Environmental problems, Analytical thinking, Patriotism, Real-life issues, Humanism, Responsibility, Contemporary problem analysis and solution*

Machine Learning-Based Approaches to Identify Key Drivers and Predict Severe Food Insecurity in Africa (1020)

Mesfin M. Ayalew*[1], Zelalem G. Dessie[1,2], Aweke A. Mitiku[1,3], Temesgen Zewotir[2]

[1]Department of Statistics, Bahir Dar University, P.O.Box 79, Bahir Dar, Ethiopia
[2]School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal, Durban, South Africa
[3]Global Change Institute, University of the Witwatersrand, Johannesburg, South Africa

*Corresponding author e-mail: m2mulu@gmail.com

Abstract

Severe food insecurity has been a major global challenge, particularly in low- and middle-income countries. Extracting knowledge and discovering insights from hidden patterns in food insecurity data through machine learning algorithms is limited. Therefore, this study employed machine learning regression algorithms, including Linear Regression, Ridge, LASSO, Elastic Net, Support Vector, Decision Tree, Random Forest, Gradient Boosting, XGBoosting, K-Nearest Neighbors (KNN), and Artificial Neural Networks (ANN) to investigate the relationships between key drivers and severe food insecurity across Africa countries from 2015 to 2021. Features were scaled using Min-Max Normalization before training the models. After spot-checking algorithms, hyperparameter tuning via grid search showed improved model performance, with XGBoosting outperforming other models. For this optimal algorithm, the permutation importance technique ranked feature importance, revealing that features had varying degrees of contributions to severe food insecurity. Through recursive feature elimination (RFE), the study identified the key drivers contributing to severe food insecurity across African countries, including inflation, climate change, unemployment, malaria incidence, GHG emissions, caloric loss, cereal import dependency, livestock production, crop production, investment inflow, energy supply, protein supply, electricity access, GDP per capita, and political stability. However, permutation importance does not provide direct insights into the direction of the relationships (positive or negative) between features and severe food insecurity. Hence, we employed SHapley Additive exPlanations (SHAP) methods to determine the direction of the relationships. Based on SHAP analysis, key features positively influencing severe food insecurity include inflation, climate change, unemployment, malaria incidence, GHG emissions, caloric loss, and cereal import dependency. Conversely, features such as livestock production, cereal crop production, investment inflows, energy supply, protein supply, electricity access, GDP per capita, and political stability are negatively associated with severe food insecurity. Considering these findings, we recommend policymakers prioritize a strong emphasis on these critical drivers when developing and implementing national prevention and intervention strategies.

Keywords: *Severe food insecurity, Machine Learning Algorithms, XGBoost algorithm, Feature Selection*

Data Science and Statistics in Turkey (1022)

Aliye Nigar Çokgezer[1*]

[1]Economics And Administrative Sciences, University Balkan University, Department Of Phd Economics, Skopje, North Macedonia

*Corresponding author e-mail: aliyecokgezer1@gmail.com

Abstract

In today's age, where the size and diversity of data are abundant, interest in data and statistics concepts, which play a big role in planning institutions, initiatives and the future, has increased in the recent period. Especially when we examine artificial intelligence, engineering, machinery and economics in the main departments, the progress in the latest technology, obtaining information and comments through data become popular, reaching definite results, increasing the interest of people and institutions on statistics and data, leading to the trainings in these departments. This orientation has increased the preference of the data scientist (data-scientist) profession over time, and in 2024 it has entered the most sought-after professions in the job market, which is 47.4%. Data scientists are interested in developing strategies for data analysis, preparing data analysis, enabling discovery, analyzing and visualizing data, creating models with data using programming languages such as Python and R, and implementing models. Data scientists are responsible for examining the complex data emerging in technology, troubleshooting problems and answering questions. The aim of this study is to contribute to the studies by drawing attention to the general field by presenting information about the data scientist profession in Turkey, taking into account what kind of fields they do business in, what they solve, what is statistics and their relationship with the graduates from this department .

Keywords*: Statistics, Data Scientist, Data Science*

Optimizing Diagnostic Accuracy with Weibull ROC curves under Measurement Errors (1026)

Danisiri Tanuja[1], Siva G[1*]

[1]Department of Mathematics, VIT-AP University, Amaravati, Andhra Pradesh, India – 522 241

*Corresponding author e-mail: siva.g@vitap.ac.in

Abstract

Accurate evaluation of the performance of diagnostic markers is essential in the medical field, as these markers play a key role in distinguishing between healthy and diseased populations. Nevertheless, measurement errors often affect these markers, leading to inaccurate results of diagnostic tests. One of the most widely used metrics for evaluating diagnostic accuracy is the Receiver Operating Characteristic (ROC) curve, which provides a true discriminative ability of diagnostic markers and classifiers. In the context of ROC analysis, the area under the curve (AUC) will get contaminated when such measurement errors are present in the data, resulting in bias and data variability. Most existing methodologies focused on estimating the accuracy of an ROC curve under the assumption of normality when the data is contaminated with measurement errors. However, we frequently encounter data that exhibit non-normal characteristics in real-life situations, making the existing methods inadequate. Hence, in this paper, we addressed the problem of estimating the accuracy of an ROC curve in the presence of measurement errors under a non-normal framework. For the study, Weibull distribution is considered, and a bias-corrected estimator is derived to account for measurement errors in ROC analysis. The proposed methodology is validated through extensive simulation studies and real datasets. Results show that the bias-corrected estimator performs well even when the data is affected by measurement errors and enhances the accuracy while minimizing bias and mean square error.

Keywords: *Bias-corrected approximation, Area under the curve, Weibull ROC curve, Measurement error, Bias, and Mean squared error.*

Optimizing Classification Accuracy in Medical and Industrial Applications Using Lomax ROC Analysis (1027)

Danisiri Tanuja, Siva G[1*]

[1]Department of Mathematics, VIT-AP University, Amaravati, Andhra Pradesh, India-522 241

*Corresponding author e-mail: siva.g@vitap.ac.in

Abstract

The area under the Receiver Operating Characteristic (ROC) curve is a fundamental metric that quantifies the discriminative ability of diagnostic markers and classifiers, particularly in distinguishing between diseased and non-diseased individuals. In medical diagnostics, these markers are often affected by measurement errors, leading to a significant impact on the accuracy of ROC analysis. The existing literature primarily focuses on estimating the accuracy of an ROC curve in the presence of measurement error under the assumption of normality. In reallife scenarios, the data often deviate from normality and exhibit non-normal data characteristics. Traditional approaches may fail to adequately address the issue of non-normal measurement errors. This paper highlights the problem of estimating the area under the curve (AUC) in the presence of measurement errors under the assumption of a non-normal distribution. For this study, we considered the Lomax distribution a heavy-tailed model, which is well suited for nonnormal data. To address the measurement errors in ROC analysis, a bias-corrected estimator is derived and validated through extensive simulation studies and real datasets. Results show that the proposed bias-corrected estimator helps in achieving true accuracy with minimum bias and minimum mean square error.

Keywords: *Lomax ROC curve, Bias-corrected estimator, Measurement error, Mean squared error, Area under the curve.*

In Silico Analysis of Hydrolytic Enzymes Produced by Particular Rumen Microorganisms (1028)

Gülşah Keklik[1*], Bahri Devrim Özcan[1], Özgür Cem Erkin[2]

[1]University of Çukurova, Faculty of Agriculture, Department of Animal Science, Adana
[2]University of Adana Alparslan Türkeş Science and Technology, Faculty of Engineering, Department of Bioengineering, Adana

*Corresponding author e-mail: gulsahkeklk@gmail.com

Abstract

Since their domestication, ruminants have served humans in almost every region of the world, providing both labor and various productive traits. Ruminants are distinguished from other animals by the fact that they host symbiotic rumen microorganisms colonized in their stomach, consisting of four compartments, which contribute to the digestion of roughages with high cellulose content by the enzymes they produce. In addition to enzymes, rumen microorganisms also produce vitamins, metabolites, and other bioactive compounds that provide additional benefits to the host animal. Nowadays, when microbiota, also known as microbiome, is popular, it is very important to investigate all these products produced by rumen microorganisms, especially enzymes, in addition to rumen microbial populations, for an economic and sustainable livestock production. Nowadays, when molecular genetics is in its golden age, studies on the identification of these microorganisms and the enzymes they produce at the molecular level have gained momentum. The analysis of data obtained from such studies using bioinformatics tools and the detailed modelling of relationships among these elements provides further insights into molecular genetics research. In this study, in silico analysis of hydrolytic enzymes such as cellulase and xylanase produced by some rumen microorganisms was performed using some bioinformatics databases including UniProt, AlphaFold, String, Expasy, ProtParam and ProtScale.

Keywords: *Rumen microorganisms, Cellulase, Xylanase, Bioinformatics tools, In silico analysis*

Financial Network Modeling with Graph Convolutional Networks: A Case Study on BIST 30 (1029)

Cemal Öztürk[1]

[1]Pamukkale University, Faculty of Economics and Administrative Science, Department of Economics, Türkiye

*Corresponding author e-mail: cemalo@pau.edu.tr

Abstract

Understanding stock market dynamics is essential for effective risk management and investment decision-making. Traditional financial models often fail to capture the intricate relationships between stocks, which can lead to suboptimal portfolio strategies. This study investigates using a Hybrid Graph Convolutional Network (Hybrid GCN) to model stock price relationships within the Borsa İstanbul 30 (BIST 30) index. By leveraging graph-based deep learning, the aim is to enhance financial network analysis and construct a more robust, data-driven investment framework that improves portfolio performance. Historical stock price data from Yahoo Finance, covering a decade, was utilized to achieve this. A correlation matrix of stock returns was computed to establish an adjacency structure, where edges indicate significant relationships between stocks. The resulting financial network was processed using NetworkX and formatted for PyTorch Geometric. A Hybrid GCN model was trained to optimize portfolio allocation by learning stock dependencies and price movements. The model's performance was benchmarked against three traditional portfolio allocation strategies: Minimum Variance Portfolio, Risk Parity Portfolio, and Market Capitalization-Weighted Portfolio, which are widely used risk-based approaches in financial markets. Results indicate that the Hybrid GCN model significantly outperforms the benchmark portfolios concerning risk-adjusted returns and overall profitability. The annual Sharpe ratio of the Hybrid GCN model was 1.0047, notably higher than the benchmark strategies, where the Minimum Variance and Risk Parity portfolios had a Sharpe ratio of 0.3830, and the Market Cap-weighted portfolio had 0.3393. The Hybrid GCN model achieved a cumulative return of 1.0938, outperforming the benchmark portfolios, which yielded 1.0260 for Minimum Variance and Risk Parity and 1.0217 for the Market Cap-weighted portfolio. Furthermore, the maximum drawdown (MDD) of the Hybrid GCN model was -7.94%, demonstrating improved downside risk control compared to the benchmark models (-9.02% for Minimum Variance and Risk Parity and -9.05% for the Market Cap-weighted portfolio). The Sortino ratio, which measures risk-adjusted returns relative to downside volatility, was 1.8342 for the Hybrid GCN, significantly higher than 0.6835 for Minimum Variance and Risk Parity and 0.5978 for the Market Cap-weighted portfolio. These findings suggest that a Hybrid GCN-based financial model can enhance portfolio performance by better-capturing stock dependencies and optimizing risk-adjusted returns. This research introduces a novel application of Hybrid Graph Convolutional Networks (Hybrid GCN) in financial modeling, demonstrating their potential to optimize portfolio allocation beyond traditional approaches. This study presents a scalable and adaptive financial forecasting and portfolio construction strategy by integrating graph-based learning. Future research could explore extending this model to broader stock indices and alternative datasets, further validating its applicability in real-world investment strategies.

Keywords: *Graph Convolutional Networks, Portfolio Optimization, Financial Network Analysis, Stock Market Prediction, Deep Learning*

Hybrid Portfolio Optimization Using Fuzzy Logic and Genetic Algorithms: A Comparative Study (1030)

Cemal Öztürk[1]

[1]Pamukkale University, Faculty of Economics and Administrative Science, Department of Economics, Türkiye

*Corresponding author e-mail: cemalo@pau.edu.tr

Abstract

This research evaluates the potential of using a Fuzzy Logic-based Genetic Algorithm (Fuzzy-GA) as an alternative to traditional portfolio optimization methods such as Markowitz, Minimum Variance, and Equal Weight. The goal is to create more balanced and efficient investment portfolios by integrating multi-criteria decision-making using fuzzy logic. We analyzed the daily closing prices of the 30 large-cap companies listed in the Dow Jones Industrial Average (DJIA) spanning from January 1, 2020, to March 10, 2025. The optimization process utilized daily logarithmic returns as input data. The Fuzzy-GA model incorporates genetic algorithm core mechanics for individual generation and crossover and mutation but includes a fuzzy inference system to evaluate portfolios. This system evaluates risk and return and diversification using fuzzy rules to give each portfolio a fitness score. This method allows for multi-objective optimization while avoiding weight combinations which makes decisions more practical. According to our findings, the Fuzzy-GA model produced the highest annual return of 12% and kept risk at a relatively low level of 15%. The combination produced the best Sharpe ratio at 1.20. The model delivered strong results in terms of maximum drawdown at 25% while attaining the highest diversification score at 1.5. The model received the highest fuzzy score (0.85) which indicated its balanced nature. The Markowitz portfolio generated a 10% return with 18% risk and achieved a Sharpe ratio of 1.00. The Equal Weight strategy generated a 9% return with 20% risk and produced a Sharpe ratio of 0.80. The Minimum Variance strategy generated the minimum risk level of 12% but produced the smallest return rate of 7%. The study demonstrates how evolutionary algorithms with fuzzy logic integration provide new solutions for managing investment challenges in real-world market conditions. Fuzzy-GA proves to be a valuable instrument for investors because it generates both significant profits and sustainable risk management, especially when markets remain unclear. Future research could explore how this method performs under varying market conditions and further investigate the potential of adaptive fuzzy rule systems to enhance their flexibility and robustness.

Keywords: *Fuzzy Logic, Genetic Algorithm, Hybrid Optimization, Portfolio Management, Multi-Criteria Decision Making*

On the Moments of the Random Walk Process with General Interference of Chance (1031)

Tahir Khaniyev[1*], Zulfiye Hanaliogu[2], Ozlem Sevinc[3]

[1]TOBB University of Economics and Technology, Faculty of Engineering, Department of Industrial Engineering, Turkey
[2]Karabuk University, Faculty of Business, Department of Actuarial Sciences, Turkey
[3]Central Bank of the Republic of Türkiye, Department of Data Governance and Statistics, Turkey

*Corresponding author e-mail: tahirkhaniyev@etu.edu.tr

Abstract

In this study, a random walk process (X(t)) with general interference of chance is considered and the ergodic moments of the process are investigated. There are some special studies in the literature on this subject. Unlike the studies in the literature, random walk process with general interference of chance, which covers a wide class including many special cases, was examined in this study. In other words, the study aims to combine many special cases in the literature under the same mathematical structure. In this study, it was proven that the process considered is ergodic under some conditions and the exact form of the ergodic distribution of the process is found. Then, the characteristic function of the ergodic distribution of the process is expressed by a boundary functional and by using this expression, the exact form of the first four moments of the ergodic distribution of the process X(t) is found. However, it has been observed that the exact formulas found are not useful in practice due to their complex mathematical structure. Therefore, the moments of the ergodic distribution of the process X(t) are expressed by the first ladder height of the random walk process, making use of the Dynkin's principle. Then, the three-term asymptotic expansions are obtained for the first two ergodic moments of the process. Finally, using the obtained results, two-term asymptotic expansions for the variance, standard deviation and coefficient of variation of the process are obtained. When the obtained asymptotic expansions are examined, it is observed that the moments of the ergodic distribution of the process are very close to the moments of the residual waiting time generated by the sequence of random variables expressing the interference of chance.

Keywords: *Random Walk Process, General Interference of Chance, Ergodic Moments, Dynkin's Principle*

Backpropagation Is Not All You Need (1033)

Selçuk Tekgöz[1*]

[1] Niğde Provincial Health Directorate, Statistics Unit, 51300, Niğde, Turkey, selcuk.tekgoz@saglik.gov.tr, ORCID NO: 0009-0007-2074-4276

*Corresponding author e-mail: selcuk.tekgoz@saglik.gov.tr

Abstract

Artificial neural networks play an important role in the field of artificial intelligence by imitating the functioning of biological nervous systems. These networks have the ability to effectively solve challenges such as classification and regression by extracting meaningful patterns from data sets. However, the performance of the networks mostly depends on the correct tuning of hyperparameters such as hidden layers, activation functions and learning rates. However, problems such as computational costs, long training times and local minima encountered in deep learning networks reveal the limitations of traditional artificial neural networks. In addition, due to the use of hidden layer structures, the "black box" problem occurs because the decision-making processes are not transparent, which reduces the reliability in some critical areas such as health and finance.

This study aims to enhance the interpretability of artificial neural networks by introducing the SingleShotClassifier model, which we developed to address the aforementioned limitations and implemented using the Python programming language.The model is designed based on the concept of Long-Term Potentiation (LTP), which plays an important role in neural network learning processes, and does not require traditional backpropagation methods. Since this innovative approach does not use hidden layers, it eliminates the "black box" structure of traditional neural networks and makes decision-making mechanisms completely transparent. For this reason, it contributes to solving ethical and security problems in the field of artificial intelligence. In addition, since no derivatives are used, it does not contain common problems such as gradient vanishing and exploding. In addition to some activation functions used in the literature, a total of 46 different activation functions developed by us have been used to produce approximately 10,000 new activation functions, and the result is achieved with high efficiency by utilizing only a single forward propagation process. Additionally, even in the absence of any activation function, comparable levels of high accuracy can still be achieved.

Widely used datasets in the literature, such as the Iris, Breast Cancer and Heart Disease datasets, were utilized to evaluate the proposed SingleShotClassifier algorithm in comparison with conventional machine learning models, including K-Nearest Neighbors (KNN), Support Vector Machine (SVM), Naive Bayes (NB), and Multi-Layer Perceptron (MLP). A 5-fold cross-validation approach was employed for all models to ensure a robust evaluation.

Based on the analysis results, the average accuracy scores obtained on the Iris dataset are as follows: SingleShotClassifier 0.993, K-Nearest Neighbors (KNN) 0.973, Support Vector Machines (SVM) 0.967, Naive Bayes (NB) 0.953, and Multilayer Perceptron (MLP) 0.980. For the Breast Cancer dataset, the calculated average accuracy values are SingleShotClassifier 0.944, KNN 0.928, SVM 0.912, NB 0.939, and MLP 0.931. Similarly, for the Heart Disease dataset, the computed average accuracy scores are SingleShotClassifier 0.848, KNN 0.747, SVM 0.703, NB 0.821, and MLP 0.820.

These findings demonstrate that the proposed SingleShotClassifier method consistently outperforms traditional classification algorithms in terms of accuracy across all tested datasets. Moreover, the results

are in alignment with additional experimental analyses conducted on various datasets, further confirming the stability and high overall performance of the proposed approach. The SingleShotClassifier not only achieves high classification accuracy but also provides significant advantages in terms of model interpretability. In this regard, the proposed method constitutes a meaningful contribution and a novel approach for neural network-based architectures.

Keywords: *Artificial Neural Networks, Explainable Artificial Intelligence, Long-Term Potentiation, Backpropagation*

Weighting and Ranking via Multi-Criteria Decision-Making Methods for a Classification Problem in Technological Investment Area (1035)

Gözde Ulu Metin[1*], Özlem Türkşen[2]

[1]ASELSAN A.Ş., Ankara, Türkiye
[2]Ankara University, Faculty of Science, Department of Statistics, Türkiye

*Corresponding author: e-mail: gozdeulumetin@gmail.com

Abstract

Multi-Criteria Decision-Making (MCDM) methods are commonly integrated with data mining methods to address real-world classification problems. The pre-processing stage of data plays a crucial role in enabling accurate decision-making. Each feature of the data may have different degrees of importance, which are called as weights. MCDM methods are employed to determine these feature weights. In this study, Analytic Hierarchy Process (AHP), CRiteria Importance Through Intercriteria Correlation (CRITIC) and ENTROPY methods are used for feature weighting. Additionally, data mining classification methods such as Logistic Regression (LR), k-Nearest Neighbors (kNN), Support Vector Machines (SVM) and Random Forest (RF) are applied to the weighted dataset for classification purposes. Several performance metrics are used to evaluate the classification methods. A decision matrix is constructed based on these performance metrics, which are considered as criteria, called Accuracy, Precision, Recall, F1-Score, AUC. Each criteria is weighted using the Integrated Determination of Objective CRIteria Weights (IDOCRIW) method. Subsequently, the rankings of the classification methods are calculated using various MCDM methods, including the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Evaluation Based on Distance from Average Solution (EDAS), Combinative Distance-based Assessment (CODAS) and Multi-Attributive Border Approximation area Comparison (MABAC). The final decision is clarified using the Copeland method, since the MCDM methods give different ranking results. For application purposes, a classification problem from the technological investment area is considered. All computations are performed using the Python PyDecision library. The results are analyzed and discussed in detail.

Keywords: *Multi Criteria Decision Making Methods, Data Mining Classification Methods, Performance Metrics, Classification Problem, Technological Investment Area.*

Acknowledgment: *This study was produced from the doctoral thesis prepared by the first author under the supervision of the second author.*

Entropy-Penalized Model Selection for Matrix-Variate Normal Mixture Models (1037)

Mostafa Tamandi*[1]

[1]Vali-e-Asr University of Rafsanjan, Faculty of Mathematical sciences, Department of Statistics, Iran

*Corresponding author e-mail: tamandi@vru.ac.ir

Abstract

We propose a novel model selection criterion for determining the optimal number of clusters in matrix-variate normal mixture models. Our approach introduces an entropy-based penalty term into the likelihood function, encouraging confident cluster assignments and mitigating overfitting in high-dimensional structured data. Through simulations and real data experiments using image matrices from the MNIST dataset, we demonstrate that our method outperforms traditional criteria such as BIC, especially in settings with ambiguous cluster structures.

Keywords: *matrix-variate normal, clustering, entropy, model selection, EM algorithm, MNIST, mixture models*

Parameter Estimation Based on Validity-Aware Gustafson Kessel Clustering (1042)

Yesim Akbas[1*], Türkan Erbay Dalkiliç[2], Serkan Akbaş[3]

[1,2,3] Karadeniz Technical University, Faculty of Science, Department of Computer Sciences, Türkiye

*Corresponding author e-mail: yesimyeginoglu@ktu.edu.tr

Abstract

Parameter estimation in regression analysis is the determination of model coefficients using statistical methods so that the relationship between the dependent variable and independent variables can be expressed mathematically. In classical regression analysis, it is assumed that the observations in the data set come from a single class. In cases where the data do not come from a single class and have different distributions, fuzzy analysis methods can be considered as an alternative to classical analysis methods in estimating the unknown parameters of the regression model. In the problem of obtaining model parameters, one of the important steps in obtaining solutions based on fuzzy logic is to determine the clusters that constitute the data set and to obtain the membership degrees that will determine the contributions of the data in these clusters to the estimation.

In this study, the DBI cluster validity index, which provides effective results in determining the optimum number of clusters in the case of ellipsoid-shaped data, was used. Depending on the number of clusters determined by this index, the membership degrees obtained with the Gustafson Kessel clustering algorithm were used as weights that determine the contributions of the observations to the model and an algorithm using these membership degrees was proposed for parameter estimation. In order to determine the effectiveness of the proposed algorithm, the algorithm was run for different data sets and compared with the classical methods available in the literature.

Keywords: *Fuzzy Clustering, Parameter Estimation, Validity Index.*

A Meta-Analysis of Animal Welfare and Personality Trait Reliability (1043)

Senol Celik[1*]

[1]Bingöl University, Faculty of Agriculture, Department of Animal Science, Unit of Biometry and Genetics, 12000, Bingöl, Türkiye

*Corresponding author e-mail: senolcelik@bingol.edu.tr

Abstract

The aim of this study is to estimate the mean value for the reliability coefficients reported in the studies involving animal welfare and various personal characteristics and to examine the sources of variation in the reliability coefficients reported in each study. In this context, a total of 25574 participants' items were examined in 23 articles published between 2018 and 2024. A reliability generalization meta-analysis study was conducted by combining the Cronbach alpha values of 23 studies that met the inclusion criteria. It was benefited from Jamovi 2.6.19 packet programs for the analysis of data. The Cronbach alpha values used in the studies were converted to transformed coefficient values and the analyses were performed under the random effects model. The mean Cronbach alpha value in the 23 studies was found to be 0.80 (95% CI: 0.73-0.87) and this result was statistically significant ($p<0.01$). According to the results, there is no publication bias in this meta-analysis study. Moderator analyses were conducted to explain the possible sources of heterogeneity among individual studies. These analysis results showed that the Cronbach alpha coefficient varied significantly depending on the item, number of options, sample content, and data collection instrument structure. Apart from the variables included in this study, other variables that are believed to have an impact on reliability can be managed and their impacts on the reliability coefficient can be investigated. In addition, suggestions are made for future studies that will reveal the personal characteristics and behavior of animals.

Keywords: *Meta-analysis, heterogeneity, bias, reliability, Cronbach's alpha*

Bibliometric Analysis of Published Studies on the Quality of Life of Laryngeal Cancer Patients (1044)

Senol Celik[1*], Nurşen Çimen[2]

[1]Bingöl University, Faculty of Agriculture, Department of Animal Science, Unit of Biometry and Genetics, 12000, Bingöl, Türkiye
[2]Ankara Etlik City Hospital ENT Department, 06000, Ankara, Türkiye

*Corresponding author e-mail: senolcelik@bingol.edu.tr

Abstract

This study conducted bibliometric analyses using data from 1000 English-language articles published between 1996 and 2025 on the quality of life of laryngeal cancer patients, sourced from the Web of Science database. The dataset includes 5369 authors from 338 journals, with 18 authors contributing as sole researchers. The publication start dates and citation counts of the journals in the dataset were evaluated, and the impact of the references was also examined. When the distribution of publications over the years was analyzed, it was found that the highest number of articles, 66, was published in 2024, followed by 63 articles each in 2020 and 2021. Three area charts were used to illustrate the relationships between articles, authors, and keywords. A word cloud analysis revealed that key concepts such as "quality of life," "head," "neck cancer," "radiotherapy," "squamous-cell carcinoma," "carcinoma," "cancer," "surgery," "radiation therapy," and "outcomes" were central to the research. The journals Head and Neck-Journal for the Sciences and Specialties of the Head and Neck, European Archives of Oto-Rhino-Laryngology, and Laryngoscope were found to be among the most influential in the field. The most prolific authors were Dietz A., Singer S., and Finizia C., while the institutions producing the most articles were The University of Texas MD Anderson Cancer Center, Leipzig University, and the University of Michigan. The countries that contributed the most to the field were, in order, the United States, China, and Germany. Based on these findings, it can be concluded that articles focusing on the quality of life of laryngeal cancer patients, described from different perspectives, may serve as a guide for researchers and practitioners in the field.

Keywords: *Bibliometrics, trend, publications, larynx, quality of life.*

Ceramic Design Optimization Through Data Mining and Artificial Intelligence: A Case Study in The Ceramics Industry (1045)

Asım Akcay[1*], Mustafa Cavus[1], Emre Oktem[1]

[1]Eskisehir Technical University, Department of Statistics, Turkiye

*Corresponding author e-mail: asimakcay97@gmail.com

Abstract

This study aims to develop a data-driven model that combines data mining and artificial intelligence (AI) techniques to support the design processes in the ceramic industry. In this context, visual features of ceramic tiles produced by Seranit Granit Seramik San. and Tic. A.Ş. over the last four years were analyzed. The dataset includes color, texture, contrast, and homogeneity values extracted from the images of previous ceramic designs using image processing techniques in the R environment. After obtaining these numerical features, a Vector Autoregression (VAR) model was applied to predict the future trends of each design parameter. The predicted values were used as input variables for generative AI tools to create new ceramic design alternatives. Thus, it has been ensured that the produced designs are compatible with both the historical design language of the company and the possible future trends of the market. The results of the study demonstrated that the proposed model can support decision-making processes in design, accelerate design development stages, and contribute to sustainable and customer-oriented production strategies. This study reveals that the integration of data mining and AI techniques into the ceramic design process provides significant potential for the digital transformation of the sector. In addition, this approach can be adapted to different sectors and design processes that include visual features.

Keywords: Human-aı Interaction, Ceramic Industry, Digital Transformation, Data Mining

Investigation of Potential Effects of Olive Seed Oil on Alzheimer's Disease (1047)

Furkan Semih Çelik[1*], Ziya Çakır[2]

[1]Tokat Gaziosmanpaşa University, Faculty of Health Sciences, Department of Nursing, 60000, Tokat, Türkiye

[2]Tokat Gaziosmanpaşa University, Faculty of Health Services Vocational School, Department of Oral and Dental Health, 60000, Tokat, Türkiye

*Corresponding author e-mail: furkansemih005@gmail.com

Abstract

Many studies on olive fruit and oil have shown that phenolic compounds with antioxidant and anti-inflammatory properties have positive effects on the nervous system. However, scientific data on the effects of olive seed oil, especially on neurodegenerative diseases, are limited. In this context, it is important to examine the role of olive seed oil in oxidative stress, inflammation and synaptic deterioration associated with the physiology of Alzheimer's disease. The results of the study may contribute to the determination of alternative and natural supportive methods for the treatment of important neurological diseases of our age, such as Alzheimer's. With this study, we aimed to examine the neurophysiological aspects of olive seed oil in Alzheimer's disease and to contribute to the scientific literature through literature reviews and research.

Keywords: *Alzheimer's disease, Olive seed oil, Neurophysiology*

Using Genetic Algorithms with Clustering for Parameter Selection in Multiple Birth Support Vector Machines (1049)

Güvenç Arslan[1*]

[1]Department of Statistics, Kırıkkale University, Türkiye,

*Corresponding author e-mail: guvenc.arslan@kku.edu.tr

Abstract

Multiple Birth Support Vector Machines (MB-SVM), which may be considered as a variant of Support Vector Machines (SVM), use non-parallel hyperplanes for classification. In addition, in MB-SVM, one optimization problem is defined for each class category. This approach results in a novel approach for multi-class classification with SVMs. It is known that this also results in reduced computational complexity. On the other hand, using one optimization problem for each class results in an increased number of parameters. Determining the appropriate values for these parameters may critically affect the performance of the classification algorithm. In this study, we investigate the use of genetic algorithms (GA) with k-means clustering for determining appropriate parameters in MB-SVM.

Keywords**:** *Support vector machines, multiple birth, genetic algorithm, k-means*

Frequently Used Statistical Methods in Agricultural Economics Research: A Review and Critical Assessment (1052)

Özge Can Niyaz Altınok[1*]

[1]Çanakkale Onsekiz Mart University, Faculty of Agriculture, Department of Agricultural Economics, Turkey

*Corresponding author e-mail: ozgecanniyaz@comu.edu.tr

Abstract

The discipline of agricultural economics necessitates the use of various statistical methods due to its complex and multidimensional structure. This study systematically examines the main statistical analysis techniques that are frequently used in agricultural economics research both globally and in Turkey. Each method is evaluated through a critical lens in terms of its application areas, advantages, and limitations. From regression analyses to time series models, and from panel data techniques to structural equation modeling, the study discusses how different approaches are employed to analyze both micro-level farmer behaviors and macro-level policy impacts.

Methods such as logistic regression and structural equation modeling, which are especially preferred in behavioral research, are noted for their strength in shedding light on decision-making processes. However, they also present limitations, particularly regarding their sensitivity to sample size and model assumptions. Furthermore, models that address sample selection bias, such as the Heckman two-step procedure, and approaches used in measuring technical efficiency—namely, parametric (SFA) and non-parametric (DEA) methods—offer solutions to common empirical challenges but also involve certain methodological complexities.

This paper emphasizes that the choice of statistical methods in agricultural economics research is not merely a technical decision, but also one that requires theoretical coherence. It aims to enhance both the analytical and methodological awareness of researchers working in the field.

Keywords: *Agricultural Economics, Statistical Methods, Regression, Panel Data, Structural Modeling.*

Use of the Jaya Algorithm in the Parameter Estimation of Three-Parameter Weibull Distribution (1053)

Emre Kocak[1*]

[1]Gazi University, Faculty of Sciences, Department of Statistics, Ankara, Türkiye

*Corresponding author e-mail: emrekocak@gazi.edu.tr

Abstract

Since the Weibull distribution is one of the best-known and widely used distributions in various disciplines, such as failure rates, reliability, and survival studies, many studies have been conducted on estimating the parameters of this distribution. Although different estimation methods have been developed for this distribution, the maximum likelihood method is the most widely used and known. Despite this, it is challenging to estimate some distributions' parameters, especially the three-parameter Weibull distribution, by maximizing the likelihood function. Various meta-heuristic methods, such as particle swarm optimization, differential evolution, and genetic algorithms, have been proposed over the years to overcome this difficulty. In this study, the Jaya algorithm, which has not been used for this distribution before, is used, and the algorithm performance is examined for different algorithm parameters, sample sizes, and swarm sizes for different skewness levels of the three-parameter Weibull distribution. A comprehensive Monte-Carlo simulation analysis is performed to examine the performance of the proposed approach, and the performances of different cases are compared in terms of the Deficiency Criterion used to test the efficiency of the methods used in parameter estimation. According to the simulation results, it is observed that the proposed Jaya algorithm gives more efficient results for some cases examined.

Keywords: *Jaya algorithm, Weibull distribution, Maximum likelihood, Parameter estimation*

Data Mining in a Three-Vector Decision Support System for Managing Innovation Development Programs of the Republic of Kazakhstan: Methodology and Practical Application (1056)

Anel Yeleukulova[1*], Assel Yeleukulova[1], Mafura Uandykova[1], Ahmed Baikhojayev[1]

[1]Narxoz University, Kazakhstan

*Corresponding Author: aneldarkhan@gmail.com

Abstract

This article presents a comprehensive data mining methodology for decision support in managing state innovation development programs of the Republic of Kazakhstan. The research is based on an original concept that combines a three-vector development paradigm (technological, monetary-financial, and socio-political) with a modular tetrad architecture "Objects-Processes-Projects-Environment." The developed approach includes integrated methods of multi-criteria analysis and scenario modeling for processing heterogeneous information sources. Special attention is paid to solving problems of cross-vector data synchronization, balancing resources between development directions, and ensuring interpretability of analytical results. The system testing results demonstrate a significant improvement in the quality of analytical support and forecasting accuracy compared to traditional approaches.

Keywords: *data mining, decision support systems, program-project management, innovative development, multi-criteria analysis, scenario modeling, three-vector development model.*

A Stacking-Based Ensemble Approach for the Diagnosis of Pediatric Lower Respiratory Tract Infections (1057)

Elif Dabakoğlu[1], Öyküm Esra Yiğit[2*]

[1]Mugla Sitki Kocman University, Research Support and Funding Office, Mugla
[2]Yildiz Technical University, Faculty of Arts and Sciences, Department of Statistics, Istanbul

*Corresponding author e-mail: oeyigit@yildiz.edu.tr

Abstract

Lower respiratory tract infections (LRTIs) are a major global health concern, particularly among children, representing one of the top causes of morbidity and mortality. Pneumonia alone is responsible for the death of over 700,000 children under the age of five annually, accounting for nearly 2,000 deaths per day. In Turkey, LRTIs rank as the second leading cause of death in children aged 0–14, with recent national health data reporting a 6.9% prevalence rate among the 0–6 age group. The two most common LRTIs in children—pneumonia and bronchitis—share overlapping symptoms, often leading to diagnostic confusion. Accurate and early differentiation between these two diseases is crucial for effective treatment and prognosis. This study implements a structured, multi-stage analytical pipeline to accurately distinguish between pediatric pneumonia and bronchitis using real-world clinical, laboratory, and radiological data collected from hospital records. The proposed diagnostic framework builds on a diverse set of base learners, including Random Forest (RF), Extra Trees (ET), Bagging Naive Bayes (BC), Extreme Gradient Boosting (XGBoost), Stochastic Gradient Boosting (SGB), and Histogram-Based Gradient Boosting (HGB). These diagnostically validated base models are subsequently integrated into a two-layer stacking ensemble, also known as a Super Learner, in which meta-models such as Elastic Net–regularized Logistic Regression and Multi-Layer Perceptrons (MLPs) are trained on the probabilistic outputs of the base classifiers. The ensemble-based approach significantly improves classification performance and demonstrates strong generalization ability, offering a promising decision-support tool for the early and accurate diagnosis of lower respiratory tract infections in pediatric populations.

Keywords: *Stacking Ensemble, Clinical Decision Support, Pneumonia, Bronchitis*

Modified Ranked Set Sampling: Simulations and Real Data Evaluation (1058)

Eda Gizem Koçyiğit[1]

[1]Dokuz Eylül University, Faculty of Science, Department of Statistics, İzmir, Türkiye.

*Corresponding author e-mail: eda.kocyigit@deu.edu.tr

Abstract

The primary objective of applying a sampling methodology is to obtain accurate and reliable estimates of population parameters while reducing the associated costs and resource requirements. Among the wide range of available sampling techniques, Ranked Set Sampling (RSS) has attracted considerable attention due to its ability to produce unbiased and efficient estimates in a cost-effective manner. This is achieved by utilizing auxiliary information or pre-existing knowledge during the sample selection process. In this study, we propose an improved sampling approach within the framework of RSS, specifically designed to enhance estimation efficiency. The proposed method modifies the initial stage of the traditional RSS procedure, which typically involves drawing a Simple Random Sample (SRS). Instead, we introduce an alternative selection strategy that replaces the conventional SRS step. This new sampling methodology is thoroughly described, and its performance is evaluated through both synthetic simulation data and real-world datasets. The strengths and limitations of the method are carefully examined. One notable limitation is the need for a population size that meets specific criteria for the technique to be applicable. Despite this, results from extensive simulation studies demonstrate that the proposed method consistently outperforms existing sampling methods, particularly in terms of mean estimation. When the required conditions for applicability are satisfied, the proposed method yields predictions with significantly lower Mean Squared Error (MSE) compared to existing methods in the literature.

Keywords: *Ranked Set Sampling, Modified Method, Monte-Carlo simulation, Estimation, Efficiency.*

An Examination of Emotional and Psychological Well-Being, Life Satisfaction, and Smartphone Addiction in Selective High Schools (1059)

Osman Cem Bektaş[1*], Kamile Şanlı Kula[2]

[1]Kırşehir Ahi Evran University, Institute of Natural and Applied Sciences, Department of Mathematics, TÜRKİYE
[2]Kırşehir Ahi Evran University, Faculty of Arts and Sciences, Department of Mathematics, TÜRKİYE

*Corresponding author e-mail: osmancembektas@gmail.com

Abstract

This study aims to investigate the emotional and psychological well-being, life satisfaction, and smart phone addiction students attending selective high schools in the central district of Kırşehir, depending on various demographic variables gender, school and grade level etc. The population of the study consists of all 9th, 10th, and 11th-grade students enrolled during the second semester of the 2022–2023 academic year in high schools admitting students through a nationwide examination. Data were collected using three validated scales: The "Stirling Children's Well-being Scale", the "Satisfaction with Life Scale"and the "Smart Phone Addiction Scale – Short Version". The Cronbach's alpha reliability coefficients for these scales in the study were 0.89, 0.84, and 0.86, respectively. The research was conducted with a total of 1192 students, of whom 698 (58.6%) were female and 494 (41.4%) were male. Among them, 1163 students (97.6%) owned a personal smartphone, while 29 students (2.4%) did not. Regarding daily smartphone use, 5.7% reported using their phones for 0–1 hours, 40.8% for 2–3 hours, 38.8% for 4–5 hours, 10.7% for 6–7 hours, and 4.0% for 8 or more hours. At the end of the study, 52.5% of the participants had low levels of psychological well-being, whereas 47.5% reported high levels. Life satisfaction was low among 51.7% and high among 48.3% of students. In terms of smart phone addiction, 613 students were identified as having low levels of addiction, while 579 exhibited high levels. There was statistically significant difference in terms of emotional and psychological well-being, life satisfaction, and smart phone addiction averages according to variables of gender, school type, perceived academic achievement, duration of daily smartphone use, mother's education level, and perceived parental attitudes.

Keywords: *Emotional and psychological well-being, Life satisfaction, Smartphone addiction, Student.*

From Speech to Schedule: A Voice-Based Restaurant Reservation System Leveraging ML and NLP (1060)

Ali Kerem Güler[1*]

[1]Protel A.Ş., Turkiye,

*Corresponding author e-mail: aguler@protel.com.tr

Abstract

This study presents a voice-based restaurant reservation system that integrates Protel A.Ş.'s Turyid Loyalty mobile application with the Check and Place table-management platform. After logging into the mobile application and selecting a venue, the user's verbal request is recorded. The recorded audio is transmitted to a machine learning (ML) based backend and converted into raw text using either the OpenAI Whisper or SpeechRecognition module. The obtained transcript is processed with the Gemini-1.5-Flash model to perform grammar correction, add punctuation, and complete missing words, thereby improving semantic coherence. The revised text is then analysed by the Gemini-2.0-Flash-Exp model, driven by an optimised prompt, to automatically extract the three parameters required for a reservation—date, time, and number of guests. Natural-language temporal expressions (e.g., "this evening," "next Tuesday") are converted into the DD/MM/YYYY format and the 24-hour clock. Contextual cues such as "evening" are examined so that "seven o'clock" is mapped to 19:00, whereas time expressions lacking contextual indicators (e.g., "ten o'clock") are, by default, assigned an evening reservation (22:00). The number of guests is parsed from numeric or verbal expressions indicating indicating the number of people. The voice-reservation service combines these extracted parameters with the name–surname and telephone information retrieved from the Loyalty backend, transmits them to the Check and Place API, and saves the reservation in the restaurant's calendar. Designed with a microservice architecture, the system delivers robust speech-to-intent conversion under real-world conditions, integrates seamlessly with existing hospitality infrastructure, reduces friction in the reservation process, and enhances the multichannel customer experience.

Keywords: *Voice-based reservation system, Machine learning, Natural language processing, Large Language Models, OpenAI Whisper, Google Gemini, Speech-to-intent.*

Efficiency Analysis of Defense Industry Companies Using Data Envelopment Analysis (1061)

İlayda Koçyiğit[1]*, Fulya Altıparmak Baykoç[1]

[1]Gazi University, Faculty of Science, Department of Industrial Engineering, Türkiye

*Corresponding author e-mail: ilayda.kocyigit@gazi.edu.tr

Abstract

Research and Development efficiency plays a crucial role in the defense sector, where innovation directly influences national security, strategic independence, and global competitiveness. Efficient allocation of Research and Development resources allows firms to maximize technological output while minimizing unnecessary costs. Therefore, assessing Research and Development performance with robust methodologies is essential to guide strategic decisions and strengthen industrial resilience.

This study aims to evaluate the Research and Development efficiency of eight leading defense industry companies in Türkiye listed in the "Ar-Ge Harcamalarına Göre En Büyük 250 Şirket" (Top 250 Companies by Research and Development Expenditures) report, published annually by Turkish Time magazine. Conducted as part of a master's thesis, the analysis covers data from 2021 to 2023 and employs Data Envelopment Analysis (DEA) to assess firm-level performance using both input-oriented CCR and BCC models. The selected input variables are Research and Development expenditure and the number of Research and Development personnel, while the output variables are the number of intellectual property assets and total revenue. DEA was preferred due to its ability to handle multiple inputs and outputs without requiring a predefined production function, which is particularly useful in evaluating Research and Development processes characterized by qualitative and quantitative variability. The use of verified, publicly available data ensures transparency and comparability. The findings of this study reveal performance disparities among companies and identify improvement opportunities for underperforming firms.

Keywords: *Data Envelopment Analysis, Defense Industry, Research and Development Efficiency, CCR Model, BCC Model*

Move Right, Feel Better: An AI-Powered Real-Time Exercise Tracking System (1062)

Songül Erdem Güler[1*], Abdulkadir Kayıklı[1]

[1]Vestel Healthcare, Türkiye

*Corresponding author e-mail: songul.erdemguler@vestel.com.tr

Abstract

This study presents a web-based artificial intelligence exercise-tracking system, grounded in real-time pose estimation, that aims to help individuals with mental health challenges develop regular physical-activity habits. The system offers a live video stream and four basic exercise options (jumping jacks, high knees, arm raises, and squats) through a web interface accessible via a domain name. When a user selects an exercise, the MediaPipe-based pose estimation model integrated into the interface runs in real time, analyzing the user's movements. The model simultaneously extracts the coordinates of the body's keypoints. These keypoints are transmitted into exercise-specific rule-based functions that compute joint angles and automatically determine the repetition count. Concurrently, the keypoints are drawn onto the live video feed, and a short GIF illustrating the correct form of the movement is displayed as a reference. In this way, users receive visual feedback to perform movements with proper posture and are encouraged to build sustainable workout habits through a micro-habit strategy that involves just a few minutes of exercise each day. The developed AI-powered interactive system thus offers an innovative solution for improving mental health and increasing physical-activity levels.

Keywords: *Artificial intelligence, Exercise tracking, Pose estimation, MediaPipe, Web-based systems.*

Linking Process Covariates to the Taguchi Capability Index C_{pm}: A Log-Linear Maximum Likelihood Framework (1063)

Sümeyra Sert[1*], Coşkun Kuş[1]

[1]Selçuk University, Faculty of Science, Department of Statistics, Turkey.

*Corresponding author e-mail: sumeyra.sert@selcuk.edu.tr

Abstract

This study introduces a novel extension of the Taguchi process capability index C_{pm} by proposing a log-linear regression model that explicitly links the index to process covariates. The proposed approach allows the practitioners to model each unit's capability index as a function of measurable factors that can influence it, such as temperature, speed, and material properties, and examines how operating conditions directly affect process variability. In this context, a maximum-likelihood estimation procedure is developed to simultaneously estimate the process mean and the covariate effects. An extensive Monte Carlo simulation study is conducted to examine the performance of the proposed procedure with the results further illustrated through a real-world application. An R-based implementation of the model is also presented, enabling practitioners to assess the impact of the controllable variables and improve process performance through detailed analysis.

Keywords: *Process capability index, Maximum likelihood estimation, Process covariates, Log-linear regression*

One of the hybrid methods; Structural Equation Modeling and Artificial Neural Networks (SEM-ANN) approach (1064)

Sanem Sehribanoglu[1*]

[1] Van Yuzuncu Yil University, Economics and Administrative Sciences, Department of Economics, Statistics Department

*Corresponding author e-mail: sanem@yyu.edu.tr

Abstract

In academic research, hybrid models offer opportunities such as higher accuracy, flexibility and interpretability by overcoming the areas where traditional methods are limited. The creation of hybrid models involves the integration of diverse methodologies in a balanced and cohesive manner. The utilisation of these models in research contexts confers numerous advantages, including high performance and accuracy, the adept processing of complex data structures, interpretability and reliability, the management of missing data or uncertainty situations, and the facilitation of interdisciplinary applications.

In recent years, hybrid models in the field of statistics have emerged as a combined use of traditional statistical methods with machine learning (ML), artificial intelligence (AI) or other computational techniques. These models have been shown to overcome the limitations of a single method and offer more powerful, flexible and interpretable results.

Structural Equation Modeling (SEM) is a statistical method that is both effective and reliable. It is used to test hypotheses and examine causal relationships, as well as to discover relationships between independent and dependent variables. Nevertheless, the utilisation of SEM is constrained in its capacity to evaluate linear relationships between variables.

Artificial neural networks (ANNs), akin to (SEMs), possess the capacity to examine linear relationships between variables and to model nonlinear relationships. However, ANN is the subject of criticism with regard to the so-called "black box" problem. This concept is predicated on the absence of a comprehensive understanding of the decision-making status of the analyses, owing to the intricacy of the operations. Due to this problem, statistical hypotheses are not appropriate for testing with ANN.

Because of this, the strengths of the two different methods have been combined. The integration of hypothesis test results derived from SEM analysis as the input to ANN analysis has enabled the rectification of the inherent limitations of both methods, thereby giving rise to a hybrid method, the SEM-ANN approach.

Keywords: *Artificial neural networks, Structural Equation Modeling, hybrit method*

Analysis of Forest Fire Risk of Ankara with Spherical Fuzzy AHP and Spherical Fuzzy TOPSIS Methods (1066)

Burcu Tezcan [1], Tamer Eren[1*]

[1]Kirikkale University, Faculty of Engineering and Natural Sciences, Department of Industrial Engineering, Turkey

*Corresponding author e-mail: tamereren@gmail.com

Abstract

Forest fires cause significant damage to ecosystems, biodiversity, and human life, leading to environmental and economic losses. Therefore, prevention and effective management of fires are vital for a sustainable environment. In this context, this study aims to make a risk assessment of the districts of Ankara with forest density under uncertainty. Ankara Forest Management Directorate has 791,676 hectares, of which 458,200 hectares is a normally closed area and 333,476 hectares is a hollow closed area. Ankara has a significant risk of forest fires due to its forested areas and dry summer climate. Managing this risk is critical for both the protection of natural ecosystems and the safety of residential areas around the city. Therefore, the application area is Ankara. Three main criteria and 9 sub-criteria that cause fires were identified. These are temperature, humidity, wind, slope, elevation, viewpoint, dry vegetation, altitude, and biomass density. Criteria weights were calculated by 10 experts using the SF-AHP method. According to the results obtained, the humidity criterion ranks first with a rate of 20.1%. The temperature criterion of 16.9% follows this criterion. Among the districts of Ankara, the districts with high forest density were selected as alternatives. These are Merkez, Beypazarı, Çamlıdere, Kızılcahamam and Nallıhan. These alternatives were ranked by the SF-TOPSIS method using criteria weights. The Center of Ankara ranks first. It provides a scientific contribution to the development of fire prevention strategies by bringing a new perspective to the literature.

Keywords: *Forest fires, SFAHP, SFTOPSIS, Ankara*

Optimization of Forest Fire Response Resources with a Goal Programming Approach: Mersin Mut Application

Burcu Tezcan [1], Tamer Eren[2*]

[1]Department of Enterprise Informatics Specialization, Cappadocia Vocational School, Cappadocia University, Nevsehir, Turkey
[2]Kirikkale University, Faculty of Engineering and Natural Sciences, Department of Industrial Engineering, Turkey

*Corresponding author e-mail: tamereren@gmail.com

Abstract

Forest fires disrupt ecological balance and economic losses. Equipment planning is essential for rapid response to forest fires. Mut district of Mersin has a dense forest cover under the influence of the Mediterranean climate, which has high temperatures and low humidity in the summer months. Due to its climatic and geographical characteristics, it is considered one of Turkey's vulnerable regions in terms of forest fire risk. Therefore, the application area is Mut. In this context, in this study, 9 regions of Mut in terms of forest fire risk were determined by considering historical forest data. An equipment allocation model was developed with the goal programming method. The model aims at balanced distribution and efficient management of available equipment. Each forest fire will be allocated as much equipment as it needs. The mathematical model was solved with the IBM ILOG CPLEX Optimization program. According to the results obtained, more equipment is allocated to regions 2 and 9 than to the other areas. This shows that the probability of fire spread in these regions is high. It also proves the effectiveness and applicability of the model.

Keywords: *Forest fires, Goal programming, Mersin*

Detecting Distributional Differences: A New Two-Sample Equality Test (1068)

A.Fırat Özdemir[1*]

[1]Dokuz Eylül University, Faculty of Science, Department of Statistics, Türkiye

*Corresponding author e-mail: firat.ozdemir@deu.edu.tr

Abstract

This study introduces the NO Test, a novel nonparametric method for comparing two independent samples by examining differences not just in central tendency but across distributional quantiles, particularly in the tails. The proposed method combines the Navruz–Özdemir (NO) quantile estimator, the Mahalanobis distance, and a percentile bootstrap approach. It evaluates whether the 2.5th, 50th, and 97.5th percentiles differ significantly between groups by examining how the vector of quantile differences is nested within the distribution of bootstrapped differences. This approach yields a global p-value for determining distributional equality.

Simulation studies covering symmetric, skewed, and heavy-tailed distributions show that the NO Test provides superior power to detect differences, especially when conventional tests like Kolmogorov–Smirnov, Anderson–Darling, or Cramér–von Mises may fail. Moreover, the test reliably maintains Type I error rates under various continuous and discrete distribution settings. Compared to other quantile-based approaches, it offers robustness even in small samples and settings with tied values.

The NO Test thus offers a powerful and flexible alternative for detecting subtle distributional differences, making it especially valuable in fields where tail behavior matters, such as finance, biomedical research, and environmental studies.

Keywords: *Nonparametric Test, Two-Sample Test, Distributional Equality, Power Comparison*

An Examination of Digital Addiction and Life Satisfaction of High School Students (1070)

İbrahim Seyhan[1*], Kamile Şanlı Kula[2]

[1]Kırşehir Ahi Evran University, Instıtute of Natural and Applied Sciences, Department of Mathematics, Türkiye
[2]Kırşehir Ahi Evran University, Faculty of Arts and Sciences, Department of Mathematics, Türkiye

*Corresponding author e-mail: ibrahim_Matematik_seyhan@hotmail.com

Abstract

This study aims to examine the digital addiction and life satisfaction of high school students in the Çankaya district of Ankara province in terms of various variables such as gender, grade level, and school. The population of the study consists of all 9th, 10th, 11th, and 12th grade students enrolled in 30 high schools (15 private and 15 public) in Çankaya, Ankara during the second semester of the 2023–2024 academic year. The main data collection scale used in the research are the "Digital Addiction Scale" and the "Satisfaction with Life Scale". The Cronbach's Alpha coefficients obtained in this study for the scales are 0.89 and 0.81, respectively. The research was conducted with 1226 students, of whom 602 (49.1%) are female and 624 (50.9%) are male. A total of 1204 students (98.2%) own a mobile phone, while 22 (1.8%) do not. Regarding screen time, 6.9% of students spend 0–1 hour, 40.2% spend 2–3 hours, 35.2% spend 4–5 hours, 11.7% spend 6–7 hours, and 6.1% spend more than 8 hours spend on computers (desktop, laptop, tablet, etc.) or phones daily. Of the students, 629 (51.3%) have low levels of digital addiction, while 597 (48.7%) have high levels. Life satisfaction is low for 47.1% of students and high for 52.9%. There was statistically significant difference in terms of digital addiction averages according to variables of gender, grade level, perceived academic achievement, daily time spent on phones and computers, mother's education level, daily study hours, and perceived parental attitudes. There was statistically significant difference in terms of life satisfaction averages according to variables of gender, grade level, school, perceived academic achievement, perceived family economic status, parental education level, regular participation in activities such as sports, music, and art, perceived parental attitudes, and mother's employment status.

Keywords: *Digital addiction, Life satisfaction, High school students*

Robust Artificial Intelligence Algorithm Based on M-Estimation Method (1072)

Elgiz Askeroğlu[1*], Fatma Zehra Doğru[2]

[1]Giresun University, Institute of Science, Department of Statistics, 28200, Giresun, Turkey
[2]Giresun University, Faculty of Arts and Sciences, Department of Statistics, 28200, Giresun, Turkey

*Corresponding author e-mail: elgiz_askeroglu@hotmail.com

Abstract

Analysing real-world data in regression analysis often comes with a unique set of challenges; outliers, autocorrelation, and heteroscedasticity can all compromise the accuracy of predictions. Traditional regression methods usually struggle with these issues, and even advanced techniques like artificial intelligence-based algorithms can also be affected by the presence of outliers and deviations in the data. To address this, our study presents a robust regression approach that combines M-estimators (using Huber and Tukey loss functions) with the Particle Swarm Optimization (PSO) algorithm. In this study, we compare this method against classical least squares and standard PSO by applying all three to a dataset on carbon dioxide emissions in Turkey, covering the years 1973 to 2022. Since carbon dioxide emissions are a major driver of global warming, uncovering their patterns is crucial for developing sound environmental and economic strategies. The dataset includes seven key indicators, such as carbon footprint, fuel and electricity consumption per capita, and the contributions of different economic sectors to GDP. We evaluate each method's performance using five metrics, focusing especially on those that are robust in the presence of outliers. Our findings highlight the strong performance of the robust PSO approach in capturing meaningful insights from complex, noisy data.

Keywords: *Artificial Intelligence, M-Estimator, Particle Swarm Optimization Algorithm, Robust.*

Stock Market and Cryptocurrency Addiction Among University Students: The Effect of Psychological and Demographic Factors (1073)

Gizem Açıkgöz[1], Malhun Sidorela Balla[1], Rana Şen Doğan[1]

[1]Manisa Celal Bayar University, Faculty Of Economics And Administrative Sciences, Econometrics, Türkiye

*Corresponding author e-mail: gacikgoz314@gmail.com

Abstract

Stock and cryptocurrency markets attract investors by offering rapid profit opportunities, but they can also encourage risky investment behaviors, challenging individuals' ability to cope with losses. For young and tech-savvy individuals, such as university students, the stock market may have an appeal similar to online games. This can be linked over time to negative psychological effects such as cognitive engagement, reduced sleep quality, anxiety, and depression.
This study was conducted with 258 university students using a survey method for data collection. The results indicate that environmental and individual factors are decisive in stock market addiction. It was found that academic disciplines affect investment habits and addiction levels. Additionally, economic conditions were found to influence addiction, with individuals having a lower income being more likely to take higher risks. Individuals with higher levels of depression and smartphone addiction were also found to have a significantly higher tendency toward stock market addiction. It is suggested that these individuals may seek psychological satisfaction through investment instruments. On the other hand, gender did not have a meaningful effect on addiction, and no significant difference was found between male and female participants.
This study aims to raise awareness of the effects of stock market addiction on young individuals and provide potential solutions in this regard.

Keywords*: Stock market addiction, cryptocurrency addiction, university students, psychological factors, demographic variables.*

Feature Selection Strategies for Accurate Energy Consumption Forecasting Using Machine Learning Algorithms (1075)

Nuri Berk Ural[1], Meral Çetin[2*]

[1]Cukurova University, Faculty of Arts and Sciences, 01250, Adana, Türkiye
[2]Hacettepe University, Faculty of Sciences, 06800, Ankara, Türkiye

*Corresponding author e-mail: nberkural@gmail.com

Abstract

This study aims to examine the impact of variable selection methods on the prediction performance of machine learning models. In multivariate and time-series datasets such as energy consumption, proper variable selection significantly improves model generalization capacity and helps prevent overfitting. For this purpose, a dataset containing temperature, humidity, and energy consumption measurements recorded every 10 minutes over a 4.5-month period was used. The data were collected from sensors placed in various rooms and on the exterior of a household, comprising a total of 19,735 observations and 28 variables.

Nine variable selection methods were applied: Correlation-Based Selection, Variance-Based Selection, Forward Selection, Backward Elimination, Stepwise Selection, Genetic Algorithms, Lasso, Ridge, and Robust Feature Selection. Using the variables selected by each method, predictive models were constructed with six machine learning algorithms: Linear Regression, Decision Trees, Random Forests, Support Vector Regression, Principal Component Analysis, and Artificial Neural Networks. The performance of the models was evaluated using metrics such as Mean Absolute Error (MAE), Mean Squared Error (MSE), and Coefficient of Determination (R^2).

The results reveal that variable selection methods have a significant impact on model accuracy. In particular, some methods yielded superior results when combined with specific algorithms. This study highlights the critical role of variable selection in machine learning workflows and offers methodological guidance for researchers working with energy data. By providing a comparative analysis of methods and models, the study contributes both to practical modeling strategies and to the broader understanding of how variable selection can influence the performance of predictive systems in complex, real-world datasets.

Keywords: *Feature selection, Machine learning, Energy consumption forecasting*

Actuarial Pricing Using Schur-Constant Models (1076)

Tuğba Aktaş Aslan[1*], Altan Tunçel[1]

[1] Kırıkkale University, Faculty of Economics and Administrative Sciences, Department of Actuarial Sciences, TURKEY

*Corresponding author e-mail: tugbaaktas@kku.edu.tr

Abstract

Schur constant models play a special role in modelling lifetimes in fields such as actuarial science and insurance. In these models, random lifetimes are defined under a certain dependence. This dependence is critical in the actuarial pricing of insurance products. In practice, actuarial pricing of life insurance products often relies on the simplifying assumption that future lifetimes of insured individuals are independent. However, this assumption may not hold in many realistic settings, such as multiple life policies or portfolios exposed to common risk factors. In order to provide a fairer pricing to the parties of the insurance contract, models that consider dependency should be preferred. For this purpose, we analyse the dependence structure in life insurance products using Schur-constant models.

In this study, we first establish a relation with proportional hazard rate (PHR) distributions and Schur-constant models. Then, we construct bivariate Schur-constant models whose marginals follow proportional hazard rate distributions, allowing for greater flexibility in modelling lifetime data. Finally, based on the relation between Schur-constant and copula models, a comparative dependence analysis is performed for life insurance. As a result, it is found that there are differences in pricing when the future lifetimes of individuals are dependent. Moreover, it is shown that it may be advantageous for the insurance company to prefer the Schur-constant model over the copula model in dependency analysis.

Keywords: *Copula, Insurance, Proportional hazard rate model, Schur-constant model*

Determination of Student Performance from Primary Education to Higher Education by Alternative Measurement and Evaluation Methods (1078)

Bahtiyar Dildeğmez[1*], Mehmet Özbaş[1]

[1]Erzincan Binali Yıldırım University, Faculty of Education, Department of Educational Sciences, Erzincan, Türkiye

Corresponding author e-mail: bahtiyar.dildegmez@erzincan.edu.tr

Abstract

It is imperative that students' performance levels are constantly measured and evaluated in a realistic way, both with their unique characteristics and with scientific methods, from the beginning of their school life. Practices that give subjectivity to measurement and evaluation from a student's point of view, objectivity and reality from a scientific point of view are process-oriented continuous measurement and evaluation methods. The purpose of this research is to reveal the functions of alternative measurement and evaluation methods from primary education to higher education, where academic cognitive teaching and the application of determining students' performance levels by using measurement and evaluation methods have begun.

The research is a scientific study designed in a descriptive model and carried out by interview method. In the study, a semi-structured interview form prepared in line with the criticisms of academicians from the field of measurement and evaluation was used. The research was carried out with 10 volunteer participants who completed their undergraduate education in different undergraduate programs of universities and studied at the graduate level. According to the findings of the research, it is emphasized that alternative assessment and evaluation methods have significant contributions to the continuous development of students both subjectively, academically and in the process. In order to increase the contribution of alternative assessment and evaluation methods to students, it is expected that teacher competencies, objective assessment skills and support for instructional technologies in teaching will increase. The research results point out that the use of alternative measurement and evaluation methods and tools is essential for student development; these methods are also extremely necessary for ensuring student-centered learning. The results obtained from the participants show that there is a need for the coordinated use of assessment and evaluation methods that provide opportunities for multi-faceted and more original development opportunities in schools, together with traditional assessment and evaluation methods and tools. The research suggests that school administrators and teachers should be skilled in using student-centered alternative measurement and evaluation methods in addition to the mass exams used for everyone in order to increase and improve students' academic success.

Keywords: *Primary Education, Higher Education, Student Performance, Alternative Assessment and Evaluation.*

Digitalization and Travel Behavior: Evidence from a Survey on Travel Behavior in Türkiye (1079)

Jülide Yıldırım[1]*, Pelin Umul[1]

[1]TED University, Faculty of Economics and Administrative Sciences, Department of Economics, Turkey

*Corresponding author e-mail:julide.yildirim@tedu.edu.tr

Abstract

This study investigates the impact of digitalization on travel decision-making, with a particular focus on online reservation tools and digital promotions. Based on survey data collected from 465 adult respondents in Türkiye, the analysis explores how digital tools shape tourism habits, influence destination preferences, and alter budgetary decisions. Participants were asked about their frequency of digital engagement in planning travel, including their use of websites, mobile applications, and social media platforms for booking and gathering information. The data also capture how digital campaigns, discounts, and personalized offers influence consumers' choices. A significant share of respondents reported using online channels as their primary means of booking, citing convenience, cost transparency, and real-time availability as key benefits. Furthermore, many indicated that price-based promotions directly affect both the timing and location of their trips. Preliminary findings indicate that younger, digitally literate individuals with higher education levels are more likely to use online platforms frequently, consistent with the existing literature on technology adoption in tourism. Environmental concern is positively associated with online booking frequency, potentially reflecting preferences for information-rich and customizable planning environments. Moreover, trust in online information significantly increases the likelihood of frequent platform use. These results contribute to the applied economics literature on tourism and digital transformation by offering empirical evidence from an emerging economy. The study emphasizes the relevance of digitalization in shaping modern travel decisions and the need for policy frameworks to enhance digital access and information reliability in tourism services.

Keywords: *Travel, Digitalizatiion, Turkey*

Statistical Literacy Levels of Prospective Graduates: The Case of Çanakkale Onsekiz Mart University[**] (1080)

Ege Dedehayır[1], Tuğba Söküt Açar[2*], Tuğçe Akça[1]

[1] Former Undergraduate Student, Çanakkale Onsekiz Mart University, Türkiye
[2]Canakkale Onsekiz Mart University, Faculty of Science, Department of Statistics, Türkiye

*Corresponding author e-mail: t.sokut@comu.edu.tr

Abstract

Statistical literacy skills play a critical role in the academic and professional success of graduating students. This study aims to determine the statistical literacy levels of final-year students (prospective graduates) in undergraduate programs based on their perceptual and cognitive skills, compare these skills, and analyze the factors influencing them. The population of the study consists of prospective graduates from the 2023-2024 academic year at Çanakkale Onsekiz Mart University, and the sample size was determined through power analysis. After preprocessing the dataset, the findings of the study were obtained from 193 students. The results revealed that there was no significant relationship between students' perceptual and cognitive skills. This lack of relationship suggests that students' perceptions of their statistical competence may not always accurately reflect their actual success levels. The study found that while students performed well in basic concepts such as sample selection and types of variables, they needed more support in advanced topics like measures of central tendency, probability, and correlation. Additionally, it was observed that the statistical literacy perceptions and cognitive skills of male and female students were largely similar. It was also determined that having previously taken a statistics course led to significant differences in students' statistical literacy perceptions and cognitive skills. However, the study found that students' general academic success did not have a significant effect on their statistical literacy perceptions or test performances. As a result, this study suggests that statistical literacy is shaped more by specific statistics education and interest and experience in the field, rather than general academic success, and that students should not only learn theoretical knowledge but also develop their confidence in applying this knowledge.

Keywords: *Statistical Literacy, Cognitive and Perceptual Skills, Prospective Graduates*

[**]: This study was supported by the Çanakkale Onsekiz Mart University Scientific Research Projects Coordination Unit [Project Number: FLÖAP-2024-4824]

Performance Comparison of Activation Functions in Breast Cancer Diagnosis: A Deep Learning Based Analysis (1081)

Ömer Faruk Nasip[1]*

[1]Tokat Gaziosmanpaşa University, Faculty of Medicine, Turkey

*Corresponding author e-mail: omerfaruk.nasip@gop.edu.tr

Abstract

Breast cancer is one of the most common types of cancer in women worldwide. According to the World Health Organization, approximately 2.3 million new cases are diagnosed each year and 685 thousand people die from the disease. Early detection can increase treatment success to over 90%. This study compares the performance of commonly used activation functions in the literature in order to contribute to the development of AI-supported systems for breast cancer diagnosis. The study uses a breast cancer dataset provided by the University of Wisconsin that includes tumor characteristics. The dataset contains 30 different pathological features (such as tumor diameter, tissue density, core shape) from 569 patients. These features were measured by expert pathologists after microscopic examinations. In this study, a 4-layer deep learning model is developed. The model has three hidden layers containing 32, 64 and 32 neurons respectively. In the output layer, the sigmoid activation function is used to calculate the probability of benign or malignant tumors. The experimental results show that the ELU activation function performs better than other activation functions. This study contributes to the development of computer-aided diagnostic systems that can be used in clinical applications. The findings reveal the potential of deep learning-based systems, especially in early-stage breast cancer diagnosis. Another important outcome of the study is to provide practical recommendations for model optimization when working with limited medical data.

Keywords: *Breast Cancer, Activation Functions, Deep Learning*

Forecasting Cardiovascular Mortality Rates in Türkiye (2025–2030): A Prophet Model Approach (1082)

Özge Pasin[1*], Senem Gönenç[2], Selman Aktaş[1], Kürşad Nuri Baydili[1]

[1]University of Medical Science, Hamidiye Medical Faculty, Biostatistics Department, İstanbul, Türkiye
[2]Ataturk University, Faculty of Science, Statistics Department, Erzurum, Türkiye

*Corresponding author e-mail: ozgepasin90@yahoo.com.tr

Abstract

This study aims to explore the theoretical foundations of the Prophet model and evaluate its potential application in health-related time series forecasting. The primary objective is to compare the forecasting accuracy of the Prophet model with an alternative technique Holt's linear trend model in predicting mortality rates attributable to cardiovascular diseases in Türkiye.

Mortality data reported by the World Health Organization(WHO) were obtained for both genders between 2009 and 2019, reflecting the number of deaths per 100,000 individuals. This dataset was used to generate forecasts for the period from 2025 to 2030 using both the Prophet and Holt models, implemented in R version 4.4.1.While Holt's method is a trend-based exponential smoothing approach that incorporates only level and trend components,Prophet is a more flexible model capable of capturing trend, seasonal, and holiday effects. Developed by Facebook,Prophet is particularly advantageous for handling missing data and modeling irregular time series patterns.

Results showed notable differences in model performance by gender. For males, the Prophet model achieved lower forecast error metrics(RMSE = 9.78; MAPE = 4.53%; MAE = 8.53) compared to females(RMSE = 11.39; MAPE = 5.69%; MAE = 8.22). Similarly, the Holt model demonstrated better predictive accuracy for males(RMSE =9.36;MAPE = 3.26%;MAE = 6.07) than for females(RMSE = 10.33; MAPE = 3.63%;MAE = 6.97).Forecasts for 2030 varied across models: Holt projected male and female mortality at 253.40 and 252.63,respectively, whereas Prophet estimated these rates as 272.71 and 274.34.

The findings indicate that the Prophet model is a suitable forecasting method for use in health-related research. The results provide critical insights for the planning of public health policies and the development of strategies to prevent cardiovascular diseases. Given its flexibility and forecasting accuracy, it is recommended that the use of the Prophet model be encouraged in future health research involving time series forecasting.

Keywords: *Time series analysis, Prophet model, Holt's linear trend model, Cardiovascular diseases*

Professional Qualification and Science Teaching Self-Efficacy Belief in Special Education Pre-Service Teachers: An Examination of the Mediating Role of Academic Motivation (1084)

Pelin Ertekin*[1], Abdullah Tunç[1]

[1]İnönü University, Faculty of Education, Special Education Department, Türkiye

*Corresponding author e-mail: pelin.ertekin@inonu.edu.tr

Abstract

This study aims to examine the mediating role of academic motivation in the relationship between professional qualification and science teaching self-efficacy belief among special education pre-service teachers. Adopting a multifactor predictive correlational design within the quantitative paradigm, the research employed path analysis since all variables in the hypothesized model were directly measurable (observed variables). The study sample consisted of 200 volunteer pre-service teachers enrolled in the first through fourth years of a Special Education Teacher Education Program (67% female, 33% male). Data were collected using the Professional Qualification Scale for Pre-service Teachers (Sarpakaya and Altun, 2021) (Cronbach's alpha = 0.94), the Science Teaching Self-Efficacy Belief Instrument (Bıkmaz, 2002) (Cronbach's alpha = 0.85), the Academic Motivation Scale (Karagüven, 2012) (Cronbach's alpha = 0.67–0.87), and a Personal Information Form. The normality of the data was assessed, and assumptions of multicollinearity, singularity, and linearity were checked before testing the hypothesized model. Correlation analysis was conducted to determine the direction and strength of relationships, while the mediating role of academic motivation was tested using bootstrapping. The research data were analysed using SPSS Statistics 21.0 and the PROCESS Macro. Specifically, the PROCESS Macro developed by Hayes was utilized to conduct a serial multiple mediation analysis. Descriptive and inferential statistical analyses were performed with a significance level set at $\alpha = 0.05$. The study is expected to reveal that academic motivation plays a critical role in enhancing both professional competence and science teaching self-efficacy among special education pre-service teachers, offering valuable insights for the improvement of teacher education programs.

Keywords: *Special Education, Professional Qualification, Science Teaching Self-Efficacy, Academic Motivation, Mediation Analysis*

Examination of Supervised Machine Learning Algorithms in Employee Turnover Prediction (1086)

Melisa Dikici[1], Gökçe Sabriye Hörük[1], Deniz Efendioğlu[1*]

[1]Ankara Yıldırım Beyazıt University, Ankara, Türkiye

Corresponding author e-mail: defendioglu@aybu.edu.tr

Abstract

This study classifies employee layoffs into two types: voluntary and involuntary. The financial effects of voluntary turnover are highlighted in organizations. The ability to predict redundancies is a very important aspect of employee retention strategies for any organization. Supervised machine learning algorithms, which are more accurate, were analyzed in the context of employee turnover prediction. The literature review identified some of the key parameters that influence turnover, which include working conditions, job satisfaction, management support, pay fairness, and career opportunities. The algorithms to be studied will include Logistic Regression, Decision Trees, Random Forests, Support Vector Machines (SVM), K-Nearest Neighbors (KNN), and Naive Bayes. Random Forest showed the best accuracy; hence, it is recommended for complicated datasets. Logistic Regression, though less accurate, is simple and interpretable, hence useful for strategic decision-making. This study henceforth highlights that the choice of algorithms should be fully aligned with data structure and organizational priorities for better human resource management and reduction in turnover ratio.

Keywords: *Employee Turnover, Machine Learning, Supervised Algorithms, HR Analytics.*

Airline Evaluation in Europe Based on Spherical Fuzzy TOPSIS Approach (1087)

Sema Akin Bas[1*]

[1]Yildiz Technical University, Faculty of Arts and Science, Department of Mathematics, İstanbul, Türkiye

*Corresponding author e-mail: akins@yildiz.edu.tr

Abstract

User evaluations of flight experiences are an essential element that strengthens airline competition so that companies can sustainably improve service quality. In this study, low-cost airlines in Europe have been evaluated utilizing the data collected through the Skytrax Airline Reviews website, where airline passengers articulated their opinions about their airline experience. In light of the alterations in passenger demands post-COVID-19 pandemic, reviews published within the last four years, from 2020 to 2024, were taken into account. The gathered data include passengers' quantitative evaluations of airline service quality features by scoring 1-5. The attributes rated by passengers in the data, including seat type and passenger type categories, are cabin crew service, Wi-Fi service, ground service, value for money, seat comfort, and inflight entertainment. However, the assessment of such attributes often involves uncertainty and subjectivity. Since fuzzy logic-based approaches provide more flexible solutions, thereby uncertainty in human nature was reflected in the model with spherical fuzzy numbers. Spherical structures were employed to systematically integrate degrees of truthiness, falsity, and indeterminacy into the model, hence enhancing the realism of decision-making processes. With a total of 2119 comments and ratings, four teen low-cost airlines were assessed and ranked with spherical fuzzy TOPSIS. The findings acquired can guide decision-makers in the aviation industry and strategic options for passengers.

Keywords: *Spherical Fuzzy TOPSIS, Airline Evaluation, Real Data Set*

An Application of Spherical Fuzzy TOPSIS for Measuring Healthcare Service Performance (1088)

Sema Akin Bas[1*]

[1]Yildiz Technical University, Faculty of Arts and Science, Department of Mathematics, Istanbul, Türkiye

*Corresponding author e-mail: akins@yildiz.edu.tr

Abstract

The healthcare sector, which must adapt to changing human expectations with advancing technology, plays an essential role in the living standards of individuals and the socioeconomic status of countries. In this context, healthcare organizations cannot ignore patient satisfaction in order to maintain sustainable competitiveness. This study evaluates patient satisfaction with healthcare services using the spherical fuzzy TOPSIS approach, focusing on the quality of healthcare services. Furthermore, spherical fuzzy sets have been employed to represent subjectivity inside the model better realistically. In this study, survey data have been utilized from emergency room patients at a foundation university hospital in Istanbul. The hospital management collected data from patient satisfaction surveys conducted in accordance with Ministry of Health guidelines between 2019 and 2022. The acquired information comprises responses to nine teen survey questions from 2,092 patients utilizing the hospital's emergency department services. The assessment performed at the study's completion identified and ranked the areas where patients reported the highest satisfaction and discontent concerning the institution. As a result of the analysis, the highest satisfaction point was that the patients could easily reach the health personnel when they needed it. Another comment obtained as a result of the study was that the information given about the post-discharge process at the exit from the emergency department was sufficient and explanatory. On the other hand, patients expressed concerns about the knowledge and competence of nurses, which stood out as the most critical point of dissatisfaction.

Keywords: *Spherical Fuzzy TOPSIS, Patient Satisfaction, Healthcare Industry*

Performance Comparison of Clustering Algorithms on Different Datasets (1089)

Mahmut Selim Boyracı[1]*

[1]Karadeniz Technical University, Department of Statistics and Computer Sciences, Turkey

*Corresponding author e-mail: selimboyraci@gmail.com

Abstract

This research provides an exhaustive assessment of how K-Means, DBSCAN, and Hierarchical Clustering algorithms perform across various datasets that present different attributes like size, density, and shape. The performance of various algorithms was examined through different datasets like Iris, Yeast, MNIST, Customer Segmentation, make_blobs, and make_moons to reveal their strengths and weaknesses which provide useful insights for practical applications.

Experiments showed that K-Means achieved high performance on spherical and well-separated clusters while DBSCAN demonstrated superior capability when dealing with non-linear and irregular clusters using datasets like make_moons. Hierarchical Clustering achieved results that were evenly distributed between interpretability and overall performance. DBSCAN requires careful parameter tuning and Hierarchical Clustering demands significant computational power were identified as drawbacks. The study's methodology follows a systematic approach that includes data preprocessing steps before algorithm implementation along with performance metric selection, data analysis and statistical validation of results. The performance evaluation examined three key factors which included accuracy, time complexity and scalability while Python libraries were used to maintain the analyses reproducibility. The research results highlight that data characteristics should guide the choice of clustering algorithms while offering method selection advice for sectors like market segmentation, bioinformatics, and image recognition. The research fills an important void in clustering algorithms literature while establishing a foundation for upcoming studies.

Keywords: *Performance Evaluation, Data Preprocessing, K-Means, DBSCAN, Hierarchical Clustering.*

Future Trends in Global Barley Production: A Time Series Analysis for the 2025–2035 Period (1090)

Nami Kaan Kızılcan[1*], Başar Altuntaş[1]

[1]Kırşehir Ahi Evran University, Faculty of Agriculture, Department of Agricultural Economics, Türkiye

* Corresponding author e-mail: nami.kizilcan@ahievran.edu.tr

Abstract

Barley, which holds a significant share among cereal crops, is a strategic commodity not only for the feed and food sectors but also for the beverage industry. In Turkey, barley adapts well to local climatic and soil conditions and plays a notable role in the agricultural production system in terms of both production volume and usage diversity. With the expansion of the livestock sector, barley has become increasingly important in the compound feed industry due to its high energy value and good digestibility. In the food industry, barley is utilized in the production of both traditional and functional foods thanks to its richness in dietary fiber. In recent years, the rising trend toward healthy eating has led to increased consumption of barley in the form of flour, bran, and whole grains.

The primary objective of this study is to produce forecasts for the 2025–2035 period by utilizing production data from 1961 to 2023 for the world's leading barley-producing countries. For each country, the most appropriate time series model (ARIMA) was identified, and future production projections were made accordingly. The findings indicate a geographical shift in barley production power, moving from the West toward the East and into the Southern Hemisphere. While countries such as Turkey, Russia, and Australia are expected to increase their share of production, traditional producers like the United States, Germany, and Canada are projected to see a decline. Turkey, in particular, is anticipated to assume a more prominent position in global production in the coming years. The results of the study demonstrate that ARIMA modeling—a widely used time series analysis method—serves as a practical tool for agricultural production planning. Furthermore, the outcomes provide strategic insights for policymakers, entrepreneurs, and relevant stakeholders in shaping marketing plans and strategies.

Keywords: *Agricultural marketing strategy, agricultural policy, production planning, forecasting, time series*

Fuzzy Weighted k-out-of-n:G Systems: A Cost-Reliability Optimization Framework (1091)

Kadir Sarıkaya[1]

[1] Tokat Gaziosmanpaşa University, Faculty of Engineering and Architecture, Industrial Engineering Department, Türkiye

*Corresponding author e-mail: ksarikaya74@gmail.com

Abstract

This study proposes a novel fuzzy weighted k-out-of-n:G system model to address reliability optimization problems where component contributions are uncertain and heterogeneous. Unlike classical reliability systems that assume binary (working/failed) states with unit weights, our framework incorporates two distinct component types characterized by triangular fuzzy weights: Type 1 ($\tilde{w}_1 = (w_1^L, w_1^M, w_1^U)$) and Type 2 ($\tilde{w}_2 = (w_2^L, w_2^M, w_2^U)$). The system operates successfully when the total fuzzy weight of functioning components meets or exceeds a threshold $\tilde{k} = (k^L, k^M, k^U)$, generalizing the conventional k-out-of-n:G paradigm to account for real-world imprecision in performance metrics.

Building on Zimmermann's fuzzy linear programming (FLP) approach, we develop a multi-objective optimization model that simultaneously: (1) minimizes total system cost $C = c_1n_1 + c_2n_2$, and (2) satisfies the fuzzy reliability constraint $\sum\tilde{w}_1X_i + \sum\tilde{w}_2Y_j \geq \tilde{k}$ through a maximized satisfaction level λ. The methodology transforms fuzzy weight aggregation into deterministic equivalents using α-cut techniques and membership function optimization, enabling computationally efficient solutions.

A comprehensive case study demonstrates the model's application to a solar microgrid design problem, where photovoltaic panels exhibit weather-dependent output variations (Type 1: 2-4 kW, Type 2: 3-5 kW) and grid stability requires 10-20 kW power generation. The FLP solution ($n_1 = 3$, $n_2 = 2$, $\lambda = 0.85$) achieves optimal cost-reliability balance at 31 monetary units, with sensitivity analysis revealing system robustness to ±15% weight fluctuations. Practical implementation guidelines are provided for energy systems and communication networks.

The research contributes both theoretically and practically by: (i) extending k-out-of-n theory to fuzzy-weighted components, (ii) developing a computationally tractable FLP solution, and (iii) offering design insights for engineers managing uncertain resource allocations. Comparative analysis shows 18-22% cost reduction over conservative deterministic approaches while maintaining equivalent reliability levels.

Keywords: *Fuzzy Reliability Systems, Weighted k-out-of-n:G, Zimmermann's FLP, Triangular Fuzzy Numbers*

Robustness of Frühwirth-Schnatter Bayesian hybrid (Finite Mixture Modelling and Markov Switching Modelling) Method on the IMF Gross Domestic Product Data Set (1092)

Selim Dönmez[1*]

[1]Eskisehir Osmangazi University, Faculty of Science, Statistics, Turkey

*Corresponding author e-mail: sdonmez3@gmail.com

Abstract

The application of the finite mixture modelling and Markov switching modelling was introduced in the book named"Finite Mixture and Markov Switching Models" written by Sylvia Frühwirth-Schnatter which provides a complete analytic approach to data types like time-series data and longitudinal data. We present an application to the IMF Gross Domestic Product change percentage longitudinal data corresponding to the specified country groupings in the IMF web site in which the modelling carried by the Matlab R2017a software provides a valid, robust but an unsuccessful or difficult to interpret conclusion. The application is implemented in steps. The first step is to transform the data set into a data set with discrete values which is a necessary preparation step. The discrete values are obtained according to a principle that if the following year's GDP percentage change value of the group is bigger than the present year's GDP percentage change value of the group, it would obtain a discrete value of 1 which means that there's an acceleration in the GDP percentage change in the following group at that year. The groups in the IMF data set were named as World, Advanced Economies, Euro Area, Emerging and developing Asian, Emerging and developing Europe, Latin America and the Caribbean, Middle East and Central Asia, Sub-Saharan Africa. The Frühwirth-Schnatter Bayesian hybrid method produces a cluster of groups when applied to the data set and returned the conclusion that there can be only one cluster in which the groups can belong to. The application to the data set had produced a robust conclusion in which if the aforementioned principle was changed significantly, the findings would resist the change in the principle. Therefore, we have two contradictory problems regarding the conclusion of the hybrid method being near universal and the critique of the algorithmic workflow. As a final note, the Bayesian method of Sylvia Frühwirth-Schnatter can be transformed mathematically into a method that provides easier to interpret conclusions.

Keywords: *Bayesian Method, Gross Domestic Product, Finite Mixture Modelling, Markov Switching Modelling*

Compromise Ranking of Pareto Optimal Solutions Using Copeland Method with Multi-Criteria Decision-Making Approach (1094)

Özlem Türkşen[1], Mehmet Ünver[2*]

[1]Ankara University, Faculty of Science, Department of Statistics, Türkiye
[2]Ankara University, Faculty of Science, Department of Mathematics, Türkiye

*Corresponding author e-mail: munver@ankara.edu.tr

Abstract

A multi-response optimization problem can be considered as a multi-objective optimization (MOO) problem. Artificial Intelligence (AI) optimization algorithms are commonly used to solve the MOO problem. Solution set of the MOO problem is composed with Pareto optimal solutions which are non-dominated alternative solutions. It is not easy to define a compromise solution among these optimal solutions. In this study, three multi-response experimental data sets are taken into consideration from the literature as MOO problems for which Pareto optimal solutions were obtained via AI optimization algorithms called NSGA-II. A comprehensive evaluation is conducted on the ranking of the Pareto optimal solutions. The non-dominated solutions are subsequently ranked using five different multi-criteria decision-making (MCDM) methods: EDAS, CODAS, MABAC, TOPSIS, and COPRAS. Spearman's rank correlation coefficients are calculated to investigate the degree of agreement of these MCDM methods. The results reveal that some MCDM methods produce rankings with notably low mutual correlations, indicating inconsistency in decision preferences. To overcome this issue, the Copeland method is utilized to generate a consensus ranking based on the obtained five individual orderings. Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) metrics are used to validate the ranking preference of the Pareto optimal solutions.

Keywords: *Pareto optimal solutions, AI optimization, Multi-Criteria Decision Making (MCDM) Methods, Copeland Method.*

A Global Comparative Statistical Perspective on Cancer Types Using GLOBOCAN Data: An Integrative Approach with Bioinformatics (1097)

Gülşah Keklik[1*]

[1]Istanbul Technical University, Eurasia Institute of Earth Sciences, Department of Ecology and Evolution, Istanbul

*Corresponding author e-mail: gkeklik@itu.edu.tr

Abstract

Cancer is one of the most pressing global public health challenges, responsible for millions of deaths annually and exhibiting considerable variation across countries, regions, and cancer types. Accurate and comprehensive analysis of global cancer data is essential for identifying patterns, understanding disparities, and informing policy and clinical strategies. The GLOBOCAN database, maintained by the International Agency for Research on Cancer (IARC), provides up-to-date estimates on cancer incidence, mortality, and prevalence across 185 countries and for 36 cancer types. This study will employ GLOBOCAN data to conduct a comparative statistical analysis of all major cancer types worldwide.

Descriptive and inferential statistical methods will be applied to evaluate age-standardized incidence and mortality rates, with a particular focus on comparing Türkiye's data with global averages. Regional disparities, temporal trends, and cancer-specific burdens will be explored using multivariate analyses, regression modeling, and clustering techniques. To further enhance interpretation, bioinformatics approaches will be integrated to examine potential risk factors—such as environmental exposures, genetic predispositions, and behavioral determinants—where supportive datasets are available.

The combined use of statistical and bioinformatics tools is expected to yield a multidimensional view of cancer epidemiology, allowing for the generation of hypotheses regarding etiology, early detection, and prevention strategies. This interdisciplinary approach will contribute to the broader effort of reducing the global cancer burden through evidence-based and data-driven insights.

Keywords: *Cancer Epidemiology, GLOBOCAN, Statistical Modeling, Bioinformatics, Global Health*

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A Probabilistic Analysis of Multi-Item Shipment Consolidation (1099)

M. Ali Ülkü[1*]

[1]Department of Management Science and Information Systems, Faculty of Management, Dalhousie University, Halifax, NS, Canada

*Corresponding author: ulku@dal.ca

Abstract

A powerful logistics policy, shipment consolidation, involves transporting several small loads on the same vehicle. This paper proposes a new cost model that explicitly presents a probabilistic analysis of multi-item shipment consolidation (MISCL), focusing on optimizing logistics costs while meeting delivery constraints. Via myopic analysis, the effects of load make-up on the dispatch decisions by private freight carriage are explored. Analytical and simulation-based results reveal key parameters influencing cost efficiency, such as demand variability and shipment-policy thresholds. The findings offer actionable insights for MISCL to enhance operational efficiency, logistics service offerings, and supply chain performance through data-driven strategies under uncertainty.

Keywords: *Analysis, Probability, Operations Research, Logistics, Consolidation*

A Robust Meta-Fuzzy Approach for Bivariate Correlation Analysis (1100)

Erdinç Karakullukcu[1*]

[1]Karadeniz Technical University, Faculty of Science, Department of Computer Sciences, Turkey

*Corresponding author e-mail: erdinc.karakullukcu@ktu.edu.tr

Abstract

Pearson's correlation, a common measure of bivariate association, is constrained by its requirement for normally distributed data and high sensitivity to outliers, particularly in small samples. This sensitivity can lead to inaccurate or misleading conclusions, especially when dealing with real-world data that often deviates from ideal conditions. While several robust correlation methods have been developed to address these issues, identifying the most suitable one for a specific dataset remains complex. The performance of these methods can vary significantly depending on the underlying data characteristics. This study introduces a meta-fuzzy function approach to mitigate these limitations and provides a more reliable measure of bivariate association. The proposed method integrates multiple correlation techniques through a combined model, leveraging the strengths of each. Specifically, it employs Fuzzy c-means (FCM) clustering to group these techniques based on their performance. FCM allows for a flexible grouping, recognizing that correlation methods may exhibit varying degrees of effectiveness across different data subsets. The Xie-Beni index is used to determine the optimal number of clusters, ensuring that the grouping effectively captures the nuances of method performance. By weighting and combining the most effective correlation methods for a given dataset, the meta-fuzzy function adaptively selects a high-performing model. This adaptive selection process enhances the robustness of the correlation analysis. Rigorous evaluation using both simulated (normal, non-normal, and outlier-contaminated) and real-world datasets demonstrates that this method surpasses individual techniques, achieving the lowest mean absolute percentage error (MAPE) across all scenarios. Consequently, it offers a robust and adaptive solution for bivariate correlation analysis, providing a more reliable tool for researchers and practitioners.

Keywords: *Bivariate Correlation, Robustness, Meta-Fuzzy Functions, Fuzzy Clustering*

The Relationship Between Team Statistics and League Rankings in the Turkish Süper League (1101)

Toprak Kıraç Taylan[1*], Özgül Vupa Çilengiroğlu[2], Ümit Kuvvetli[3]

[1] Dokuz Eylül University, The Graduate School of Natural and Applied Sciences, Data Sciences, 35390, İzmir, Türkiye
[2] Dokuz Eylül University, Faculty of Sciences, Statistics, 35390, İzmir, Türkiye
[3] İzmir Bakircay University, Faculty of Economics and Administrative Sciences, Business Administration, 35665, İzmir, Türkiye

*Corresponding author e-mail: toprakkirac.taylan@ogr.deu.edu.tr

Abstract

Today, football is known as the most popular team sport. In football matches, coaches, players, and club management implement various methods to achieve development and success. In the literature, factors related to teams and players are analyzed using different statistical methods in studies examining the outcomes of football matches, whether won or lost. However, identifying the factors that influence overall success in a league, rather than focusing solely on individual match outcomes, is highly valuable for shaping team strategies throughout the season. For teams aiming for goals such as winning the championship, qualifying for European competitions, or avoiding relegation, analyzing the teams that achieved these goals in previous seasons, in terms of both on-field and off-field factors, can serve as a crucial guide. This study categorized teams competing in the Turkish Süper Lig between the 2014/2015 and 2023/2024 seasons into five clusters based on their success levels. Then, the off-field variables (such as squad quality, number of managerial changes, average squad age, etc.) and on-field match statistics (such as average number of shots per home game, home possession rate, average number of goals per away game, etc.) of the teams within these clusters were compared. The results indicate a strong relationship between team performance indicators and end-of-season success levels. The findings are particularly striking in identifying critical indicators for teams with specific goals within the league.

Keywords: *Football, League Rankings, Performance Indicators*

Air Quality Classification Based on Meteorological Data Using Machine Learning (1103)

İremsu Menekşe[1], Öznur Özaltın[2*]

[1]Hacettepe University, Institute of Science, Department of Actuarial Science, Türkiye

*Corresponding author e-mail: oznurozaltin@hacettepe.edu.tr

Abstract

Air pollution, exacerbated by rapid industrialization, population density, and resource exploitation, poses a global threat, resulting in millions of premature deaths and serious public health issues each year. Continuous air quality monitoring and accurate estimation are, therefore, crucial for public health and environmental policy. This study classifies air quality levels using meteorological data, including latitude, longitude, temperature, wind speed, pressure, humidity, cloudiness, visibility, and ultraviolet intensity as features. Unlike many machine learning-based studies on air quality estimation that rely on pollutant concentrations (CO_2, SO_2, NOx), this approach utilizes these meteorological features, providing a practical alternative when pollutant data is scarce. Data preprocessing included handling missing values and outliers, eliminating low-correlation variables, and standardization. Random Forest, Extreme Gradient Boosting (XGBoost), Light Gradient Boosting Algorithm (LightGBM), and Category Boosting (CatBoost) algorithms are employed for classification, with Grid Search optimization and the Synthetic Minority Oversampling Technique (SMOTE) to address class imbalance. Model performance is evaluated using various metrics. The results demonstrate successful air quality classification using only meteorological variables, highlighting their strong indicative value.

Keywords: *Mortality, Machine Learning, Air Quality Classification, SMOTE, Hyperparameter Optimization.*

Damage Detection of Building Structures Based on Artificial Intelligence Algorithms (1104)

Öznur Özaltın[1*], Nursel Koyuncu[2]

[1] Hacettepe University, Faculty of Science, Department of Actuarial Science, Türkiye
[2] Hacettepe University, Faculty of Science, Department of Statistics, Türkiye

*Corresponding author e-mail: oznurozaltin@hacettepe.edu.tr

Abstract

In the insurance industry, accurately and efficiently assessing property damage following a major earthquake is essential for accelerating claims processing. This study introduces an innovative framework that employs deep learning and machine learning methodologies to automate post-disaster damage assessment using image data. By analyzing images of affected structures, pre-trained hybrid Convolutional Neural Network (CNN) architectures are utilized to extract deep image features indicative of varying levels of structural building damage. To optimize the assessment process further, feature selection via sampling techniques has been implemented, the most relevant and significant attributes. These selected features are then classified through a machine learning algorithm to determine the level of damage. For insurance companies, this novel approach offers a solution by significantly reducing the time and resources required for manual damage inspections. It can enable rapid and objective evaluations, allowing insurers to make decisions more swiftly and eliminating delays in claim settlements. Ultimately, this framework enhances operational efficiency, boosts insured satisfaction, and plays a pivotal role in post-earthquake recovery by ensuring timely financial support for affected individuals.

Keywords: *Artificial Intelligence, Building Damage Detection, Feature Selection, Insurance Industry, Sampling Theory.*

Insurance Fraud Classification via Feature Selection Method and Machine Learning Algorithms (1105)

Öznur Özaltın[1], Övgücan Karadağ Erdemir[2*]

[1,2] Hacettepe University, Faculty of Science, Department of Actuarial Science, Türkiye

*Corresponding author e-mail: ovgucan@hacettepe.edu.tr

Abstract

Fraud detection in the insurance industry remains a critical and complex challenge, particularly due to the high dimensionality of data and class imbalance commonly present in real-world datasets. This study presents a comprehensive machine learning-based framework that integrates data preprocessing, feature selection, and hyperparameter optimization to improve the accuracy and reliability of fraud detection systems. Specifically, the Synthetic Minority Oversampling Technique (SMOTE) is applied to address class imbalance, ensuring that minority class instances are sufficiently represented during model training. A variety of machine learning models are evaluated within this framework. Each model undergoes five-fold cross-validation and grid search-based hyperparameter tuning to mitigate overfitting and ensure robustness. Among the tested configurations, the one involving the top important features selected yields the highest fraud detection rate. The results emphasize the effectiveness of combining feature selection with careful tuning and data balancing strategies. Although these techniques require considerable computational resources, they substantially enhance model generalizability and practical applicability. The proposed methodology not only improves detection performance but also offers a scalable approach for deploying machine learning solutions in the insurance sector. Future work may involve exploring alternative feature selection algorithms and applying the approach to larger, more diverse datasets.

Keywords: *Fraud Detection, Feature Selection, Insurance Sector, Machine Learning.*

NEET in Europe: Gender-Based Spatial Panel Data Analysis (1108)

Eda Selin Ilikkan Basdogan[1,2*], Elif Yıldırım[1,3]

[1]Hacettepe University, Faculty of Science, Department of Statistics, Turkey
[2]Ministry of Labour and Social Security, Directorate General of Labour, Statistics Department Turkey
[3]Konya Technical University, Rectorate, Department of Statistics and Quality Coordinator, Turkey

*Corresponding author e-mail: edaselinilikkan@gmail.com

Abstract

Nowadays, the deprivation of young people from education, employment or vocational training has serious consequences not only at the individual level, but also at the social and economic level. The NEET (Not in Education, Employment or Training) rate, which defines this situation, indicates the insufficient integration of young people into the labor market and is considered an important indicator of social exclusion and economic losses. High NEET rates may have multidimensional consequences in the long term, such as idle labor force potential, damage to social cohesion and increased public expenditure. In this context, analyzing the determinants of the NEET phenomenon by taking gender-based and spatial differences into account is of great importance for policy-making processes. Therefore, in this study, the variables that are thought to affect the NEET rate in European countries are analyzed with a spatial panel data model with both a general and gender-based approach to examine the spatial dependency structure across countries. According to the model results, it is found that, regardless of gender, youth unemployment and inactive people rate significantly increases the NEET rate and educational participation rate has a decreasing effect. Moreover, it is found that only social exclusion increases the male NEET rate for males and the early school leaving rate increases the female NEET rate for females. While direct spatial effects are limited in the models for total population and women despite cross-country similarities, significant spatial interactions are found in the model for men. Accordingly, while the male NEET rate is more sensitive to spatial interactions, NEET rates for women and total population are more determined by country-specific endogenous dynamics. The results showed that policies to reduce NEET rates should be shaped by education, youth employment and gender-based approaches and emphasized that spatial approaches can contribute to policy design.

Keywords: *NEET, Spatial panel regression, Gender-based approach, European countries, Youth unemployment*

Rethinking Vocational Skills in the Age of AI: Evidence from Turkiye (1109)

Ahmet Kurnaz[1], İdil Ayberk[2*]

[1] Çanakkale Onsekiz Mart University, Faculty of Political Sciences, Political Science and Public Administration, Turkiye
[2]Ankara Medipol University, School of Administrative and Social Sciences, Management Information Systems, Turkiye

*Corresponding author e-mail: idil.ayberk@ankaramedipol.edu.tr

Abstract

The dynamics of labor markets are experiencing significant changes due to technological advancements driven by the rise of automation and artificial intelligence. Funded by The Scientific and Technological Research Council of Turkey (TÜBİTAK) and conducted in collaboration with the Turkish Statistical Institute (TÜİK), this study examines the changing labor demand for vocational and technical high school graduates in Turkey between January 2015 and July 2024. Using a dataset of 1.54 million online job advertisements collected from open sources, we employ a hybrid methodology that combines natural language processing (NLP) techniques, large language models (LLMs), and deep learning to map job postings to ISCO-08 occupational codes. The study analyzes educational requirements stated in job advertisements to identify occupational demand for vocational and technical education graduates. It also explores the demand for specific skills, including digital, software, green, social, and cognitive, as well as the certification requirements to provide a better understanding of the qualifications required for the job. By linking educational fields from vocational training programs with occupational classifications in job postings, the study aims to explore the link between educational backgrounds and labor market needs. The study will, therefore, provide evidence to support efforts to improve the responsiveness of vocational education to evolving labor market dynamics and enhance labor market monitoring.

Keywords: *Automation, Digital Skills, Skill Mismatch, LLMs*

Forecast Refinements via Similar Trajectories and the Traffic Flow Forecasting Problem (1111)

Ümit Işlak[1], Elif Yılmaz[2], İlker Arslan[3*], Tuna Çakar[4]

[1]Boğaziçi University, Department of Mathematics, Turkey
[2]University of Université de Neuchâtel, Institut d'informatique, Switzerland
[3]MEF University, Dept. of Electrical and Electronics Engineering, Turkey
[4]MEF University, Dept. of Computer Engineering, Turkey

*Corresponding author e-mail: arslanil@mef.edu.tr

Abstract

The purpose of this study is to develop a hybrid model that combines the method of similarity of trajectories with machine learning (ML) algorithms. The proposed approach is analyzed in the context of traffic flow prediction, and experimental results indicate that it significantly outperforms the individual models. Although the method of similarity of trajectories has been extensively popularized in economics, it has since been adapted to various other fields involving time series analysis. In particular, it has been successfully applied in different forms to traffic flow prediction. While there are several variations of the method, it generally involves identifying the most similar past trajectories based on recent observations and then appropriately combining the corresponding candidate forecasts. In contrast, the present study aims to develop a hybrid model for point forecasting by integrating similarity-based methods with ML algorithms. The approach relies on residual correction, where the errors from a preliminary prediction are modeled again to improve forecasting accuracy. In our experiments, we tested two hybrid configurations: one in which the initial model is an ML algorithm followed by correction residual using the method of similar trajectories, and another in which the order is reversed. The experiments use the PEMS traffic dataset, focusing on time series corresponding to traffic flow. The results show that the hybrid model consistently yields more accurate forecasts than the individual models and other benchmarks. Additionally, following the literature, we experimented with hybrid models where one component is an economic model such as ARIMA, and the results remained competitive. Finally, statistical tests were conducted to assess the significance of the improvements, which yielded positive evidence.
Acknowledgement: This study has been supported by the Scientific and Technological Research Council of Turkey (TÜBİTAK) under the Grant Number 124F023.

Keywords: *Similar Trajectories, Hybrid Model, Traffic Flow Forecasting, Statistical Testing, Econometric models.*

Percentile based Control Charts for Monitoring Energy Consumption (1117)

Derya Öztürk[1*], Nursel Koyuncu[2]

[1]Hacettepe University, Faculty of Science, Department of Statistics, TÜRKİYE
*Corresponding author e-mail: derya_ozturk @hacettepe.edu.tr

Abstract

This study evaluates the effectiveness of percentile-based control charts, proposed as alternatives to traditional quality control cart methods, under non-normal data distributions. Two distinct scenarios are analyzed using real energy consumption data obtained from a white goods manufacturing process.

In the first scenario, a joint control chart structure, capable of simultaneously monitoring the process mean and variability, is constructed based on a percentile-driven design. To ensure the chart's reliability under both in-control and out-of-control conditions, an optimization-based approach is applied.

In the second scenario, a percentile-based Shewhart median control chart, which is expected to be more responsive to skewed data structures, is developed and examined.

The findings demonstrate that percentile-based approaches offer a more robust process monitoring capability, particularly when compared to conventional methods. These results contribute to the integration of more flexible and data-sensitive tools in modern quality control systems.

Keywords: *percentile-based control charts, quality monitoring, process control, skewed data*

Data-driven Forecasting in Sustainability-Oriented Economic Decision Processes: The Case of BIST Sustainability Index (1121)

Kadriye Nurdanay Öztürk[1],[2*], Öyküm Esra Yiğit[3]

[1]Yildiz Technical University, Graduate School of Science and Engineering, Department of Statistics, PhD Program in Statistics, Istanbul
[2]Bilecik Şeyh Edebali University, Faculty of Science, Department of Statistics and Computer Sciences, Bilecik
[3]Yildiz Technical University, Faculty of Arts and Sciences, Department of Statistics, Istanbul

*Corresponding author e-mail: kadriye.ozturk@bilecik.edu.tr

Abstract

Sustainability, along with environmental and social responsibilities, is playing an increasingly important role in economic decision-making. Understanding and predicting how sustainability-based financial indicators behave is therefore of strategic importance for both investors and policymakers. The Borsa Istanbul (BIST) Sustainability Index, which stands out in this context in Türkiye, aims to provide investors with the performance of companies with sustainable business models traded on Borsa Istanbul. In this study, a machine learning-based approach is adopted to predict the movements of the BIST Sustainability Index. In addition to technical indicators, gold prices, Brent oil prices and exchange rate changes were used as macroeconomic variables in the modeling process. Support Vector Machines (SVM), Histogram Gradient Boosting (HGB) and Extreme Gradient Boosting (XGBoost) algorithms were applied separately and the learning capacity, overall performance and classification ability of each model were analyzed in detail. In order to create a more balanced and stable learning structure by minimizing the limitations of the individual models, the Voting ensemble model, which combines the predictions of these three algorithms, is used and the performance outputs obtained are comparatively evaluated. The results show that the Voting classifier outperforms the individual machine learning models in terms of accuracy, precision, recall, F1 score and generalization ability. These results show that ensemble models, which can produce more reliable predictions by combining the strengths of different algorithms, can be considered as an important alternative, especially in sustainability-oriented financial data analysis. The study also supports that machine learning techniques can play an important role in the interpretation of complex economic indicators and decision support processes.

Keywords: *Sustainability Index, Machine Learning, Ensemble Method.*

Improving Airport Efficiency Through IoT: A Study on Smart Airport Systems (1122)

Mohamed Ahmed Ahmed[1*]

[1]Ibn Haldun University, İstanbul, Türkiye

[*]Corresponding Author e-mail: ahmad-mad@hotmail.com

Abstract

Recent technological advancements, particularly the Internet of Things (IoT), have the potential to revolutionize airport operations and enhance service delivery. This study hypothesizes that IoT implementation significantly improves operational efficiency, passenger experience, and airport security. The research explores four primary domains of airport operations: airside operations, ground operations, billing and invoicing, and information management. In airside operations, IoT technologies such as sensors and connected devices are used for monitoring runways, taxiways, and apron areas, which enhances situational awareness and supports decision-making. For ground operations, RFID, drones, and surveillance systems are employed to detect and manage activities efficiently, improving safety and reducing operational costs. In billing and invoicing, IoT enables automation and accuracy, minimizing errors and disputes and boosting customer trust. For information management, real-time data sharing improves coordination, streamlines check-in and baggage handling, and enhances the overall passenger journey. From a security perspective, IoT supports real-time monitoring of people and equipment, enhances threat detection, and strengthens coordination among security personnel. However, several implementation challenges persist. These include cybersecurity risks, privacy concerns, integration difficulties with legacy systems, high installation and maintenance costs, and vendor interoperability issues.

The findings highlight that despite its transformative potential, IoT remains underutilized in the airport sector. Greater focus on strategic implementation, standardization, and cybersecurity is essential to fully leverage IoT technologies for operational and security gains.

Keywords: *Internet of Things (IoT), Smart Airports, Airport Operations, Operational Efficiency*

Comparison of Count Regression Models and Machine Learning Techniques for Predicting Cigarette Consumption (1123)

Elifsu Ütük[1*], Deniz Özonur[2]

[1]Gazi University, Graduate School of Natural and Applied Sciences, Department of Statistics, 06500, Ankara, Türkiye
[2]Gazi University, Faculty of Science, Department of Statistics, 06500, Ankara, Türkiye

*Corresponding author e-mail: elifsuutuk@gmail.com

Abstract

Understanding the determinants of cigarette consumption is critically important for shaping effective public health policies, as smoking remains a major risk factor for numerous chronic diseases and preventable deaths worldwide. This study compares the traditional count regression models and machine learning techniques in predicting cigarette consumption. The dependent variable is the number of cigarettes smoked per day, while the explanatory variables include a range of sociodemographic and health-related factors such as gender, education level, marital status, employment status, income level, general health status, tobacco and alcohol use, and presence of chronic diseases. The analysis is based on microdata from the 2016 Turkey Health Survey, conducted by the Turkish Statistical Institute. This study employs a range of count regression models, including Poisson, Negative Binomial, Zero-Inflated Poisson, Zero-Inflated Negative Binomial, as well as Poisson and Negative Binomial Hurdle models. In addition, flexible machine learning algorithms such as Random Forest and XGBoost are also utilized for comparative analysis. All models are applied to both training and test datasets, and their performances are evaluated using Akaike Information Criterion (AIC), Root Mean Squared Error (RMSE), and Mean Absolute Error (MAE). The results indicate that zero-inflated and hurdle models outperform others, particularly in datasets characterized by excess zeros and overdispersion. Machine learning methods, on the other hand, better adapt to the complexities within the data and provide high-accuracy predictions.

Keywords: *Count Regression Models, Machine Learning, Cigarette Consumption, Turkey Health Survey*

Housing Price Prediction Based on Automated Machine Learning: A Case Study on Three Major Cities in Turkey (1125)

Hakan Ergün[1*], Prof. Dr. Ersoy Öz[2]

[1] Yildiz Technical University, Institute of Science, Department of Statistics, Istanbul, Turkey
[2] Yildiz Technical University, Faculty of Arts and Sciences, Department of Statistics, Istanbul, Turkey

*Corresponding author e-mail: haakanergun@gmail.com

Abstract

In this study, automated machine learning (AutoML) techniques were used to forecast housing prices in Turkey's three major metropolitan areas: Istanbul, Ankara, and Izmir. The 2024 housing sale prices were standardized in US dollars according to exchange rates at the corresponding dates to capture economic fluctuations. Separate datasets were prepared for each city, incorporating property-specific features (e.g., square footage, number of rooms, building age) and regional location data. During preprocessing, missing values were imputed and outliers were detected and corrected.

In the modeling phase, the AutoML platform automatically tested and compared different algorithmic approaches—such as decision trees, ensemble methods, and support vector machines. Prediction accuracy was evaluated using Mean Absolute Error (MAE), Mean Squared Error (MSE), and Root Mean Squared Error (RMSE). For hyperparameter tuning, AutoML's automated optimization routines employed both grid search and random search strategies to efficiently explore the parameter space, with k-fold cross-validation ensuring robust selection of the best parameter combinations. This approach delivered reliable performance on both training and test datasets.

The findings demonstrate that AutoML approaches offer rapid, flexible, and data-driven forecasting solutions in the complex and dynamic real estate market. This work provides a solid foundation for developing more precise decision support systems for public agencies and private enterprises in the real estate sector.

Keywords: *Housing Price Prediction, Automated Machine Learning, Parameter Optimization*

Branches: Important Parts of Trees and Potential Uses (1126)

Göksu Sirin[1*]

[1] Tokat Gaziosmanpaşa University, Almus Vocational School, Department of Forestry and Forest Products, 60150, Tokat, Türkiye

* Corresponding author e-mail: goksu.sirin@gop.edu.tr

Abstract

There are approximately 4 billion hectares of forest land worldwide and it is known that about two-fifths of the current biomass potential is used. Branch wood biomass is an important resource that is not sufficiently utilized for many forest industry sectors. Branch wood from prunings or dead trees is not used as primary raw material for the industry and is considered as residues of tree biomass. It has been reported that branch pieces constitute a significant part of the wood volume and can increase the yield by about 60%. In this study, an assessment of wood biomass flows and potential branch production in different regions of the world was made. Data were taken from ForesSTAT datasets, UN statistics and various reports for countries with full access to international statistics on forestry production. Estimations of unrecorded production are also available using data on industrial roundwood extraction volumes from FAO statistics. This database allows to analyze trends and patterns of wood biomass flows, its share in total primary energy consumption, its relationship with population, forest area and forest productivity. Using forestry data, the amount of marketable branch wood volume is estimated. Data obtained on branch wood/log volumes can be quantitatively determined according to statistical analysis methods as maximum, minimum and average values and mean square deviation and variation coefficients according to the marketable branch wood characteristics of various species. The results show that the average branch wood volume produced is important enough as it will increase the yield in commercial productions and help reduce the high depletion rate.

Keywords: *Branch Wood, Principal Component Analysis, Residual Wood, Volume Modeling*

A Study on Health Expenditures in Turkey (1127)

Abdulkerim Güler[1]

[1]Sivas Cumhuriyet University, Şarkışla School of Applied Sciences, Department of Information Systems and Technologies, Türkiye.

*Corresponding author e-mail: aguler@cumhuriyet.edu.tr

Abstract

Health expenditures refer to the spending made by governments, societies, and individuals for health services. Health expenditures indicate the level of social welfare in a country and the importance of public service. Factors such as disease prevention and quality of life are important for protecting and improving health. In countries or societies with adequate and quality health services, people's quality of life also increases. The importance of health is also evident in other areas such as economy, workforce, etc. Social justice and the elimination of inequalities are also possible through public health expenditures. For example, in our country, individuals in society are protected by universal health insurance. This study aims to analyze Health Expenditures in Turkey between 1999-2023, including criteria such as Total health expenditure (Million TL), Health expenditure per capita (TL), Ratio of total health expenditure to gross domestic product (%), Ratio of general government health expenditure to total health expenditure (%), and Ratio of private sector health expenditure to total health expenditure (%). According to available data, health expenditures have been in a continuous upward trend since 1999, with this rate of increase accelerating in recent years. It shows that person-centered costs of health services have increased, the ratio of health expenditures to GDP has fluctuated over the years but generally shows an increasing trend, indicating that the share of the economy allocated to the health sector has increased. Moreover, while the share allocated by the government to health expenditures is increasing day by day, this situation is decreasing in the private sector. It is also understood from this situation that the government's role in health financing is significant. It is highly positively associated with total health expenditure and per capita health expenditure.

Keywords: *Health expenditures, Health services, Correlation analysis.*

Data-Driven Decision Making for Research Assistant Employment in Higher Education: An Application for the Ankara University (1128)

Özlem Türkşen[1*], Evrim Ağaçdelen[2], Ümit Can İnözü[1,2], Ebru Onbaşılar[2]
Kaan Orhan[3], Necdet Ünüvar[3]

[1]Ankara University, Faculty of Science, Department of Statistics, Türkiye
[2]Ankara University, Dean of Research, Türkiye
[3]Rectorate of Ankara University, Türkiye

*Corresponding author e-mail: turksen@ankara.edu.tr

Abstract

Data-driven decision-making plays an important role in enhancing the effectiveness of administrative processes in higher education. The employment of research assistants is of critical importance for ensuring the sustainability of research activities within universities. In this context, it is essential to determine the optimal number of research assistants needed in each faculty and department, and to prioritize employment based on objective and measurable criteria. This study aims to facilitate data-driven decision-making in higher education at Ankara University (AU) through the application of Multi-Criteria Decision-Making (MCDM) methods. The ranking of research assistant employment is done by using several MCDM methods for the AU. Data set is obtained from various data sources, e.g. Web of Science database, several academic, administrative parts and coordinatorships of the AU. The obtained data is well-organized with preprocessing stage by using statistical explaratory data analysis. Units of the data is considered as all the departments of the AU in the fields of science, health and social. The data is considered as decision matrix in which criteria are defined quantitatively. The MCDM methods are applied for two aims: (i) Weighting, and (ii) Ranking. The criteria are weighted by using IDOCRIW (Integrated Determination of Objective CRIteria Weights) method objectively. The units are ranked through TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution), COPRAS (COmplex PRoportional ASsessment), EDAS (Evaluation based on Distance from Average Solution) and MABAC (Multi-attributive Border Approximation Area Comparison) according to the weights. Due to difference of the TOPSIS, COPRAS, EDAS and MABAC ranking results, Copeland Method is applied to these ranks to get final ranking list. It is seen from the final ranking results that Copeland ranking list can be taken into consideration for the employment of research assistants at the AU.

Keywords: *Statistical Explaratory Data Analysis, Multi-Criteria Decision Making (MCDM) Methods, Ranking, Research Assistant Employment, Higher Education*

The Application of Unsupervised Machine Learning Algorithms to the Clustering of Universities Worldwide in Relation to Sustainability Rankings (1129)

Gulcan Petricli[1*]

[1]Bursa Uludag University, Inegol Business School, Business Administration, Quantitative Methods Department, Turkey

*Corresponding author e-mail: gulcanp@uludag.edu.tr

Abstract

Sustainability rankings are crucial in evaluating universities' environmental and social commitments, but their clustering based on shared sustainability characteristics have been barely investigated. This research aims to address three key questions: how universities are clustered based on their sustainability ranking data, what institutional characteristics correlate with different sustainability clusters, and the strategic implications of these clusters for institutional planning and policy. The UI GreenMetric World University Rankings data for 2022 was analyzed twice by the k-prototype clustering algorithm, employing Huang's distance measure and Gower distance separately. The data was partitioned into different levels of clusters, 2 to 10, and evaluated with 25 different internal clustering indices. The results indicated two partitions, with the first cluster consisting of countries with better performance on all indicators, higher mean and median values, and the second cluster varying in performance, lower mean and median values. Turkey's universities are observed in both clusters, while most of them are in the first cluster. The findings are inconclusive, and the inquiry is still under progress. A more thorough methodology will be used to ascertain the optimal split of nations. This research methodically addresses the analytical process, distinguishing it from the majority of existing literature, and the anticipated results will facilitate the identification of both weaker and stronger nation groups, essential indicators, benchmarking prospects, and prospective partnerships for universities.

Keywords: *GreenMetric, Sustainability, Universities, k-Prototype Clustering*

Development of a Machine Learning-Based Risk Score for Predicting Adverse Outcomes in Rhabdomyolysis (1130)

Ömer Faruk Karakoyun[1], Fulden Cantaş Türkiş[2*], Yalçın Gölcük[3], Mehmet Reha Yılmaz[3], Burcu Kaymak Gölcük[4]

[1]Muğla Training and Research Hospital, Emergency Department, Türkiye
[2]Muğla Sıtkı Koçman University, Medical Faculty, Division of Biostatistics, Türkiye
[3]Muğla Sıtkı Koçman University, Medical Faculty, Division of Emergency Medicine, Türkiye
[4]Muğla Training and Research Hospital, Biochemistry Laboratory, Türkiye

*Corresponding author e-mail: fuldencantas@mu.edu.tr

Abstract

Accurate early risk prediction in rhabdomyolysis is challenging due to clinical heterogeneity and unpredictable outcomes. This study aimed to develop a machine learning-based prognostic model to predict the composite outcome of renal replacement therapy or 90-day mortality in emergency department patients with serum creatine kinase ≥1000 U/L. Using a retrospective dataset of 1,031 patients (2019–2024), key predictors were identified through feature selection and modeled using regularized logistic regression. Model performance was evaluated with 5-fold cross-validation and 1000 bootstrap iterations. The final model achieved strong discrimination (AUC: 0.861; 95% CI: 0.824–0.898) and was transformed into an interpretable clinical score (Muğla Score). A cut-off of ≥4 points yielded 97% negative predictive value. This study demonstrates how machine learning methods can be adapted into clinically useful, transparent tools for risk stratification in heterogeneous populations.

Keywords: *Logistic regression, Clinical Risk Score, Rhabdomyolysis*

Identification Of Key Predictive Biomarkers For mTOR Inhibitor Response In Leukemia Cell Lines Using Multi-Omics Database Mining And Statistical Modeling (1131)

Erhan Aptullahoglu[1,2,*]

[1]Bilecik Şeyh Edebali University, Faculty of Science, Department of Molecular Biology and Genetics, Türkiye
[2]Bilecik Şeyh Edebali University, Biotechnology Application and Research Centre, Türkiye

*Corresponding author e-mail: erhan.aptullahoglu@bilecik.edu.tr

Abstract

The PI3K/AKT/mTOR signaling pathway plays a critical role in the pathogenesis and progression of hematologic malignancies by regulating cellular processes such as survival, proliferation, metabolism, and resistance to therapy. Despite the growing interest in targeting mTOR as a therapeutic strategy, clinical responses to mTOR inhibitors remain highly variable among patients with leukemia and lymphoma, highlighting the urgent need for predictive biomarkers to guide treatment decisions. In this study, we employed an integrative bioinformatics and statistical modeling approach to identify molecular determinants of mTOR inhibitor sensitivity across a panel of 20 leukemia cell lines. Key components of the mTOR pathway—RICTOR, PTEN, AKT, PIK3CA, TSC1/2, and DEPTOR—were selected based on functional relevance and previous literature. Gene and protein expression data were compiled from the Human Protein Atlas and DepMap, while mutation profiles were obtained from COSMIC and the Sanger Cell Line Project. Drug sensitivity data for commonly used mTOR inhibitors, including rapamycin, everolimus, and AZD8055, were integrated from publicly available pharmacogenomic datasets. Statistical analyses were conducted using SPSS to identify associations between molecular alterations and drug response. Preliminary results suggest that elevated RICTOR expression and PTEN loss are key predictors of increased sensitivity to mTOR inhibition, particularly in T-cell acute lymphoblastic leukemia (T-ALL) and acute myeloid leukemia (AML) models. These findings provide a basis for future functional validation using next-generation mTORC inhibitors in preclinical models and lay the groundwork for biomarker-driven precision medicine approaches in the treatment of hematologic cancers.

Keywords: *mTOR inhibitors, RICTOR, PTEN, Hematological cancers.*

Performance Evaluation of k-means Clustering Method on High Dimensional Low Sample Size Datasets (1132)

Gülşah Kılıç[1*], Necla Gündüz[2]

[1]Gazi University, Institute of Science, Department of Statistics, Turkiye
[2]Gazi University, Faculty of Science, Department of Statistic, Turkiye

*Corresponding author e-mail**:** gulsah.kilic@gazi.edu.tr

Abstract

In recent years, increasing attention has been directed toward the analysis of high-dimensional low sample size (HDLSS) datasets, particularly in the context of genomic data. These datasets are characterized by a number of variables that far exceeds the number of observations ($p \gg n$), leading to significant methodological and computational challenges. One of the major consequences of this structure is the diminished reliability of distance-based measures, which hinders the effectiveness of clustering algorithms and complicates the separation of distinct groups within the data.

In this study, beyond the inherent challenges of HDLSS data structures, scenarios involving both the presence of outliers and the contamination of the dataset with observations from different distributions are considered, and the performance of the k-means clustering algorithm—one of the multivariate methods—is comprehensively examined under these complex conditions. The algorithm was first applied to real-world genomic cancer datasets with high dimensionality and low sample sizes. The clustering results were evaluated using a comprehensive set of external and internal validation indices, including the Adjusted Rand Index (ARI), Dunn index, Silhouette index, and Calinski-Harabasz index, to assess the consistency and quality of the clustering structure.

Moreover, a series of simulation studies were conducted to systematically examine the algorithm's robustness under contamination scenarios. Specifically, synthetic HDLSS datasets were generated from the normal distribution, and controlled proportions of observations drawn from alternative distributions were introduced to simulate contamination. The k-means algorithm was then applied under each scenario, and its performance was assessed using the same set of validation metrics. All simulation experiments were implemented using the R programming language.

The results of this study provide empirical insights into the limitations of traditional clustering methods in high-dimensional contaminated environments and emphasize the importance of robust validation frameworks for reliable cluster detection in HDLSS contexts.

Keywords: *High-dimensional low-sample-size datasets (HDLSS), K-means clustering, Outliers, Contamination, Clustering validation index*

The $C^{\#}_{pmk}$ Index under the Chris–Jerry Distribution: Estimation, Comparative Analysis and Application to Engineering Dataset (1134)

Erdem Cankut[1*], Kadir Karakaya[1]

[1]Selçuk University, Faculty of Science, Department of Statistics, Konya, Türkiye

*Corresponding author e-mail: erdem.cankut@selcuk.edu.tr

Abstract

Under the non-normal conditions, traditional process capability indices such as C_p, C_{pk}, C_{pm}, and C_{pmk} are generally preferred when the underlying distribution follows a normal distribution. This study presents a comprehensive assessment of the process capability index $C^{\#}_{pmk}$, which is considered a robust index compared to existing non-normal alternatives, based on the flexible lifetime distribution known as the Chris–Jerry distribution. The $C^{\#}_{pmk}$ index is estimated by the maximum likelihood method, using the invariance property of maximum likelihood estimators. A Monte Carlo simulation study with 10,000 replications is conducted for various combinations of parameter values and target, lower and upper specification settings to evaluate the maximum likelihood estimator. For each scenario, the maximum likelihood estimator of $C^{\#}_{pmk}$ is evaluated in terms of bias, mean square error, average absolute bias, and mean relative error. Additionally, the $C^{\#}_{pmk}$ index is examined with respect to the non-conforming rate. To demonstrate its practical applicability in quality control, a real measurement dataset is also analyzed.

Keywords: *Process capability indices, Non-normal processes, Quality control, Measurment dataset*

Comparative Analysis of Meta-Learners in Stacking Ensemble for Imbalanced Classification (1135)

Adil Kılıç[1*], **Özge Gürer**[2]

[1]Kırıkkale University, Faculty of Engineering and Natural Sciences, Department of Statistics, Turkey
[2]Ankara University, Faculty of Science, Department of Statistics, Turkey

*Corresponding author e-mail: adilkilic@kku.edu.tr

Abstract

Ensemble learning is an approach that combines multiple machine learning models to build a more robust and accurate predictive system (Polikar, 2006). This approach principally aims to improve prediction performance by aggregating or learning from the outputs of diverse models. Recently, ensemble learning has gained popularity for imbalanced classification problems, as they can capture complex patterns that may not be identified by a single model (Bi & Zhang, 2018; Rithani et al., 2025). In this study, the 'Vehicle Silhouettes' dataset from the UCI machine learning repository is utilized for empirical evaluation and the original multi-class target variable is transformed into a binary classification problem exhibiting class imbalance (Dua & Graff, 2017). To overcome this issue, a stacking ensemble learning approach is implemented. In doing so, first the dataset is divided into training and testing sets for training the stacking ensemble model. Then, in order to mitigate the adverse effects of class imbalance, the majority class in the training data is split into several subsets such that their number is determined in accordance with the imbalance ratio, and each of them is combined with the minority class to obtain a different balanced sub-training set. Subsequently, each of these balanced data sets is used to train a different base model in the ensemble. It should be noted that nested cross-validation is employed during the training process to prevent data leakage. For the meta-learner, which integrates the predictions of the base models, eight different algorithms are employed and their performances are compared with respect to the commonly used evaluation metrics, including Area Under the ROC Curve (AUC), F1-score, precision, and recall. As a result, multi-layer perceptron and logistic regression show overall superior performance compared to other methods.

Keywords: *Imbalanced classification, Stacking ensembles, Meta-learner, Machine learning*

Statistical Inference for the Kumaraswamy-Log-Logistic Distribution based on Ranked Set Sampling with an Application to Real Data (1136)

Özge Gürer[1*], Adil Kılıç[2], Birdal Şenoğlu[1]

[1]Ankara University, Faculty of Science, Department of Statistics, Turkey
[2]Kırıkkale University, Faculty of Engineering and Natural Sciences, Department of Statistics, Turkey

*Corresponding author e-mail: otanju@ankara.edu.tr

Abstract

The log-logistic (LL) distribution is often applied in reliability, medical, economic and environmental studies, as data in these fields typically exhibit noticeable right skewness. However, in many real-world applications, more flexibility is required to adequately capture varying tail behaviors and distributional shapes. The Kumaraswamy-Generalized (KumG) family, introduced by Cordeiro and De Castro (2011), generalizes classical continuous distributions by incorporating two additional shape parameters. When the LL distribution is used as the base, the resulting Kumaraswamy-LL (KumLL) distribution offers increased adaptability in modelling varying levels of skewness, kurtosis and tail behavior, see De Santana et al. (2012). This study addresses the estimation of the location and scale parameters of the KumLL distribution under the assumption that the shape parameters are known. To improve estimation accuracy, ranked set sampling (RSS) is employed as a cost-efficient alternative to the commonly used simple random sampling (SRS). RSS, first proposed by McIntyre (1952), uses auxiliary information to rank units before the actual measurement is performed. Therefore, it results in more representative samples and more efficient estimators. However, the likelihood equations under RSS do not have closed-form solutions. In order to overcome this problem, the modified maximum likelihood (MML) methodology proposed by Tiku (1967) is utilized in this study. MML estimators have explicit forms, are robust to outliers and are asymptotically equivalent to maximum likelihood (ML) estimators. The performance of the proposed estimators is compared with the traditional ML estimators obtained under SRS with respect to bias and mean squared error criteria via a comprehensive Monte Carlo simulation study. Results indicate that RSS based MML estimators outperform their counterparts especially for larger set sizes. A real data application is also conducted to demonstrate the practical implementation of the proposed methodology.

Keywords: *Ranked set sampling, Kumaraswamy Log-logistic distribution, Parameter estimation, Modified likelihood, Monte Carlo simulation*

Predictive Models and Decision Support Systems in Recruitment Processes (1137)

Samet Kaya[1*], Esin Köksal Babacan[2]

[1]The Scientific and Technological Research Council of Türkiye, ANKARA
[2]Ankara University, Department of Statistics, ANKARA

*Corresponding author e-mail: sametky_1301@hotmail.com

Abstract

Modern human resources processes are becoming increasingly complex, highlighting the need for more objective and data-driven decision-making mechanisms in recruitment. This study aims to develop decision support systems for recruitment processes using machine learning and data mining techniques. In this research, retrospective data obtained from job postings published by TÜBİTAK Human Resources will be analyzed to develop a predictive model that estimates the likelihood of a candidate's success in the recruitment process. Machine learning algorithms such as Random Forest, XGBoost, AdaBoost, CatBoost, and Support Vector Machines (SVM) will be utilized. Model performance will be evaluated using metrics such as accuracy, recall, precision, F1-score, and AUC-ROC.
The findings of this study will contribute to making recruitment processes more objective and providing data-driven support to human resources managers. This model is expected to serve as a significant reference for improving recruitment processes in both public and private sector organizations, with TÜBİTAK as a primary application area.

Keywords: *Human Resources, Artificial Intelligence, Machine Learning, Decision Support Systems, Recruitment Processes*

Mapping the Research Landscape: Artificial Intelligence Applications in Recruitment– A Bibliometric Analysis (1138)

Samet Kaya[1*], Esin Köksal Babacan[2]

[1]The Scientific and Technological Research Council of Türkiye, ANKARA
[2]Ankara University, Department of Statistics, ANKARA

*Corresponding author e-mail: sametky_1301@hotmail.com

Abstract

The integration of artificial intelligence (AI), machine learning (ML), large language models (LLMs), and natural language processing (NLP) into human resources (HR) and organizational management is reshaping modern recruitment and leadership processes. This study aims to provide a comprehensive bibliometric analysis of research trends at the intersection of AI technologies and HR practices.
Using the Web of Science database, publications containing keywords such as "artificial intelligence," "ChatGPT," "machine learning," "deep learning," "generative AI," "natural language processing," "LLM," "NLP," "video interview," "recruitment," "CV," "leader," "organization management," and "human resource" were retrieved. Bibliometric indicators including publication trends, leading authors, top journals, collaboration networks, keyword co-occurrence, and citation analysis were explored. R Shiny applications were employed to perform dynamic visualization and interactive analysis of the data.
This study contributes to mapping the evolving research landscape at the intersection of AI and human resource management and identifies potential gaps and future directions for academic and applied research.

Keywords: *Human Resources, Artificial Intelligence, Natural Language Processing, Bibliometric Analysis, Recruitment Processes*

Local Risk Patterns in Traffic Accidents in Türkiye: A Simultaneous Analysis of Provinces and Accident Factors Using the Biclustering Method (1143)

Ahmet Kocatürk[1], Ayşenur Akın Vargeloğlu[2*], Yaprak Arzu Özdemir[2]

[1]Erzurum Technical University, Rector's Offices, Turkey
[2] Gazi University, Faculty of Science, Department, Department of Statistics, Turkey

*Corresponding author e-mail: aysenurakin@gazi.edu.tr

Abstract

This study analyzes traffic accidents that occurred in 2023 across 81 provinces in Türkiye, taking into account the conditions under which these accidents took place. The primary aim of the research is to identify patterns of similarity among provinces in terms of structural, environmental, and behavioral factors contributing to traffic accidents.

The dataset consists of 98 variables, each representing the proportion of accidents occurring under specific conditions. These variables can be grouped under categories such as settlement type, road type and surface, speed limit, road geometry, intersection type, visibility conditions, collision type, accident type, and the presence of traffic and lighting infrastructure. Prior to the analysis, the data were transformed into binary format to allow for the identification of more distinct and meaningful patterns. For each variable, values above the 75th percentile were coded as "1" and those below as "0". This threshold was intentionally selected to ensure that only conditions associated with high accident rates were represented in the clusters, thereby enhancing the specificity and interpretability of the resulting patterns

A biclustering method was applied to the dataset, and the Bimax algorithm was selected for this purpose. Bimax was chosen due to its suitability for binary data structures and its ability to generate distinct and well-separated clusters. The biclustering approach enables the simultaneous clustering of both provinces and accident-related variables, allowing the discovery of meaningful two-dimensional patterns rather than unidimensional groupings. In this way, specific accident factors that become prominent concurrently in certain provinces are effectively revealed.

As a result of the analysis, provinces exhibiting similar accident conditions were grouped together into meaningful clusters. For instance, some clusters were characterized by factors such as open-air environments, highways, and daytime conditions, while others were defined by variables such as intersection types, road surface characteristics, or collision types. These findings may contribute to the development of traffic safety policies tailored to individual provinces and support the design of preventive strategies aligned with localized risk profiles.

Keywords: *Traffic accidents, Biclustering, Local risk patterns, Bimax algorithm*

A Novel Fuzzy MCDM Framework for E-Waste Management: Integrating CRITIC–MABAC with Continuous Function-Valued Intuitionistic Fuzzy Sets (1145)

Büşra Aydoğan[1*], Mehmet Ünver[1]

[1] Ankara University, Faculty of Science, Department of Mathematics, Ankara Türkiye

*Corresponding author e-mail: aydoganb@ankara.edu.tr

Abstract

Multi-Criteria Decision Making (MCDM) constitutes a set of mathematical and analytical methods that aim to determine the best or most appropriate option in decision problems with multiple conflicting criteria. By introducing systematicity and objectivity into the decision-making process, MCDM is a valuable tool for addressing complex decision-making challenges. The Criteria Importance Through Intercriteria Correlation (CRITIC) method, a prevalent approach in decision-making, utilizes statistical analysis to determine the weights of the criteria, while the Multi-Attributive Border Approximation Area Comparison (MABAC) method assesses the relationship between each alternative and the criteria-based boundary region. The present study puts forth a proposal for the integration of these two methods with fuzzy set theory, thereby creating a new decision environment. Addressing the pressing global concern of electronic waste (e-waste) in this novel context entails the implementation of a sophisticated set approach, such as Continuous Function-Valued Intuitionistic Fuzzy Sets, in conjunction with a pioneering theoretical contribution. This integrated study provides a novel perspective on fuzzy-MCDM.

Keywords: *Continuous Function-Valued Intuitionistic Fuzzy Sets, MABAC, CRITIC, Cross-Entropy, E-waste*

Cancer Diagnosis With RGB Image to Hyperspectral Conversion (1146)

Göker Berkay Bilecen[1]*, Aylin Alın[2], Durmuş Özdemir[3], Hayriye Mine Antep[4]

[1]Dokuz Eylul University, Faculty of Science, Department of Statistics, Türkiye
[2]Dokuz Eylul University, Faculty of Science, Department of Statistics, Türkiye
[3]Izmir Institute of Technology, Faculty of Science, Department of Chemistry, Türkiye
[4]Dokuz Eylul University, Faculty of Science, Department of Chemistry, Türkiye

*Corresponding author e-mail: gokerberkay.bilecen@deu.edu.tr

Abstract

Cancer remains one of the most common causes of death worldwide, and early diagnosis has a direct impact on the success rate of treatment. Therefore, the development of early detection methods is of great importance, both in extending patient survival and in reducing healthcare costs. In recent years, rapid advances in medical imaging technologies, artificial intelligence, and especially deep learning-based algorithms have made it possible to automatically extract meaningful features from images and significantly improve diagnostic accuracy.

Hyperspectral imaging (HSI) offers much more detailed information compared to traditional RGB imaging methods, thanks to its wide spectral range and high spatial resolution, enabling the detection of biochemical differences in tissues. This capability provides a significant advantage, particularly in detecting cancerous tissues at early stages with high accuracy. However, the widespread use of hyperspectral imaging faces several challenges due to the high cost of HSI systems and the large size of the data they produce. Additionally, processing such data requires more computational power and more complex models compared to RGB images.

In this study, a deep learning approach was developed to predict hyperspectral spectral bands from RGB images using the Microscopic Hyperspectral Choledoch Dataset, which contains microscopic hyperspectral images related to Choledoch cancer. Within this scope, a U-Net-based architecture was employed to synthesize hyperspectral images with a reduced number of bands from RGB inputs. Subsequently, the synthesized hyperspectral data were used to classify cancerous and healthy tissues. Furthermore, different deep learning-based classification methods were compared, and their performance on hyperspectral data was analyzed in detail. The results demonstrate that hyperspectral images predicted from RGB data can be used with high accuracy for the early detection of bile duct cancer, suggesting that this approach offers a more efficient and cost-effective alternative.

Keywords: *Cancer Dıagnosıs, Rgb Reconstructıon, Hyperspectral Imagıng, Deep Learnıng*

Comparison Of Different Machine Learning Algorithms For Weight Prediction (1147)

Ahmet Sinan Güler[1*], Yalçın Tahtalı[2]

[1]Tokat Gaziosmanpaşa Universty, , Faculty of Agriculture, Department of Animal Science, Turkey
[2] Tokat Gaziosmanpaşa Universty, , Faculty of Agriculture, Department of Animal Science, Turkey

*Corresponding author e-mail: sinanguler2568@gmail.com

Abstract

In this study, the predictive performance of machine learning algorithms, including Random Forest (RF), Extreme Gradient Boosting (XGBoost), LightGBM, Gradient Boosting Machine (GBM), and Support Vector Machine (SVM), was compared for weight estimation in Romanov lambs. For this purpose, data were collected from 50 Romanov lambs born to 285 Romanov ewes raised in the Niksar district of Tokat province. The dataset includes measurements of withers height, rump height, body length, chest depth, chest circumference, chest width between the shoulders, and live weight during the growth period. The predictive performance of the models was evaluated using Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and the Coefficient of Determination (R^2). The results indicated that the GBM algorithm exhibited the best performance and highest accuracy in predicting the live weight of Romanov lambs, with the lowest MSE (0.728), RMSE (0.995), and the highest R^2 (0.980). Additionally, RF, XGBoost, LightGBM, and SVM algorithms also demonstrated high predictive performance, yielding effective results in live weight estimation.

Keywords: *Machine Learning, XGBoost, LightGBM, GBM, Random Forest*

Robust Feature Selection with the Boruta Algorithm: Simulation and Real-World Applications across Low- and High-Dimensional Settings (1148)

Hulya Yurekli [1*], Reza Arabi Belaghi [2]

[1]Yildiz Technical University, Department of Statistics, Istanbul, Türkiye
[2]Swedish University of Agricultural Sciences, Department of Energy and Technology, Uppsala, Sweden

*Corresponding author e-mail: hyurekli@yildiz.edu.tr

Abstract

Variable selection in high-dimensional data is a critical step for improving model interpretability and predictive accuracy. This study evaluates the performance of the Boruta algorithm - a feature selection method that employs random forest models within a wrapper framework to identify all variables associated with the outcome - across a wide range of simulation scenarios and a real-world application. The simulation design systematically varied the sample size, number of predictors (reflecting both low- and high-dimensional settings), strength of association between predictors and outcome, and the degree of inter-variable correlation. In scenarios where influential variables had strong effects, Boruta identified relevant features with high accuracy and minimal inclusion of irrelevant ones. As dimensionality increased and the association between predictors and outcome weakened, Boruta continued to perform robustly, particularly in identifying features with complex or non-uniform patterns. In low-dimensional scenarios, the algorithm maintained reliable performance, with accuracy improving as sample size increased. Boruta also demonstrated resilience to multicollinearity, consistently selecting informative features across both highly and moderately correlated predictor spaces. To complement the simulation findings, Boruta was applied to a real-world, high-dimensional dataset representative of applied data science challenges. The algorithm successfully identified a compact and interpretable subset of features with strong discriminatory power, as demonstrated by high area under the curve (AUC) values in downstream classification tasks. Overall, this study highlights Boruta's strengths as a flexible, non-parametric feature selection approach that performs effectively under a variety of data conditions. The results offer practical guidance for researchers working with structured, noisy, and complex datasets in domains such as biomedicine, education, and the social sciences.

Keywords: *Feature Selection, Boruta Algorithm, Predictive Modeling, Variable Importance, Wrapper Methods*

How Do Medical School Interns Plan Their Future? (1149)

Erdogan Asar[1*], Yunus Emre Bulut[2]

[1] University of Health Sciences, Gulhane Faculty of Medicine, Department of Medical Informatics, Ankara, Türkiye
[2] University of Health Sciences, Gulhane Faculty of Medicine, Department of Public Health, Ankara, Türkiye

*Corresponding author e-mail: erdo6718@gmail.com

Abstract

Objective: When physicians are students, their career plans and specialty preferences after graduation are issues that occupy their minds a lot and are a turning point in their professional lives. This study aims to reveal the career plans and specialty preferences of students studying at the Faculty of Medicine and those who have just started clinical education, and the factors that affect these preferences.

Materials and methods: The study included 4th-year students (n=254) studying at the Health Sciences University Gülhane Faculty of Medicine in the 2022-2023 Academic Year. A Survey Form consisting of four sections (general information, medical school preference status, specialization and working life preference status, Ten-Item Personality Scale) was applied to the students face-to-face or via the web.

Results: The first place among students' career plans after graduation was "being a physician in a public institution". The students' desire to receive specialization training after graduation was very high. In their specialization preferences, branches related to Surgical Medical Sciences were the most preferred branches. This was followed by Internal Medical Sciences branches. The least desired branches were observed to be branches in the Basic Medical Sciences Department. It was observed that "working conditions and violence" and "professional satisfaction and career opportunities" were important factors in the preference of the Surgical Medical Sciences Department (OR=0.395 and OR=2.391, respectively; $p<0.05$). "Professional satisfaction and career opportunities" was found to be the most important factor in the preference of the Internal Medical Sciences Department (OR=2,084, $p<0.05$). The branches in which students most want to receive specialization training are "skin and venereal diseases", "ear, nose and throat diseases", "plastic reconstructive and aesthetic surgery", "eye diseases", and "cardiology". The most important factors in determining students' preferences for specialization after graduation were found to be that the branch "makes the student happy" and "professional satisfaction".

Conclusion: Participants mostly want to work as physicians in public institutions after graduation. Students mostly prefer the Department of Surgical Medical Sciences, and while "working conditions and violence" affect the preference in a decreasing way, "professional satisfaction and career opportunities" affect the preference in an increasing way. The preference rates of the 5 most preferred branches are very close to each other. It is thought that the use of research results in decision-making processes will be beneficial for decision-makers.

***Keywords**: Medical Students, Career Plan, Specialization Choice.*

Count-Based Time Series Modeling of Maritime Accidents in High-Risk Shipping Zones: Evidence from the Sea of Marmara and the Istanbul Strait (1151)

Ömer Önder[1*], Öyküm Esra Yiğit[2], Kaan Ünlügençoğlu[3]

[1]Yildiz Technical University, Faculty of Arts and Sciences, Department of Statistics, Turkey
[2] Yildiz Technical University, Faculty of Arts and Sciences, Department of Statistics, Turkey
[3] Yildiz Technical University, Faculty of Naval Architecture and Maritime, Department of Naval Architecture and Marine Engineering, Turkey

*Corresponding author e-mail: omer.onder@std.yildiz.edu.tr

Abstract

Maritime transport, which accounts for over 80% of global trade volume, is a critical pillar of the world economy. The increasing density of maritime traffic—particularly in high-risk transit zones such as the Sea of Marmara and the Istanbul Strait—has led to a rise in both the frequency and severity of maritime accidents. Understanding the temporal dynamics and underlying determinants of these accidents is essential for improving safety standards and informing effective preventive policies. This study statistically investigates the temporal patterns of monthly maritime accidents in the Sea of Marmara and the Istanbul Strait during the period from October 2016 to May 2023 using count data regression models. Given the discrete and sparse nature of accident counts, generalized linear models (GLMs) based on Poisson and Negative Binomial distributions are employed. To account for temporal autocorrelation, these models are extended with autoregressive structures incorporating AR(1) terms. The modeling framework includes time-sensitive covariates such as exponential moving averages, polynomial trends, and lagged accident counts. Predictive performance is evaluated using multiple error and fit metrics, and the models are validated through comprehensive diagnostic tests addressing overdispersion, residual autocorrelation, and distributional assumptions. Among the four models tested, the AR(1)-Poisson model yielded the best overall statistical performance and diagnostic validity, successfully capturing the temporal structure of the data. The findings reveal that maritime accidents follow systematic temporal patterns rather than occurring randomly, underscoring the need for autoregressive count modeling and targeted, time-aware safety interventions in maritime traffic management.

Keywords: *Maritime Transport, Maritime Accidents, Sea of Marmara, Istanbul Strait, Count Data Regression*

Forecasting Monthly Production By Fabric Type In A Textile Company Using Time Series Analysis (1152)

Beyza Asena Çakıroğlu[1], Ersoy Öz[2]*

[1]Yildiz Technical University, Graduate School of Science and Engineering, Department of Statistics, Türkiye
[2]Yildiz Technical University, Faculty of Arts and Sciences, Department of Statistics, Türkiye

*Corresponding author e-mail: asena.cakiroglu@gmail.com

Abstract

This study aims to forecast the monthly production quantities by fabric type in a textile company. The data set to be used in the study includes variables such as production quantity, material type, unit of measurement, and date (on a year/month basis). The data set consists of a total of 12 different material types. These data will be grouped by material type, unit of measurement, and time variable to create time series specific to each category. Through these series, it is aimed to analyze past production behavior and make forward-looking predictions.
Based on the production data from 2022, 2023, and 2024, it is planned to generate production forecasts for the first quarter of 2025 using the Python programming language. In this process, time series analysis methods will be utilized; in particular, seasonal and trend components will be considered in order to improve model accuracy. The performance of the models is planned to be evaluated using error metrics such as Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), and Root Mean Square Error (RMSE). It is also planned that the selection of the final model will be made based on the structural characteristics of the data and prediction performance. If necessary, the forecasts obtained will be compared using different models and the most suitable one will be selected.
In addition to numerical accuracy, the study is expected to provide operational benefits through the developed models. Accordingly, it is aimed to achieve stock optimization, improve raw material procurement processes, manage production capacity more efficiently, and respond more quickly to customer demands. The integration of forecasts with the company's SAP-based ERP system is also considered, which would allow forecast outputs to directly support production planning and enhance decision-making in real-time.

Keywords*: Tıme Series Forecasting, Textile Industry, Production Plannıng,*

Evaluation of the Performance of BRICS-T Countries in the Context of the Economic Freedom Index (1153)

Bilal Saraç[1], Çağlar Karamaşa[2*]

[1]Anadolu University, Faculty of Economics and Administrative Sciences, Department of Quantitative Methods, Türkiye
[2]Anadolu University, Faculty of Economics and Administrative Sciences, Department of Quantitative Methods, Türkiye

*Corresponding author e-mail: ckaramasa@anadolu.edu.tr

Abstract

Since 1995, the Heritage Foundation has presented the factors that directly contribute to the economic freedom and prosperity of the international community in detail through the Economic Freedom Index (EFI). While the index evaluates the progress or decline of countries around the world, it focuses on key indicators of economic well-being such as economic growth, poverty reduction, longevity and health, as well as environmental protection. Through the index, the degree of economic freedom is relatively calculated as a significant factor in national development and prosperity on a global scale, and countries are ranked accordingly. Rankings of economic freedoms provide critical insights for countries, scholars, policymakers—in short, all stakeholders—to understand the impact of the measured criteria on economic growth. These rankings can guide the development of solutions to issues such as poverty and economic contraction faced by societies. Furthermore, through cross-country comparisons, differences in economic freedoms can be identified, and much can be learned about how to improve economic welfare and development. For these reasons, assessing the economic freedoms of countries holds vital importance. In this study, the economic freedom levels of the BRICS countries along with Turkey for the year 2025 are determined. Due to the presence of multiple indicators in the index and the involvement of multiple countries in terms of economic freedoms, the MEREC and WENSLO integrated AROMAN approach as Multi-Criteria Decision-Making (MCDM) techniques has been preferred. While the objective MCDM methods MEREC and WENSLO were used to determine the weights of the evaluation criteria, the AROMAN approach was employed to rank the BRICS-T countries.

Keywords: *Economic Freedom Index, BRICS-T, MEREC, WENSLO, AROMAN*

Parametric Mortality Modelling: A Comparative Analysis Based on Mortality Data in Türkiye, 2009–2023 (1154)

Funda Karaman[1*]

1 Hacettepe University, Department of Actuarial Science, Turke

*Corresponding author e-mail: fundakaraman@hacettepe.edu.tr

Abstract

With the rapid aging of the population, mortality modeling and projection have become increasingly important, especially for institutions facing several challenges such as:

• Escalating social security expenditures for governments,

• Forecasting future cash flows and valuing liabilities,

• Determining capital requirements and ensuring sustainability for annuity providers and pension schemes.

This study aims to provide evidence-based recommendations for decision-makers, actuaries, and public administrators on the selection of appropriate mortality models by age group for long-term mortality projections in Turkey.

Using mortality data from Turkey between 2009 and 2023 obtained from the Turkish Statistical Institute (TURKSTAT), we evaluated the performance of a set of parametric mortality models. The models analyzed include traditional forms like Gompertz, Makeham, Weibull, and Logistic, as well as age-sensitive models that focus on late-life mortality such as Perks, Beard, Kannisto, Heligman-Pollard, and Siler. The statistical analyses were conducted using the R programming language and relevant statistical packages.

Model performance was assessed using maximum likelihood-based criteria and residual analysis methods, including Root Mean Square Percentage Error (RMSPE), sign tests, and normality tests. Results reveal that Kannisto and Beard models perform best in capturing old-age mortality patterns, whereas the Siler model offers a better overall fit across all age groups.

Keywords: *mortality, insurance, demography, applied statistics, actuarial science*

Predicting the Survivability of Software Start-ups in Technoparks (1155)

Basak Apaydın Aysar[1]*, Mehmet Yılmaz[2]

[1]Ankara University, Faculty of Science, Department of Statistics, Türkiye
[2]Ankara University, Faculty of Science, Department of Statistics, Türkiye

*Corresponding author e-mail: bapaydin89@gmail.com

Abstract

Technoparks are essential ecosystems that foster innovation and support entrepreneurs in developing technology-based products and services. This support is particularly critical for software start-ups, making it important to identify the factors affecting their survival. This study aims to predict the survival likelihood of software start-ups in technoparks using machine learning methods. The dataset includes 21 variables related to company demographics and operational fields. Feature selection was conducted using Lasso, Mutual Information, and Random Forest techniques. To address class imbalance, SMOTE was applied. Among the three feature selection methods, Random Forest (95.42% accuracy) yielded the best results. Using the features selected by this method, several machine learning classification algorithms (K-Nearest Neighbors (KNN), Decision Trees, Random Forest, XGBoost (Extreme Gradient Boosting), Gradient Boosting, Support Vector Machines (SVM)) were tested to predict firm activity status. Gradient Boosting emerged as the most effective algorithm, achieving 95.77% accuracy, a 96.44 F1 score, and a ROC AUC of 0.9826. Feature importance analysis showed that the "Number of Ongoing Projects" was the most significant predictor of survival, followed by "Duration of Stay in TDZ," "Total Research and Development Support Amount," and whether the company was an "Incubated Firm". These findings confirm that machine learning can effectively predict start-up survival in technoparks. The results also highlight the importance of active project management and sustained Research and Development engagement. Therefore, policymakers and technopark administrators should consider enhancing support programs focused on project management and Research and Development funding. Entrepreneurs, in turn, can improve their chances of success by optimizing project portfolios and leveraging incubation opportunities.

Keywords: *Technopark, Start-up, Machine Learning, Survivability, Feature Selection*

Measuring Productivity Change in the Turkish Non-Life Insurance Sector: A Malmquist Index Approach (2010–2023) (1156)

Funda Karaman[1*]

1 Hacettepe University, Department of Actuarial Science, Turke

*Corresponding author e-mail: fundakaraman@hacettepe.edu.tr

Abstract

This study aims to evaluate the productivity changes of 21 non-life insurance companies operating in Turkey between 2010 and 2023 by employing the Malmquist Total Factor Productivity (TFP) Index method. Using panel data, the analysis decomposes productivity change into efficiency change (catching up) and technological change (innovation frontier shift). The input variables considered include total operating expenses and number of employees, while outputs are defined as net earned premiums and total technical profits.

Data Envelopment Analysis (DEA) was used under the assumption of variable returns to scale, and annual Malmquist indices were computed to track temporal dynamics. The findings reveal that while some firms achieved technical efficiency gains, technological progress remained stagnant in the post-pandemic period. Notably, firms with foreign capital showed more consistent improvements in total factor productivity. The study concludes with policy implications regarding digital transformation and regulatory flexibility.

Keywords*: Malmquist Index, Productivity, Efficiency, Non-Life Insurance, DEA, Türkiye, Total Factor Productivity*

Investigation of Unemployment Hysteria with Nonlinear Unit Root Tests: The Case of Turkey (1158)

Emre Ürkmez[1*]

[1]Recep Tayyip Erdogan University, Faculty of Economics and Administrative Sciences, Department of Economics, Turkey

*Corresponding author e-mail: emre.urkmez@erdogan.edu.tr

Abstract

This study aims to test whether the unemployment hysteria hypothesis is valid or not in Turkey by using monthly data for the period between January 2010 and March 2025. For this purpose, Fourier-ADF and Fourier-LM unit root tests, which have the capacity to detect nonlinearities and potential structural breaks, were applied to the seasonally adjusted unemployment rate series. According to the unit root test results, the Fourier components are not statistically significant. This indicates that the unemployment rate variable does not contain non-linear trends or structural breaks. Then, the classical ADF unit root test was applied to the unemployment rate series. The findings of the ADF unit root test indicate that the unemployment rate series is non-stationary and contains a unit root. The presence of this unit root in the series reveals that shocks in the unemployment rate have permanent rather than temporary effects and supports the unemployment hysteresis hypothesis in this context. In other words, negative shocks in the labor market persist in the long run and the unemployment rate does not automatically return to its equilibrium level. This finding suggests that the ability of the labor market to return to natural equilibrium is limited. This result emphasizes the necessity of active employment policies and structural reforms rather than passive policies in combating unemployment. The findings of the study reveal that the effects of economic policies on employment should be evaluated with longer-term perspectives.

Keywords: *Unemployment, Hysteresis, Turkey, Nonlinear, Unit Root.*

Investigation of The Mean Reversion in Stock Prices: The Case of BIST-100 Index (1160)

Emre Ürkmez[1*]

[1]Recep Tayyip Erdogan University, Faculty of Economics and Administrative Sciences, Department of Economics, Turkey

*Corresponding author e-mail: emre.urkmez@erdogan.edu.tr

Abstract

This study aims to examine the mean reversion feature of stock prices for Turkey between January 2010 and April 2025. For this purpose, the logarithmic price series of the Borsa Istanbul 100 (BIST-100) index are analyzed using monthly data. Lagrange Multiplier (LM) Fourier unit root test developed by Enders and Lee (2012) is applied to determine the stationarity level of the series. Compared to standard unit root tests, this test stands out by taking into account the effects of structural breaks in the series more precisely. The findings reveal that the logarithmic price series of the BIST 100 index is not mean-reverting in the analyzed period. In other words, it is concluded that stock prices do not tend to return to a constant equilibrium level in the long run. When evaluated in the context of the efficient market hypothesis, which is one of the financial market theories, this indicates that the logarithmic price series of the BIST-100 index exhibits a random walk. This finding suggests that past price data are insufficient to predict future price movements and therefore the market is weakly efficient. The results question the effectiveness of models based on the mean reversion assumption in terms of investment strategies.

Keywords: *Stock Prices, Mean Reversion, BIST-100, Nonlinear, Unit Root.*

Causality Relationship between Chicken Meat and Red Meat Prices in Turkey: Time Series Analysis (1161)

Samed Gür[1*], Hayriye Sibel Gülse Bal[2]

[1]Tokat Gaziosmanpasa University, Faculty of Agriculture, Department of Agricultural Economics, Tokat, Turkey
[2]Tokat Gaziosmanpasa University, Faculty of Agriculture, Department of Agricultural Economics, Tokat, Turkey

*Corresponding author e-mail: samedgur60@gmail.com

Abstract

The aim of this study is to determine the causality relationship between red meat prices and chicken meat prices in Turkey. The causality relationship is analysed using Toda-Yamamoto Test. The research material consists of 192 monthly red meat and chicken meat prices data for the period January 2008 - December 2023. ADF and PP unit root tests are applied and it is concluded that the series contain unit root at the level. This result shows that our series contain unit root at level. However, it is found that there is no unit root in first differences. The lag length of the series is 2 months and there is no changing variance and autocorrelation. According to the results of the study, a significant causality relationship was found from chicken meat prices to red meat prices at 10% level of accuracy. However, no significant relationship was found from red meat prices to chicken meat prices. It can be said that the increase in red meat prices is a function of the increase in chicken meat prices. These results suggest that changes in red meat prices may be a function of increases in chicken meat prices. In this context, it can be said that in order to ensure price stability, policy measures should be strengthened to increase chicken meat production and reduce import dependency.

Keywords: *Toda-Yamamoto, Causality Analysis, Meat Prices*

A Macroeconomic Perspective on Türkiye's Climate Crisis Risk: Decision Tree Analysis (1163)

Mervenur Ünver[1,*], Şahika Gökmen [1,2]

[1]Hacı Bayram Veli University, Econometrics Department, Ankara, Türkiye
[2]Uppsala University, Statistics Department, Uppsala, Sweden

*Corresponding author e-mail: mervenur.unver@ogr.hbv.edu.tr

Abstract

Climate change poses a significant threat on a global scale, encompassing not only environmental but also socio-economic and macroeconomic dimensions. This issue is addressed at the Paris Climate Conference through the concept of the "Green Swan," which refers to sudden, unpredictable, and serious economic impacts that may arise from environmental risks. Taking these risks into account, countries create scenarios and develop policies to guard against possible economic and climate crises. Therefore, a better understanding of nations' climate-related vulnerabilities and risk factors is crucial for shaping their future under today's conditions. When the literature in this field is examined, there is no study that focuses on Türkiye in this context especially one employing machine-learning algorithms. Based on this motivation, the present study aims to identify the main factors affecting climate-related risk levels by using Türkiye's macroeconomic and environmental indicators for 1995 – 2022. The study first assessed Türkiye's climate vulnerability by considering the Notre Dame Global Adaptation Index (ND-GAIN) and carbon dioxide (CO_2) emissions, creating a four-level risk classification (unstable, low risk, medium risk, and high risk). Next, the relationships between these risk levels and various socio-economic and environmental indicators were modelled with a decision-tree analysis. The analysis revealed the most important variables affecting climate-vulnerability levels in Türkiye and evaluated their temporal effects. In this context, the study is expected to contribute to a better understanding of Green Swan events in Türkiye. The findings can guide the formulation of climate policies and sustainable-development strategies.

Keywords: *Decision Tree, Machine Learning, Climate Risks, ND-GAIN, Green Swan*

Creating an Interactive Dashboard for Türkiye's Human Development Index with the EurostatRTool (1165)

Fethi Saban Özbek[1*], Antonio Grosso[2], Rosa Ruggeri Cannata[2]

[1]Turkish Statistical Institute[1], Social Statistics Department, Türkiye
[2]Eurostat, National accounts methodology; Standards and indicators, Luxembourg

*Corresponding author e-mail: fethiozbek@tuik.gov.tr

Abstract

The Human Development Index (HDI) and its sub-indices are key indicators for assessing the socio-economic progress of countries. This study aims to develop an interactive dashboard to disseminate Türkiye's Human Development Index and its sub-indices, including the Life Expectancy Index, Education Index, and Income Index, for the years 2018 to 2022. To achieve this goal, we utilized the EurostatRtool, an R package designed to facilitate the dissemination of European statistics. The EurostatRtool offers robust capabilities for accessing, processing, and visualizing statistical data, making it an ideal choice for this study. The dashboard leverages EurostatRtool's functions to provide an interactive and user-friendly interface, enabling users to explore the trends and patterns of the HDI and its components over the specified period.

The dashboard offers various interactive visualizations, including time series graph, and comparative bar chart, to better understand changes in the indices. Additionally, users can access detailed information and metadata for each index, facilitating deeper insights into the factors influencing Türkiye's human development trajectory. This interactive dashboard serves as a practical tool for researchers, policymakers, and stakeholders to evaluate and monitor Türkiye's socio-economic progress within the context of HDI. By integrating dynamic visualization with comprehensive data access, the dashboard enhances the accessibility and usability of human development statistics. This study highlights the potential of EurostatRtool in developing data dissemination platforms that support evidence-based decision-making and promote transparency in socio-economic reporting.

Keywords: *Data Visualization, EurostatRtool, Human Development Index (HDI), Interactive Dashboard*

[1]*The opinions and contents of the article remains the responsibility of the authors, not of the Turkish Statistical Institute.*

A Family of Log-Type Estimators Under Ranked Set Sampling Method (1168)

Merve Yazla*[1,2], Nursel Koyuncu [2]

[1]Etlik City Hospital, Emergency Medicine, Ankara, Turkey
[2]Hacettepe University. Department of Statistics, Ankara, Turkey

*Corresponding author e-mail: merveyazla@gmail.com

Abstract

Body fat percentage is a critical biomarker in evaluating individuals' metabolic health. However, direct measurement techniques, such as underwater weighing, are often expensive and impractical for routine use. Consequently, there is an increasing demand for estimation methods that leverage easily accessible anthropometric variables. This study investigates the performance of various estimators for predicting body fat percentage using abdominal circumference as an auxiliary variable. Ranked Set Sampling (RSS) was employed as the sampling method due to its cost-effectiveness, improved efficiency, and enhanced estimation precision. In addition to conventional estimators, ratio, regression, ratio-cum-product, and logarithmic-type estimators were analyzed. Special emphasis was placed on the log-type and regression-log-type estimators introduced by Bhushan and Kumar (2020). Theoretical properties of the proposed family of log-type estimators were derived under optimal conditions, and their performance was assessed through comparative analysis using real-world data. The evaluation was based on the mean squared error (MSE), and findings indicated that the proposed estimators substantially minimized MSE compared to existing alternatives. These results suggest that log-type estimators integrated with the RSS framework offer a robust and efficient approach for accurately estimating key health indicators such as body fat percentage.

Keywords: *Ranked Set Sampling, Body Fat Percentage, Abdominal Circumference, Log-Type Estimator, Estimation Performance*

Mortality Modelling Under Long Term Effects Of Catastrophic Events (1169)

Beyza Taşbaş[1], Selin Özen[2]

[1]Ankara University, Graduate School of Natural and Applied Sciences, Department of Actuarial Sciences , Turkey
[2]Ankara University, Faculty of Applied Sciences, Department of Actuarial Sciences, Turkey

*Corresponding author e-mail: btasbas@ankara.edu.tr

Abstract

This study presents the modeling of mortality using the Lee-Carter model and the development of future projections. The importance of selecting appropriate time series models in mortality forecasting is emphasized, particularly for long-term actuarial applications such as pension planning, insurance pricing, and public health policy. The analysis is conducted based on annual population data of Turkey from 2009 to 2021, grouped by age and gender. One of the key components of the model, mortality time index, is modeled using various time series approaches, and the results are compared to evaluate forecasting performance. The findings contribute to identifying which time series model is more suitable for long-term mortality forecasting, specifically the COVID-19 effects are included. In this study, model comparisons are carried out by applying the Random Walk with Drift, ARIMA models, and ARFIMA models. The model parameters are estimated, and model selection criteria such as AIC and BIC values are computed. Special attention is given to the ARFIMA model due to its ability to capture long-memory dependencies in time series data, which is particularly relevant in the presence of structural shifts such as pandemics. The comparative analysis clearly demonstrates that the ARFIMA model provides the most accurate and robust projections, particularly in capturing long-memory behavior and structural changes in mortality trends.

Keywords: *Mortality Models, Lee-Carter Model, RWD, ARFIMA, Long Term Effects.*

Electricity Consumption and Manufacturing Industry Index: An Empirical Analysis Based on Monthly Industrial Consumption Data (1170)

Burcu Mestav[1]*, Fikret Bartu Yurdacan[2]

[1]Çanakkale Onsekiz Mart University, Faculty of Arts and Sciences, Department of Statistics, Türkiye
[2]Ege University, Institute of International Computer Science, Türkiye

*Corresponding author e-mail: burcu.mestav@comu.edu.tr

Abstract

In this study, the relationship between monthly electricity consumption of industrial subscribers and the manufacturing industry production index in Türkiye is investigated. The motivation behind the study stems from the idea that electricity consumption patterns may serve as a high-frequency indicator for industrial production activities. The dataset includes monthly manufacturing electricity consumption figures for industrial-profile subscribers as reported by EPİAŞ (Energy Exchange Istanbul), alongside seasonal and calendar-adjusted industrial production index values published by TÜİK (Turkish Statistical Institute). The analysis covers the period from January 2020 to December 2024, including both free consumers and non-entitled industrial users. A time series correlation analysis was conducted in R to explore whether electricity usage reflects variations in the manufacturing industry index. Results demonstrate a statistically significant and positive association between electricity consumption and industrial output, suggesting that monthly industrial electricity usage can serve as a real-time proxy for production performance. These findings offer practical implications for economic forecasting, energy policy planning, and industrial trend monitoring.

***Keywords*:** *Industrıal Electrıcıty Consumptıon, Manufacturıng Index, Tıme Serıes Analysıs, Energy Economıcs, Industrıal Productıon*

Standardization in Elevator Systems: Call and Display Panel Design Compatible with CANopen Communication Protocol (1173)

Eyup Sayin[1*], Murat Topuz[1], Bayram Yarim[1], Muhammet Fatih Aslan[2], Akif Durdu[3]

[1] Butkon Asansör San. Tic. A.Ş., Research and Development, Konya, Turkiye
[2] Karamanoglu Mehmetbey University, Faculty of Engineering, Electrical and Electronics Engineering, Karaman, Turkiye
[3] Konya Technical University, Faculty of Engineering, Electrical and Electronics Engineering, Konya, Turkiye

*Corresponding author e-mail: sayineyup@gmail.com

Abstract

Since each of the elevator motherboards produced in Turkey today uses its own special communication protocol, the panels must be adapted separately to each system, and this leads to serious time and cost loss in the production process. For this purpose, this study aims to ensure that elevator control cards developed by different manufacturers work in harmony with common call and display panels. In this context, a call and display panel compatible with the CANopen Lift (CIA-417) profile, one of the international standards, has been designed. The embedded software infrastructure was created with the CANopenNode open source library and the system's error analysis was performed with the CAN Analyzer. In addition, the sustainability and usability of the system was increased with the architecture that allows software updates and parameter changes to be made remotely. The prototype developed at the end of the project was tested in pilot elevators and successful results were obtained. With this study, a domestic solution that will reduce Turkey's dependence on imports has been presented. In addition, the way will be paved for CIA-417 compatible panel production.

Keywords: *CIA-417, CANopen, Elevator, CAN Bus.*

Latent Similarity Clustering of Video Games Based on Euclidean Distance and PCA (1175)

Diana Bratić[1*]

[1]University of Zagreb, Faculty of Graphic Arts, Department of Printing Processes, Croatia

*Corresponding author e-mail: diana.bratic@grf.unizg.hr

Abstract

This paper presents a multi-criteria similarity analysis of video games using quantitative variables from an available dataset. The research includes the following variables: user rating, number of recommendations, average playing time (overall and in the last two weeks), and percentage of positive reviews. The research aims to develop a similarity model for games in a multidimensional space defined by these attributes and to identify patterns and groupings based on their quantitative profiles. The data was standardized to ensure comparability across variables with different scales. Euclidean distance was used to measure similarity between games, as it is intuitively interpretable in real space: the distance between two games is calculated as the square root of the sum of squared differences across all dimensions. This metric enables accurate positioning of games within the attribute space and forms the basis for hierarchical clustering. Principal component analysis (PCA) was applied to reduce dimensionality and facilitate visual interpretation of the results.

Preliminary findings indicate the existence of several stable clusters, including games with high ratings and recommendations but relatively short playing time, as well as a group of games played extensively but rated lower by users. These combinations suggest distinct usage patterns and perceived value, which are not directly aligned with traditional categories such as genre or publisher. The approach presented in this study can serve as a foundation for structuring large-scale game datasets and as a starting point for developing classification and recommendation algorithms based on objective rather than subjective product characteristics.

Keywords: *Video Games, Latent Similarity, Euclidean Distance, Clustering, PCA*

A Data-Driven Approach to QoS and Energy Optimization in 6G Using Non-Terrestrial Networks (1176)

Metin Öztürk[1*]
[1]Ankara Yıldırım Beyazıt University, Faculty of Engineering and Natural Sciences, Electrical and Electronics Engineering, Türkiye
*Corresponding author e-mail: metin.ozturk@aybu.edu.tr

Abstract

Mobile data traffic has been continually and exponentially growing over the years, stretching the limits of cellular wireless communication systems. In order to alleviate the increasing data traffic, various solutions have been introduced in the fifth generation of cellular communication systems (5G), including opening up a new spectrum (i.e., millimeter-wave frequencies), network densification, and massive multiple-input multiple-output (mMIMO) systems, to name a few. One common point in these solutions is to enhance the network capacity to accommodate more data traffic. Considering that the sixth generation (6G) is expected to be even more challenging in terms of data traffic due to the support for more bandwidth-hungry applications and connecting more devices (e.g., Internet of everything), there is an urgent need to utilize the available resources more efficiently. This study investigates the spatio-temporal dynamics of a cellular communications environment using a real call detail record (CDR) dataset released by Telecom Italia for Milan, Italy, where the city of Milan is divided into 10,000 square-shaped grids and telecommunication activities (i.e., call, messaging, and Internet) are recorded every 10 minutes. After showcasing the spatio-temporal dynamics in a cellular communication environment using the above-mentioned dataset, the impact of non-terrestrial networks (NTN), especially high-altitude platform stations (HAPS), is explored to reduce the load on ultra/over-utilized base stations (BSs), which subsequently improves the quality of service (QoS). HAPS is a stratospheric vehicle, and once deployed with a BS, it can serve as a flying BS to complement terrestrial cellular communication networks. Additionally, by using the capacity of NTN components strategically and efficiently, some under-utilized BSs can be put into sleep mode to save energy. Therefore, the approach proposed in this study increases QoS and energy efficiency simultaneously in 6G networks.

Keywords: *6G, Spatıo-Temporal, Qualıty Of Servıce, Energy Efficıency, Statıstıcs*

Performance Evaluation of Artificial Intelligence Optimization Algorithms for Nonlinear Model Parameter Estimation through Multi-Criteria Decision Making Methods (1178)

Murat Açıkgöz[1*], Özlem Türkşen[2]

[1]Ankara University, Institute of Science, Department of Statistics, Ankara, Türkiye
[2]Ankara University, Faculty of Science, Department of Statistics, Ankara, Türkiye

*Corresponding author e-mail: muratacikgoz@live.com

Abstract

Nonlinear regression models are widely used in pharmacokinetics to model the dynamics of biological processes. Two-Compartment Model is a key tool for understanding drug distribution and elimination in the body. Accurate parameter estimation is critical for model reliability. Traditional methods have limitations, prompting the use of Artificial Intelligence (AI)-based optimization algorithms as alternatives. However, evaluating algorithms based on a single performance metric can lead to incomplete analyses. Thus, Multi-Criteria Decision-Making (MCDM) approaches, considering multiple criteria such as solution quality and computational efficiency, are necessary. This study systematically analyzes the performance of twelve AI optimization algorithms applied to parameter estimation of the Two-Compartment Model using MCDM methods. The dataset comprises multidimensional performance outputs, including error metrics, computation time, and the number of iterations required for convergence. In the MCDM application, IDOCRIW method was employed to determine the relative importance of performance metrics (criteria) in a data-driven, objective manner, independent of decision-maker subjectivity. Subsequently, these objective weights were used to rank the algorithms (alternatives) with three MCDM methods, called COPRAS, EDAS, and MABAC, which are based on different theoretical foundations. Then, Copeland method was applied to integrate potentially differing rankings from the MCDM methods through pairwise comparisons, yielding a robust final performance ranking. The results demonstrate that MCDM methods provide a balanced evaluation of solution quality and computational efficiency. This study highlights that the MCDM methods offer an objective and systematic decision-support tool about choosing AI optimization algorithms for nonlinear model parameter estimation.

Keywords: *Multi-Criteria Decision Making, Artificial Intelligence Optimization Algorithms, Performance Evaluation, Nonlinear Regression, Two-Compartment Model*

Using Statistical Moments in Hierarchical Machine Learning for Estimation of Birefringence in Mode-Locked Fiber Laser Systems (1181)

Hasan Arda Solak[1], Şeyma Koltuklu[1], Sueda Turgut[1], Mahmut Bağcı[1]*

[1] Marmara University, Faculty of Business Administration, Department of Management Information Systems, Türkiye

*Corresponding author e-mail: mahmut.bagci@marmara.edu.tr

Abstract

Adaptive control and self-tuning of mode-locked fiber laser systems is an interesting topic in applied optics. Cavity birefringence varies randomly and significantly affects the mode-locking performance of fiber laser. In addition, the birefringence value in the laser cavity cannot be measured directly. Therefore, rapid and accurate detection of the birefringence value is critical for the adaptive control and self-tuning of fiber laser systems. In this study, from a new perspective, the birefringence value is determined (estimated) by hierarchical implementation of supervised machine learning algorithms. Unlike previous studies, instead of using the laser pulse energy directly, the energy evolution is recorded and a separate time series is obtained for each case of birefringence. The four statistical moments (mean, variance, skewness and kurtosis) of these time series are used as input variables (features) in the machine learning algorithm. When the obtained results are compared with the results of previous studies, it is seen that the birefringence value can be estimated with higher accuracy in a short time with the hierarchical approach. More accurate classification of birefringence increases efficiency of algorithms that enable adaptive control and self-tuning of mode-locked fiber laser systems. Consequently, the study contributes to the advancement of mode-locked fiber laser technology by enhancing performance in various industrial and scientific applications, enabling broader and more efficient use of laser systems.

Keywords: *Mode-locked fiber lasers, Birefringence, Hierarchical machine learning, Adaptive laser systems.*

An Explainable AI-Based Model for Customer Churn Prediction in the Banking Sector (1182)

Latif Yasir Kızılçelik[1*], Buse Erken[1], Fadime Ceren Avcı[1], Yiğit Emre Yılmaz[1], Gözde Ulutagay[1]

[1]Ege University, Faculty of Science, Department of Statistics, 35040, Izmir, Türkiye

*Corresponding author e-mail: yasirdevop@gmail.com

Abstract

In today's highly competitive banking industry, customer retention has become a critical challenge for financial institutions. Customer churn not only results in financial losses but also damages brand trust and long-term client relationships. This study aims to develop a predictive framework enriched with Explainable Artificial Intelligence (XAI) methods to enhance transparency in churn analysis. Traditional machine learning models, such as Decision Tree, Random Forest, and XGBoost, are known for their high predictive capabilities. However, these models often act as black boxes, leaving decision-makers with limited understanding of the reasoning behind the predictions. In this context, our research utilizes SHAP (Shapley Additive Explanations) and LIME (Local Interpretable Model-Agnostic Explanations) to interpret model behavior at both the global and local levels. Using a dataset of 10,000 retail banking customers, we conducted comprehensive preprocessing, including data cleaning, feature selection, and encoding. The models were evaluated using key performance metrics such as Accuracy, Precision, Recall, F1-score, and ROC-AUC. Among the models tested, XGBoost achieved the highest classification performance. More importantly, SHAP and LIME revealed critical churn predictors, including customer tenure, account balance, credit card usage, and active status. These insights enabled a more transparent decision-making process and allowed business units to implement targeted retention strategies. Our findings confirm that the integration of XAI not only improves interpretability but also boosts stakeholder confidence and regulatory compliance. This research contributes to the growing need for responsible AI in banking by demonstrating how interpretability and predictive accuracy can coexist. Ultimately, the study emphasizes the importance of building trustworthy AI systems that provide actionable and transparent insights to support human-centric decisions in customer management.

Keywords: *Customer churn, Banking, Machine learnig, Explainable AI, Artificial intelligence*

Mplus Jamovi Amos and Spss Programs in Partial Mediation Model: A Simulation Study (1183)

Murat Yıldırım[1*]

[1]Tokat Gaziosmanpaşa University, Faculty of Economics and Administrative Sciences, Department of Business Administration, Türkiye

*Corresponding author e-mail: murat.myildirim@gop.edu.tr

Abstract

This study aims to investigate the performance of Mplus, Jamovi, Amos and Spss programs, which are widely used for partial mediation model, with a certain sample size and categorical data. In line with the purpose of the study, R program was used to generate data for partial mediation model. The three path coefficients of the model are pre-assigned as real values with a 0.6, b 0.7 and c path 0.3 and there is no other pre-assigned value. Sample size selection is 100, 200, 300, 400 and 500 units. In the established model, X latent variable has 4, M latent variable has 3 and Y latent variable has 5 observed variables. All observed variables have 5-categorical structure. The model was analyzed in all programs; direct effects, indirect effects and fit values were obtained. The estimation techniques of the programs are WLSMV, DWLS, ML and OLS, respectively. The closeness of the programs to the previously assigned values as performance criteria was intended to be examined. For this situation, Mean Absolute Bias- MAB and Relative Bias- RB values were used. The findings obtained were as follows. In all programs, the direct effect from the latent variable X to the latent variable Y was found to be insignificant in sample sizes of 100 and 200 units, and all other path coefficients were found to be significant. In this case, the existing partial mediation model was transformed into a full mediation model. This result indicates that the sample size of 100 and 200 units is insufficient for model studies on partial mediation relationship. While all programs had overestimation in the sample size of 100 units, underestimation was obtained in sample sizes increasing with 200 units. All programs had quite good fit values in all volumes. According to MAB and RB values, Spss indicates that its use for categorical data is problematic with its quite high deviation results. In other programs, MAB and RB values of 300 units sample were found to be good and the best performance was provided by Mplus program with WLSMV estimation technique. The next performance ranking after Mplus program was Jamovi and Amos.

Keywords: *Partial Mediation Model, Simulation Study, Categorical Data.*

The Effect of Youth Centre Participation on Leisure Satisfaction, Leisure Motivation and Happiness: A Log-linear Modelling Approach (1184)

Funda Erdugan[1*], Sevgi Yurt Öncel[2], Ebru Aytekin Acar[3]

[1,2]Kırıkkale University, Faculty of Engineering and Natural Science, Department of Statistics, Türkiye
[3]Şanlıurfa Youth Center, Türkiye

*Corresponding author e-mail: ferdugan@kku.edu.tr

Abstract

Youth centres are institutions that aim to help young people make effective use of their leisure time, guide them towards various social, cultural and sporting activities, support their personal development, provide mentoring and raise awareness against harmful habits. These centres aim to promote the physical, mental and social well-being of young people and contribute to their holistic development. The aim of this study was to identify how the way young people spend their leisure time affects their happiness and quality of life, to raise awareness of the social role of youth centres, and to provide practical recommendations for practitioners regarding leisure activities. Using survey data and scale scores collected from participants attending youth centres, the relationships between leisure satisfaction, leisure motivation and happiness levels were analysed using log-linear modelling. According to the results, individuals who reported never participating in activities were 8 times more likely to have low leisure satisfaction compared to those with high satisfaction. Those who reported participating 'sometimes' were 2.4 times more likely to have low satisfaction. These results suggest that the frequency of participation in activities has a significant impact on leisure satisfaction. Furthermore, those with low leisure satisfaction were 4.3 times more likely to have low levels of happiness than those with high levels of happiness, suggesting a positive relationship between leisure satisfaction and happiness. In addition, among those with low leisure satisfaction, the proportion of people who 'never' participated in activities was 5.2 times higher than those who 'often' participated. These findings show that active participation in the activities offered by youth centres contributes to more fulfilling leisure experiences and consequently to higher levels of happiness among young people. In this context, young people with high levels of leisure satisfaction and happiness are fundamental building blocks for a healthy, productive and high quality society.

Keywords: *Leisure Satisfaction, Leisure Motivation, Happiness, Log-linear Model*

Machine Learning-Driven User-Specific Cellular Network Orchestration with Multi-Tier Network Architecture (1186)

Metin Öztürk[1*]

[1]Ankara Yıldırım Beyazıt University, Faculty of Engineering and Natural Sciences, Electrical and Electronics Engineering, Türkiye

*Corresponding author e-mail: metin.ozturk@aybu.edu.tr

Abstract

Static cellular communication orchestration is inefficient, though simple and straightforward, as it treats all users the same; however, each user has diverse and distinctive characteristics and requirements. Additionally, cellular communication networks have been under great pressure due to the increasing number of connected devices—especially since the Internet of Things (IoT) concept has become pervasive—and emerging high data rate applications, including virtual/augmented reality, holographic communications, and autonomous driving. Hence, the efficient and effective use of network resources is vital. Therefore, dynamic and user-specific network orchestration is beneficial from both user and network perspectives. In this study, a multi-tier cellular communication network is considered, and users are clustered using the k-means machine learning algorithm. First, user requirements such as latency, data rate, and mobility are taken as inputs to the developed k-means algorithm. Then, the algorithm clusters users based on these inputs, followed by assigning them to tiers of the network that best match their requirements. In this way, users are better served by their most suitable network components, while those components are utilised more efficiently. Furthermore, the integration of multi-layer clustering is also investigated in this study, where a second-layer clustering using the k-means algorithm is applied to assign users to different slices within their designated network tier (as determined by the first-layer clustering), in order to further enhance network efficiency. Therefore, this work leverages multi-tier cellular communication networking by taking advantage of non-terrestrial networks (NTN) and multi-layer clustering with the k-means algorithm, providing full-scale, efficient network orchestration. This is an important step toward the next generation of cellular communication networks, such as the sixth generation (6G), which is envisioned to connect significantly more devices with greater heterogeneity compared to legacy networks.

Keywords: *Clustering, 6G Networks, Telecommunications, Non-Terrestrial Networks, Machine Learning*

Optimal Product Mix And Resource Allocation In Furniture Manufacturing Using Linear Programming: A Case Of Furniture Company (1188)

Çağdaş Yıldız[1*], Âdem Tüzemen[1]

[1]İktisadi ve İdari Bilimler Fakültesi, İşletme Bölümü, Üretim Yönetimi ve Pazarlama AD., Tokat Gaziosmanpaşa Üniversitesi, Türkiye

*Corresponding author e-mail: cagdasyildiz60@gmail.com

Abstract

Increasing competitive conditions and changing customer demands in the furniture industry necessitate efficient use of production resources and optimal planning of product mix. In this study, a linear programming model is presented for profit maximization and balanced product mix objectives of a furniture manufacturing company. The model aims at optimal allocation of limited resources such as CNC machine time, labor hours, MDF, and fabric for three main product groups: sofa sets, dining tables, and wardrobes. The mathematical model, presented using LINGO software, optimizes the company's weekly production planning. In the study, minimum demand and maximum capacity constraints were defined for each product, and all resource limitations were included in the model. According to the optimization results, a production plan providing a weekly profit of 91,700 TL was obtained. This plan envisions the production of 17 sofa sets, 7 dining tables, and 6 wardrobes. Resource utilization analysis showed that CNC machine and labor hours were utilized at a rate of 99.2%, indicating these resources as the main limiting factors for production. MDF utilization rate was calculated as 92.7%, while fabric utilization rate was 96.9%. Product mix analysis revealed that sofa sets constitute 64.9% of the total profit. As a result of the study, strategic recommendations were presented to increase the company's resource utilization efficiency and achieve a more balanced product mix. The presented model can be adapted to other companies in the furniture industry and provides a scientific approach to production planning and resource allocation decisions.

Keywords: *Furniture Manufacturing, Linear Programming, Optimal Product Mix, Resource Allocation, Production Planning, Profit Maximization.*

Multi-Period Production And Inventory Planning In Textile Industry: A Case Study Of Textile Company (1189)

Çağdaş Yıldız[1*], Âdem Tüzemen[1]

[1]İktisadi ve İdari Bilimler Fakültesi, İşletme Bölümü, Üretim Yönetimi ve Pazarlama AD., Tokat Gaziosmanpaşa Üniversitesi, Türkiye

*Corresponding author e-mail: cagdasyildiz60@gmail.com

Abstract

Seasonal demand fluctuations and variable raw material prices in the textile industry increase the strategic importance of production and inventory planning. In this study, a mathematical model was developed to optimize the production and inventory planning for a three-month period for a Textile, a company operating in Turkey. The model aims to minimize regular time, overtime, and inventory holding costs for three main product groups: T-shirts, trousers, and shirts. The mathematical model, solved using LINGO software, optimizes the company's three-month (June-August 2025) production and inventory planning. In the study, demand forecasts, labor requirements, minimum inventory levels, and maximum storage capacity constraints were defined for each product. According to the optimization results, an optimal production and inventory plan with a total cost of 2,137,450 TL was obtained. This plan envisions the production of 7,800 T-shirts, 4,200 trousers, and 4,650 shirts. Capacity utilization analysis showed that regular time capacity was utilized at a rate of 100%, while overtime capacity was used only for T-shirt production in August at a rate of 55%. Inventory level analysis revealed that minimum inventory levels were maintained for T-shirts and shirts, while high inventory levels were kept for trousers in June and July in preparation for high demand in August. Cost analysis showed that 91.3% of the total cost consists of regular time production costs, 7.4% consists of overtime costs, and 1.3% consists of inventory holding costs. As a result of the study, strategic recommendations were presented regarding the company's production capacity management, inventory optimization, cost reduction, and demand management. The presented model can be adapted to other companies in the textile industry and provides a scientific approach to production and inventory planning decisions suitable for seasonal demand fluctuations.

Keywords: *Textile Manufacturing, Multi-Period Planning, Inventory Management, Production Planning, Cost Minimization, Linear Programming.*

The Impact of Artificial Intelligence Integration on Labor Productivity: Panel Data Analysis (1190)

Menekse Cicek[1],

[1]İzmir Demokrasi Üniversitesi,İktisadi ve İdari Bilimler Fakültesi, İktisat Bölümü, Türkiye

*Corresponding author e-mail: menekse.cicek@idu.edu.tr

Abstract

Labor productivity is considered to be the most important indicator of social development and one of the main determinants of economic growth. Today, one of the most effective tools for increasing labor productivity is the digitalization process. This process reduces labor costs, automates production processes and ultimately increases output. The effects of digitalization on labor markets have become even more important in recent years as rapidly developing technologies have transformed the work environment. Digital technologies, especially artificial intelligence, allow employees to focus on more value-added jobs by freeing them from routine tasks. In this context, the effects of the integration of artificial intelligence technologies into production processes on labor have become an important research area in recent years. In line with these explanations, the aim of this study is to empirically test the effects of artificial intelligence applications on labor productivity using the panel data analysis method. In the study, data obtained from high-income countries according to the World Bank's income group classification between 2010 and 2023 were used, fixed effects and random effects models were compared, the appropriate model was selected with the Hausman test and the model was estimated with robust estimators. The findings show that variables representing artificial intelligence have a statistically significant effect on productivity.

Keywords: *Artificial intelligenc, Labor productivityi, Panel data analysis*

Determinants of Income Related Health Inequality: Evidence from a Cross-Sectional Study in Türkiye (1192)

Hatice Isık[1*]

[1]Hacettepe University, Faculty of Science, Department of Statistics, Ankara, Türkiye

*Corresponding author e-mail: haticeyeniay@hacettepe.edu.tr

Abstract

Socio-economic health inequalities are a global challenge for public policy. This study aims to assess the extent of health inequality in Turkey, and to analyze the contributions of its determinants, with the goal of informing future policy decisions. The data for this study come from Income and Living Conditions Survey (Cross-Sectional) 2022, which is a national data set, conducted by Turkish Statistical Institute. After excluding cases with default values on related important items in this study the final analyses included 39328 individuals. The independent variables included demographic factors, socio-economic characteristics, and health-related determinant, while the dependent variable was self-rated health. Demographic factors included gender, age, and marital status; socio-economic characteristics included educational level, employment status, and annual income; and the health-related determinant was a variable indicating whether the respondent had been diagnosed with a chronic disease. The logistic regression model was used to analyze the influencing factors of self-rated health. Furthermore, the Erreygers concentration index was used to measure income-related health inequality. A decomposition analysis was performed to determine the contribution of each variable to inequality. The logistic regression results showed that socio-economic factors (income, education and employment status), demographic factors (age and marital status), and the health-related determinant (chronic disease) were the main determinants of health inequality. The negative income-related concentration index (–0.1763; 95% CI: –0.1868 to –0.1658) for poor self-rated health indicates a significant pro-rich inequality in health among residents. In the decomposition analysis, most of the observed health inequality was attributable to the health-related determinant, specifically the presence of chronic disease, which accounted for 65% of the total inequality. The second most significant contributor was income, highlighting the role of socio-economic disparities in shaping health outcomes.

Keywords: *Health Inequality, Concentration Index, Income Related Inequality, Self-Rated Health*

Adaptive Weighted Least Squares Approach in Regression Analysis (1193)

Ecenur Erkci[1*], Meltem Ekiz[2]

[1]Gazi University, Graduate School of Natural and Applied Sciences, Statistics, Turkey
[2]Gazi University, Faculty of Science, Statistics, Turkey

*Corresponding author e-mail: ece6800@icloud.com

Abstract

This study investigates the performance of the Adaptive Weighted Least Squares Ratio (AWLSR) estimator, a robust method designed to address the challenges posed by heteroscedasticity and outliers in regression analysis, particularly when the number of outliers in the dataset is high. Classical Ordinary Least Squares (OLS) estimation becomes inefficient and unreliable under violations of homoscedasticity and the presence of extreme values. To overcome these issues, the AWLSR method, developed as an adaptive version of the Least Squares Ratio (LSR) estimator, assigns weights based on the variance of residuals. Through extensive Monte Carlo simulation studies, various levels of heteroscedasticity and outlier ratios were incorporated to compare the effectiveness of AWLSR with OLS and other robust estimators. Results indicate that AWLSR consistently yields lower Mean Squared Error (MSE) and bias values, even under severe data contamination. These findings highlight the potential of AWLSR as a powerful alternative in practical data analysis when traditional assumptions do not hold.

Keywords: *Robust Regression, Heteroscedasticity, Outliers, Adaptive Weighted Least Squares Ratio, Simulation Study*

Multivariate Anomaly Mapping in Video Games: A Mahalanobis Distance Approach (1194)

Diana Bratić[1*]

[1]University of Zagreb, Faculty of Graphic Arts, Department of Printing Processes, Croatia

*Corresponding author e-mail: diana.bratic@grf.unizg.hr

Abstract

This paper addresses the detection of anomalous video games by analyzing disparities between key variables, including the game's price, the number of recommendations, the percentage of positive and negative reviews, and the number of rated owners. The research aims to develop a sophisticated model that identifies games significantly deviating from expected patterns based on multidimensional relationships between these variables. Advanced statistical techniques were employed for outlier detection, including z-scores, which quantify deviations from the mean, interquartile ranges (IQR), which pinpoint extreme values within the data distribution, and Mahalanobis distance, which allows for anomaly detection by incorporating correlations between variables and multidimensional differences. Utilizing a covariance matrix, this method ensures precise identification of outliers, even in complex datasets with multiple correlated variables.

The detected anomalies were confirmed through appropriate visualization techniques, which enable the clear identification of exceptional cases and patterns that deviate from the expected distributions. These visualizations deepen the understanding of the anomalies, allowing for the formulation of critical questions regarding the credibility and balance of attributes such as price, ratings, recommendations, and ownership, as well as potential latent market anomalies.

Preliminary results suggest the existence of several notable outliers, including games with exceptionally high prices coupled with poor ratings, and games that have an extraordinarily high number of recommendations despite low average playtime. These findings point to potentially disruptive patterns in market dynamics, potentially stemming from marketing manipulation or inadequate recommendation systems that fail to reflect the true quality of the game. Identifying these anomalies lays the groundwork for further research aimed at reducing recommender bias and optimizing product rating systems, focusing on objective rather than subjective product characteristics.

Keywords: *Video Games, Anomaly Mapping, Outlier Detection, Mahalanobis Distance*

The Role of Smart City Theories in Enhancing Traffic Flow: The Case of Tirana (1196)

Veranda Syla[1*], Algenti Lala[1], Ardit Dervishi[1]

[1] Polytechnic University of Tirana, Faculty of Information Technology, Department of Electronics and Telecommunications, Tirana, Albania

*Corresponding author e-mail: vsyla@fti.edu.al

Abstract

Urban traffic congestion poses growing challenges to mobility, environmental quality, and public health, particularly in rapidly expanding cities like Tirana. This paper presents a theory-based literature review of smart city frameworks, namely the Internet of Things (IoT), Intelligent Transportation Systems (ITS), data-driven urbanism, AI-driven traffic control, and sustainable urban mobility theory as applied to traffic management. Drawing on global case studies, the review highlights how these frameworks improve traffic efficiency, integrate public transportation, and reduce emissions. The paper contextualizes these theories within Tirana's traffic landscape, marked by high car dependency, limited transit alternatives, and rising air pollution. It discusses the applicability of interventions such as adaptive signal control, real-time traffic monitoring, and integration of electric bus rapid transit (e-BRT). The study concludes that while technology is essential, it must be coupled with sustainability-oriented policies to transform Tirana's transport system. Recommendations include prioritizing data collection via IoT, deploying adaptive signals and transit signal priority, and using AI for predictive traffic control. Challenges such as funding, technical capacity, and public acceptance are noted. A phased approach starting with smart pilot corridors, expanding through evidence-based planning, and supported by citizen engagement is proposed. The findings emphasize that a combined strategy leveraging smart city technologies and sustainable mobility principles can help Tirana build a smarter, cleaner, and more efficient urban transport network.

Keywords: *AI Traffic Management, Intelligent Transportation Systems (ITS), Smart City Frameworks, Sustainable Urban Mobility, Traffic Flow Optimization*

Forecasting Volatility In Wind Power Productıon: A Comparatıve Analysıs Of Real-Tıme Data And R-Based Models (1197)

Fikret Bartu Yurdacan[1]*, Gül Boztok Algın[1]

[1]Ege University, International Computer Institute, Türkiye

*Corresponding author e-mail: bartuyurdacan@gmail.com

Abstract

This study aims to explore the forecasting behavior and volatility patterns of wind power production in Türkiye by utilizing high-frequency real-time data. The dataset includes 10-minute interval production and forecast values from the Wind Power Monitoring and Forecasting Center (RİTM), covering the period between January 1, 2021 and January 1, 2025. The forecast values are complemented by quantile-based intervals (Q5–Q95) reflecting uncertainty bands. The primary focus of this research is to examine whether there is a consistent deviation between forecasted and realized generation values and to what extent these deviations can signal volatility in wind energy production. Particular attention will be given to systematic bias—especially in underforecasting—and to the identification of high-variance time intervals. In the second phase of the study, time series forecasting models such as ARIMA, ETS, and Prophet will be implemented using R. The goal is to evaluate and compare the performance of these models against the RİTM-provided forecasts. Error metrics including MAE, RMSE, and MAPE will be used to assess model performance. The study is expected to provide insights into both the reliability of operational wind forecasts and the feasibility of alternative model-based approaches for short-term wind energy prediction.

Keywords: *Wind Energy Forecasting, Volatility Analysis, Forecast Error, Tıme Serıes Modelıng, R Programmıng*

Modeling Risk and Behavioral Causes in Traffic Accidents: Application of LSTM, K-Means and Single Linkage Clustering Method in Turkey (1198)

Özge Ustahüseyin[1*], Esra Nur Eviren[1], Mehmet Yılmaz[1], Cemal Atakan[1]

[1]Ankara University, Faculty of Science, Department of Statistics, Turkey

*Corresponding author e-mail: ozgeustahuseyin@gmail.com

Abstract

In this study, data on traffic accidents occurring in 81 provinces in the area of responsibility of the General Command of Gendarmerie in Turkey between 2020 and 2025 are analysed and the risk levels of accidents and the causes of accidents due to driver behaviors are modelled with a multi-method approach. Firstly, the K-means clustering analysis was applied to the annual traffic accident data. The K-means clustering algorithm is predicated on the requirement for the number of clusters (k) to be predetermined. In this study, the objective was to determine the optimal grouping to reflect the natural structure of the data. This was achieved by determining the number of clusters. The number of clusters was determined as k=3, in accordance with the research objectives and data structure. The analysis yielded a classification of provinces as "high risk", "medium risk" and "low risk" according to their accident density. In consequence of the clustering process, a total of ten provinces were designated as high-risk provinces. In the study, driver defects causing fatal and injury traffic accidents, as well as the types of vehicles involved in accidents, were clustered using the single linkage algorithm. The single linkage algorithm is one of the hierarchical clustering methods. Finally, time series forecasting was performed with the Long Short-Term Memory (LSTM) model to model the fatal-injury traffic accident data in the clustered high-risk provinces. In this context, the performance of the model was analyzed using MAE, MAPE, RMSE and R^2 measures. The results demonstrated that the model exhibited a high degree of explanatory power. Furthermore, forecasts of 12-month fatal-injury traffic accident data for high-risk provinces were calculated.

Keywords: *Traffic Accident, Cluster Analysis, K-means, Single Link Method, LSTM*

Investigation of Latent Profiles According to Mathematics Anxiety, Mathematical Effort-Persistence and Self-Efficacy Variables Using PISA 2022 Data (1201)

Ecenaz Killik[1,*], Mahmut Sami Koyuncu[2], Muhammed Recai Türkmen[3]

[1]Afyon Kocatepe University, Institute of Science, Turkey
[2]Afyon Kocatepe University, Faculty of Education, Turkey
[3] Afyon Kocatepe University, Faculty of Education, Turkey

*Corresponding author e-mail: ecenazkillik@gmail.com

Abstract

PISA is the Programme for International Student Assessment. PISA is an international research program conducted by the OECD to evaluate the knowledge and skills of students at the age group of 15. In PISA research; students' maths literacy, science literacy, and reading skills are assessed. In this study, it was aimed to model the individual differences of students from Turkey participating in the PISA 2022 application according to mathematics anxiety, mathematical effort- persistence, and mathematical reasoning- 21st-century skills self-efficacy variables, to reveal their latent profiles and to examine their property. A quantitative research method and correlational research model were used in the study. The data of the study were obtained from the international database of OECD. The data of 7250 students studying in Turkey were used in the study. Microsoft Excel, SPSS, and Jamovi programs were used for analyses. In the study, students' scores from the mathematics anxiety, mathematical effort- persistence mathematical reasoning- and 21st century skills self-efficacy variables from the 2022 PISA student survey were used. As a result of the latent profile analysis carried out during the study, it was concluded that the students were classified into three homogenous subgroups. Profile 1 has a medium level of 21st-century skills - mathematical reasoning self-efficacy, a high level of mathematics anxiety, and a medium level of mathematics effort - persistence. Profile 2 has medium level of 21st-century skills - mathematical reasoning self-efficacy, medium level of mathematics anxiety, and medium level of mathematics effort – persistence. Profile 3 has medium level of 21st-century skills - mathematical reasoning self-efficacy, medium level of mathematics anxiety, and medium level of mathematics effort - persistence skills. Profile 2 and Profile 3 were very similar with no difference.

Keywords: *Latent profile analysis, Perseverance-effort, Mathematics anxiety, Self-efficacy, PISA.*

Forecasting the Production of Milk and Milk Product with ARIMA Models: An Interactive R Shiny Application (1202)

Fethi Saban Özbek[1*]

[1]Turkish Statistical Institute, Social Statistics Department, Türkiye

*Corresponding author e-mail: fethiozbek@tuik.gov.tr

Abstract

In this study, monthly data on different milk and selected dairy products were used to forecast future production quantities through ARIMA models. These forecasts are presented via a user-friendly and visualized tool developed on the R Shiny platform. The interactive application allows users to generate forecasts for the selected product group: collected cow's milk, butter and ghee, cheese made from cow's milk, yoghurt (including plain, strained, and flavored yogurts), and drinking milk (including plain, flavored, and packaged raw milk). During the modeling process, the most appropriate ARIMA model for time series analysis was automatically selected using AIC and BIC criteria, taking seasonality into account.

Based on the model estimations, and taking January 2025 as the reference point, the production levels are projected to increase over the subsequent 12 months as follows: collected cow's milk by 1.88%, butter and ghee by 2.61%, cheese produced from cow's milk by 2.54%, yoghurt by 4.35%, and drinking milk by 5.03%. Furthermore, the 24-month ahead forecasts indicate more pronounced growth, with respective increases of 4.46%, 7.08%, 5.63%, 7.63%, and 7.90% in the aforementioned product categories.

Keywords: *ARIMA, Forecasting, Milk and milk products, R Shiny*

Comparison of the Performance of Naive Bayes and Decision Tree Methods in the Classification of Alzheimer's Disease (1203)

Özlem Bezek Güre[1*]

[1]Batman University, Health Services Vocational School, Medical Documentation and Secretarial Program, Batman, Turkey

* Corresponding author e-mail: obezekgure@gmail.com

Abstract

Alzheimer's disease is a disorder due to nerve cell damage in the brain. As the disease progresses, cognitive functions deteriorate, leading to memory loss. Alzheimer's disease ranks among the leading causes of death worldwide (Mahamud et al., 2025). Early and accurate prediction is critical. Data mining methods are assumption-free techniques that can uncover hidden relationships among variables with high accuracy. This study compares the performances of the Naive Bayes and Decision Tree methods, two popular data mining approaches. The Alzheimer's Disease Dataset available on Kaggle was used. This dataset comprised 2,149 individuals, of whom 1,389 (64.6%) were non-patients and 760 (35.4%) were patients; 1,061 (49.4%) were male and 1,088 (50.6%) were female. The dataset included 33 variables related to disease status, demographic information, lifestyle factors, medical history, clinical measurements, and cognitive and functional assessments. Disease status was treated as the dependent variable, and the remaining 32 variables were treated as independent variables. In the absence of missing data and due to the differing scales of the variables, the data were first normalized. The dataset was then split into 70% training and 30% testing sets. Accuracy, F1 score, precision, and recall were used as performance metrics. A 10-fold cross validation procedure was applied to avoid overfitting. Analyses were conducted using Orange, a free Python based platform. The results showed that classification accuracy was 94.1% for the Decision Tree method and 89.3% for Naive Bayes. For the Decision Tree method, the F1 score, precision, and recall were all 0.941. For Naive Bayes, the F1 score was 0.891, and both precision and recall were 0.893. These findings indicate that the Decision Tree method outperformed Naive Bayes in classification performance. According to the information gain feature selection method, the most important factor influencing Alzheimer's risk was functional assessment.

Keywords: (*Alzheimer's disease, Naive Bayes, Decision Tree, Data Mining)*

Bibliometric Analysis of Graduate Theses in Big Data (1205)

Hanife Şahin[1*], Yelda Güçlü[2], Fatih Kaleci[3]

[1]Necmettin Erbakan University, Office of Big Data, Turkey
[2]Necmettin Erbakan University, Office of Big Data, Turkey
[3]Necmettin Erbakan University, Ahmet Keleşoğlu Faculty of Education, Math and Science Education, Turkey

*Corresponding author e-mail: shinhanife@gmail.com

Abstract

In recent years, increasing data production with the development of technology has led to the popularization of big data. This situation has attracted the attention of different branches of science. It has become important to analyze big data, make it meaningful and make inferences. In this study, it is aimed to present a perspective by analyzing the theses in the field of big data. It has been observed that academic studies on big data have shown a significant increase since 2012. In this context, theses containing the keyword "big data" between 2012 and 2025 from the databases of the Council of Higher Education National Thesis Center (YÖKTEZ) and ProQuest Dissertations & Theses Global (excluding Turkey) were scanned. Bibliometric analysis of the theses obtained from YÖKTEZ; their distribution according to the universities, departments, keywords, software used, and years were reported. A national - international comparison was made with the theses obtained from ProQuest Dissertations & Theses Global database. As a result of the findings, the university that wrote the most theses in the field of big data in Turkey is Istanbul Technical University, the institute that wrote the most theses is the Institute of Science, and the department that wrote the most theses is the department of computer engineering. In keywords, "data", "big", "analysis", "machine", "learning", "artificial", "mining", "deep" are the most used words. "python" ranks first as the software language used. Traditional statistical, data pre-processing and reporting software such as "spss", "excel" are still widely used. Software such as "matlab" and "r" have found a place in the field of big data. The presence of "apache", "hadoop" and "spark" is directly related to distributed and large-scale data processing. The presence of the words "digital", "social" and "media" both nationally and internationally indicates that data is obtained from social media and digital environments.

Keywords: *Big Data, Bibliometric Analysis, Literature Review, ProQuest, YÖKTEZ*

Determinants of Theft Crime Patterns in Rural parts of Türkiye with GAMLSS and Machine Learning (1206)

Müge Borazan[1*], Serpil Aktaş Altunay[2], Erdem Erciyes[3]

[1]Gendarmerie and Coast Guard Academy, Department of Security Sciences, Türkiye
[2]Hacettepe University, Faculty of Science, Department of Statistics, Türkiye
[3]İstanbul Provincial Gendarmerie Command, Türkiye

*Corresponding author e-mail: muge.borazan@jsga.edu.tr

Abstract

The aim of this study is to model the determinants of theft crime patterns in rural parts of Türkiye using statistical and machine learning approaches. The study utilizes records of 5,121 theft incidents that occurred between 2012 and 2021 under the jurisdiction of gendarmerie authorities in the rural provinces of Tunceli, Bingöl, Muş, and Elazığ. Theft incidents are categorized under eighteen different types. For each incident, variables such as province, district, and the specific location of the crime, along with the date, time, population, number of patrols, number of photo capture and surveillance cameras, road conditions, number of gendarmerie personnel per capita, internal migration rates, and other contextual factors were analyzed for their effects on theft crime counts. Since crime counts are shaped by complex socio-economic and environmental dynamics, it is essential to consider the distributional characteristics of the data and both linear and non-linear relationships when developing predictive models. In this context, the Generalized Additive Models for Location, Scale, and Shape (GAMLSS), which are well-suited for modelling count data, were employed. GAMLSS offers a flexible framework that models not only the mean but also the variance, skewness, and kurtosis of the distribution as functions of explanatory variables. In this way, it provides a powerful alternative to classical Poisson and Negative Binomial models, especially for complex crime datasets. To enhance the robustness of the modelling process, machine learning algorithms with high predictive accuracy such as Random Forest and Extreme Gradient Boosting (XGBoost) were also applied. In addition, a trend analysis was conducted to examine the temporal evolution of theft crimes over the years. The findings demonstrate the methodological applicability of the GAMLSS approach in modelling rural crime data, which are often limited in terms of quantity and quality. From the perspective of practitioners, the results offer evidence-based insights for resource allocation and the development of strategic decision-support systems for law enforcement organizations. In conclusion, this study emphasizes the necessity of jointly evaluating accuracy, flexibility, and interpretability in the prediction of crime counts, and highlights the complementary strengths of statistical and machine learning-based methods.

Keywords: *GAMLSS, Theft Crimes, Count Data, XGBoost, Random Forest*

Predicting of Severity of COVID-19 using Machine Learning Algorithms (1208)

Muslu Kazım Körez[1], Dilek Ergün[2*], Recai Ergün[3]

[1]Selcuk University, Faculty of Medicine, Department of Biostatistics, Turkey
[2]Selcuk University, Faculty of Medicine, Department of Pulmonary Disease, Turkey
[3]Selcuk University, Faculty of Medicine, Department of Pulmonary Disease, Turkey

*Corresponding author e-mail: mkkorez@gmail.com

Abstract

The coronavirus disease 2019 (COVID-19) pandemic caused by the SARS-Cov2 virus has become the greatest health worldwide. Predicting the severity of the disease at an early stage can help reduce healthcare costs for patients and the burden on physicians. The aim of the study is to predict severity of COVID-19 by investigating various machine learning algorithms using demographical and clinical characteristics and laboratory findings. Additionally, feature importances in each model and the influences of features in the severity prediction of ML models have been evaluated. A total of 979 patients diagnosed with COVID-19, 506 of whom were classified as severe disease and 473 as mild disease, were included in this study. The performance of ML models has been evaluated by evaluation metrices such as accuracy, precision, sensitivity, specificity, AUC and F1-score. The ML models showed similar classification performance, but the highest accuracy (75.41%), sensitivity (78.29%) and F1-score (77.10%) were observed for the Neural Network (NN) model, when training was performed with 14 features, which is age, comorbidity, CRP, LDH, albumin, troponin, neutrophile count, WBC, glucose, urea, total protein, AST, D-dimer and lymphocyte count, selected by using feature selection technique, BORUTA. Age, LDH and CRP were determined as the most importance features of severity of COVID-19. We recommend physicians age, comorbidity, CRP, LDH, albumin, troponin, neutrophile count, WBC, glucose, urea, total protein, AST, D-dimer and lymphocyte count for potential progression to severe conditions among COVID-19 patients.

Keywords: *Machine Learning, COVID-19, Prediction, Classification*

A Secure and Automated Web Data Pipeline for Open Data Platform of Central Bank of Türkiye (1210)

Cengiz Tosun[1]*

[1]Central Bank of Republic of Türkiye, Data Governance and Statistics Department, Data Analytics and Visualization Division, Istanbul, Türkiye

Corresponding author e-mail: cengiz.tosun@tcmb.gov.tr

Abstract

The purpose of this paper is to present the studies conducted by Central Bank of Türkiye's (CBRT) Data Governance and Statistics Department in order to collect data from various sources automatically, arranging this data set in a secure and analytical format and transferring them into a data base for the use of end users of EVDS which is the open data platform of CBRT.

The study is composed of three phases:

- *Web Scraping*
- *Data Cleaning*
- *Data Publishing*

On the first phase, web scraping, the internet pages, API end points or third-party platforms are verified so as to define data need. In institutions which have web service system, data can be easily automatically obtained by API while for others the data can be obtained by using Selenium Library. By using Phyton based Selenium library, scanner automation is realized.

The subsequent phase focusses on data preparation. Raw data is ingested into a Pandas-based workflow for validation, cleansing, normalization and enrichment. This process includes schema verification data type conversions and the enforcement of integrity constraints to ensure data quality and consistency.

In the final phase, the cleansed datasets are mapped to the target schema and integrated into the organizations data platform. Automated orchestrations tools, such as Apache Airflow, schedule regular updates and manage data workflows. The processed data is then prepared for publication at the the open data platform of CBRT which is called EVDS, facilitating access for researchers, analysts and policymakers.

By automating repetitive data collection tasks with Selenium, this architecture achieves significant time and resource efficiencies. Where API access is available, RESTful methods are preferred for their performance and efficiency; otherwise, Selenium-based automation offers a practical alternative. The published datasets are readily integrable with machine learning models and statical methods, supporting advanced analyses and predictive modeling on robust data foundations.

Keywords: *Web Scraping, Selenium, Python, Data Preparation, EVDS*

An Application of Geometric Process for Multi-System Repair Process Modeling (1211)

Ömer Altındağ[1*], Halil Aydoğdu[2]

[1]Bilecik Şeyh Edebali University, Department of Statistics and Computer Sciences, Turkey
[2]Ankara University, Department of Statistics, Turkey

*Corresponding author e-mail: omer.altindag@bilecik.edu.tr

Abstract

Most systems are repaired after a failure if the repair cost is lower than the cost of replacing the product. It is important for a manufacturer to model the repair process of a repairable system probabilistically in order to estimate the lifetime repair cost of the product. The repair process of a repairable system is generally modeled using a counting process. A counting process is used to represent consecutive failures of the product after each repair, allowing the repair process to be modeled probabilistically. For most repairable systems, operating times tend to decrease due to accumulated wear. In such cases, the repair process should be modeled using a monotonic counting process. One important monotonic counting process model is the geometric process. Let $X_1, X_2, \ldots$ *be non-negative random variables representing the inter-arrival times of consecutive events, then the process* $\{X_k,\ k = 1,2, \ldots\}$ *said to be a geometric process with trend parameter* $a > 0$ *if the random variables* $a^{k-1}X_k$, $k = 1,2, \ldots$ *are independent and identically distributed with a general distribution function* F*. The geometric process has been widely used in repair and replacement problems due to its ease of implementation for modeling monotonic processes. Most of the existing studies in the literature focus on modeling the repair process of a single system. In this study, we consider the geometric process model for the repair processes of multiple systems and compare it with other competing models for multi-system repair process modeling.*

Keywords: *geometric process, multi-system, repair process*

Investigation of Pollutant Movement Modeled with Advection-Diffusion-Dispersion Equations Using Numerical and Artificial Intelligence Methods (1215)

Ahmet Selim Yavuz[1], Hande Uslu Tuna[1]

[1]Yıldız Technical University, Mathematics, Türkiye

*Corresponding author e-mail: selim.yavuz1@std.yildiz.edu.tr

Abstract

Environmental pollution is a growing threat to ecosystems and human health due to the uncontrolled release of pollutants into air, water and soil. Understanding how these pollutants spread in natural environments is essential for effective prevention and control. The study mainly includes a partial differential equation which has been used to model pollutant transport in environmental media. The equation captures the physical processes including advection, diffusion, dispersion and reaction. Its comprehensive structure makes it essential for accurately simulating pollutant dynamics in some medium such as water, air, and soil systems. In this study, contaminant concentrations have been modeled using conventional numerical methods, especially finite difference approaches. These methods have inherent coding complexity, long computational times and inherent limitations. To address these limitations, the results of numerical methods were compared with artificial intelligence-based approaches.

Keywords: *Advection-diffusion-dispersion, Pollutant spreading, Numerical methods, Artificial intelligence methods*

Simulation and Statistical Validation of Entropy-Based Fuzzy and Neutrosophic Models (1216)

Nihal İnce[1*], Sevil Şentürk[2]

[1,2] Eskisehir Tecnical University, Faculty of Science, Department of Statistics, Turkey

*Corresponding author e-mail: nihalince23@gamil.com

Abstract

In this study, we propose a novel framework for fuzzy entropy optimization using two generalized entropy-based approaches: the Generalized Maximum Fuzzy Entropy Method (GMax(F)EntM) and the Generalized Minimum Cross Fuzzy Entropy Method (GMinx(F)EntM). These methods are developed based on the principles of entropy maximization and cross-entropy minimization under moment vector constraints. The resulting distributions—referred to as GMax(F)Ent and GMinx(F)Ent distributions—are tailored to model uncertainty in fuzzy environments more effectively than classical counterparts.To evaluate the performance and robustness of the proposed methods, simulation studies were initially conducted using Gaussian fuzzy numbers. The methodology was then extended to Neutrosophic numbers, which incorporate three parameters: truth (T), indeterminacy (I), and falsity (F), providing a richer framework for representing uncertainty in complex systems. Statistical performance metrics including fuzzy entropy measure, Chi-square statistic, Root Mean Square Error (RMSE), and coefficient of determination (R^2) were employed to assess the model quality. ANOVA results indicated statistically significant differences between the two distribution *types ($p < 0.01$), and these findings were further validated through post-hoc Tukey's HSD tests. Visual comparisons using ggplot2 and boxplots were also provided to support interpretability.Overall, this study contributes a generalized and flexible entropy-based modeling approach for fuzzy and Neutrosophic data. The proposed framework demonstrates strong performance and interpretability, offering potential for applications in soft computing, decision support systems, and uncertainty modeling. The integration of Neutrosophic logic further enhances its relevance to real-world scenarios involving partial or conflicting information.*

Keywords: *Entropy-based distributions, Fuzzy data analysis, Uncertainty modeling, Neutrosophic numbers, Simulation study.*

Balancing Workloads in Store Renovations: A Case Study from a Shopping Mall (1217)

Zülal Demirci*[1], Mustafa Avcı[2*]

[1]İzmir Bakırçay University, Graduate Education Institute, Türkiye
[2]İzmir Bakırçay University, Faculty of Engineering and Architecture, Department of Industrial Engineering, Türkiye

*Corresponding author e-mail: zulaldemirci2@gmail.com

Abstract

In this study, the renovation processes of stores located in a shopping mall are examined with a focus on workforce assignment and workload balancing. The workforce consists of both in-house and outsourced workers, each with different skill sets related to specific job types such as painting, mechanical workshop and woodworking. Each renovation task falls under one of these categories and requires workers with the appropriate skills to perform the job effectively. The problem is formulated as an optimization model aiming to distribute renovation workloads fairly among available workers. The model considers key constraints such as skill compatibility between workers and tasks, task deadline and worker availability throughout the project timeline. For data handling, Microsoft Excel is used to manage inputs and organize the necessary parameters. IBM ILOG CPLEX Optimization Studio is employed to solve the mathematical model and find optimal task assignments. The main objective is to achieve an efficient allocation of workers to tasks, minimizing workload imbalances and improving the use of available human resources. By optimizing task distribution, the model supports more balanced planning and enhances operational efficiency in large-scale renovation projects. The proposed approach offers practical value by integrating data management with mathematical optimization. It serves as a useful framework for similar resource allocation problems in project management and facility operations.

Keywords: *Workforce Assignment, Workload Balancing, Optimization Model, Renovation Projects, Resource Allocation*

A Legal and Economic Assessment of the Alignment Between Regional Wage Levels and Living Income in Agriculture: The Case of Seasonal and Permanent Employment (1218)

Cem Gül[1], Bilge Gözener[1], Mehmet Can Kaya[1*]

[1]Tokat Gaziosmanpaşa University, Faculty of Agriculture, Departmant of Agricultural Economics, Turkey
2Tokat Gaziosmanpaşa University, Faculty of Agriculture, Departmant of Agricultural Economics, Turkey
[3]Özyeğin University,Faculty of Law,, Turkey

*Corresponding author e-mail: cem.gul4820@gop.edu.tr

Abstract

In this study, the average regional wages paid to seasonal and permanent workers employed in the agricultural sector in Türkiye were examined, and the alignment of these wage levels with living income thresholds was evaluated. The analysis was based on the Turkish Statistical Institute's (TÜİK) data for the period 2010–2024 on "average daily wages paid to seasonal workers" and "average monthly wages paid to permanent workers." These indicators were compared with the hunger and poverty thresholds reported by Türk-İş, as well as the minimum wage data published by the Ministry of Labour and Social Security.

The findings reveal that seasonal workers in many regions are employed at wage levels even below the hunger threshold, while permanent workers also remain below the poverty line. Within this context, regional distributions were presented using basic statistical methods and graphical illustrations; ratio analyses were conducted, and trends in wage levels over time were examined.

Moreover, based on the data obtained, the legal status of seasonal and permanent workers was compared within the framework of labour law. Legal evaluations were carried out with reference to the relevant legislation (Labour Law No. 4857), Supreme Court case law, and key issues such as social security notification obligations, severance and notice pay entitlements, and annual leave rights. In particular, the lack of social security coverage and contractual insecurity faced by seasonal workers were analyzed through concrete examples.

In this study, statistics were not only used as a technical tool of analysis, but also as an explanatory instrument that enhances the socio-economic visibility of agricultural workers and quantitatively exposes social inequalities. In this respect, the research adopts an interdisciplinary approach that bridges statistics, law, and economics, offering an original contribution to the existing literature.

Keywords: *Agricultural Labor, Regional Wage Statistics, Labor Law, Wage Levels, Seasonal Employment*

Assessing the Performance of Empirical Distribution Function Based Goodness of Fit Tests for Weibull Distributed Data (1220)

Zahir Hajizada[1*], İlhan Usta[1]

[1]Eskisehir Technical University, Faculty of Science, Statistics, Turkey

*Corresponding author e-mail: zahirhajizada@ogr.eskisehir.edu.tr

Abstract

The Weibull distribution is extensively employed in various fields due to its flexibility in modeling a broad range of data patterns. Because of its widespread application, accurately assessing whether empirical data conform to a Weibull distribution is essential for ensuring the validity of statistical analysis. Accordingly, goodness-of-fit (GoF) tests are frequently applied to evaluate the consistency between observed data and the theoretical Weibull model. However, the performance of these tests may vary depending on factors such as sample size or the nature of deviation from the assumed distribution, and thus they may not provide reliable results in all cases. To address this, in this study, the performance of commonly used GoF tests based on empirical distribution function (EDF) is examined for the Weibull distribution in terms of Type I error rates and statistical power through a comprehensive Monte Carlo simulation. The comparison includes widely used tests in the literature: the Kolmogorov-Smirnov, Cramer-von Mises, Anderson-Darling, Kuiper, and Watson tests. In addition, critical values for these tests are obtained empirically. The findings indicate that the Anderson–Darling test demonstrates superior performance compared to the other methods considered. These results are expected to offer practical guidance for both researchers and practitioners in selecting appropriate goodness-of-fit tests when evaluating Weibull distributional assumptions.

Keywords: *Weibull distribution, EDF type Goodness-of-Fit test, Critical value, Type I error, Power of test*

Forecasting of Meat Production in Azerbaijan Using ARIMA and LSTM Models (1221)

Zahir Hajizada[1*], Hanefi Gezer[1]

[1]Eskisehir Technical University, Faculty of Science, Statistics, Turkey

*Corresponding author e-mail: zahirhajizada@ogr.eskisehir.edu.tr

Abstract

The livestock sector holds strategic importance in terms of sustainable development, rural employment generation, and ensuring food security. Accordingly, many countries implement agricultural and livestock policies to increase meat production and reduce dependence on imports. This study aims to forecast based on annual production data of different types of meat in Azerbaijan. Annual production data covering the period from 1992 to 2023 were used, and separate time series were constructed for each meat category. To predict future production trends, two different forecasting models were compared: the classical model of AutoRegressive Integrated Moving Average (ARIMA) and the deep learning-based Long Short-Term Memory (LSTM) neural network. The predictive performance of both models was evaluated using some metrics. The findings indicate that the ARIMA model provides stable and interpretable forecasts, whereas the LSTM model demonstrates superior predictive accuracy by capturing nonlinear patterns more effectively. This study presents a data-driven approach that can contribute to the development of policies by the Ministry of Agriculture and Livestock, aimed at improving domestic meat production and reducing dependency on imports, thus informing strategic planning in Azerbaijan.

Keywords: *LSTM, ARIMA, Meat production, Forecasting*

Estimation of the Exponential Distribution Parameters Based on an Improved Adaptive Type-II Progressive Censoring with Binomial Removals (1222)

Hanefi Gezer[1]*, İlhan Usta[1]

[1]Eskisehir Technical University, Faculty of Science, Statistics, Turkey

*Corresponding author e-mail:hgezer@eskisehir.edu.tr

Abstract

An improved adaptive Type-II progressive censoring scheme (IAT-II PCS) has recently been proposed to ensure that the duration of the experiment does not exceed a pre-specified threshold. In this context, the present study focuses on parameter estimation for the Exponential distribution under the IAT-II PCS framework, where the number of units removed at each failure time follows a binomial distribution, which provides a more realistic representation of life-testing experiments encountered in practical applications. Both maximum likelihood and Bayesian estimation methods are employed to estimate the scale parameter of the exponential model and the binomial removal probability. A comprehensive simulation study is carried out to assess the performance of the proposed estimators under various censoring schemes. The obtained results demonstrate that Bayesian estimators, particularly those incorporating informative priors, outperform their classical counterparts in terms of mean squared error under IAT-II PCS with binomial removals.

Key words*: Improved adaptive Type-II progressive censoring, Binomial removal, Maximum likelihood estimation, Bayesian estimation, Exponential distribution*

Estimation of Rayleigh Distribution Characteristics under an Improved Adaptive Type-II Progressive Censoring Scheme with Random Removals (1223)

Hanefi Gezer[1]*, İlhan Usta[1]

[1]Eskisehir Technical University, Faculty of Science, Statistics, Turkey

*Corresponding author e-mail:hgezer@eskisehir.edu.tr

Abstract

The Rayleigh distribution plays a significant role in reliability and lifetime analysis, particularly in contexts where failure rates increase over time. On the other hand, the improved adaptive Type-II progressive censoring scheme (IAT-II PCS) has attracted considerable attention due to its ability to limit experiment duration, offering greater flexibility in practical applications. Accordingly, this study investigates the estimation of the parameter and reliability measures of the Rayleigh distribution under the IAT-II PCS with randomly determined removals, providing a more realistic framework for real-world situations in which the number of units removed at each failure time cannot be fixed in advance. Within this framework, both maximum likelihood and Bayesian estimators are derived. A Monte Carlo simulation study is carried out to evaluate the performance of these estimators under various random censoring scenarios. The results demonstrate that Bayesian estimators based on informative priors yield better performance in estimating the model parameter and reliability characteristics under the IAT-II PCS, where the number of units removed at each stage is randomly determined.

Keywords: *Improved adaptive Type-II progressive censoring, Random removal, Maximum likelihood estimation, Bayesian estimation, Rayleigh distribution characteristics*

Investigation of the miR-124a Gene Expression in Latent Autoimmune Diabetes of Adults (LADA) (1224)

Sura Edanur Sağlam[1], Mustafa Altınkaynak[2], Gülşah Yenidünya Yalın[2], Ayşe Kubat Üzüm[2], İlhan Satman[2], Deniz Kanca Demirci[1*]

[1] Haliç University, Faculty of Fine Arts, Department of Molecular Biology and Genetics, Türkiye
[2]Istanbul University, Istanbul Faculty of Medicine, Department of Internal Medicine, Türkiye

*Corresponding author e-mail: denizkanca@halic.edu.tr

Abstract

Diabetes mellitus (DM) is a chronic disease prevalent worldwide, classified into type 1 diabetes (T1DM), type 2 diabetes (T2DM), gestational diabetes and other causes. Latent autoimmune diabetes of adults (LADA), a subtype of T1DM, is characterized by slow immüne-mediated damage to pancreatic β-cells and is often misdiagnosed as T2DM due to late onset and insulin requirements.

microRNAs (miRNAs) are single-stranded, non-coding RNAs that regulates gene expression post-transcriptionally and are involved in several cellular processes, including the cell growth, apoptosis, differentiation and the development of cancer. miR-124a plays a crucial role in the regulation of β-cell developments. This study aims to investigate the role of miR-124a levels in LADA developments.

For this purpose, after miRNA isolation from blood, cDNA synthesis from miRNA via reverse transcriptase PCR, and qPCR from cDNA were conducted to assess the gene expression levels of miR-124a. The expression levels were Compared between LADA patients and healty individuals.Then, the genetic data of the study group was analyzed statistically together with clinical and biochemical data and its contribution to the role of LADA in immune pathogenesis was evaluated.

Based on the findings obtained in this study, it was determined that in the LADA patient group, age ($p<0,001$), HbA1c ($p<0,001$), and phosphorus levels ($p<0,001$) were significantly higher, While albumin levels were significantly lower ($p=0,011$) Compared to the control group. Evaluation of hyperlipidemia ($p=0,015$) were independently and significantly associated with disease developments. In the gene expression analyses, although no statistically significant difference was found in miR-124a expression levels between the patient and control groups ($p=0,205$), a positive and statistically significant correlation was detected between miR-124a expression levels and HDL levels in the patient group ($r=0,324$, $p=0,032$). These results suggest a potential role of miR-124a in the immunopathogenesis of LADA and indicate the need for further studies with larger sample sizes.

Keywords*: miR-124a, Biomarkers, LADA, Gene expression*

The Impact of Choice Overload on University Selection and Student Satisfaction in Türkiye (1225)

Ashkan Badiozzamani Tari Nazari[1], Erkan Isikli[2*]

[1] Istanbul Technical University, Faculty of Management, Industrial Engineering Department, Türkiye
[2]Istanbul Technical University, Faculty of Management, Industrial Engineering Department, Türkiye

*Corresponding author e-mail: isiklie@itu.edu.tr

Abstract

When faced with many options, decision-makers may have difficulty making decisions, experience increased cognitive load, and regret their choices. This phenomenon, called choice overload, has attracted researchers in consumer behavior and economics, but less so in education, especially in countries with an educational system like that of Türkiye. This study aims to inform higher education policy in Türkiye by examining how choice overload affects university applicants during undergraduate program selection. To investigate this, a five-section questionnaire was developed based on a comprehensive literature review and expert input. It was administered to undergraduate students in Türkiye using convenience sampling between October 2024 and May 2025. The questionnaire included a series of demographic questions (e.g., gender, age, place of birth), the ultra-short version of the Big Five Personality Scale, several questions that can characterize the participants (e.g., current program, high school major, exam ranking) and their university selection process (e.g., number of programs and universities listed, alignment with high school major), factors that may have influenced their decision (e.g., pre-existing preferences, influence of others, external support, and prior information). Additionally, the survey assessed students' satisfaction with their current programs and the degree of regret they felt about their choices. The dataset, consisting of 259 participants (134 women), was cleaned and analyzed using several statistical techniques. The study examined the direct and indirect effects of choice overload on student satisfaction, with variables such as gender, academic year, and exam ranking considered as potential moderators. This research contributes to the literature on decision-making in education and to all constituents of the Turkish higher education system, as it provides recommendations for the creation and implementation of strategies that can reduce the effect of choice overload in the university selection process.

Keywords: *Choice Overload, University Selection, Student Satisfaction, Mediator.*

Investigation of the Effect of miR-155 Rs767649 T>A Polymorphism on miR-155 Gene Expression in Latent Autoimmune Diabetes in Adults (LADA) (1226)

Farid Nuriyev[1], Gülşah Yenidünya Yalın[2], Ayşe Kubat Üzüm[2], Mustafa Altınkaynak[2], İlhan Satman[2], Deniz Kanca Demirci[1]

[1] Haliç University, Faculty of Fine Arts, Department of Molecular Biology and Genetics, Türkiye
[2] İstanbul University, Istanbul Faculty of Medicine, Department of Internal Medicine, Türkiye

*Corresponding author e-mail: denizkanca@halic.edu.tr

Abstract

Diabetes mellitus is a chronic metabolic disease characterized by hyperglycemia. Rare forms of diabetes such as Latent autoimmune diabetes in adults (LADA) is an unique form of diabetes that has characteristics of both type 1 and type 2 diabetes and prone to misdiagnosis as well. Therefore, different additional parameters are needed to facilitate the diagnosis of rare forms of diabetes. MicroRNAs are non-coding, single-stranded RNA molecules that function as post-transcriptional regulators of gene expression and controls cellular mechanisms. miR-155 plays a role in the pathogenesis and complications of diabetes.

In this study, we aimed to investigate the relationship between miR-155 gene expression and miR-155 Rs767649 (T>A) polymorphism in 50 LADA patients and 62 healthy control subjects. miR-155 Rs767649 polymorphism are analyzed by SNP probe assay and miR-155 and the housekeeping gene-(RNU6B) expression levels are analyzed by Real-time PCR. Statistical analyses were performed by SPSS v.24.0 and Shapiro-Wilk, Mann-Whitney U, and Student's tests were applied.

When the general characteristics of the patient and control groups were examined, TT genotype was not observed and, age ($p<0.001$) and phosphate ($p<0.002$) were found to be higher in the patient group compared to the control group, while albumin ($p=0.013$) and eGFR ($p=0.064$) were found to be higher and significant in the control group compared to the patient group. When the effect of miR155 expression levels on LADA pathogenesis was examined by correlation analysis, age ($p=0.045$) and IAA20 ($p=0.042$) levels were positively correlated with increased miRNA expression and were higher, while eGFR ($p=0.045$) was negatively correlated and lower. Our findings suggest that miR155 may play an active role in LADA genetics and may add some parameters as diagnostic criteria. Therefore, further studies with larger sample sizes are required for stronger findings.

Keywords: *LADA, obesity, diabetes, SNP, miR-155, Rs767649*

Forecasting Hourly Wind Power Using Deep Learning Models vs XGBoost (1227)

Fatma Başoğlu Kabran[*], Kamil Demirberk Ünlü[2]

[1]İzmir Institute of Technology, İzmir
[2]Atilim University, Faculty of Engineering, Industrial Engineering, Ankara

*Corresponding author e-mail: fatmakabran@iyte.edu.tr

Abstract

Using a comparative study of several state-of-the-art machine learning and deep learning models, this paper seeks to predict hourly wind power generation at the Karaburun Wind Power Plant. Wind energy production forecasting is accurate enough to maximize energy dispatch, strengthen grid reliability, and raise market efficiency. Aiming 1 to 10 hours ahead, the forecasting job is treated as a univariate multi-step time series prediction problem. Among the assessed models are a gradient boosting machine using eXtreme Gradient Boosting (XGBoost), a Neural Basis Expansion Analysis (N-BEATS) architecture, a Long-Short Term Memory (LSTM) network with multi-head attention, and a WaveNet-based convolutional neural network. Extensive hyperparameter tuning was done to find the most efficient forecasting method. Across all deep learning models, parameters including window size, dilation rates, kernel sizes, number of filters, dropout ratios, and dense layer units were varied and tested. XGBoost was optimized for the machine learning method using multi-output regression, lag-based features, and data normalization. Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), and Coefficient of Determination (R^2) metrics across all forecast horizons were used to assess each model. Experimental findings show that across almost all forecast horizons, the XGBoost model consistently beat all other architectures, attaining the lowest MAE and RMSE and the highest R^2. Especially when backed by engineered lag features and appropriate data scaling, this emphasizes the power of gradient boosting in capturing short-term temporal dependencies in structured, univariate time series data.

Keywords: *Wind power forecasting, deep learning, time series, multi-step forecasting, Karaburun*

A Systematic Review of Big Data Analysis Methods and Application Areas (1228)

Muhammed Cağrı Aksu[1], Yunus Emre İgaç[2*]

[1] Artvin Çoruh University, Faculty of Engineering, Department of Computer Engineering, 08000, Artvin, Türkiye
[2] Artvin Çoruh University, Graduate School of Education, Department of Statistics, 08000, Artvin, Türkiye

*Corresponding author e-mail: cagriaksu@artvin.edu.tr

Abstract

Big data analytics offers significant opportunities for businesses and researchers in today's digital age. The aim of this study is to systematically review recent academic publications on big data analysis methods and their application domains in order to provide an overview of the current state of the field. A comprehensive literature review was conducted following the PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analyses) guidelines. Based on predefined selection criteria, 200 publications from 2019 to 2025 were analyzed, examining the big data analytics methods employed, the sectors of application, and the research gaps noted in the literature. The findings indicate that the most commonly used big data analysis methods are data mining, machine learning (especially deep learning), time series analysis, and big data frameworks (such as Hadoop and Spark). Big data analytics is most frequently applied in sectors such as manufacturing, healthcare, environment, energy, and finance. However, the literature reveals significant research gaps in the context of big data analytics, particularly regarding data security and privacy, data integration, and data quality. Additionally, areas such as ethics, real-time analytics, and the integration of the Internet of Things (IoT) with big data require further investigation. This systematic review contributes to the academic literature by uncovering current trends and gaps in the field, while for practitioners it provides guidance by highlighting which big data methods are prominent in different industries and which areas require improvement. In conclusion, big data analytics is a rapidly evolving field with applications across a wide range of domains, and addressing the identified gaps is crucial for the advancement of both research and practice.

Keywords: *Big Data, Data Mining, Machine Learning, Application Areas, Systematic Revıew*

Statistics of the Collection of Tax Arrears in Romania in the Post-Accession Period to the European Union (1229)

Marius Fandly[1]*

[1]National Agency of Fiscal Administration, Bihor County Administration of Public Finances Oradea, Romania

Corresponding author e-mail: fandlymarius@yahoo.com

Abstract

Between 2008 and 2024, Romania experienced significant fluctuations in collecting tax arrears, generated by economic crises, fiscal reforms, and digitalization efforts. Although this period was characterized by many challenges, strategic fiscal measures helped Romania make significant strides in improving tax collection efficiency and addressing outstanding debts. In the first period (2008–2015), global financial crisis and domestic economic instability led to a decrease in tax revenues. In 2015, the National Agency for Fiscal Administration (ANAF) reported an accumulated stock of tax arrears owed by large taxpayers of RON 5.9 billion, representing 12.8% of the Large Taxpayer Office's total collections. The overall national tax debt amounted RON 19.25 billion, meaning 20% of collections. In the years 2016–2019, ANAF implemented several digitalization measures aiming to reduce VAT fraud and improve tax collection efficiency. These efforts contributed to enhanced voluntary compliance and more effective tax administration. The last five years (2020–2024) were a real success story. In July 2024, ANAF achieved a historic revenue collection of over RON 50 billion, attributed to intensified fiscal controls and digitalization. To further address arrears, the Government introduced a tax amnesty, aiming to collect overdue debts, which targeted RON 71.8 billion in arrears, with expectations of recovering 10–15% of this amount. By November 2024, taxpayers who settled their principal debts were eligible for the cancellation of interest and penalties. My paper presents the evolution of this process in terms of numbers, highlighting the economic factors underlying it.

Keywords: *Romania, Taxation, Arrears, Fiscal Reform.*

Factors Influencing Students' Acceptance and Use of Generative Artificial Intelligence in Higher Education (1230)

Merve Yıldız[1*]

[1]Karadeniz Technical University, Faculty of Economics and Administrative Sciences, Department of Management Information Systems, Trabzon/Türkiye

*Corresponding author e-mail: merve.yildiz@ktu.edu.tr

Abstract

This research focuses on examining the factors influencing the acceptance and utilization of GAI among university students. In this context, the research is based on the Unified Theory of Acceptance and Use of Technology (UTAUT) model, which provides a strong theoretical framework by integrating elements from multiple theories and is widely applied in educational contexts to understand technology adoption behaviors. In the study, the key constructs (performance expectancy, effort expectancy, social influence, and facilitating conditions) in the UTATUT model on students' behavioral intentions and usage behaviors regarding GAI were addressed. Collected from 238 students enrolled in various discipline and using GAI tools for learning purposes, the self-reported data was analyzed using the Partial-Least Squares Structural Equation Modeling (PLS-SEM) method to confirm the model's validity and reliability. The results indicated that performance expectancy, effort expectancy, and social influence significantly influenced behavioral intention, and effort expectancy predicted university students' behavioral intention to use GAI with the highest relative weight. Also, facilitating conditions and behavioral intention were found to have significant impact on actual use behavior. Based on the iteration of the UTAUT model, this research provides theoretical implications regarding the critical factors influencing the acceptance and integration of GAI in higher education. These implications are considered important in guiding policymakers, educators, and researchers to develop strategies for GAI practical applications.

Keywords: *Generative artificial intelligence, UTAUT, Technology acceptance, PLS-SEM, Higher education*

Examination of Factors Affecting First Calving Age in Some Dairy Cattle Breeds Using Regression Tree Analysis (1231)

Samet Hasan Abacı[1*], Ertugrul Kul[2], Ayla Sevim Satılmış[3], Ali Ayhan Barut[4]

[1]Ondokuz Mayis University, Faculty of Agriculture, Department of Animal Science, Samsun, Türkiye
[2]Kırşehir Ahi Evran University, Faculty of Agriculture, Department of Animal Science, Kırşehir, Türkiye
[3]Kırşehir Ahi Evran University, Institute of Science, Animal Science Department, Kırşehir, Türkiye
[4]Samsun Animal Science Association, Samsun, Türkiye

*Corresponding author e-mail: samet.abaci@omu.edu.tr

Abstract

The aim of this study is to determine the effects of some factors on the first calving age (FCA) of 73674 dairy cows of breeds including Simmental, Holstein, Brown Swiss, and their crossbreds raised in Kastamonu province, using regression tree analysis. The study utilized records of 41914 first-lactation Simmental, 15489 Simmental crossbreds, 8561 Holstein, 976 Holstein crossbreds, 4787 Brown Swiss, and 1947 Brown Swiss crossbred cows. The effects of breed, calving year (2009–2022), and calving season (spring, summer, autumn, winter) on FCA were examined. The data analysis was performed using the CHAID algorithm.

In the regression tree analysis, breed was identified as the most important factor influencing FCA, and it was divided into four subgroups. These subgroups consist of Simmental, Holstein, Simmental and Brown Swiss crossbreds, and Brown Swiss and Holstein crossbreds. Additionally, the effects of calving year were significant within these groups.

For the Simmental breed, the influence of calving year was divided into 13 subgroups, with the FCA of cows calving in 2009 being 847 days, and those calving in 2021 being 924 days. In the Holstein breed, calving year was divided into 10 subgroups, with FCA of 826 days in 2009 and 729 days in 2020. For Simmental and Brown Swiss crossbreds, calving year effects were divided into 8 subgroups, and FCA after 2020 was 912 days. For Brown Swiss and Holstein crossbreds, calving year was divided into 7 subgroups, and cows calving after 2017 had an FCA of 916 days. Additionally, seasonal effects were found to be significant in some years. The CHAID analysis showed that the most important factor was breed, followed by calving year and season. Significant differences in FCA were observed across different breeds and years. The results indicate that all examined factors have significant effects on FCA.

Keywords: *Simmental, Holstein, Brown Swiss, First calving age*

The Impact of Climate Variables on Wheat Yield in Turkey (1232)

Melike Tekin[1*], Gülistan Erdal[1]

[1]Tokat Gaziosmanpaşa Üniversitesi, Ziraat Fakültesi, Tarım Ekonomisi Bölümü, Türkiye

*Corresponding author e-mail: melike.tekin6215@gop.edu.tr

Abstract

In recent years, global warming and climate change have been affecting agricultural production both directly and indirectly. Rising temperatures, changing precipitation patterns, and extreme weather events have significant impacts on the yield of many crops. In Turkey, which is located in a semi-arid climate zone, this variability becomes even more pronounced. Staple cereal crops such as wheat are particularly sensitive to fluctuations in temperature and precipitation, and climatic anomalies occurring during critical growth stages can lead to substantial yield losses.This study aims to examine the effects of climate variables on wheat yield in Turkey. The dataset covers the period from 2004 to 2024. Wheat yield (Y, kg/ha) was used as the dependent variable, while climate-related independent variables (Xi) included temperature (°C), average precipitation (mm), the Southern Oscillation Index (SOI), and the Oceanic Niño Index (ONI). The SOI and ONI indicators employed in the study provide important parameters for understanding the impact of large-scale climatic oscillations on crop yields. In this context, ocean-atmosphere interactions such as the El Niño–Southern Oscillation (ENSO), which operate on a global scale, create significant regional and temporal variability in agricultural production.The results of the multiple linear regression analysis show that the model is statistically significant, with a high coefficient of determination (R^2 = 64.3%). Among the climate variables, temperature and SOI were found to be statistically significant. In particular, SOI was determined to have a strong and positive effect on wheat yield. The analysis results reveal that climate indicators play a critical role in wheat productivity. The findings of this study offer valuable insights for the development of sustainable agricultural policies and the formulation of adaptation strategies in response to climate change.

Keywords: *Wheat yield, climate variables, regression analysis, Southern Oscillation Index (SOI), Oceanic Niño Index (ONI).*

Improving the Efficiency of Tax Collection Mechanisms by Using Artificial İntelligence-based Tools (1234)

Marius Fandly[1], Zoltan Zakota[2*]

[1]National Agency of Fiscal Administration, Bihor County Administration of Public Finances Oradea, Romania

[2]University of Debrecen, Hungary

*Corresponding author e-mail: zzakota@gmail.com

Abstract

There are many ways to improve the efficiency of tax collection mechanisms using artificial intelligence (AI)-based tools. Because it can process vast amounts of data, recognize patterns, and make predictions, it can significantly improve the accuracy and speed of collection processes. Its algorithms can analyse past tax returns, transaction data, social media activity, and other relevant information to identify high-risk taxpayers who are more likely to attempt tax fraud. They can automate the process of reviewing tax returns, comparing them with other available data, and highlighting potential discrepancies or irregularities. Algorithms can contribute to a better understanding of taxpayer behaviour and willingness to pay, and through data analysis, identify factors that influence payment morale, thereby developing more personalized and effective collection strategies. AI-based chatbots and virtual assistants can answer taxpayers' frequently asked questions, send reminders about payment deadlines, and help with payment plans, thereby reducing the customer service burden on tax authorities and improving taxpayer satisfaction. At the same time, they can process and analyse tax collection data faster and more efficiently, creating more detailed reports and statistics that help managers make better decisions and optimize processes. AI-based tools are already being used worldwide in various areas, such as risk assessment and fraud detection, automated customer service and information, or automated tax filing and data processing. The aim of our study is to present, based on the available literature, the industry and the most well-known such tools, the experiences gained so far in this field, and the expected trends.

Keywords: *Artificial İntelligence, Taxation, Collection Efficiency, Fiscal Reform.*

Effective Classification of Phishing Data Using Regularized Extreme Learning Machines (1236)

Ömer Özbilen[1*], Murat Genç[2]

[1]Mersin University, Faculty of Education, Department of Primary Mathematics Teaching, Türkiye
[2]Tarsus University, Faculty of Economics and Administrative Sciences, Department of Management Information Systems, Türkiye

*Corresponding author e-mail: ozbilen@mersin.edu.tr

Abstract

Phishing attacks pose a significant threat to online security, necessitating robust detection methods. This study comprehensively evaluates the performance of regularized Extreme Learning Machine (ELM) models for identifying phishing websites. Utilizing a dataset of 11,054 samples with 30 features, we compared standard ELM with Ridge, Lasso, and Elastic Net regularization approaches. The models were tested across sin, tansig, poslin, leaky_relu, and elu activation functions, with hidden neuron counts of 256, 512, 1024, 2048 and 4096. Performance was assessed using 5-fold cross-validation, calculating metrics such as accuracy, sensitivity, specificity, precision, and F1 score. Majority voting across all ELM models achieved an impressive accuracy of 96.76% with 4096 neurons. Furthermore, majority voting with the top three performing models yielded an accuracy of 96.67% and an AUC of 0.966, indicating excellent discriminative capability. ROC curve analysis further confirmed the superior classification performance of the best models. These findings demonstrate that regularized ELM models offer a fast, reliable, and high-performing solution for phishing detection. This work contributes to the advancement of machine learning-based security systems for combating phishing threats.

Keywords: *Phishing Detection, Extreme Learning Machine, Regularization, Majority Voting, Machine Learning*

A Snapshot of Bibliometric Research in Emergency Medicine: Trends, Collaborations, and Research Impact (1237)

Atilla Özdemir*[1,3] Merve Yazla[2,3], Onur Toka [3]

[1]Süleyman Demirel University, Faculty of Education, Department of Mathematics and Science Education, Isparta, Türkiye
[2]Etlik City Hospital, Emergency Medicine, Ankara, Türkiye
[3]Hacettepe University, Department of Statistics, Ankara, Türkiye

*Corresponding author e-mail: atillaozdemir@sdu.edu.tr

Abstract

In recent years, bibliometric analysis has emerged as a vital tool for understanding the intellectual and collaborative structure of emergency medicine research. This study aims to evaluate bibliometric studies conducted in the emergency medicine domain, using the R-based bibliometrix package for comprehensive data visualization. A total of 97 articles were initially retrieved from the Web of Science Core Collection using a combination of controlled terms related to "emergency medicine" and "bibliometric analysis." After applying inclusion criteria and eliminating records with missing metadata, 60 eligible articles were included. These articles were analyzed through descriptive indicators (annual production, authorship, source impact), scientific collaboration metrics (co-authorship, country-level networks), and conceptual structures (keyword co-occurrence and clustering). Findings revealed a steady increase in bibliometric publications post-2015, with core contributions emerging from the USA, China, and Spain. Journals such as the American Journal of Emergency Medicine and Emergencias were identified as Bradford Zone 1 sources. Author productivity adhered to Lotka's Law, with most contributors having only one publication. Co-word analysis identified prominent concepts such as "h-index," "scientometrics," and "visualization," while trend topic mapping showed a methodological shift from citation metrics to semantic evaluations. Network and historiographic analyses revealed growing collaborative clusters, especially among European and Asian institutions. Visualization tools like network plots, conceptual maps, and timeline graphs were generated using bibliometrix and ggplot2. The study's findings underscore the growing sophistication of bibliometric methodologies in emergency medicine and their value in shaping policy and guiding future research efforts.

Keywords: *Emergency Medicine, Bibliometric Analysis, Scientific Visualization, Research Collaboration, Publication Trends*

Enhancement of the Lee-Carter Model with Machine Learning (1238)

İremsu Menekse[1], Başak Bulut Karageyik[2*],

[1]Hacettepe University, Graduate School of Science and Engineering, Department of Actuarial Science, Türkiye
[2] Hacettepe University, Faculty of Science, Department of Actuarial Science, Türkiye

*Corresponding author e-mail: iremsumenekse@hacettepe.edu.tr

Abstract

Mortality modeling plays a fundamental role in actuarial science, particularly in forecasting future insurance liabilities, determining life insurance premiums, and projecting social security expenditures each of which is essential for the long-term financial sustainability of both public and private institutions. Among the many models developed for mortality forecasting, the Lee–Carter model remains one of the most widely applied due to its interpretable structure and ability to represent age-specific mortality trends over time. However, its rigid parametric formulation often falls short in capturing nonlinear dynamics, sudden demographic shifts, and high variability especially in older age groups or within smaller populations.

To address these limitations, this study proposes a hybrid framework that integrates traditional mortality modeling with machine learning (ML) techniques. Specifically, mortality data disaggregated by age and gender from the Turkish Statistical Institute (TURKSTAT) are used to estimate and forecast mortality rates. The initial projections generated by the Lee–Carter model are refined by modeling and correcting the residuals using tree-based ML algorithms, including Random Forest, LightGBM, XGBoost, and RPART.

Model performance is evaluated using standard error metrics. Results show that the hybrid approach significantly improves predictive accuracy. By leveraging the strengths of both parametric and nonparametric methods, this methodology offers a more robust and flexible framework for mortality forecasting, with practical implications for insurance pricing, reserve setting, and public policy planning.

Keywords: *Mortality Modelling, Lee-Carter Model, Machine Learning Algorithms*

An Turkey's Biodiesel Potential and the Economics of Related Agricultural Products (1239)

Berra Kocakoç[1]*, Prof. Dr. Gülistan Erdal[1]

[1]Tokat Gaziosmanpaşa University, Faculty of Agriculture, Departmant of Agricultural Economics, Turkey

*Corresponding author e-mail: berra.kocakoc6620@gop.edu.tr

Abstract

With the rapid increase in the world population and the intensification of industrialization, the need for energy to sustain human life is also increasing day by day. As the use of energy resources has reached a significant level today, environmental pollution, greenhouse gas emissions, and the resulting climate change, along with the limited capacity of raw materials and energy sources, have led people to seek alternative energy sources.

Renewable energy sources are becoming increasingly important due to their low cost, minimal environmental impact, and very low levels of greenhouse gas emissions. These renewable energy sources include solar, wind, geothermal, and biomass energy.

Biomass is an energy source derived from all natural materials of non-fossil origin, including plant- and animal-based matter. The most important characteristics of biomass are its environmental friendliness and sustainability.

Biodiesel is a biomass-derived energy source in liquid form. It is obtained through processing oils derived from oilseed crops as well as used cooking oils.

This study examines the production quantities and cultivation areas of vegetable oils used in biodiesel production, along with the import and export statistics of these products in Turkey. Based on these data, the potential of biodiesel production and its evaluation have been assessed.

Keywords: *Biodiesel, Oilseed Crops, Biomass Energy*

Designing a Composite Internal Migration Index for Actuarial Modeling (1240)

Birgül Özkan[1], Doğanur Acınan[1], Başak Bulut Karageyik[1]*

[1]Hacettepe University, Faculty of Science, Department of Actuarial Sciences, Ankara, Türkiye

*Corresponding author e-mail: basakbulut@hacettepe.edu.tr

Abstract

This study investigates the actuarial implications of internal migration within the broader context of catastrophic risk insurance. In recent years, the increasing frequency and severity of catastrophic events—such as earthquakes, floods, and extreme weather—have emphasized the need for more dynamic and socially responsive models in insurance risk assessment. Internal migration, a persistent demographic phenomenon in Turkey, significantly reshapes regional population structures, thereby influencing local vulnerability, exposure levels, and insurance demand.

To capture these complex interactions, the study develops a composite index that integrates internal migration statistics with a range of macroeconomic and environmental indicators. These include energy consumption, gross domestic product (GDP), consumer price index (CPI), housing price index, and air pollution metrics (PM2.5, PM10, SO_2, NO_2, NO, and CO etc.). The proposed index aims to provide a statistically meaningful and multidimensional representation of regional risk levels that can be utilized in actuarial modeling and catastrophe risk forecasting.

Generalized linear models (GLMs) are employed to analyze the relationships between the composite index and key outputs. Through this modeling framework, the study evaluates how individual indicators contribute to risk variation and quantifies their influence on insurance-based financial outcomes.

The empirical analysis focuses on data from the province of Ankara, providing a practical case study in a high-migration urban setting. The modeling framework utilizes observed data from the years 2010 to 2024, enabling a real-world evaluation of the proposed index. The findings provide a valuable tool to assess not only the impact of internal migration but also the combined influence of key economic and environmental factors, thereby enhancing actuarial prediction capabilities and supporting data-driven decision-making in risk analysis.

***Keywords*:** *Internal Migration, Catastrophic Risk Insurance, Composite Index, Generalized Linear Models (GLM), Actuarial Modeling,*

Moving Beyond Significance: Understanding and Reporting Effect Sizes in Applied Linguistics Research (1241)

Öznur Semiz[1*]

[1]Karadeniz Technical University, Faculty of Letters, Department of English Language and Literature, Turkey

*Corresponding author e-mail: oznursemiz@ktu.edu.tr

Abstract

In applied linguistics research, the overreliance on p-values as the primary indicator of "significant" results often overshadows the importance of effect size, which provides a measure of the magnitude of the difference between groups or association between variables. Recent methodological critiques within the social sciences—and increasingly in applied linguistics research—have called for a shift beyond simplistic, dichotomous thinking around statistical outcomes, a movement referred to as a 'methodological turn' and characterized by heightened 'methodological awareness'. This presentation critically examines the limitations of statistical significance as the primary method for determining the meaningfulness of results and advocates for the inclusion of alternative or complementary approaches, such as effect size interpretation, confidence intervals, and the consideration of practical significance. Reporting effect sizes alongside p-values enhances both the transparency and practical relevance of research findings. Recognizing the practical importance of effect size enhances the interpretability of findings and enables meaningful comparisons across studies in applied linguistics. This presentation advocates for an approach to statistical analysis that balances methodological precision with practical relevance, urging applied linguists to look beyond p-values by incorporating measures of effect size and confidence intervals, reported alongside significance testing, to yield more meaningful interpretations of research findings.

Keywords: *Statistical significance, Effect size, Methodological turn, Applied Linguistics.*

Investigation of Fat Mass and Obesity-Associated (FTO) Gene rs9939609 Polymorphism in Indviduals with Type 2 Diabetes (1243)

Sara Guliyeva[1], Meryem Merve Ören Çelik[2], Mustafa Altınkaynak[3], Gülşah Yenidünya Yalın[3], Ayşe Kubat Üzüm[3], İlhan Satman[3], Deniz Kanca Demirci[1]

[1] Haliç Üniversitesi, Fen-Edebiyat Fakültesi, Moleküler Biyoloji ve Genetik Bölümü, İstanbul, Türkiye
[2] İstanbul University, Istanbul Faculty of Medicine, Department of Internal Medicine, Division of Public Health
[3]İstanbul University, Istanbul Faculty of Medicine, Department of Internal Medicine, Türkiye

*Corresponding author e-mail: denizkanca@halic.edu.tr

Abstract

Diabetes mellitus, characterized by hyperglycemia, obesity and dyslipidemia, is a chronic metabolic disease caused by reduced/insufficient insulin action/signalling and insulin resistance. T2DM arised from impaired glucose homeostasis, β-cell dysfunction and insulin resistance in addition to genetic and environmental factors. Variants of the Fat Mass and Obesity Associated-(FTO) gene is associated with diabetes and obesity by regulating energy balance, food intake and energy expenditure.

Therefore, we aimed to observe the effect of the FTO rs9939609-(T>A) gene variant on the development of T2DM and its relationship with obesity parameters. With this aim, after DNA isolation, FTO-rs9939609-SNP was analyzed with SNP-probe assay. Genetic findings were statistically evaluated with Student's T-test, ANOVA and regression analysis to correlate with clinical and biochemical parameters of the study groups.

When the clinical and biochemical findings of the study groups were examined, age($p<0.001$), waist-circumference($p<0.001$), pulse-pressure($p=0.054$), pulse($p<0.017$), HbA1c($p<0.001$), creatinine($p<0.010$), vitamin-B12($p<0.018$), vitamin-D($p<0.001$), calcium($p<0.001$) and phosphorus($p<0.001$) were higher in patients compared to controls, while E-GFR($p<0.001$), LDL-cholesterol($p<0.001$) and HDL-cholesterol($p<0.010$) were higher in controls compared to patients. In the regression analysis performed according to genotypes, folic acid, LDL-Cholesterol, triglyceride and phosphorus were found to be associated with disease pathogenesis.

When the effect of genotype distribution on diagnostic parameters was examined in the patient group, vitamin B12($p=0.018$), folic acid($p=0.026$), triglyceride($p=0.037$), AST($p=0.058$) and albumin($p=0.005$) levels were found to be associated with TA, TA, AA, TT and AA genotypes, respectively, suggesting that the minor-A allele may have an effect on the development of the disease. In the control group, age($p=0.022$), hip-circumference($p=0.008$), triglyceride($p=0.006$) and TSH($p<0.001$) levels were found to be associated with TA genotypes and high, while the T allele was evaluated as protective.

These findings indicate that FTO-rs9939609 SNP is effective on obesity related parameters in patients with T2DM. However, the study can be repeated in larger sample groups to increase statistical significance.

***Key Words**: T2DM, obesity, FTO, rs9939609, polymorphism.*

Bayesian Estimation under Different Loss Functions of $P(X > Y)$ for the Topp-Leone Distribution (1244)

Asuman Yılmaz[1*]

[1]Van Yüzüncü Yıl University, Faculty of Economics and Administrative Sciences, Department of Econometrics, 65080 Van, TURKEY

*Corresponding author e-mail:asumanduva.@yyu.edu.tr

Abstract

The reliability of a system is commonly evaluated using the stress-strength model, which is expressed as $R = P(X > Y)$. In this context, the parameter R represents the reliability parameter. Also, X and Y are independent random variables. Y is the random stress, and X is the random strength. In this study, we examined the Bayesian parameter estimation of $P(X > Y)$ under different loss functions based on the Topp Leone distribution. Bayesian inference is an attractive framework in estimation problems and has attracted great attention in recent years. Also, in Bayesian parameter estimation, the choice of loss function plays an important role. Therefore, we used three different loss functions. For the Bayesian computation, Lindley and Gibbs sampling methods are considered. A Monte Carlo Simulation study is carried out to compare the performance of the various estimation methods developed in this study.

Keywords: *Topp-Leone distribution, Bayesian inference, Loss function, Monte Carlo simulation study.*

Examining The Impact Of Twitter Sentiment On Bitcoin Price Movements (1245)

Musab Talha Akpınar[1*], Muhammed Fatih Özer[1]

[1]Ankara Yıldırım Beyazıt University, Ankara, Turkey

Corresponding author e-mail: mtakpinar@aybu.edu.tr

Abstract

Cryptocurrency markets, characterized by high volatility and rapid price fluctuations, are particularly sensitive to shifts in investor sentiment. Social media platforms like Twitter play a crucial role in shaping market perceptions, as they provide real-time insights into the collective emotions and expectations of investors. This study aims to investigate the influence of Twitter sentiment on Bitcoin price movements, focusing on how changes in online sentiment correlate with short-term market trends. To achieve this, Bitcoin-related tweets collected over a specified time period will be analyzed using natural language processing (NLP) techniques to categorize them as positive, negative, or neutral. Sentiment analysis will be conducted using advanced language models, such as FinBERT, which is fine- tuned for financial contexts, or GPT-4, known for its broad contextual understanding and linguistic accuracy. These sentiment scores will then be aligned with historical price data to assess the impact of sentiment fluctuations on Bitcoin price movements.The effectiveness of the proposed approach will be evaluated using standard classification metrics, including accuracy, precision, recall, and F1-score. Additionally, the study will explore the effects of sudden sentiment shifts on short-term price volatility, providing valuable insights for traders and market analysts.This research seeks to offer a more comprehensive understanding of the dynamic relationship between social media sentiment and cryptocurrency price behavior, potentially enhancing investment decision-making in rapidly changing markets.

Keywords: *Natural Language Processing, Cryptocurrency,, Sentiment Analysis, GPT-4, Twitter Data*

Price Forecasting of ELUS Stock Data Using Machine Learning and Time Series Methods (1249)

Harun Benli[1*], Peri Güneş[2]

[1,2] R&D Department of İnfina Software Development, Istanbul, Turkey
[2]İstanbul Gelişim University, Faculty of Engineering, Computer Department, Turkey
[1]Middle East Technical University, Faculty of Engineering, Industrial Engineering Department, Turkey

*Corresponding author e-mail: hbenli@infina.com.tr

Abstract

In today's digital and data-driven economy, the agricultural sector is swiftly transforming, directly impacting production, financial integration, and price discovery mechanisms. Barley, an essential commodity influencing various industries including feed manufacturing and beverage production, is particularly significant for precise price forecasting. This study focuses on Turkey's ELÜS (Elektronik Ürün Senedi) system within TÜRİB (Türkiye Ürün İhtisas Borsası), where barley is securely stored in licensed warehouses and electronically traded.

By evaluating and comparing time series and machine learning models, this research examines how data-driven methodologies can enhance barley price predictability, thereby supporting producers and investors in cost planning, hedging strategies, and financial risk management. The findings aim to increase market depth and strengthen stakeholder competitiveness within the barley supply chain, emphasizing the agriculture-finance nexus as a critical driver of economic value creation.

Through detailed analysis of national-level barley ELÜS data, this study highlights the importance of accurate, transparent pricing mechanisms in bolstering rural economies, optimizing resource allocation, and accelerating agriculture's digital transformation. Insights derived from the research provide valuable implications for agricultural commodities broadly, offering guidance on best practices, informed policy development, and innovative market structures. Ultimately, this work underscores the potential of digital financial integration to transform agricultural markets, improve decision-making capabilities, and ensure sustainable economic growth.

Keywords: *Stock Data, Elüs, Türib, Prıce Forecastıng, Agrıculture*

Performance Evaluation of Supervised Machine Learning Algorithms for Classification on Credit Risk Data (1252)

Talha Ceylan[1*], Abdullah Yalçınkaya[2]

[1, 2]Ankara University, Faculty of Science, Department of Statistics, Türkiye

*Corresponding author e-mail: talhacey@gmail.com

Abstract

In this study, we have analyzed credit risk data containing some features of cardholders from a major bank in Taiwan from 2005. This dataset includes 24 characteristics, such as payment information, demographic factors, credit information, bill statements, etc., of 30,000 card clients. The aim of the study is to bring about accurate predictive models by analyzing the variables that determine whether a cardholder is classified as credible or non-credible based on their risk factors. To do this, we have utilized supervised machine learning algorithms, such as Logistic Regression, Nearest Neighbors, Support Vector Machines, Decision Trees, Random Forest, AdaBoost, Gradient Boosting, Naive Bayes, etc. First, after the data pre-processing, we have used feature selection methods to solve problems such as multicollinearity in the data. Then, we have randomly divided the data into two parts. The first and second parts have been used for training the model and testing the prediction success, respectively. We have obtained fitted models according to supervised machine learning algorithms. For classification algorithms, hyperparameter values that give the best accuracy value have been identified by using the Grid Search technique. Also, we have used the K-fold Cross Validation procedure to evaluate the modeling performance of the algorithms. Finally, we have compared the models in terms of the classification accuracy measures.

Keywords: *Data Science, Machine Learning, Financial Data, Classification, Accuracy*

Piecewise Regression Analysis of Healthcare Access: Evaluating Breakpoints in Türkiye's Universal Health Coverage System (1253)

Mehtap Cakmak Barsbay[1]

[1]Ankara University, Faculty of Health Sciences, Healthcare Management Department, Türkiye

*Corresponding author e-mail: mehtapcakmak@gmail.com

Abstract

Due to policy changes, seasonal variations and structural healthcare reforms, actual healthcare expenditures may have non-linear trends and standard linear models may be insufficient to describe these variations. The aim of this study is to apply piecewise regression analysis to examine the relationship between time and hospital billing in the Turkish healthcare system. Breakpoints where significant changes occur in cost patterns are analysed using piecewise regression. These changes may be linked to government funding adjustments, changes in reimbursement models, patient admission trends, or pricing regulations.

The findings emphasize the importance of adaptive financial planning for hospitals, especially within Turkey's universal health insurance system. This research contributes to health management strategies by helping policy makers and hospital managers to improve budget forecasting, pricing mechanisms and patient care resource allocation. By integrating data-driven policy insights, this approach can contribute to transparency and sustainability in hospital spending across Türkiye.

Keywords: *Piecewise regression, healthcare utilization, universal health coverage.*

A Bibliometric Analysis of Publications in the Field of Statistics: A Comparative Study of Turkiye and the World (1257)

Fırat Çınar[1*]

[1]Mersin University, Faculty of Engineering, Department of Food Engineering, Turkiye

*Corresponding author e-mail: firatcinar@mersin.edu.tr

Abstract

Bibliometric analysis is a quantitative method used to assess and measure the impact and relevance of scientific research publications. Additionally, research in the field of statistics in Turkiye is compared and interpreted with the global situation in the same field. Bibliometric analyses were performed by extracting the metadata of the publications from the WoS database using VOSviewer and Rstudio (bibliometrix package) programs. According to the word clouds obtained as a result of the co-occurrence analysis of the keywords used by the authors in their publications, the prominent topics in the field of Statistics in Turkiye are "estimator", "multicollinearity", "maximum likelihood", while "markov chain monte carlo", "causal inference" and "maximum likelihood" are the most frequently used keywords in the world. In addition, when we look at the network maps obtained; there is a high relationship between the keyword pairs "causal inference-machine learning", "bayesian estimation-max.likelihood estimation". Looking at the thematic maps obtained, the terms that are driving the field in Turkiye are "linear regression", "computational approach test", and "robustness", while in the rest of the world they are "bayesian inference" and "clustering". While the keywords "monte carlo", "average run length", "bias" and "multicollinearity" constitute niche themes globally, "clustering", "linear mixed model" and "blup" are the most prominent niche themes in Turkiye. In the countries analysis unit in the co-authorship analysis type, among the top 5000 most cited publications in the world in the last 5 years, the top 5 countries that contributed the most to the "Statistics & Probability" disciplines were the USA, China, England, Canada and Germany, while Turkiye ranked 20th among 97 countries in this field. Turkiye has bilateral relations with 47 different countries, with the strongest bilateral ties with the USA. In the last 5 years, the most cited institutions in the world have been Stanford, Pennsylvania and Oxford universities. Ranked from highest to lowest, the top five universities from Turkiye on this list are Hacettepe, Cankırı Karatekin, Ankara, Cukurova and Marmara Universities. In addition, prominent academic journals and authors in the field of statistics were identified.

Keywords: *Statistics, Bibliometric Analyses, VOSviewer, Rstudio*

Machine Learning-Based Management of Loan Delinquency Stage (1258)

Şahin Nicat[1*], Buse Sena Vardar[1]

[1]Koçfinans, Research and Development, Turkiye

*Corresponding author e-mail: snicat@kocfinans.com.tr

Abstract

Accurately predicting the stages of credit delinquency is crucial for early risk detection and effective management of collection processes in the banking and finance sector. This paper presents a machine learning-based approach to estimate the probability of delinquent loans transitioning to a higher delinquency bucket. The proposed model is trained on features derived from various data sources, including payment behavior, credit history, customer demographics, and loan details. These features are carefully selected through a combination of statistical analysis and domain expertise to ensure their predictive power.

The model utilizes advanced machine learning algorithms, including LightGBM, to capture complex patterns in customer behavior and loan characteristics. Extensive data preprocessing is applied, including handling missing values, scaling, and feature engineering, to enhance model accuracy. Feature importance is evaluated using SHAP values, highlighting critical factors influencing the predictions.

The model's performance is evaluated using standard metrics such as accuracy, precision, recall, and AUC-ROC, demonstrating its effectiveness in predicting delinquency transitions. Additionally, a comparative analysis with traditional statistical methods reveals the superiority of the machine learning approach in handling complex relationships and non-linear patterns.

This approach provides significant advantages in dynamically assessing credit risk, developing early warning systems, and improving risk management strategies. Financial institutions can use the model's predictions to optimize collection strategies, reduce losses, enhance customer management, and allocate resources more efficiently.

Keywords: *Financial Risk Analysis, Risk Management, Credit Risk Prediction, Machine Learning, Credit Delinquency*

A Generalized Counting Process For Risk Theory: The Pólya-aeppli Process (1260)

Derya Çalışkan[1], Mustafa Hilmi Pekalp[2*]

[1]Ankara Üniversitesi, Uygulamalı Bilimler Fakültesi, Aktüerya Bilimleri, Türkiye
[2]Ankara Üniversitesi, Uygulamalı Bilimler Fakültesi, Aktüerya Bilimleri, Türkiye

*Corresponding author e-mail: deryacaliskan18@gmail.com

Abstract

This study introduces the Pólya-Aeppli process as a generalization of the classical homogeneous Poisson process and analyzes it within the context of risk theory. The investigation begins by constructing a risk model where the number of claims is modeled using the Pólya-Aeppli counting process. This model offers a more flexible framework compared to traditional risk models by incorporating non-independent increments and a parameter that controls clustering. Notably, the model accounts for the possibility of multiple simultaneous claims. The Pólya-Aeppli distribution and its associated renewal process are thoroughly defined, with key properties such as Laplace-Stieltjes transforms and expected values derived. The resulting theoretical foundation serves as a basis for examining ruin probabilities in non- classical insurance models.

Keywords: *Pólya-Aeppli, Risk Model, Counting Process, Delayed Renewal Process*

Changing Audience Preferences in The Film Industry With The Pandemic (1261)

Aybala Hilal Yüksel[1*], Nihat Gümüş[2]

[1]Ibn Haldun University, School of Graduate Studies, Big Data and Business Analytics, Türkiye
[2]Dr. Öğr. Üyesi, Ibn Haldun University, School of Business, Department of Management, Türkiye

*Corresponding author e-mail: aybala.yuksel@stu.ihu.edu.tr

Abstract

This study examines the economic impacts of the COVID-19 pandemic on the global film industry and the shifts in audience preferences. Data from over 900 films released between 2007 and 2023 were analyzed, categorizing the films into three periods: Pre-Pandemic (2007–2019), Pandemic (2020–2021), and Post-Pandemic (2022–2023). The dependent variable used was the total global box office revenue as a ratio of production budget (Budget Recovered). The study not only explores the pandemic's impact on economic success but also investigates the structural changes within the film industry before and after the pandemic. Key variables included genre, budget level (low–medium–high), quality (critic score), and popularity (audience score). To analyze the distribution of these variables across the periods and their relation to economic performance, one-way ANOVA and Chi-Square tests were conducted. Where ANOVA results indicated significant differences, group comparisons were further examined using Tukey's HSD test. A literature review revealed that COVID-19 led to both supply- and demand-side shifts in the industry. Accordingly, the study focuses not only on economic indicators but also on audience behavior and production strategies. The suspension of content production, closure of cinemas, and the rise of digital platforms during the pandemic accelerated structural transformation in the industry, influencing film genres, budget preferences, and target audiences. Findings show that economic success significantly declined during the pandemic, with a partial recovery observed in the post-pandemic period. Notable changes in preferred film genres, budget levels, and quality-popularity ratings were also identified in the post-pandemic era. Adopting a comparative analysis approach based on statistical differences, the study offers significant insights into how external shocks like the pandemic affect content production, audience behavior, and economic outcomes.

Keywords: *Box Office, Film Industry, COVID-19 pandemic, Audience*

A Comparison of the Statistical Attitudes of University Students Studying in Ivory Coast and Those Studying in Turkey (1262)

Aıcha Kanoute[1*] Taner Tunç[2]

[1,2]Ondokuz Mayıs University Faculty of Sciences Department of Statistics

*Corresponding author e-mail: 22280554@stu.omu.edu.tr

Abstract

In this study, in addition to revealing the demographic characteristics of students studying at universities in both Türkiye and Ivory Coast, the statistical attitudes of undergraduate students studying at universities in Turkey and Ivory Coast were examined to reveal differences and similarities. In this context, in addition to demographic characteristics, the Statistical Attitude Scale developed by Tunç et al. (2014) was used in the questionnaire form prepared for the study. The inclusion of some demographic factors thought to affect statistical attitudes in the questionnaire form used in the study is also important in terms of revealing the different interactions of different cultures on the same subject, which is the aim of the study. According to the results of the study, statistical attitudes generally differed significantly by country in terms of demographic variables ($p<0.05$). The love/interest scores of female students in Türkiye are significantly higher than those of female students in Ivory Coast ($p<0.05$). The love/interest scores of students between the ages of 18-23 in Türkiye are significantly higher than those of students in Ivory Coast ($p<0.05$). The love/care scores of students in Türkiye whose mothers have secondary education are significantly higher than those in Ivory Coast ($p<0.05$). The love/care scores of students in Turkey whose fathers have undergraduate and associate degrees are significantly higher than those in Ivory Coast ($p<0.05$). No significant difference was found between the other levels of demographic variables ($p>0.05$).

Keywords: *Statistics Attitudes Scale (SAS), Explanatory and confirmatory factor analysis, statistical attitude, Ivory Coast, Benefit Importance factor*

Determining The Causal Relationship Between Egg Prices and Egg Feed Prices in Turkey by Toda-Yamamoto Test (1264)

Vahit Cem Tüzemen[1], Lütfi Bayyurt[2*]

[1]Atatürk University, Faculty of Agriculture, Department of Agricultural Economics, Türkiye
[2]Tokat Gaziosmanpasa University, Faculty of Agriculture, Department of Animal Science, Tokat, Turkey

*Corresponding author e-mail:lutfi.bayyurt@gop.edu.tr

Abstract

This study aims to determine the causality relationship between egg prices and egg feed prices in Turkey. For this purpose, it is aimed to determine the relationship between egg price (units/TL) and egg feed price (tons/TL) in Turkey between January 2016 and March 2025. In the study, unit root tests were used to test the stationarity of the series and it was determined that the series were stationary at first difference. In order to conduct the Toda-Yamamoto causality test, the appropriate VAR model was constructed by considering Schwarz (SC). The lag length of the VAR Model was re-determined as 1 lag length by considering the Schwarz Information Criterion (SC) and Hannan-Quinn Information Criterion (HQ). According to the results of the Toda-Yamamoto causality test, the wald test between feed prices and egg prices is 7.618144 and the probability value is 0.0222, while the wald test from egg prices to feed prices is 0.428747 and the probability value is 0.8070. According to all results, there is a causality relationship from egg prices to feed prices, while there is no causality from feed prices to egg prices. An increase or decrease in egg prices does not affect egg feed prices.

Keywords: *Toda-Yamamoto, Causality, Egg Prices, Egg Feed Prices*

Personalized Product Recommendations In Cosmetic E-commerce Using A Two-tower Model (1265)

Suat Tuncer[1], Ayşe Kızılay[2], Zeynep Şahin[2], Kübra Abalı[3], Büşra Tosun[2*], Jay Nimish Patel[3], Tuna Çakar[3*]

[1]MEF Üniversitesi, Bilişim Teknolojileri Yüksek Lisans Programı, İstanbul, Türkiye
[2]Eve Mağazacılık A.Ş., Ar-Ge Merkezi, İstanbul, Türkiye
[3]SST TEK A.Ş., Ar-Ge Merkezi, İstanbul, Türkiye
[4]MEF Üniversitesi, Bilgisayar Mühendisliği Bölümü, İstanbul, Türkiye

*Corresponding author e-mail: cakart@mef.edu.tr

Abstract

This study comparatively analyzes two-tower retrieval models for personalized product recommendations on e-commerce platforms within the cosmetics sector. Recommendation systems are critically important for enhancing user experience and engagement. Using the TensorFlow Recommenders (TFRS) framework, this thesis evaluates the performance of a basic embedding model based solely on user and product IDs against a more advanced model incorporating semantic content features such as product title and category. The models were trained on implicit purchase data and evaluated using the FactorizedTopK metric within a nearest neighbor approach. The research assumes a warm-start scenario, emphasizing the importance of semantic context in embedding spaces. Results showed that the model utilizing only ID-based embeddings performed effectively for frequently interacted products but showed limitations in recommending new or infrequently interacted items. Conversely, the inclusion of semantic features enabled the model to better generalize, particularly improving recommendation accuracy in cold-start scenarios. Finally, a hybrid model combining the strengths of both approaches was proposed, demonstrating superior recommendation performance, particularly in extensive product catalogs. The integration of semantic and behavioral data for product representation is identified as a critical factor enhancing the effectiveness of personalized recommendation systems.

Keywords: *Tavsiye Sistemleri, Makine Öğrenmesi, İki Kuleli Modelleme, Geri Getirme Modelleri, İçerik Tabanlı Filtreleme.*

Development of Economic Intelligence Systems in the Context of Global Crisis and Its Economic Impacts (1266)

Fatıma Mammadova[1*]

[1]Eskişehir Osmnagazi University, Faculty of Economics and Administrative Sciences, Department of Finance

*Corresponding author e-mail: mammadovafatima09@gmail.com

Abstract

The global economic crisis and the associated high costs have revitalized academic and policy interest in "early warning indicators" of crises. This paper provides evidence on the effectiveness of proposed economic intelligence systems in predicting severe recessions and crises in countries around the world. An economic intelligence system is a broad concept defined as a set of coordinated actions involving research, processing and dissemination of useful information. In this study, macroeconomic indicators of countries with and without economic intelligence systems are analyzed in terms of variations in performance during periods of stability and periods of economic volatility or crisis in order to predict global economic crises in Asia, Europe and the United States. The findings reveal that macroeconomic and institutional variables are valuable indicators in predicting crises. In times of global crises, the inability of countries to effectively monitor data on their economic performance significantly complicates crisis management processes. This research investigates the impact of economic crises in some countries on other countries in the process of globalization and the development of an economic intelligence system for the early detection of such crises. The 2008 Financial Crisis and the COVID-19 crisis, as well as the sharp impact of the global economic crises on the world economy, provide the clearest examples of how fragile these crises are. This research argues that the acceleration of globalization and increasing economic crises have made economic intelligence activities at the national level strategic in terms of crisis prevention and management, based on the ability to collect and evaluate data and information.

Keywords: *Crises, Economic Intelligence Systems, 2008 Financial Crisis, COVID-19 Crisis*

Predicting Customer Churn In The Cosmetics Industry Using Machine Learning: A Random Forest And Borderlinesmote Approach (1267)

Tuna Çakar[1], Hamza Gözükara[2], Ayşe Kızılay[3], Zeynep Şahin[3], Müberra Şen[3], Cansu Soysal[3], Jay Nimish Patel[4]

[1]MEF Üniversitesi, Bilişim Teknolojileri Yüksek Lisans Programı, İstanbul, Türkiye
[2]Eve Mağazacılık A.Ş., Ar-Ge Merkezi, İstanbul, Türkiye
[3]SST TEK A.Ş., Ar-Ge Merkezi, İstanbul, Türkiye
[4]MEF Üniversitesi, Bilgisayar Mühendisliği Bölümü, İstanbul, Türkiye

*Corresponding author e-mail: cakart@mef.edu.tr

Abstract

This study presents a machine learning-based model developed to predict customer churn for a cosmetics company. The dataset utilized encompasses online and physical shopping records from the years 2022-2024. Extensive data preprocessing steps were applied to manage missing data, filter outliers, and generate new customer behavior metrics such as shopping frequency, total spending, and diversity of purchased products and stores visited. Notably, the diversity of product types purchased and store visits exhibited a negative correlation with customer churn rates. Due to the imbalanced nature of the dataset, iterative sampling techniques combined with the BorderlineSMOTE method were implemented over defined shopping activity periods to achieve balanced data. In the model development phase, several algorithms including Random Forest, Logistic Regression, K-Nearest Neighbors (KNN), AdaBoost, Extra Trees, and Decision Trees were evaluated. The models demonstrated optimal and most significant performance on a subset of the dataset, where users exceeding 99.9 percentile thresholds in metrics like total shopping count and total expenditure, and users with more than ten shopping instances, were selected. Additionally, this subset excluded anomalous users with unusually frequent shopping activities (over 30 purchases within a month). Performance evaluation metrics such as accuracy, precision, recall, and F1-score indicated that the Random Forest algorithm delivered the best performance, achieving an average accuracy of 86%, with balanced prediction capabilities for both classes. Findings from the study underline the superior performance and effective classification capabilities of the Random Forest algorithm compared to other machine learning methods in predicting customer churn. These results emphasize the significant contribution of machine learning approaches in developing strategies to enhance customer loyalty and proactively manage churn. Furthermore, the study suggests that continuous data enrichment and model optimization are critical for improving model performance in future research.

Keywords: *Customer Churn, Cosmetics Industry, Machine Learning, Random Forest, BorderlineSMOTE.*

A Comparative Mortality Analysis Using the Lee-Carter Model: Evidence from the UK, Bulgaria, and Belarus (1268)

Gizem Özgüden[1], Övgücan Karadağ Erdemir[2*], Başak Bulut Karageyik[3]

[1,2,3] Hacettepe University, Faculty of Science, Department of Actuarial Science, Turkiye

*Corresponding author e-mail: ovgucan@hacettepe.edu.tr

Abstract

While numerous models have been developed for mortality analysis, the Lee-Carter model remains one of the most prominent due to its balance of simplicity and interpretability. Widely applied in demographic studies and mortality forecasting, the model's strength lies in its ability to capture age- and time-specific mortality dynamics effectively. In this study, the Lee-Carter model is applied to compare mortality trends in three countries—the United Kingdom, Bulgaria, and Belarus—using high-quality data obtained from the Human Mortality Database (HMD). These countries are deliberately selected to reflect significant contrasts in healthcare systems, socioeconomic conditions, and cultural settings. The analysis focuses on age- and gender-specific mortality rates, modeled over time within a consistent parametric framework. By situating the mortality profiles within their broader demographic, institutional, and geographic contexts, the study aims to explore the multifaceted drivers of mortality variation. The findings suggest that the United Kingdom, with its advanced healthcare infrastructure and high living standards, consistently exhibits lower mortality rates. In contrast, Bulgaria and Belarus are characterized by relatively higher mortality, influenced by factors such as limited access to medical services, lower economic development, and social disparities. The analysis also highlights how gender and age interact differently across countries, revealing patterns shaped by both biological and structural determinants. This study underscores the importance of linking statistical modeling with contextual interpretation in understanding international mortality differentials.

Keywords: *Mortality Modeling, Lee-Carter Model, Human Mortality Database, Age- and Time-Specific Mortality Dynamics*

Personalized Product Recommendations Using Matrix Factorization On Large- scale Retail Data (1269)

Tuna Çakar[1], Mehmet Talha Bozdoğan[2], Hamza Gözükara[2], Ayşe Kızılay[3], Zeynep Şahin[3], Kübra Abalı[3], Büşra Tosun[3], Jay Nimish Patel[4]

[1]MEF Üniversitesi, Bilgisayar Mühendisliği Bölümü, İstanbul, Türkiye
[2]Mindsane Ltd. Şti., İstanbul, Türkiye
[3]Eve Mağazacılık A.Ş., Ar-Ge Merkezi, İstanbul, Türkiye
[4]SST TEK A.Ş., Ar-Ge Merkezi, İstanbul, Türkiye

*Corresponding author e-mail: talhabozan@mindsane.net

Abstract

In this study, personalized recommendation systems were developed using three matrix factorization- based algorithms—Alternating Least Squares (ALS), LightFM, and Singular Value Decomposition (SVD)—on large-scale retail transaction data. The dataset comprises over 85 million transaction records, involving approximately 1.63 million unique users and more than 16,000 products. Initially, implicit feedback from user-product interactions was modeled using the ALS algorithm, achieving Precision@10 of 6.88% and MAP@10 of 3.29% after 15 iterations. An ALS model developed with PySpark on the same data measured an RMSE of 8.26 in the test set. The MAPE value was 79.68%, primarily driven by the prevalence of low-value transactions and values near zero, resulting in proportionally high error rates. However, this did not significantly affect the model's ability to generate accurate recommendation rankings. The LightFM algorithm generated hybrid recommendations supported by content-based information such as product category, brand, and name. Precision@10 was calculated as 3.65%, while Recall@15 reached 5.60%. Additionally, the AUC (0.8199) and MRR (0.1446) metrics indicated that the model performed well in ranking-focused tasks. The average similarity score of recommended products to actual preferences in the test set was 0.437, indicating positive outcomes concerning content consistency. The SVD algorithm was applied to classify users' purchasing tendencies, achieving a very low RMSE of 0.0063 and demonstrating highly accurate predictions. Although each algorithm offers different advantages, all three provided valuable contributions in generating personalized recommendations, particularly with large-scale retail data. This analysis highlights that user habits can be transformed into meaningful recommendations through appropriately structured data and method selection.

Keywords: *Matrix Factorization, Alternating Least Squares (ALS), LightFM, Singular Value Decomposition (SVD), Personalized Recommendation Systems.*

Predicting Default Risk In Cheques Using Machine Learning Models: An Analysis Of Individual And Corporate Customers (1270)

Kerem Kaya[1], Alperen Sayar[2], Seyit Ertuğrul[2], Tuna Çakar[3]

[1]Mindsane Ar-Ge ve İnovasyon Ltd. Şti., İstanbul, Türkiye
[2]TAM Finans A.Ş., Ar-Ge Merkezi, İstanbul, Türkiye
[3]MEF Üniversitesi, Bilgisayar Mühendisliği Bölümü, İstanbul, Türkiye

*Corresponding author e-mail: cakart@mef.edu.tr

Abstract

The aim of this study is to develop machine learning models to predict default risk in cheque payment processes, focusing on both individual and corporate customer profiles. Utilizing financial characteristics of cheque transactions and time-based interactions, the research targets effective classification performance by applying preprocessing techniques that address class imbalance, as well as optimization methods. Previous literature on bounced cheque predictions has integrated conventional classifiers such as logistic regression, random forest, and gradient boosting machines with feature engineering and scaling steps. Additionally, methods such as undersampling and weighting are commonly employed to handle class imbalance. The dataset comprises various attributes including customer transaction amounts, credit limits, risk levels, and cheque histories. Missing values were imputed using mean and mode methods, outliers were treated with the interquartile range (IQR) method, and the data was standardized using min-max scaling. After defining the target variable related to cheque defaults, class balance was ensured through undersampling. Time-based transformations converted historical bounced cheque occurrences (past 30, 60, 90, 180, 210, 240, 270, 300, 330, and 360 days) into customer-specific features and added these to the dataset. Hyperparameters of the models were optimized using Optuna. For individual customers, the best F1 scores achieved were 0.8575 for Random Forest, 0.8432 for Logistic Regression, and 0.8561 for XGBoost. For corporate customers, the performance was generally superior, with Random Forest achieving an F1 score of 0.9038, Logistic Regression 0.8996, and XGBoost leading with an F1 score of 0.9115. The results demonstrated that the developed XGBoost model delivered the highest performance overall. The study concluded that time-based features and undersampling significantly improved classification accuracy, while Optuna-based hyperparameter tuning enhanced model stability. Future research is recommended to explore additional artificial intelligence methods (e.g., deep learning) and more refined strategies for class imbalance.

Keywords*: Bounced Cheque Prediction, Machine Learning, Feature Engineering, Optuna, XGBoost.*

Predicting Customer Churn In Video Streaming Services Using Machine Learning: Random Forest And Catboost Applications (1271)

Hamza Gözükara[1]*, Erkan Kara[2], Yasemin Kurtçu Meşe[3], Ayşenur Yıldız[3], Emir Obalı[3], Tuna Çakar[4]

[1]MEF Üniversitesi, Bilgisayar Mühendisliği Lisans Bölümü, İstanbul, Türkiye
[2]Demirören Medya A.Ş., İstanbul, Türkiye
[3]D-Smart, Ar-Ge Merkezi, İstanbul, Türkiye
[4]MEF Üniversitesi, Bilgisayar Mühendisliği Bölümü, İstanbul, Türkiye

*Corresponding author e-mail: gozukarah@mef.edu.tr

Abstract

In this study, a predictive model based on machine learning techniques was developed to forecast customer churn in video streaming services. The dataset comprises user login data, content viewing information, subscription details, and content features. During data preprocessing, customer churn was defined by considering subscription renewal status and renewal duration. Missing values in the dataset were handled by imputation using mean values. Feature engineering included metrics such as user login frequency, viewing durations, and content diversity. The model development employed algorithms such as Random Forest, CatBoost, XGBoost, Logistic Regression, K-Nearest-Neighbors, and Gradient Boosting. However, a significant class imbalance in the target variable was identified prior to model training. To address this imbalance, the Borderline-SMOTE technique was applied, yet the resulting model still exhibited a bias towards the majority class (users who renew subscriptions). Primary reasons for this class imbalance include the dataset's limited historical scope, which may not cover multiple subscription periods adequately, and inherent subscription behaviors of users. The model results indicate that the CatBoost algorithm achieved the highest overall accuracy of 85%. However, this high accuracy predominantly reflects correct predictions of the majority class, with comparatively lower performance for the minority class (users who do not renew subscriptions). The model's performance was rigorously assessed through training-test accuracy plots, classification reports, and hyperparameter optimization. Metrics such as F1-score and recall, which are particularly meaningful for imbalanced classes, clearly indicate the need for improvement, especially regarding the minority class. The study findings suggest that features such as content diversity and total usage duration are negatively correlated with customer churn. These insights are instrumental in developing effective strategies to reduce churn.

Keywords: *Customer Churn, Machine Learning, Video Streaming Services, Random Forest, CatBoost.*

Hybrid Approaches Based On Machine Learning And Deep Learning For Predicting Agricultural Commodity Prices In Turkey (1272)

Tuğçe Ata[1], Mert Çeküç[2], Peri Güneş[1], Tuna Çakar[2]

[1]İnfina Yazılım A.Ş., Ar-Ge Merkezi, İstanbul, Türkiye
[2]MEF Üniversitesi, Bilgisayar Mühendisliği Bölümü, İstanbul, Türkiye

*Corresponding author e-mail: tugceata00@gmail.com

Abstract

This study presents a comprehensive and comparative analysis of advanced machine learning (ML) and deep learning (DL) methodologies aiming to effectively predict price fluctuations in Turkish agricultural commodity markets. Due to the inadequacy of traditional price forecasting methods in capturing sudden market changes, there is a growing demand for more precise and sophisticated prediction models. Accordingly, a comprehensive dataset has been compiled from various sources such as the Turkish Mercantile Exchange (TÜRİB), Turkish Statistical Institute (TÜİK), Central Bank, and Turkish State Meteorological Service. The dataset includes price indices, exchange rates, meteorological data, and international market information. During data preprocessing, missing values were filled using the KNearest Neighbors (KNN) algorithm, outliers were managed through the interquartile range (IQR) method, and standardization procedures were performed. The modeling process utilized classical regression techniques such as Ridge, Lasso, and Elastic Net, as well as advanced machine learning models like XGBoost and LightGBM, and deep learning techniques including Long Short-Term Memory (LSTM), Sequential models, and N-BEATS. Hyperparameter optimization for the models was conducted using Grid Search and Randomized Search methods, and validation was performed through time-series cross-validation techniques. Results demonstrated that traditional regression methods inadequately captured non-linear market dynamics, whereas XGBoost and LSTM models exhibited superior performance. Particularly, hybrid models combining XGBoost and LSTM delivered higher predictive accuracy compared to individual models. Additionally, a user-friendly web-based interface was developed to facilitate practical application of the proposed models. The outcomes of this research support decision-making processes in agricultural markets, contributing significantly to economic sustainability.

Keywords: *Agricultural Commodity Price Prediction, Machine Learning, Deep Learning, Hybrid Models, Time Series Analysis.*

Analyzing Happiness Index by using Fuzzy Clustering (1275)

Suzan Kantarcı Savaş[1*], Kevser Tüter Şahinoğlu

[1]Department of Econometrics, Faculty of Economics and Administration Sciences, Kırklareli University, 39000, Kayali Campus, Kırklareli Türkiye

*Corresponding author e-mail: suzan.kantarci@klu.edu.tr

Abstract

Happiness represents a multifaceted concept that reflects the well-being of individuals and societies across psychological, social, and economic dimensions. Since 2012, the World Happiness Report has provided valuable insights by compiling data on various indicators—including GDP per capita, social support networks, life expectancy, personal freedom, generosity, and corruption perceptions. These variables collectively form the basis of the Happiness Index, which is now widely recognized as a significant metric for assessing national development and quality of life.

The question of what truly drives happiness has been central to scholarly debate since Richard Easterlin's influential study in 1974, which demonstrated that higher income levels do not necessarily correlate with increased happiness. This finding has prompted researchers to explore broader determinants of happiness that go beyond mere economic performance.

In recent years, the application of advanced analytical tools, such as machine learning, has opened new avenues. Clustering and prediction offer a means to uncover complex patterns and relationships within multidimensional data. Particularly, fuzzy logic techniques capable of handling uncertainty and vagueness in human-related data have been integrated with traditional algorithms (e.g., Fuzzy c-means, to enhance algorithm's performance.

This study aims to analyze global happiness levels by using fuzzy clustering approach. It is applied to identify groups of countries with similar happiness profiles. In the future researches, these clusters will serve as a foundation for further predictive analysis, allowing us to model and interpret the diverse pathways through which happiness is shaped across different nations.

Keywords: *Happiness index, fuzzy clustering, unsupervised learning.*

Integrated Automation System Based on Optical Character Recognition and Semantic Interpretation in Invoice and Receipt Processing (1276)

Ahmet Yıldız[1], Mert Güvençli[1], Tuna Çakar[2*]

[1]Apsiyon Inc., R&D Center, İstanbul, Türkiye
[2]MEF University, Computer Engineering Department, İstanbul, Türkiye

*Corresponding author e-mail: cakart@mef.edu.tr

Abstract

This study aims to develop an integrated automation system designed to automate invoice and receipt processing. By combining web scraping, optical character recognition (OCR), and semantic interpretation technologies, the system significantly reduces manual interventions in document processing while enhancing processing accuracy. Data extraction from HTML-formatted documents using libraries such as BeautifulSoup and lxml achieved an accuracy rate of 98%. For parsing dynamic JavaScript content, the Selenium library was employed, attaining an accuracy rate of 96%. PaddleOCR technology was utilized for processing visually formatted documents, and through Turkish language optimizations, the character recognition accuracy reached 95%. High success rates were notably achieved even with low-resolution documents and those featuring complex backgrounds. In the semantic interpretation stage, the Transformer-based MiniLM-L6-v2 model was implemented, successfully categorizing document content with a 90% accuracy rate. Additionally, inconsistencies in category matching were effectively identified as anomalies at an 85% success rate, enabling timely user interventions. Consequently, the integrated automation system developed has markedly improved operational efficiency in document processing and significantly contributes to digital transformation objectives. Tested under real business conditions, this system presents a comprehensive approach, serving as a valuable reference for both academic literature and practical applications. Future research is recommended to evaluate the system across various document types and more extensive datasets.

Keywords: *Optical Character Recognition (OCR), Semantic Interpretation, Web Scraping, PaddleOCR, Transformer Models.*

Explaining Patient Satisfaction with Demographic Factors: The Case of Central Anatolia Region (1277)

Kübra Durukan[1*], Kübra Uçarsu[2]

[1]Kırıkkale University, Faculty of Engineering and Natural Science, Department of Statistics, Turkey
[2]Kırıkkale University, Science Institute, Department of Statistics, Turkey

*Corresponding author e-mail: kubraba@gmail.com

Abstract

Patient satisfaction is considered as the counterpart of the customer-oriented service approach, which is prominent in many sectors, in the field of health. Satisfaction in health services is important in terms of increasing service quality, improving hospital conditions, and strengthening patient-healthcare personnel communication. The aim of this study is to model the satisfaction levels of patients receiving outpatient services from public hospitals in the Central Anatolia Region with demographic factors. The study used data obtained from 17.851 patients, consisting of women and men between the ages of 18 and 65, who received services from public hospitals in the Central Anatolia Region between 2018 and 2023. The Multinomial Logistic Regression (MLR) model was applied in the analysis of the data; the analyses were performed with the SPSS 26 package program. As a result of the study, it was determined that the interactions of gender and age groups had significant effects on the level of patient satisfaction. In particular, the probability of showing "Medium" level satisfaction was found to be significant in women between the ages of 18 and 49 and men between the ages of 18 and 41. In addition, men aged 26–33 are approximately twice as likely to have a "Low" satisfaction level as men aged 58–65. At the regional level, the probability of low satisfaction is 1.47 times higher in Aksaray than in Ankara, while it is approximately twice as likely in Çankırı, Eskişehir and Niğde. These findings suggest that patient satisfaction is affected not only by individual demographic factors but also by regional differences, thus indicating that these differences should be taken into account in the planning of healthcare services.

Keywords: *Outpatient Satisfaction Scale, Multinomial Logistic Regression, Central Anatolia Region*

Forecasting Extra Virgin Olive Oil Prices by using Recurrent Neural Networks (1278)

Suzan Kantarcı Savas[1]

[1]Department of Econometrics, Faculty of Economics and Administration Sciences, Kırklareli University, 39000, Kayali Campus, Kırklareli Türkiye

*Corresponding author e-mail: suzan.kantarci@klu.edu.tr

Abstract

Olive oil is an important agricultural food product. In agricultural food product sector, the official market for the negotiation of future contracts for olive oil in the world is important. The market situation is so important for the producers, exporters and importers. It is important to forecast prices to balance supply and demand in future periods of time. This study aims to forecast the extra virgin olive oil prices for four European Union (EU) countries (Spain, Italy, Greece, Croatia) by using recurrent neural network algorithm. The prediction models of future prices aims to increase the global benefits of the sector. The data is provided from Agricultural and Rural Development of European Commision. Model performances are analyzed by using statistical methods.

Keywords: *Olive Oil Price, Recurrent Neural Network, Forecasting.*

Neural Correlates of Cognitive Load in Drivers During Simulated Working Memory Tasks (1279)

Yavuz Yıldız[1], Buket Çınar Gelir[1], Nihal Er[1], Türkay Şahin[2], Esin Tuna[2], Tuna Çakar[2*]

[1]DHL Türkiye Supply Chain, İstanbul, Türkiye
[2]MEF University, İstanbul, Türkiye

*Corresponding author e-mail: cakart@mef.edu.tr

Abstract

This study investigates the neural correlates associated with cognitive load in drivers during working memory tasks within a simulation environment. The primary objective is to comprehend the emotional and cognitive states of drivers by integrating neurometric, biometric, and telemetric data. The experimental protocol consists of four distinct phases: a safe-drive phase followed by three increasingly complex n-back tasks (0-back, 1-back, and 2-back), which systematically elevate cognitive load. The methodology employs extensive multimodal data collection, including EEG/ERP recordings from 32 channels at 500 Hz, facial activation coding, eye blink and movement tracking, telemetry data (over 160 variables sampled at 10 Hz), and response data gathered via steering wheel and pedals. Artifacts such as head movements and eye blinks were carefully managed through dedicated artifact removal techniques, including the application of notch filters. Analysis focuses on neurometric indices such as attention, arousal, engagement, interest, memory retention, mental workload, and positiveness, extracted from EEG signals across different frequency bands (delta, alpha, beta, gamma). The obtained findings indicate that as cognitive load increased across the n-back tasks, drivers' accuracy diminished from 0-back to 2-back, accompanied by increased incorrect responses and missed targets, highlighting the substantial cognitive demands placed on the drivers. Future work aims to utilize these insights for the practical enhancement of driver safety protocols by real-time monitoring of cognitive states, potentially enabling interventions to mitigate accident risks and improve driver well-being and operational safety.

Keywords: *EEG/ERP, Working Memory, Cognitive Load, Driver Safety, Neurometrics.*

Prediction Of Climate Change-induced Floods Using Artificial Intelligence And Quantum Learning Methods (1280)

Tuna Çakar[1*], Özgüray Aydın[2], Eymen Berkay Yorulmaz[2], Hasan Hüseyin Yurdagül[2], Adem Seller[2], Hatice Özdemir[2]

[1]Mef University, İstanbul, Turkiye
[2]Universal Yazılım A.Ş., İstanbul, Turkiye
*Corresponding author e-mail: cakart@mef.edu.tr

Abstract

In this study, an innovative model based on machine learning and quantum learning methods has been developed to more accurately predict the increasing flood risks induced by climate change. Hydraulic data from two different basins at the Kömürcüler Station located on the Salarha Stream in Rize Province were utilized to construct both regression and classification models. Linear Regression (LR) and eXtreme Gradient Boosting (XGBoost) algorithms were employed for regression analysis, while Random Forest (RF), Gradient Boosting (GB), Support Vector Machines (SVM), and Naive Bayes (NB) methods were used for classification modeling. Additionally, the Pennylane quantum learning algorithm was integrated into the model to enhance the performance of traditional models. Mean Absolute Percentage Error (MAPE) was used to evaluate the regression models, whereas accuracy, precision, and sensitivity metrics were applied for classification models. It was observed that for short-term predictions, the LR algorithm exhibited lower error rates compared to XGBoost, while error rates for both algorithms increased as the prediction horizon extended. Among classification models, RF and GB algorithms achieved near-perfect results, and the Pennylane quantum algorithm demonstrated higher accuracy in certain scenarios compared to classical models, highlighting its potential as a robust alternative. Consequently, the developed models effectively contribute to flood prediction and could support flood risk management and the development of early warning systems. The combined use of machine learning and quantum learning methods holds promise for more reliable disaster management applications in the future.

Keywords*: Flood Prediction, Climate Change, Machine Learning, Quantum Learning, Hydrological Modeling.*

Comparative Analysis Of Product Recommendation Systems To Enhance Upselling In B2b Operations (1281)

Tuna Çakar[1], Samim Erinç[2], Besna Ruken Tarhan[2], Damla Doğru Babacan[2]

[1]Mef University, İstanbul, Turkiye
[2]Insider A.Ş., Ar-Ge Merkezi, İstanbul, Turkiye

*Corresponding author e-mail: besna.tarhan@useinsider.com

Abstract

This study performs a comparative analysis of various recommendation systems aiming to automatically identify upselling opportunities in global B2B operations. Currently, upselling opportunities are determined through manual and non-standard methods, resulting in time-consuming processes that disregard regional dynamics. Consequently, this negatively impacts process efficiency and recommendation quality. Four methodologies were tested in the context of the Turkish region and the fashion retail sector: rule-based systems, association analysis based on the Apriori algorithm, deep learning utilizing Softmax classification, and item-based collaborative filtering. Rule-based systems and the Apriori algorithm exhibited low success rates due to their inability to account for regional and sectoral variations. The deep learning model based on Softmax achieved limited success with an accuracy rate of 29.6%, primarily due to inadequate generalization performance. Item-based collaborative filtering emerged as the most successful method. This approach identifies similarities between products based on the historical purchasing behavior of users, employing cosine similarity, achieving an accuracy rate of 90%. This method demonstrates the potential for increasing monthly recurring revenues (MRR) by approximately 30%. The model significantly enhances operational efficiency and recommendation quality by automating existing processes. In conclusion, the item-based collaborative filtering method is determined as the most suitable recommendation system for upselling suggestions within Insider's B2B operations. Future research aims to enhance recommendation system performance through the application of broader datasets across various regions and sectors.

Keywords: *Recommendation Systems, Upselling, Collaborative Filtering, Deep Learning, Operational Efficiency*

Performance Review of OECD Countries' Merchant Fleets (1282)

Maruf Gögebakan [1*], Tarık Erdoğmuş[1]

[1]Bandırma Onyedi Eylül University, Maritime Faculty, Department of Maritime Business Administration, Turkiye

*Corresponding author e-mail: mgogebakan@bandirma.edu.tr

Abstract

Countries' ship fleets play a crucial role in foreign trade via maritime routes. Countries with larger and newer fleets capture a greater share of maritime trade, contributing to their economies. A country's fleet directly affects both international trade and logistics efficiency. This study evaluates the trade fleet performance of OECD countries between 2014-2024 using multi-criteria decision-making (MCDM) methods, considering three variables: the number of ships, total carrying capacity (DWT), and the average age of ships. The number of ships represents the fleet size, while carrying capacity reflects the fleet's cargo transport potential, and the average age indicates the fleet's modernity. The importance weights of these variables were determined using the Entropy method, and fleet performances were ranked using the TOPSIS and VIKOR methods. These rankings were combined with the Borda method to produce the final performance ranking. Greece ranked highest due to its large fleet and high carrying capacity. Japan and Germany followed with large fleets and modern ships. South Korea, the United States, and the United Kingdom also showed strong performance, with high carrying capacity and low average ship age. Denmark, Turkey, and Switzerland were in the middle ranks, while the Netherlands, Belgium, and Italy ranked lower. Countries with lower performance tended to have smaller, older, and lower-capacity fleets, which negatively impacted trade and logistics efficiency. In conclusion, fleet performance differences are primarily influenced by carrying capacity, fleet size, and the average age of ships. Fleets with higher capacity, newer ships, and larger sizes demonstrated better performance.

Keywords: *Marıtıme Fleet Performance, Mcdm, Entropy, Topsıs, Vıkor*

XAI-Enhanced Deep Learning for RNA-Seq Classification: Comparing MLP and KAN (1284)

Ferdi Güler[1], Melih Ağraz[2*]

[1,2]Giresun University, Institute of Science, Department of Statistics, Türkiye

*Corresponding author e-mail: melih.agraz@giresun.edu.tr

Abstract

The classification of high-dimensional RNA-Seq cancer data poses significant challenges, particularly when working with limited sample sizes. This study presents an explainable artificial intelligence (XAI)-enhanced deep learning framework for classifying chromophobe renal cell carcinoma (KICH) using data obtained from The Cancer Genome Atlas (TCGA). Following preprocessing steps such as normalization, transformation, and dimensionality reduction, robust feature engineering was performed using Boruta, Random Forest (RF), and Principal Component Analysis (PCA) techniques. To address class imbalance and enhance model performance, three data augmentation strategies—linear interpolation, SMOTE, and MixUp—were applied. Two deep learning models, Multi-Layer Perceptron (MLP) and the Kolmogorov–Arnold Network (KAN), were compared across various experimental scenarios. The KAN model achieved the highest classification accuracy of 99.90% with linear interpolation, while the MLP model also performed strongly, reaching 99.45% accuracy with SMOTE. Moreover, all augmentation strategies clearly improved model performance across all evaluation metrics. To further interpret the results, explainable AI tools such as SHAP and LIME were applied in 20 experimental runs to identify the most influential genes in the classification task. Genes such as NAT2, MAPK15, and IRX2 were frequently identified, underscoring the biological relevance of the model outputs. These findings demonstrate that careful preprocessing, appropriate augmentation strategies, and interpretable deep learning approaches can substantially enhance classification accuracy in RNA-Seq datasets, even under constrained sample conditions. Furthermore, the KAN model emerges as a computationally efficient and interpretable alternative for biomedical applications.

Keywords: *RNA Sequence, Deep Learning, Explainable Artificial Intelligence, Data Augmentation, Kolmogorov-Arnold Network.*

Determination of Attitudes of Agricultural Engineer Candidates Towards Digitalization in Agriculture by Factor Analysis: Tokat Gaziosmanpaşa University Example (1285)

Asena Ertürk[1*], Didem Doğar[1], Adnan Çiçek[1]

[1]Tokat Gaziosmanpaşa University, Faculty of Agriculture, Agricultural Economy, Turkey

*Corresponding author e-mail: asenaerturk14@gmail.com

Abstract

The aim of this study is to determine the attitudes of Tokat Gaziosmanpaşa University Faculty of Agriculture students towards digitalization in agriculture and to analyze the factors affecting these attitudes. The data of the study were collected from 208 3rd and 4th grade students through a survey in the 2024-2025 academic year. Exploratory factor analysis was applied to the data obtained with a Likert-type scale consisting of 23 items. According to the factor analysis results, the scale was divided into three main dimensions: "Perception of the Benefits and Education of Digital Agriculture", "Economic and Environmental Benefits of Digital Agriculture" and "Concerns and Need for Support for Digital Agriculture". While the students largely approached the contributions of digital agriculture such as efficiency, cost reduction and environmental sustainability positively, they expressed concerns about issues such as technology dependency, high costs and reduced employment. In the study, the KMO value was found to be 0.950 and the Cronbach's Alpha value was found to be 0.971, and it was determined that the data set was extremely suitable for factor analysis and the scale was reliable. The research results reveal that training in digital agricultural technologies should be increased in agricultural faculties and support mechanisms should be developed for small-scale producers in the digitalization process..

Keywords: *Digital agriculture, Digitalization in agriculture, Factor analysis, Tokat*

Implications of Normality Assumptions For Strategic Planning and Policy Issues in Higher education. A Case Study of the Business Education in the Turkish Higher Educational System (1286)

E. Abdülgaffar Ağaoğlu[1*]

[1]Yalova University, Department of Industrial Engineering, Yalova, Turkey

*Corresponding author e-mail: agaoglu_ag@yahoo.com

Abstract

It is very well known that properly educated population is the fundamental determinant of societal welfare and economic growth. Turkey has invested tremendously in the Institutions of Higher Education. Since 1985 Turkish Foundation Universities are participating side by side with previously monopolic public university system with much higher magnitudes in this process of investment. Presently foundation universities occupy a proportion of nearly 40%; making it necessary to be carefully and strategically evaluated on "Within the peer groups", as well as on "Between the peer groups" basis. We attempt to cluster up and elaborate the basic determinants of the "Turkish Higher Education System". Beginning with emphasis upon the student selection characteristics of the University Entrance Examination System (Called ÖSYS), we try to capture the determinants of foundation and public universities which make them distinct peer groups, and then pickBusiness education in the system.

The primary focus of this study is on the overall population of all business departments, which renders the minimization of any "Validity" and "Reliability" problems for the models used. The study begins with Dummy Regression Analysis to discriminate between Business Departments of state versus foundation universities. On the basis of these primary findings, the study moves further to focus upon educational strategy planning, which is accomplished through the testing of Normality hypothesis, asserting that no normality exist either within the individual groups, nor between them as a single group. The study concludes that The Two Groups" discriminate themselves on the basis of "Entrance Scores"; "Number of Seats Available"; "Number of Candidates Placed"; "Number of Students per instructor"; and "Level of Tuition", thus rendering "Economies of Scale" as well as "Economies of Scope" problems inherited within the overall system. Both individual systems are statistically not consistent within themselves indicating the presence of micro-level planning issues. The study concludes with crucial lessons on efficiency and effectiveness which need urgent attention.

Keywords: *Turkish Higher Educational System; Business and Economics Education, Normality as a Signalling Model; Educational Strategy and Policy Making*

The Allegorical Poem "Sohbatul-asmar" by Muhammad Fuzuli in Bibliographic Indexes

Fidan Nasirova[1*]

[1]Azerbaijan National Academy of Sciences Institute of Oriental Studies after acad. Z.M.Bunyadov, Baku, Azerbaijan

*Corresponding author e-mail: nasirova1975@list.ru

Abstract

The works by one of the phenomenal figures in the history of philosophical thought of the Near and Middle East Muhammad Fuzuli (1498-1556) not only marked the beginning of a new stage in the social and poetic thoughts of the Azerbaijani people with its profound ideas and poetic beauty, but also became a source of inspiration and ideas for hundreds of his successors.

We have also been inspired to choose Muhammad Fuzuli's allegorical poem "Sohbatul-asmar" ("Conversation of Fruits") as a subject for a research in the context of bibliographic collections and have attempted to clarify the extent of references to the poem and researches based on statistical indicators.

It has to be noted that the numerous studied resources allow us to state, that compared to other creative works by Muhammad Fuzuli this allegorical poem was left aside of fundamental research. In our opinion, the reason behind the formation of such an abandoned attitude toward this poem is highly hidden also in the doubts and hesitations about whether it is the product of Fuzuli's pen or not. As is known, this allegorical poem by Fuzuli was not included in the complete works published neither in Turkey, nor in the Tashkent, and the fact that the poem belongs to the great poet was first declared by the literary scholar Amin Abid in 1926 in his article "An Unresearched Work by Fuzuli" in the journal "Education and Culture" ("Maarif ve medeniyyet"). By this, those who confirm and deny that the poem belongs to Fuzuli were split on 2 different fronts, presenting their own arguments and facts, scientific evidences and proofs. To find a justification of what was said, we addressed to the poet's artistic heritage compiled by A. Khalafov (1958), F. Mammadov (1996), J. Gahramanov and A. Khalilov (1996), and the systematized search resources-bibliographic collections, which are a comprehensive indicator of the research and studies on it. However, the dynamic development in Azerbaijani Fuzuli studies revealed that these resources are already outdated and do not reflect the real state. Finally, we had to dwell on the more modern "Bibliography of Muhammad Fuzuli" compiled by A. Khalilov (2011). The studies showed that only 5 out of 974 manuscripts, only 33 out of 1509 published works by Fuzuli, and only 15 out of 1884 studies (articles) related to the artistic heritage of the great poet mentioned the this work. Including the literature in Russian language, only 57 out of 4737 most diverse works of writing on Fuzuli's creativity and related studies cover the poem, what serves as a self-evident confirmation of the above-mentioned indifferent attitude toward poem "Sohbatul-asmar".

Keywords: *Muhammad Fuzuli, Fuzuli bibliography, Sohbatul-asmar, Allegorical poem*

Ratio Analysis with Fieller's Theorem and a Python Application (1287)

Derviş Topuz[1*], Selçuk Tekgöz[2]

[1]Niğde Ömer Halisdemir University, Niğde Zübeyde Hanım Health Services Vocational School, Department of Medical Services and Techniques, Niğde/TURKEY
[2]Niğde Provincial Health Directorate, Statistics Unit, Niğde/TURKEY.

*Corresponding author e-mail: topuz@ohu.edu.tr

Abstract

In research, the ratio of two measured quantities is a commonly analyzed and important statistical metric. However, when the denominator is small, calculating confidence intervals for such ratios becomes more challenging, and the reliability of the results may decrease. Furthermore, the appropriate method to use in such cases is often unclear. Fieller's theorem provides more accurate and reliable confidence intervals for the ratio of two means especially when the normality assumption of the ratio's distribution is not satisfied.

The aim of this study is to explain the fundamental principles of the Fieller method and to demonstrate the related statistical calculations step by step through a practical application using a sample dataset in the Python programming language.

Keywords: *Fieller's theorem, Confidence ınterval, Python*

Data Analysis and Visualization of TÜİK User Satisfaction Survey (1289)

Ahmet Akın Atasoy[1*], Mert Mesut Selçuk[1]

[1]Turkish Statistical Institute, Türkiye

Corresponing Author e-mail: ahmet.atasoy@tuik.gov.tr

Abstract

This study presents an innovative approach to analyzing user satisfaction patterns for a web platform through the implementation of an automated data pipeline and visualization system. The research utilizes postgraduate database sources to extract, manipulate, and analyze user satisfaction data automatically through SQL environments, with results visualized in Qlik Sense dashboards.

The methodology involves automated data extraction from the database via scheduled jobs running daily at 4:00 AM, followed by systematic processing and visualization of multiple satisfaction metrics. User responses are measured on a scale ranging from "Not Satisfied At All" to "Very Satisfied," with particular focus on questions 8, 9, 10, and 11 that assess specific aspects of user experience.

Our analysis reveals significant insights into user satisfaction levels across various demographic segments, including age groups, education levels, and user categories. The visualizations demonstrate that 52.83% of users report general satisfaction with the platform, with usage patterns varying significantly based on frequency (from daily use to less than once per year) and purpose of visit.

The data pipeline created for this research enables real-time analysis of user behavior and satisfaction metrics, allowing stakeholders to make data-driven decisions for website improvement. This automated approach eliminates manual data processing, reducing the potential for human error while providing consistent and timely insights.

This study contributes to the growing field of automated data analytics by demonstrating how integrated systems can transform routine survey data into actionable visualizations for ongoing service improvement.

Keywords: *Data Visualization, Data automation, Qlik Sense, SQL,*

Macro Variables in the New Economic Era: Examining the Relationship between Consumer and Producer Price Indices (1291)

Cem Baş[1*], Abdullah Çağrı Uçucu[1]

[1]Turkish Statistical Institute, 06100, Ankara, Türkiye

*Corresponding Author e-mail: cembas@metu.edu.tr

Abstract

The Consumer Price Index (CPI) and Producer Price Indices (PPI) aim to measure changes in the general and lower levels of prices. These index data, which are considered as macro variables, enable the determination of fiscal policies. In this study, the relationships between CPI and PPI rates will be analyzed at both macro and sub-factor levels. The causality relationship between CPI and PPI rates is analyzed by applying the Augmented Dickey Fuller (ADF) unit root test and Granger Causality Tests.

Keywords: *CPI, PPI, Augmented Dickey Fuller (ADF), Granger Causality Test*

Analysis of Consumer Price Index and Domestic Producer Price Index on Moving Averages (1294)

İlhami Mintemur[1*]

[1]Turkish Statistical Institute, Türkiye

*Corresponding Author e-mail: ilhamimintemur@gamil.com

Abstract

Moving averages are important indicators frequently used in financial and statistical analyses. By taking the Moving averages are important indicators frequently used in financial and statistical analyses. By calculating the average of a dataset over a specific period, they help reduce fluctuations and assist in identifying trends. This method, based on historical data, is especially significant in analysing economic indicators. The most common types of moving averages are:

- *Simple Moving Average (SMA)*
- *Exponential Moving Average (EMA)*
- *Weighted Moving Average (WMA)*

In this study, the Consumer Price Index (CPI) and the Domestic Producer Price Index (D-PPI) were analysed using short-term (5- and 10-month) and long-term (50- and 100-month) moving averages.

Short-Term Analysis: Both indices are currently close to their short-term averages, and a downward trend below the averages is projected.

Long-Term Analysis: Current CPI and D-PPI values are significantly higher than their long-term averages. The D-PPI shows a greater deviation. There is no observed tendency for either index to approach long-term averages.

As of December 2024, the D-PPI is 0.92% above the 5-month EMA and 3.37% above the 10-month EMA, indicating a short-term convergence trend. The CPI is 2.40% and 6.96% above the same averages, respectively.

***Keywords**: TÜFE, Yİ-ÜFE, SMA, MAC, WMA*

Analysis of Child Victimization, Pushed into Crime and Missing (Found) Incidents: A Markov Chain Application (1297)

Onur Can[1*], Sibel Atan[2]

[1]Turkish Statistical Institute, Social Statistics Department, Türkiye
[2]Hacı Bayram Veli University, Department of Econometrics, Türkiye

*Corresponding author e-mail: onur__can@windowslive.com

Abstract

The right of children to live a safe and happy life is one of the fundamental responsibilities of societies. However, millions of children worldwide and throughout history face various risks. Understanding the temporal trends of child-related incidents is crucial for developing preventive and protective policies. This study aims to examine child victimization, pushed into crime, and missing (found) cases based on official statistics and predict their future trends. For this purpose, a Markov chain analysis was applied using historical data on these cases.

Markov chains allow for predicting a system's behavior both in the short and long term by examining the transition probabilities between states. Additionally, they enable the calculation of the average time required for the process to return to the same state after starting from any given state.

The study utilized data from the 'Juvenile Statistics Received Into Security Unit' to construct the Markov chains. Initially implemented in 27 provinces in 1996, the 'Juvenile Statistics Received Into Security Unit' dataset was expanded to include all 81 provinces by 2007. Accordingly, the variables in the model were divided into two periods: 1996–2006 and 2007–2023. Annual variations in the reasons for children's referrals to security units were modeled as two distinct transition states (increase and decrease) within the Markov chain's state space. Since the first dataset did not cover all regions of Türkiye, it served as a prior estimate, providing an opportunity to evaluate the predictive power of the Markov chain analysis. The results from the initial dataset successfully predicted incident trends in the second dataset.

The second analysis revealed that for victimized and pushed into crime, the number of incidents is likely to continue increasing long-term, with an 81.3% probability, and the average recurrence time for these increases is approximately 1.2 years. The fact that decrease periods for these groups last longer than five years indicates the persistence of upward trends. In contrast, the situation for missing (found) children is more positive, with a decrease duration of three years and a 33.3% probability of improvement. This study aims to contribute an evidence-based framework to the literature by analyzing children's interactions with security units.

Keywords: *Child Victimization, Pushed Into Crime, Missing (Found) Children, Markov Chain*

How to Improve Quality of Official Statistics? (1299)

Serdar Cihat Gören[1*]

[1]Turkish Statistical Institute, Methodology Department, Türkiye

*Corresponding author e-mail: scgoren@gmail.com

Abstract

Quality refers to the level at which the users' satisfaction and needs are met by the product or service provided. While quality is an unattainable point of perfection, it is an ongoing concept that can be developed with the goal of continuous improvement. Like many products and services, the quality of official statistics is also directly related to the level of user satisfaction and how well their needs are met.

To improve the quality of official statistics, many international principles and standards have been adopted. These principles ensure that official statistics are produced of higher quality and made accessible to users. Additionally, just like any other product or service, official statistics should also be continuously improved by identifying areas for development and creating action plans for their enhancement.

This study examines the concepts of quality and official statistics. It also explores international principles and standards in the context of improving the quality of official statistics and provides examples of best practices for enhancing quality.

Keywords: *Quality, Official Statistics, Users, Relevance*

The Process of Statistical Usage in International Political Economy: Opportunities and Challenges (1301)

Yusuf Girayalp Atan[1*]

[1]Turkish Statistical Institute, Istanbul Regional Directorate, European Coordination Group Directorate, Short-Term Business Statistics Team, Türkiye

*Corresponding author e-mail: yusufgirayalp.atan@tuik.gov.tr

Abstract

International political economy represents a complex and multidimensional field where economic relationships intertwine closely with political dynamics in an increasingly globalized world. In this context, statistical methods provide policymakers with objective data, establishing an effective and scientific foundation for policy-making processes. The application of statistics is expanding in critical areas such as the analysis of international trade, evaluation of foreign direct investments, forecasting global financial crises, and measuring political risks. However, due to the inherent nature of international political economy, this process comes with certain significant opportunities and challenges.

The primary objective of this research is to identify opportunities that could enhance the effectiveness of statistical usage in international political economy and to thoroughly define the challenges that impede this process, while offering concrete solutions. It is argued that accurate and appropriate statistical analyses enable the creation of objective, consistent, and effective policies in international politics, though various challenges limit this potential.

Initially, the research outlines the detailed processes involved in using statistics within international political economy, examining each stage clearly—data collection, analysis, interpretation, and reporting. Subsequently, the opportunities generated by statistical methods in international economic and political contexts are evaluated. Specific advantages, such as enhancing policy effectiveness and objectivity, enabling international comparisons, and creating early-warning systems, are illustrated with concrete examples.

Conversely, the research addresses significant challenges, including issues with data quality, inconsistencies in cross-country data, risks of data manipulation, and complexities inherent in statistical methods. These challenges are examined through international datasets and applied examples. Case studies analyzing both successful and unsuccessful examples are conducted to provide findings that serve as guidance for policymakers.

Finally, concrete recommendations are proposed to increase the efficiency of statistical usage in international political economy. These suggestions include standardizing data collection practices, expanding international education and capacity-building programs, enhancing transparency, and making statistical models more comprehensible to decision-makers. Conclusively, the study provides a comprehensive evaluation of statistical applications within international political economy, offering forward-looking perspectives for future research and policy-making efforts.

Keywords: *International Politic Economy, Statistical Analysis, Data Quality, Politic Risk, Data Standardization*

FULL-TEXT PRESENTATIONS

Impact of the Informal Economy on the Efficiency and Productivity of Pakistan's Agricultural Sector (1016)

Jovera Shakeel[1], Iman Attique[1], Munazza Nadir[1]

[1]Lahore University of Management Sciences, Baluchistan, Pakistan

*Corresponding author e-mail: jovera.shakeel02@gmail.com

Abstract

According to the Ministry of Finance, more than 40 percent of Pakistan's GDP is attributed to the informal sector. Nearly 75 percent of Pakistan's working-age population is employed in the informal sector, according to the Labour Force Survey (2020-2021). The widespread persistence of the informal sector has several manifestations in the country's agricultural economy. This study analyses the impact of the informal economy on agricultural productivity in Pakistan by applying Stochastic Frontier and Principal Component Analysis models using the Pakistan Standards of Living Measurement (PSLM) farm-level data collected in 2014, 2016, and 2019. It is the first regionally and nationally representative study of the informal economy's impact on agricultural indicators using the country's largest dataset. These findings show, as prior literature has suggested, that farms utilizing formal economic relations, including better working employment contracts, more access to proper credit resources, and better irrigation systems, produce higher yields than farms that operate within informal structures. In addition, crop diversification and resource allocation were found to be significant in raising the efficiency of agriculture. But there is also a geographical dimension to productivity - some of the agro-climatic regions are lagging consistently implying a case for focused attention.

Keywords: *Agricultural Productivity, Farm Efficiency, Cropping Patterns, Agro-climatic Zones, Crop Diversification, Informal Economy, Stochastic Frontier Analysis, Resource Allocation*

INTRODUCTION

Pakistan's informal economy significantly involves the country's labour force and economic activities, particularly in agriculture. The sector shapes the livelihoods of millions of individuals. Therefore, a crucial attempt to scrutinize how the informal economy affects Pakistan's agricultural productivity and efficiency was required. This provides insights for formulating policies that foster growth and development across the country. Given that the informal economy limits access to formal credit, modern technologies, and structured labour practices, we expect that it negatively impacts agricultural productivity and efficiency in Pakistan.

Context

Approximately 80% of Pakistan's workforce is engaged in the informal economy, which encompasses a broad spectrum of economic activities ranging from small-scale farming to street vending and domestic services (International Labour Organization n.d.). A World Bank report valued the informal economy at $457 billion in 2022, accounting for 35.6% of the country's GDP-PPP level.

The majority of people live on farms, which is typical of economies relying on agriculture. There is a slight bias towards informal work in rural areas, but it accounts for almost 69% of employment in urban areas. Numerous factors can be attributed to the growth of this industry. Lack of regulation is one way in which the informal sector differs from the formal economy. In particular, agriculture is made up of a large number of unregistered businesses that also hire seasonal workers and local farmers who toil in unofficial conditions without oversight.

This widespread informality has significant effects on the region's productivity and wealth and it presents both opportunities and challenges for development initiatives meant to promote economic growth. Due to a lack of formal employment opportunities informal employment plays a crucial role in offering gainful employment in economically poor areas. This phenomenon is most evident in rural settings since the government cannot support crucial public utilities such as construction, health services, or education without experiencing a massive loss owing to unpaid taxes. This is evident in the low level of investment in modern farming techniques that reduce productivity and profitability.

These services are crucial in enhancing the production and productivity of agriculture. In addition, low pay, unreliable employment, and unfavourable working conditions are frequently associated with informal agricultural labour. As a result, the industry becomes less efficient which in turn threatens the economy's growth prospects. Barriers such as insufficient capitalization, insufficient infrastructure, and insufficient capacity building also hinder endeavours in this area. In particular and with a reference to productivity and efficiency, this paper will discuss the effects of Pakistan’s informal economy on the country’s agriculture sector.

Literatüre Review

Pakistan's agricultural efficiency and productivity are linked to access to formal credit. In Khyber Pakhtunkhwa, the availability of formal financing significantly impacts the productivity of strawberry farmers, according to Iqbal, Niaz, & Munir (2018). A 15% increase was showcased in their study when viewing agricultural productivity for farmers with access to official loans compared to those relying on informal financing systems. This finding is consistent with many studies reiterating the positive impact of formal credit on agricultural productivity. For instance, Benjamin et al. (2014) highlighted how casual agricultural businesses struggle to access programs funded through unofficial means, leading to reduced productivity. A 2021 study on cotton production in Pakistan found that farmers using formal labour contracts and financing achieved a 25% higher yield per hectare compared to those reliant on informal labour and credit sources. The study also indicated that farmers who opted for informal lending mechanisms faced interest rates that were 30% higher than those who had access to formal credit. This created a significant challenge for such farmers when deciding to reinvest in technologies that would improve their output but decrease their profit margins. The importance of credit was further underscored by Mushtaq A. Khan and Abid Burki (2013) by providing evidence on how companies with easier access to formal credit were able to invest more in technology and training, which is shown to boost productivity and reduce inefficiencies.

The level of education attained by farmers is also a crucial factor influencing agricultural productivity. The research by Saeed et al. (2021) indicates that farmers with secondary education achieved a 10% higher output than those who had primary or no formal education. This remained consistent across research on agriculture; it was found that higher education is linked with increased productivity and efficiency. Educated farmers have a deeper understanding of modern agricultural technologies and methods, facilitating better farm operations optimization.

Two key elements influencing agricultural efficiency are farm size and farming experience. According to Saeed et al., farmers with more than ten years of experience outperformed their less experienced counterparts by 12% (2021). This study supports the idea that seasoned farmers are better qualified to oversee farm operations effectively. However, the study did note that larger farms with lower levels of formal education might be less productive. This implies that to get the most output informal knowledge alone might not be sufficient.

The utilization of informal labour methods is a significant factor influencing productivity in Pakistan's agricultural sector. The employment of informal labour while flexible can result in inconsistent output and low quality as highlighted by research on cotton production in 2021. 70% of cotton production workers

were not part of the formal labour force and their productivity decreased by 20% in comparison to formal labour practices. In formal labour contracts, there was a 25% increase in yield per hectare. Unreliable prices caused by unofficial market activity were blamed for farmers' decreased capacity to invest and unstable revenue. This also prevented farmers from investing in new technology and resulted in a 10% annual decline in average income. This finding which applies to the agriculture sector states that informal labour practices in the manufacturing sector lead to inefficiencies Mushtaq and Burki argue that formalising labour contracts can improve both productivity and the use of available resources.

Literature highlights how resource utilization is endangered by unregulated market conditions and informal labour practices. Saeed et al. indicate that depending solely on unofficial lending channels and informal information results in increased inefficiencies. The absence of formal agreements worsened the financial instability of farmers, leading to a 15% increase in default rates in informal credit transactions. Benjamin et al. (2014) emphasize that informal agricultural enterprises limit their access to financial services and training programs by not registering or filing taxes, perpetuating inefficiencies.

Informal labour practices and unregulated markets pose a threat to resource utilization, as demonstrated by research. Saeed et al. point out that increased inefficiencies in 2021 resulted from reliance on unofficial lending channels and informal information. The 15% rise in default rates in informal credit transactions due to the absence of formal agreements further exacerbated farmers' financial instability. Benjamin et al. noted that unregistered agricultural enterprises' failure to file taxes restricts their access to financial services and training opportunities, contributing to inefficiencies. According to academic studies, extensive policy interventions are necessary to address the inefficiencies arising from the informal economy. The agriculture sector would benefit from expanded formal credit availability, improved farmer education and training programs, and the development of formal labour contracts. Additionally, implementing equitable pricing policies and regulated markets could potentially mitigate the adverse impacts of informal practices on the overall performance and productivity of this sector. Understanding labour dynamics, family size, and regional differences is essential for evaluating productivity.

Incorporating these variables into efficiency models can provide valuable insights into the factors influencing productivity and inefficiency in Pakistan's agricultural sector, especially because no prior nationally and provincially representative research of this kind has been conducted. Therefore, this paper aims to investigate how the informal economy affects productivity and efficiency in Pakistan's agricultural sector using the largest available dataset in Pakistan, the Pakistan Standards of Living Measurement (PSLM) survey.

MATERIAL AND METHODS

Our data is taken from the Pakistan Standards of Living Measurement (PSLM) survey, a comprehensive regular study with economic and social indicators at the provincial and district levels, to evaluate farm production in relation to the informal economy. We analyzed farming households using data from three different years (2014, 2016, and 2019). With the use of this longitudinal technique, we could track changes and patterns in agricultural productivity and efficiency over time.

We specifically looked at the Household Integrated Income & Consumption Survey (HIICS) for these years, focusing on the farm and household portions. We compiled relevant data into the STATA software and our primary raw variables included land ownership status, total land owned, land rent, acres rented out, rent received, land value, and various costs related to agricultural operations (e.g., seeds, fertilizers, pesticides, utilities, labour, equipment rent, and other expenses). Additionally, household identifiers like province and region were also part of the dataset. Others like household asset index to get a proxy of income, government support in terms of BISP receiving, and cropping combinations, if used on the farm, were added.

We observed that a significant portion of our data had values of 0, and we addressed this by creating agro-climatic zones in our data. The zoning framework we have used in our study is based on an established classification of agro-climatic regions rooted in the work of Pickney (1989), further developed by subsequent studies, including the Asian Development Bank (2005) report. This zoning excludes Gilgit Baltistan, FATA, and AJK due to their limited agricultural practices and the unavailability of relevant data. We focus on nine distinct agro-climatic zones, which are defined as follows:

1. Other KPK
2. Low-Intensity Punjab KPK
3. Rainfed Punjab Islamabad
4. Mixed Punjab
5. Rice-Wheat Punjab
6. Cotton-Wheat Punjab
7. Rice-Other Sindh Balochistan
8. Cotton-Wheat Sindh
9. Other Balochistan

We placed districts from the PSLM dataset for three years into their respective zones. Dummy variables were created for each of these zones, labelled 1 through 9, allowing for detailed analysis across the different regions. This zoning method allows us to analyze the impacts of agro-climatic conditions on the informal economy and productivity across Pakistan's diverse agricultural landscape. A detailed listing of districts and their zonal categories can be found in Figure 1.2 of the appendix. Some districts had farms that did not report a particular input entirely so an average could not be taken, hence the zonal average was substituted. Then, we only kept entries that had cultivated land and reported harvesting crops for our analysis. This yielded 13,462 total observations. This ensured that our dataset was ready for subsequent analysis.

Equation One

$$ln(y)=\beta 0+\beta 1ln(x1)+\beta 2ln(x2)+\beta 3ln(x3)+\beta 4ln(x4)+\beta 5ln(x5)+\beta 6ln(x6)+\epsilon$$

where

y is the value of production per acre (real).

x1 represents the total land under cultivation during the last Rabbi and Kharif seasons reported in acres.

x2 represents labour per acre of cultivated land. It consists of the sum of the following elements: freight, transportation, commission, insurance, storage, etc. charges, payments to permanent labour, and payments to casual and other labour.

x3 represents the cost invested in seeds/plants (including delivery charges) per acre of cultivated land.

x4 represents the cost of pesticides, chemical fertilizers, and farm yard manure (including delivery charges) per acre of cultivated land.

x5 represents the cost of utilities: water, electricity, and all other fuel charges per acre of cultivated land.

x6 represents the cost of all types of taxes, rent of equipment, animals (tractor, thresher, bullock, etc), rent paid for land during the last Rabbi and Kharif season, and any other miscellaneous expenses per acre of cultivated land.

ϵ is an error term accounting for variability in the dependent variable y.

We then logged each x variable for our log-linear analysis for the stochastic frontier. The stochastic frontier analysis (SFA) is an economic modelling method that measures the efficiency of production units, and farms in this case, by estimating the gap between observed output and the maximum possible output called

the production frontier, given the set of inputs including land, labour, and fertilizers. The approach is ideal for our study to understand how much output deviation from the frontier is due to inefficiency versus random shocks or external factors.

Equation Two

$$ui = \delta 0 + \gamma 1z1 + \gamma 2z2 + \gamma 3z3 + \gamma 4z4 + \gamma 5z5 + \gamma 6z6 + \delta 1z7t1 + \delta 2z7t2 + \delta 3z7t3 + \delta 4t2 + \delta 5t3 + wi$$

where

wi is the inefficiency term, which is modelled as a function of the z variables.

z1 is a binary variable representing whether the respondent rented any agricultural land on a cash basis in the last Rabbi and Kharif season.

z2 is a dummy constructed from a continuous variable that accounts for acres of irrigated cultivated land. The dummy represents whether the respondent's land was irrigated.

z3 is a dummy constructed from a continuous variable that accounts for annual income from the Benazir Income Support Programme (BISP). Hence the dummy represents whether the respondents received BISP or not. The BISP variable itself has been used as a proxy for government subsidy.

z4 represents the number of individuals who are working as 'family workers', yielding total family workers per household/farm.

z5 represents the total family size per household/farm.

z6 is a dummy representing the region of the farm, rural or urban.

z71, z72, z73, and z74 are dummies generated to represent the provincial location of the farm - KPK, Punjab, Sindh, and Balochistan, respectively.

z8 is a continuous variable for the asset index. To construct the asset index, we first gathered data on key financial assets, including net savings, value of precious metals, stocks, and loans. We then collected information on various land holdings such as agricultural land, non-agricultural land, and buildings. After merging these datasets based on household identifiers, we applied Principal Component Analysis (PCA). PCA reduced the data's complexity by identifying the main components of variation. The first principal component was used to create the asset index, providing a comprehensive measure of household wealth.

RESULTS

After running the Stochastic Frontier, we were able to view the efficiency level for each farm. Its summary statistics for each year are displayed in Table 1, including the efficiency of the highest and lowest-performing farms.

Table 1. Farm efficiencies for the years 2014, 2016, and 2019

-> year = 2014

Variable	Obs	Mean	Std. dev.	Min	Max
efficiency	4,490	.7366658	.1168753	.000023	.9476124

-> year = 2016

Variable	Obs	Mean	Std. dev.	Min	Max
efficiency	3,324	.7728129	.098837	.0000516	.9473826

-> year = 2019

Variable	Obs	Mean	Std. dev.	Min	Max
efficiency	5,648	.7541043	.1134422	.0000415	.9480948

Source: These statistics have been generated from the data we compiled using the PSLM dataset from years 2014, 2016, and 2019.

Table 2: Summary statistics for overall efficiency over the three years: 2014, 2016, 2019

Efficiency

Percentiles		Smallest		
1%	.3509277	.000023		
5%	.5265461	.0000415		
10%	.6140913	.0000458	Obs	13,462
25%	.711805	.0000516	Sum of wgt.	13,462
50%	.7804276		Mean	.7529075
		Largest	Std. dev.	.1120356
75%	.8262767	.9473826		
90%	.8581576	.9476124	Variance	.012552
95%	.8739734	.94802	Skewness	-1.878813
99%	.9059548	.9480948	Kurtosis	8.55659

Source: These statistics have been generated from the data we compiled using the PSLM dataset from the years 2014, 2016, and 2019

To provide an in-depth view of how efficiency was distributed, we observed the data as provided in Tables 3, 4, and 5. For 2014, the data has a broad distribution for the observations, and the mean efficiency lies at 0.78. Negative skewness indicates that there are a significant number of observations with very low efficiency. On the other hand, the presence of high-performing farms in the top percentiles indicates the optimal use of resources. For 2016, the efficiency data shows an overall improvement in comparison to 2014. There has been an increase in mean and median efficiency scores alongside a reduction in variance,

suggesting that farms are making better use of their resources. The continued presence of negative skewness suggests that the farms with low efficiency could benefit from targeted intervention. For 2019, while some entities have maintained efficiency, indicated by the stability in the 75th and 99th percentile, there has been a decline in mean and median efficiency. This suggests that some farms are encountering new problems and have failed to maintain the improvements made in the previous years.

Table 3: Summary statistics for 2014

Efficiency

	Percentiles	Smallest		
1%	.3154516	.000023		
5%	.5166641	.0000529		
10%	.5973497	.0000559	Obs	4,490
25%	.6884258	.0001127	Sum of wgt.	4,490
50%	.761897		Mean	.7366658
		Largest	Std. dev.	.1168753
75%	.8129072	.9373819		
90%	.8497123	.9416275	Variance	.0136598
95%	.8677024	.9465716	Skewness	-1.897747
99%	.9109662	.9476124	Kurtosis	9.352191

Source: Data has been self-generated in STATA using the PSLM dataset from 2014

Table 4: Summary statistics for 2016

Efficiency

	Percentiles	Smallest		
1%	.4206568	.0000516		
5%	.571726	.073893		
10%	.6490835	.1818228	Obs	3,324
25%	.738066	.183447	Sum of wgt.	3,324
50%	.7958149		Mean	.7728129
		Largest	Std. dev.	.098837
75%	.8371228	.9330864		
90%	.8663852	.934897	Variance	.0097688
95%	.881142	.9431141	Skewness	-1.911596
99%	.9060862	.9473826	Kurtosis	8.664771

Source: Data has been self-generated in STATA using the PSLM dataset from 2016

Table 5: Summary statistics for 2019

Efficiency

	Percentiles	Smallest		
1%	.3492424	.0000415		
5%	.5166441	.0000458		
10%	.6050869	.0001734	Obs	5,648
25%	.7158904	.0820836	Sum of wgt.	5,648
50%	.7851052		Mean	.7541043
		Largest	Std. dev.	.1134422
75%	.828218	.9357161		
90%	.8590165	.9375362	Variance	.0128691
95%	.872419	.94802	Skewness	-1.825903
99%	.9009422	.9480948	Kurtosis	7.559

Source: Data has been self-generated in STATA using the PSLM dataset from 2019

The analysis of farm efficiency vis-a-vis their periods of 2014, 2016, and 2019 depicts the constant patterns in the determinants of productivity and the huge differences between efficient and inefficient farms. A striking theme emanating from the data is that efficient-style farms thrive on input investments, cropping strategies, and irrigation access, while the lowly efficient varieties bear the burden of lack of diversification, inadequate finances, and poorly advanced management practices.

High-performing farms have one of their major defining characteristics in the engagement of quality inputs, namely labour, seeds, and fertilizers. In the three years on average, farms with higher-than-average efficiency consistently afforded more resources for the hiring of skilled labour, the purchase of quality seeds, and a broad level of fertilization. On average, 85% of top-performing farms significantly spent on labour management that they claimed to elevate productivity. In a similar vein, 75% of farms spent relatively more money on pesticides and soil nutrition because this is an indication that technical agronomic best management practices are efficient. While this was not the case for the lower-performing farms, which couldn't reach the required investment levels due to budget constraints, their reliance on informal credit also prevented them from investing adequately in productivity-enhancing inputs.

The accessibility of irrigation defines another very important determinant of farm efficiency. High performers display a strong tendency towards consistent, properly managed irrigation, with over 90% of the highest farms relying on irrigated land as opposed to rain-fed agriculture. This underlines how important water availability is to both yield stability and overall efficiency. Contrarily, we see that a large number of inefficient farms depended on seasonal rains only, thereby making them extremely susceptible to the vagaries of climate and drought-like conditions. Particularly apparent was the constant irrigation gap in Sindh and Balochistan; as a result, farmers in these regions recorded very low efficiency scores, thereby necessitating specific irrigation infrastructure investments in these areas.

Crop diversification also emerged as an important factor in determining efficiency levels. Efficient farms, which diversified their crop rotations by practising wheat-cotton and wheat-rice rotations, outperformed those practising scientific monoculture. At least 40% of the best-performing farms were involved in wheat-rice combinations whose remaining 35% grew other crops in addition to a staple wheat, providing more evidence for the hypothesis that timely crop rotations can minimize soil degradation by enhancing soil fertility and pest resistance. In contrast, the worst-performing farms were mostly dominated by monoculture, which cultivated at least one or two crops in a very long growing season. The challenge of

efficiency was further aggravated by crop monoculture, contributing to lower soil productivity, poor resistance to pest infestations, and lower long-term yield sustainability.

Another key distinguishing factor between a high-performing farm and a low-performing farm is its integration into the formal economic structures. Farms accessing institutional credit, the structure of labour contracts, and government assistance programs had largely higher degrees of efficiency. A notable trend among high-performing farms was active credit in a positive formal way; given the possibility of obtaining credit as a reinvestment decision, farmers by far had better equipment usage and greater amounts of fertilizers applied and irrigation systems installed. Except for lower-performing farms, where most turned to informal sources for credit—contingent on a dire reliance on informal credit due to high interest rates, their reinvestment options in agricultural development remained limited. Again, the many low-performing farms employed casual unskilled labour rather than structured employment contracts, resulting in considerable inconsistency in labour quality and productivity.

Regional disparities played a critical role in farm efficiency. Farms located in Punjab's Cotton-Wheat and Rice-Wheat agro-climatic zones signified consistent high farm efficiency. These advantages were generated by good soil fertility conditions, accessible markets, and much-established infrastructure. This is in stark contrast to Sindh and Balochistan, where farm efficiency scores have consistently ranked lower due to climatic challenges aggravated by weak financial support systems and a lack of irrigation facilities. Interestingly, certain areas in Khyber Pakhtunkhwa (KPK) showed interesting improvements in farm efficiency, thereby implying that localized adaptations and sustainable practices could partly compensate for otherwise challenging conditions.

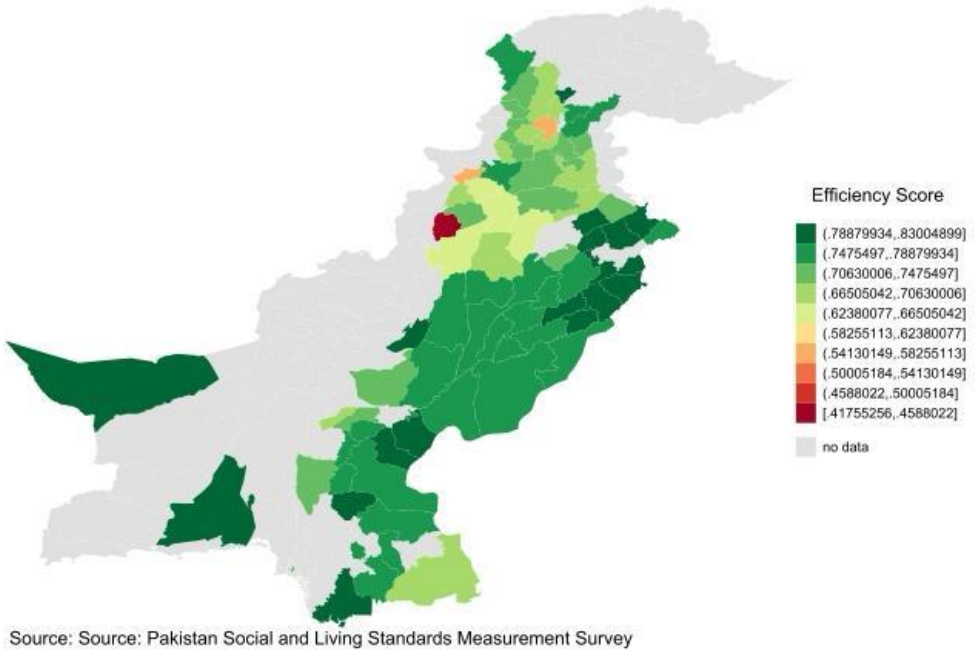

Figure 1: Farm Efficiency Scores Across Districts in Pakistan

The figure above presents farm efficiency scores across districts in Pakistan, derived from the Pakistan Social and Living Standards Measurement Survey. The efficiency scores are visually represented through a gradient, with darker green shades indicating higher farm efficiency and red shades representing lower efficiency. Punjab's Cotton-Wheat and Rice-Wheat agro-climatic zones exhibit high-efficiency levels, aligning with prior analysis that links better infrastructure, irrigation access, and formal credit availability

to improved productivity. Conversely, districts in Sindh show lower efficiency scores, reinforcing the argument that inadequate irrigation facilities, lack of access to structured financial markets, and climatic constraints significantly hinder agricultural productivity. The spatial disparities in farm efficiency emphasize the need for region-specific policy interventions to enhance productivity in low-performing areas.

While there were fluctuations in the trend of efficiency through the years, the basic drivers of good agricultural productivity prevailed: greater investment in inputs, consistent assured provision of irrigation, crop diversification, and integration into formal financial and labour structures. That said, inefficient farms constantly struggled with little diversification, weak financial power, weak irrigation, and informal participation in the economy. This justifies the necessity of policy interventions widening financial access, improvement of water management measures, and enhancement of sustainable farming institutes.

Limitations

The research paper has certain limitations. First, the dataset covers only three non-consecutive years, limiting our ability to observe long-term trends and potentially overlooking the effects of certain policy changes or external factors. Secondly, before analysis, a large number of missing values in the dataset were substituted with mean values. Even though it is required, this method could skew the data in favour of the average and hide more subtle findings.

We encounter difficulties with the variables as well. The influence of government support on agriculture may not be fully captured by using BISP as a proxy in the lack of a clear metric for subsidies. We are also limited in our research of the impact that formal credit plays in improving efficiency because there are no variables to quantify access to this important element in agricultural output. Lastly, there are no statistics on weather patterns or associated details provided by the PSLM study. Even though we established zones per national weather trends, this presents a challenge when assessing the agricultural industry, as the weather has a substantial impact on productivity.

CONCLUSION

Policy Recommendation

To improve agricultural efficiency and productivity in Pakistan, decision-makers need to implement a focused and region-specific strategy. The results of this research reveal that informality in land tenure, access to credit, and labour setups adversely affect farm productivity, especially in low-performing agro-climatic regions.

Localized Agricultural Assistance Initiatives

Efficiency levels differ greatly among agro-climatic zones, with Punjab's Cotton-Wheat and Rice-Wheat areas outperforming Sindh and Balochistan, which face challenges like inadequate irrigation, limited market integration, and climatic limitations. Focused actions ought to encompass:

- Sindh and Balochistan (Zones 7, 8, 9): Development of irrigation infrastructure, such as solar-powered tube wells and community-operated water storage systems, to lessen reliance on unpredictable rainfall.
- Khyber Pakhtunkhwa & Low-Intensity Punjab (Zones 1 & 2): Advancement of climate-resilient crop types and rainwater collection systems to enhance yield in areas with limited water resources.
- Punjab's Cotton-Wheat and Rice-Wheat Areas (Zones 5 & 6): Enhanced funding for research aimed at improving crop rotations and reducing reliance on monoculture.

Formalization of Land and Credit Sectors

Farms that have access to formal credit consistently do better than those that depend on informal financing, but numerous farmers in informal land setups continue to be shut out from credit systems. To close this divide:

- Restructuring land ownership and tenure systems is necessary to enhance productivity. Stable land tenure and favourable rental arrangements are vital, alongside implementing land reforms to support formal land markets and resolve disputes. A land titling initiative ought to be established to legitimize land ownership, enabling farmers to utilize land as security for loans.
- Microfinance models tailored to specific regions should be implemented to provide smallholder farmers in less productive areas with low-interest loans.
- A guarantee program for small farmers in high-risk regions (Sindh & Balochistan) needs to be established to ensure that financial entities provide loans even under difficult circumstances.

Increase in Market Linkages and Infrastructure

A significant obstacle to productivity in informal agriculture is the limited access to organized markets and supply chains. Tackling this entails:

- Decentralized processing centres in Balochistan and KPK to help small farmers collectively gather products, lower transport expenses, and obtain improved prices.
- Mechanisms for price stabilization for farmers in fluctuating markets, guarantee equitable returns on high-input crops like cotton and sugarcane.
- Investment in digital trading platforms allows farmers to avoid exploitative middlemen and connect directly with buyers.

Encouraging Crop Variety and Eco-friendly Agriculture

Monoculture continues to be a significant concern, as wheat-rice rotations prevail and lead to soil degradation. To encourage sustainable variety:

- Incentives for crop diversification ought to be implemented, providing financial assistance for farms that grow diverse crops (e.g., wheat-cotton or wheat-legumes). Investing in water management infrastructure, such as drip irrigation systems financed through low-interest loans, will ensure reliable water sources, boost crop yields, and stabilize farm output in water-scarce areas.
- In Punjab's high-intensity areas, subsidies for precision agriculture technologies should be provided to guarantee efficient input utilization while preventing soil degradation.
- The government needs to enhance agricultural extension services by offering farmers education on multi-cropping methods and soil conservation practices.

Enhancing Rural Employment Markets & Social Safeguards (such as the BISP)

Farmers using formal labour contracts show much greater efficiency compared to those depending on informal labour. Even where this awareness is present in Pakistan, informal employment continues to be common, resulting in decreased productivity. To tackle this:

- A system for registering seasonal labour must be established to guarantee that agricultural labourers obtain organized pay and social protections.
- Programs for skill development should be initiated, concentrating on mechanized agriculture, effective irrigation management, and contemporary pest control methods.

Reforms in Water Management and Irrigation

Since more than 90% of productive farms depend on irrigation, enhancing water accessibility is a vital policy focus. We propose the following:

- Development of water conservation infrastructure (e.g., check dams and lined channels) in dry regions like Balochistan and interior Sindh.

- Subsidies for smart irrigation aimed at high-efficiency systems such as drip irrigation and laser land levelling in the Rice-Wheat and Cotton-Wheat regions of Punjab.
- Cooperatives of farmers operating in a decentralized manner oversee common irrigation resources, minimizing the misallocation and waste of water.

Way Forward

Future research ought to set an emphasis on longitudinal studies that monitor the effects of market and agricultural policy changes over time on farm efficiency, to increase agricultural output and deepen our understanding of the subject. A better perspective can also be achieved by integrating qualitative information from farmer interviews with quantitative data. Lastly, evaluating the success of current agricultural policies will provide information on their real-world effects, directing the creation and modification of new policies in the future.

CONCLUSION

This research analyses the effect of the informal economy on Pakistan's agricultural sector efficiency and productivity. To achieve these objectives, the study has used nationally and regionally representative data from the PSLM survey and applied methodologies like Stochastic Frontier Analysis and Principal Component Analysis. Such analytical observations show that issues of informality in the land, labour, and credit markets imply adverse impacts on productivity and cost recovery particularly in areas with limited adoption of these technologies and physical modalities.

The study supports our hypothesis that informality yields lower productivity in the agricultural sector while access to capital and modern innovative agricultural technologies, as well as crop management techniques, yield noticeably higher performance. Despite its limitations, including gaps in longitudinal data and incomplete variables for government support and credit access, the research underscores the urgent need to address the structural inefficiencies in the agricultural sector caused by informality. Policymakers must focus on initiatives such as formalizing the agricultural economy, enhancing access to credit and subsidies, improving rural infrastructure, and promoting sustainable farming practices.

In conclusion, the findings of this study call for a holistic approach to reforming Pakistan's agricultural sector. By addressing the inefficiencies perpetuated by informality, fostering regional equity, and equipping farmers with the tools and knowledge needed to optimize productivity, Pakistan can significantly enhance its agricultural output and contribute to broader economic development.

Appendix

Exhibit A1. Summary Statictics for Dataset

VARIABLES	(1) N	(2) mean	(3) sd	(4) min	(5) max
wheat_rice	13,462	0.252	0.434	0	1
sugarcane	13,462	0.239	0.426	0	1

wheat_other	13,462	0.561	0.496	0	1
crop_num	13,462	2.735	1.260	0	8
x1	13,462	5.611	9.918	1	322.5
x2	13,462	4,823	7,128	32.25	192,000
x3	13,462	3,964	4,210	28.45	136,542
x4	13,462	9,304	8,194	60	221,881
x5	13,462	5,185	5,759	41.67	81,925
x6	13,462	8,981	15,334	21.50	833,333
y	13,462	69,644	58,747	1	1.451e+06
zone	13,462	4.975	2.684	1	9
id	13,462	6,732	3,886	1	13,462
t	13,462	2.086	0.864	1	3
ln_y	13,462	10.90	0.808	0	14.19
ln_x1	13,462	1.224	0.935	0	5.776
ln_x2	13,462	6.154	3.352	0	12.17
ln_x3	13,462	7.745	1.560	0	11.82
ln_x4	13,462	8.548	1.805	0	12.31
ln_x5	13,462	4.596	4.164	0	11.31
t1	13,462	1.086	0.864	0	2
ln_x6	13,462	8.356	1.689	0	13.63
z1	13,462	0.134	0.341	0	1
z2	13,462	0.816	0.388	0	1
z3	13,462	0.138	0.345	0	1
z4	13,462	1.085	1.378	0	20
z5	13,462	7.482	3.629	1	55
z6	13,462	0.928	0.258	0	1
z71	13,462	0.196	0.397	0	1
z72	13,462	0.453	0.498	0	1
z73	13,462	0.271	0.445	0	1
z74	13,462	0.0802	0.272	0	1
t_2	13,462	0.247	0.431	0	1
t_3	13,462	0.420	0.494	0	1
z8	13,462	-0.0696	0.830	-4.014	25.89
efficiency	13,462	0.753	0.112	2.30e-05	0.948

Source: Data has been self-generated in STATA using the PSLM dataset from years 2014, 2016, and 2019.

Exhibit A2. Districts Classification into Zones

Zone	**Districts Within Zone**
1 Other KPK	Chitral, Upper Dir, Lower Dir, Swat, Shangla, Bonair (Buner), Malakand, Kohistan, Mansehra (Mansehra), Batagram, Abbottabad, Haripur, Tor Garh (Tor Ghar), Mardan, Swabi, Charsada (Charsadda), Peshawar, Nowsehra (Nowshera), Kohat, Hangu, Karak, Bannu, Lakki Marwat, Tank, Bajur, Khyber, Mohmand, Kurram, Orakzai, South Waziristan

2 Low-Intensity Punjab-KPK	D. I. Khan (Dera Ismail Khan), Bhakhar (Bhakkar), Mianwali, D. G. Khan (Dera Ghazi Khan), Rajanpur, Layyah, Muzaffar Garh (Muzaffargarh)
3 Rain-Fed Punjab-Islamabad	Attock, Rawalpindi, Jehlum, Chakwal, Islamabad
4 Mixed Punjab	Sargodha, Khushab, Faisalabad, Chiniot, Jhang, T.T. Singh (Toba Tek Singh), Okara
5 Rice-Wheat Punjab	Gujranwala, Hafizabad, Gujrat, Mandi Bahuddin, Sialkot, Narowal, Lahore, Kasur, Sheikhupura, Nankana Sahib
6 Cotton-Wheat Punjab	Sahiwal, Pakpattan, Vehari, Multan, Lodhran, Khanewal, Bahawalpur, Bahawalnagar, Rahim Yar Khan
7 Rice-Other Sindh-Balochistan	Jacobabad, Kashmore, Shikarpur, Larkana, Shahdadkot, Dadu, Jamshoro, Badin, Thatta, Sujawal (Sijawal), Karachi Central
8 Cotton-Wheat Sindh	Sukkur, Ghotki, Khairpur, Nowshero Feroze, Nawabshah, Hyderabad, Tando Allah Yar, Tando Muhammad Khan, Matiari, Sanghar, Mir Pur Khas (Mirpurkhas), Umer Kot (Umerkot), Tharparkar, Malir, Shaheed Benazirabad
9 Other Balochistan	Nushki, Sherani, Quetta, Pishine (Pishin), Qilla Abdullah, Loralai, Barkhan, Musa Khel (Musakhel), Kalat, Qilla Saifullah, Zhob, Sibbi (Sibi), Ziarat, Dera Bugti, Bolan/ Kachhi (Kachhi), Jaffarabad, Nasirabad/ Tamboo (Nasirabad), Jhal Magsi, Lasbela, Lehri, Panjgur, Sohbatpur

Source: 'Assessment of Farmers' Vulnerability to Climate Change in Agro-Climatic Zones of Pakistan: An Index-Based Approach

Exhibit A3. Statistics for Farms

Category	**Top Performing 20 Farms (2014)**	**Bottom Performing 20 Farms (2014)**
Cropping Patterns		
Wheat-Cotton	4 farms	0 farms
Wheat-Rice	2 farms	0 farms
Wheat-Sugarcane	4 farms	0 farms
Wheat-other	7 farms	2 farms

Crop number	1-5	0-2
X Variables		
X1 (Mean: 6.761)	6 farms above mean	7 farms above mean
X2 (Mean: 4811.52)	17 farms above mean	5 farms above mean
X3 (Mean: 3815.69)	13 farms above mean	7 farms above mean
X4 (Mean: 9520.44)	15 farms above the mean	4 farms above mean
X5 (Mean: 5216.31)	10 farms above the mean	6 farms above mean
X6 (Mean: 8686.06)	7 farms above mean	7 farms above mean
Z Variables		
Z1 (Mean: 0.121; Std Dev: 0.326)	3 farms above mean	0 farms above mean
Z2 (Mean: 0.822; Std Dev: 0.383)	18 farms above mean	9 farms above mean
Z3 (Mean: 0.135; Std Dev: 0.341)	3 farms above mean	3 farms above mean
Z4 (Mean: 1.043; Std Dev: 1.328)	4 farms above mean	2 farms above mean
Z5 (Mean: 7.528; Std Dev: 3.584)	12 farms above mean	12 farms above mean
Z6 (Mean: 0.955; Std Dev: 0.207)	17 farms above mean	17 farms above mean
Agro-climatic Zones		
Zone 1	2 farms	11 farms
Zone 2	1 farm	3 farms
Zone 3	2 farms	0 farms
Zone 4	1 farm	2 farms
Zone 5	3 farms	2 farms
Zone 6	11 farms	2 farms

Source: Data has been self-generated in STATA using the PSLM dataset from 2014

Category	**Top Performing 20 Farms (2016)**	**Bottom Performing 20 Farms (2016)**
Cropping Patterns		

Wheat-Cotton	3 farms	0 farms
Wheat-Rice	0 farms	1 farm
Wheat-Sugarcane	4 farms	0 farms
Wheat-other	5 farms	11 farms
Crop number	1-6	1-5
X Variables		
X1 (Mean: 16.875)	2 farms above mean	7 farms above mean
X2 (Mean: 25410.66)	6 farms above mean	7 farms above mean
X3 (Mean: 12155.48)	4 farms above mean	10 farms above the mean
X4 (Mean: 21448.81)	7 farms above mean	4 farms above mean
X5 (Mean: 12469.78)	9 farms above mean	11farms above mean
X6 (Mean: 62003.57)	2 farms above mean	8 farms above the mean
Z Variables		
Z1 (Mean: 0.25; Std Dev: 0.4442617)	5 farms above mean	1 farm above mean
Z2 (Mean: 0.9; Std Dev: 0.3077935)	18 farms above mean	6 farms above mean
Z3 (Mean: 0.1; Std Dev: 0.3077935)	2 farms above mean	6 farms above mean
Z4 (Mean: 0.85; Std Dev: 1.663066)	8 farms above the mean	7 farms above mean
Z5 (Mean: 8.1; Std Dev: 2.989455)	8 farms above the mean	7 farms above mean
Z6 (Mean: 0.85; Std Dev: 0.3663475)	17 farms above mean	16 farms above mean
Agro-climatic Zones		
Zone 1	5 farms	6 farms
Zone 2	2 farms	8 farms
Zone 3	0 farms	2 farms
Zone 4	0 farm	2 farms
Zone 5	2 farms	1farms
Zone 6	2 farms	1 farm

Zone 7	2 farms	0 farms
Zone 8	1 farm	0 farms
Zone 9	6 farms	0 farms

Source: Data has been self-generated in STATA using the PSLM dataset from 2016

Category	**Top Performing 20 Farms (2019)**	**Bottom Performing 20 Farms (2019)**
Cropping Patterns		
Wheat-Cotton	1 farm	0 farms
Wheat-Rice	8 farms	0 farms
Wheat-Sugarcane	1 farm	0 farms
Wheat-other	8 farms	4 farms
Crop number	1-7	0-2
X Variables		
X1 (Mean: 7.352)	3 farms above the mean	5 farms above the mean
X2 (Mean: 21647.42)	6 farms above the mean	9 farms above the mean
X3 (Mean: 8773.591)	3 farms above the mean	5 farms above the mean
X4 (Mean: 34221.94)	6 farms above the mean	8 farms above the mean
X5 (Mean: 9860.706)	13 farms above the mean	8 farms above the mean
X6 (Mean: 19067.78)	4 farms above the mean	5 farms above the mean
Z Variables		
Z1 (Mean: 0.1; Std Dev: 0.3077935)	2 farms above the mean	0 farms above the mean
Z2 (Mean: 0.9; Std Dev: 0.3077935)	18 farms above the mean	7 farms above the mean
Z3 (Mean: 0.05; Std Dev: 0.2236068)	1 farm above the mean	4 farms above the mean
Z4 (Mean: 0.95; Std Dev: 1.316894)	9 farms above the mean	3 farms above the mean
Z5 (Mean: 11.35; Std Dev: 11.77095)	5 farms above the mean	9 farms above the mean
Z6 (Mean: 0.95; Std Dev: 0.2236068)	19 farms above the mean	16 farms above the mean

Agro-climatic Zones		
Zone 1	4 farms	14 farms
Zone 2	1 farm	0 farms
Zone 4	0 farms	1 farm
Zone 5	9 farms	1 farm
Zone 6	1 farm	1 farm
Zone 9	5 farms	3 farms

Source: Data has been self-generated in STATA using the PSLM dataset from 2019

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Comparative Analysis of Cross-lingual Homograph Disambiguation Challenges in French, Azerbaijani, and Turkish (1023)

Shafa Gurbanova[1*]

[1]Student of PhD at the Azerbaijan National Academy of Science French Langue and Literature teacher

*Corresponding author e-mail: shafagurbanova5@gmail.com

Abstract

This study explores the challenges of homograph disambiguation in French, Azerbaijani, and Turkish, focusing on their linguistic differences and computational difficulties. Homographs, words with identical spelling but different meanings, cause ambiguity in multilingual contexts. French disambiguation relies on lexical and syntactic context, Azerbaijani on morphological parsing, and Turkish on contextual and statistical analysis. Examples include "mode" (French: fashion, Azerbaijani: trend) and "çay" (Azerbaijani: tea, Turkish: river). Computational linguistics faces challenges in processing homographs due to polysemy and syntactic complexity. Understanding these issues enhances language learning, translation accuracy, and natural language processing (NLP). This study contributes to cross-lingual communication and artificial intelligence development in multilingual settings.

Keywords: *French, Azerbaijani, Turkish, Comparative Language, NLP, multilanguage*

INTRODUCTION

Languages often share words that look identical but have different meanings, known as homographs. These words can cause confusion, especially in multilingual contexts. Homograph disambiguation is crucial for language learners, translators, and artificial intelligence systems like machine translation. This study explores the challenges of cross-lingual homograph disambiguation in French, Azerbaijani, and Turkish. These languages, despite their differences, share numerous homographs due to historical, linguistic, and cultural exchanges. Understanding these challenges will enhance computational linguistics and language learning.

Example: In French and Azerbaijani, the word "mode" exists but has different meanings: "mode" in French means fashion or a way of doing something, while in Azerbaijani, "mod" (a variation) means trend.

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Theoretical Background

Homographs are words that share the same spelling but have different meanings, either within a single language or across multiple languages. In cross-lingual settings, homographs can arise due to historical borrowings, phonetic similarities, or accidental convergence. Disambiguation is the process of correctly identifying the meaning of a homograph in a given context. While human speakers rely on context, syntax, and semantics, computational systems require specific algorithms to achieve this task.

1. Theoretical Approaches to Homograph Disambiguation in French, Azerbaijani, and Turkish

Each language has developed different linguistic theories to tackle the problem of homograph disambiguation.

- French Approach (Lexical and Syntactic Contextualization): French linguistic theory emphasizes the role of syntactic structure and lexical cues in disambiguation. Structuralist linguists, such as Saussure, argue that words derive meaning from their context within a sentence. French relies heavily on determiners and agreement markers to clarify ambiguous words.

Example: The word "banc" means "bench" in one context ("Je suis assis sur un banc") but refers to a "school of fish" in another ("Un banc de poissons").

Bibliography:

1. Saussure, F. (1916). Course in General Linguistics. McGraw-Hill.
2. Grevisse, M. (2016). Le Bon Usage. De Boeck Supérieur.

- Azerbaijani Approach (Morphological Disambiguation): Azerbaijani, as an agglutinative language, uses extensive suffixes to indicate tense, number, and possession. Homographs often arise due to loanwords from Persian and Russian. Disambiguation in Azerbaijani relies heavily on morphological parsing, as suffixes clarify word function.

Example: The word "qaz" can mean "goose" or "gas," but the suffix in "qazlar" (geese) distinguishes it from "qaz borusu" (gas pipe).

Bibliography:

1. Johanson, L. (2002). Structural Factors in Turkic Language Contacts. Routledge.
2. Doğan M. (2018). Azerbaijani-Turkish Lexical Comparisons. Cambridge Scholars Publishing.

Turkish Approach (Contextual and Statistical Analysis): Turkish, another agglutinative language, shares similarities with Azerbaijani but also incorporates computational linguistic approaches. Recent research in Turkish NLP relies on statistical models and neural networks to process homographs in machine translation and text recognition.

Example: "Kol" means "arm" in one context but "branch" (of a river) in another. AI-based systems use context analysis to differentiate them.

Bibliography:

1. Göksel, A., & Kerslake, C. (2005). Turkish: A Comprehensive Grammar. Routledge.

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These theoretical frameworks highlight the unique challenges of homograph disambiguation in each language and provide insight into linguistic and computational solutions.

Homographs in French, Azerbaijani and Turkish

Each of these languages has homographs that create ambiguity in cross-lingual communication.

1-French

French, a Romance language, has homographs due to its complex conjugation system, borrowed words, and homophonic structures. Examples include:

- "Livre" (book) vs. "livre" (pound – unit of weight)
- "Porte" (door) vs. "porte" (carries – verb form)

Example: The word "pain" means "bread" in French but "pain" in English refers to suffering.

Bibliography:

1. Grevisse, M. (2016). Le Bon Usage. De Boeck Supérieur.
2. Trask, R. L. (2007). Language and Linguistics: The Key Concepts. Routledge.

2- Azerbaijani

Azerbaijani, a Turkic language, shares many loanwords with Persian, Russian, and Turkish, leading to homographs. Some examples include:

- "Dolma" (stuffed food) in Azerbaijani and Turkish, but in French, "dolma" may be an unfamiliar term.
- "Al" (red) in Azerbaijani vs. "Al" (take – imperative verb) in Turkish.

Example: The Azerbaijani word "çay" means "tea," while in Turkish, it also means "river."

Bibliography:

1. Johanson, L. (2002). Structural Factors in Turkic Language Contacts. Routledge.
2. Doğan, M. (2018). Azerbaijani-Turkish Lexical Comparisons. Cambridge Scholars Publishing.

3- Turkish

Turkish, an agglutinative language, has homographs mostly derived from historical borrowings. Some cases include:

- "Ada" (island) in Turkish vs. "Ada" (name) in Azerbaijani.
- "El" (hand) in Turkish vs. "el" (foreign) in Azerbaijani.

Example: The Turkish word "kol" means "arm," but in Azerbaijani, "kol" refers to a branch of a river.

Bibliography:

1. Göksel, A., & Kerslake, C. (2005). Turkish: A Comprehensive Grammar. Routledge.
2. Kornfilt, J. (1997). Turkish Grammar. Routledge.

Strategies for Homograph Disambiguation

Several methods can help resolve homograph ambiguity:

A. **Word Meaning Analysis:** Checking a word's definition and usage in a dictionary to clarify its meaning.

B. **Sentence Structure Examination:** Looking at how a word is used in a sentence to understand its correct interpretation.

C. **AI and Machine Learning:** Using artificial intelligence models to recognize and differentiate homographs based on context.

AI and machine learning have become essential tools in natural language processing (NLP) for tasks like homograph disambiguation. These technologies use advanced algorithms and large datasets to analyze and understand the context of words, making them valuable for differentiating homographs in various languages. Here's how AI and machine learning are used for this purpose:

1. Contextual Understanding

Machine learning models, particularly those based on deep learning (like neural networks), are trained on vast amounts of text data. These models learn to recognize patterns in how words are used in different contexts. By analyzing the surrounding words, a machine learning model can determine which meaning of a homograph is most appropriate.

*Example***:** Consider the French word *banc*:

- In the sentence "Je suis assis sur un banc," the model can recognize that "banc" refers to a "bench" because of the surrounding words like "assis" (sitting).
- In "Un banc de poissons," the model understands that "banc" refers to a "school" of fish due to the word "poissons" (fish) nearby.

2. Training Data and Corpora

For AI to accurately disambiguate homographs, it needs a large corpus of text from various domains, like books, news articles, and social media. These corpora help the model learn the diverse ways words are used. In cross-lingual homograph disambiguation, multilingual corpora are especially useful, as they contain homographs across different languages, which teaches the model how context can affect meaning in different linguistic environments.

*Example***:** A bilingual corpus of French and Azerbaijani texts could teach an AI system to differentiate words like *mode* (fashion in French) and *mod* (trend in Azerbaijani) based on context.

3. Word Embeddings and Contextualized Models

Machine learning models often use word embeddings, which are numerical representations of words in vector space. Embeddings capture semantic similarities between words, helping the model differentiate homographs. Recent advances like transformers (e.g., BERT, GPT) offer contextualized embeddings, meaning the vector representation of a word changes depending on the context in which it appears. These models are particularly effective for homograph disambiguation because they adapt to the sentence's specific context.

4. Pre-trained Models

Pre-trained models such as BERT (Bidirectional Encoder Representations from Transformers) or GPT (Generative Pre-trained Transformer) are trained on vast amounts of text before being fine-tuned for specific tasks like homograph disambiguation. These models understand the meaning of words and can make highly accurate predictions based on context. They are widely used in machine translation and text processing, making them suitable for distinguishing homographs in multilingual texts.

5. Real-time Applications

In machine translation, AI models are used to disambiguate homographs in real-time. For example, Google Translate or other translation services use these AI-powered models to choose the correct translation of homographs depending on context. Similarly, AI systems in chatbots, virtual assistants, and text analytics tools rely on these techniques to understand user input accurately.

Challenges:

- Data Availability: Training AI models requires large, annotated datasets, which may not always be available for every language or homograph.
- Cultural and Contextual Nuances: Some homographs may be highly context-dependent, requiring the model to account for cultural differences in meaning.

AI and machine learning offer powerful tools for resolving homograph ambiguity, improving both the accuracy of machine translations and the understanding of language in various multilingual contexts.

D. **Bilingual and Multilingual Data:** Training computer models with large collections of text from multiple languages to improve homograph recognition.

CONCLUSION

Cross-lingual homographs in French, Azerbaijani, and Turkish create unique challenges for both people and AI systems. These challenges come from words that look the same in different languages but have different meanings. To understand and resolve these ambiguities, both human speakers and AI systems need to focus on the context in which the word is used. For humans, recognizing the meaning of a word depends on the surrounding words in the sentence. For AI systems, advanced techniques are required to accurately determine the correct meaning of homographs based on context.

Machine learning and deep learning technologies have made great progress in helping AI understand homographs, especially in areas like machine translation and natural language processing. These technologies train AI systems on large amounts of data to recognize how words change meaning depending on context. But challenges still exist, particularly in languages like Azerbaijani and Turkish, where word structure is more complex.

In the future, research should focus on improving AI systems to better understand context and recognize homographs across different languages. At the same time, language learning strategies should be enhanced to help learners deal with homographs more easily. By improving both AI technology and language education, we can reduce confusion caused by homographs and make communication smoother in multilingual environments.

Optimal Point Forecasting Under General Loss Functions (1032)

Mahir Hasanov[1*]

[1]Istanbul Beykent University, Faculty Arts-Sciences, Department of Mathematics, Istanbul, Türkiye

*Corresponding author e-mail: mahirhasansoy@beykent.edu.tr

Abstract

We analyze the relationship between loss functions and their corresponding optimal point forecasts. In this context, we discuss Bregman loss functions, equivariant loss functions, and other commonly encountered classes. By using the concept of equivariant loss functions, we extend optimal point forecasting to a broader class of random variables. Our study addresses the theoretical aspects of these problems.

Keywords*: Expectation, Optimal point forecast, Loss function, Equivariance*

INTRODUCTION

The optimal point forecast is defined as follows:

$$arg\,min_{y\in R}\, E[L(X,y)] = y^* \quad (1.1)$$

where $L: D \times D \to R$ is a loss function, D is a decision-observation domain (see Gneiting T, 2011, p.198) and X is a random variable taking values in D. The optimal point forecast is any $y^* \in$R, that minimizes the expectation E[L(X,y)] in (1.1). In practice, the decision-observation domain usually appears as a subinterval of R^n. In this paper we will consider the case $n = 1$ and $D = R$. Many results of this article can be easily extended to a subset $D \subset R$. Note that $E[L(X,y)]$ denotes the expectation of the random variable $L(X,y)$, which is defined as follows

$$E[L(X,y)] = \int_R L(x,y)dF(x) \coloneqq \int_R L(x,y)d\mu_F(x),$$

where $F = F_X$ is the cumulative distribution function of X (for the basic concepts from probability theory, see Athreya KB, Lahiri, SN, 2006; Grimmett G, Stirzaker D, 2004; Williams D, 2001.

Loss functions play an important role in optimal point forecasts. Although the properties of loss functions vary depending on the problem (Elliott G, Timmermann A, 2008; Lehmann EL, Casella G, 1998]) they have some generally accepted properties (see Banerjee A, Guo X, Wang H, 2005; Gneiting T, 2011, p.198). In this article, we adopt the following definition of a loss function.

Definition 1.1. Let $L: D \times D \to R$, where D is a prediction-observation domain (in our case $D = R$).

L is said to be a loss function if it satisfies the following:

L1) $L(x,y)$ is continuous,

L2) $L(x,y) \geq 0$ with equality if $x = y$.

L3) $L(x,y)$ is continuously differentiable whenever $x \neq y$.

Examples of frequently encountered loss functions include Bregman loss functions, symmetric piecewise linear functions (Lehmann & Casella, 1998, p.50), asymmetric piecewise linear functions (see Raiffa & Schlaifer, 1961, p.196), and generalized piecewise linear loss functions (Gneiting, 2011).

The optimization problem (1.1) is directly related to the least squares prediction problems. The least square prediction of X given the observation Y is to find the global minimizer of

$$E[\big(X - g(Y)\big)^2$$

over all random variables $g(Y)$, where g is a Borel function. The solution to the least squares prediction problem is the conditional expectation $E[X|Y]$, i.e.,

$$arg\,min_{g(Y)}\, E[\big(X - g(Y)\big)^2 = E[X|Y]$$

(see Athreya KB Lahiri, SN, 2006; Grimmett G, Stirzaker D, 2004; Williams D, 2001).

Another key concept in optimal point prediction problems is that of equivariant loss functions (see Definition 2.1). Equivariant loss functions are used to select a specific loss function for forecast comparison within the broad class of all consistent loss functions (see Fissler & Ziegel, 2019; Gneiting, 2011; Gneiting, 2012). The main contribution of this study is to analyze optimal point forecasts using equivariant loss functions.

A brief literature review is provided here. Basic concepts of probability theory and statistics used in the article, including least squares prediction, can be found in Athreya & Lahiri (2006); Grimmett & Stirzaker (2004); and Williams (2001). Least squares prediction and maximum entropy problems are discussed from a broader perspective by Csiszár (1991).

Necessary and sufficient conditions under which the conditional expectation is the unique optimal predictor for general loss functions were provided by Banerjee A, Guo X, Wang H, 2005. Bregman loss functions play an important role in this field; see Reem, R, Reich, S, De Pierro A, 2019. Economic forecasting and optimal point forecasts have been studied by Elliott G, Timmermann A, 2008; Gneiting T, 2011; Gneiting T, 2012, and by examining different economic models (see also Koenker R, 2005; Koenker R, Bassett G 1978; Raiffa H, Chlaifer R, 1961

The paper by Lehmann EL, Casella G, 1998, dealt with point estimation in Euclidean sample spaces.

Equivariant loss functions have been studied comprehensively by Fissler T, Ziegel JF, 2019 and Koenker R, 2005, with a focus on forecast selection, forecast comparison, and forecast ranking. In particular, results on order-sensitivity and equivariance of consistent loss functions for elicitable functionals are presented in Fissler T, Ziegel JF, 2019. An inverse problem of finding L for a given y^* has been studied in Hasanov M, 2023. using a partial differential equations approach.

In this paper we study Bregman loss functions, asymmetric piecewise linear, symmetric piecewise linear and generalized piecewise linear loss functions separately. For each of these classes, we analyze both the optimal point forecasts and the set of equivariance functions corresponding to a given loss function.

RESULTS

Equivariant Loss Functions

The equivariance and invariance properties are very important in statistics, as in many branches of mathematics. Many statistical functionals exhibit equivariance property. For example, if we define $\Phi(X) = E[X]$ then $\Phi(X)$ is a linear functional on the space $L^1(\Omega)$ (the space of integrable functions) and is, therefore, equivariant under linear transformations. This means that for any random variable X and any linear map $h: R \to R$, we have

$$\Phi[h(X)] = h(\Phi[X]).$$

However, if we define $\Phi(X) := Var[X]$ then the variance is invariant under translations, that is $Var[X - c] = Var[X]$ for any $c \in R$, but scales quadratically:

$$Var[cX] = c^2 Var[X].$$

Now, we give the definition of equivariance as used in this article (see Gneiting T, 2012, p.750). Note various versions of the definition of equivariance appear in the literature (see Fissler T, Ziegel JF, 2019, Section 4).

Definition 2.1. A loss function L is said to be equivariant with respect to some class H of injections $h: R \to R$ if

$$arg\,min_{y \in R}\, E[L(h(X), y)] = h\big(arg\,min_{y \in R}\, E[L(X, y)]\big)\ for\ all\ X \quad (2.1).$$

for all random variables X, provided that both sides of (2.1) are defined. An equivalent formulation of (2.1) is

$$arg\,min_{y \in R} \int_R L(h(x), y) dF(x) = h\left(arg\,min_{y \in R} \int_R L(x, y) dF(x)\right)$$

for all cumulative distribution functions F.

In other words, if the optimal prediction for the random variable X is y^*, then for any $h \in H$ the optimal prediction for the random variable $h(X)$ will be $h(y^*)$.

We remind again that by the Lebesgue-Stieltjes integral $\int_R L(h(x), y) dF(x)$ we simply mean the Lebesgue integral $\int_R L(h(x), y) d\mu_F$.

We now state our main result regarding the equivariance of general loss functions.

Theorem 2.1. Let L be a loss function that is equivariant with respect to some H of injections $h: R \to R$. Then necessarily

$$\frac{\partial}{\partial x}\left[\frac{L_y(h(x), h(y)}{L_y(x, y)}\right] = 0, whenever\ x \neq y. \qquad (2.2)$$

Proof. By setting $arg\,min_{y \in R}\, E[L(X, y)] = y^*$ the definition of equivariance can be written as

$$\begin{cases} a) \quad arg\,min_{y \in R}\, E[L(X, y)] = y^* \\ b) \ arg\,min_{y \in R}\, E[L(h(X), y)] = h(y^*). \end{cases} \qquad (2.3)$$

Let X be a random variable, taking two values a and b with $P(X = a) = p$, $P(X = b) = q$ and $p + q + q = 1$. Since $E[L(X, y)] = pL(a, y) + qL(b, y)$, it follows from (2.3) a) that

$$E_y[L(X, y^*)] = pL_y(a, y^*) + qL_y(b, y^*) = 0. \quad (2.4)$$

Similarly, since $h(x)$ is an injection we have $P\{h(X) = h(a)\} = p$ and $P\{h(X) = h(b)\} = q$. Thus, $E[L(h(X), y)] = pL(h(a), y) + qL(h(b), y)$ and (2.3) b) implies that

$$E_y\big[L\big(h(X), h(y^*)\big)\big] = pL_y\big(h(a), h(y^*)\big) + qL_y\big(h(b), h(y^*)\big) = 0. \quad (2.5)$$

Hence, from (2.4) we deduce

$$\frac{p}{q} = -\frac{L_y(b, y^*)}{L_y(a, y^*)}$$

and similarly from (2.5)

$$-\frac{L_y(h(b), h(y^*))}{L_y\big(h(a), h(y^*)\big)}.$$

This implies

$$\frac{L_y(h(b), h(y^*))}{L_y(b, y^*)} = \frac{L_y(h(a), h(y^*))}{L_y(a, y^*)}.$$

Since $a, b, and\ p$ are arbitrary and h is an injection, the above equality shows that the function

$\frac{L_y(h(x),h(y))}{L_y(x,y)}$ does not depend on x; that is

$$\frac{L_y(h(x), h(y))}{L_y(x, y)} = C(y).$$

Thus

$$\frac{\partial}{\partial x}\left[\frac{L_y(h(x), h(y)}{L_y(x, y)}\right] = 0, whenever\ x \neq y.$$

Optimal Prediction Under Bregman Loss Functions

Bregman loss functions are defined as follows (see Banerjee A, Guo X, Wang H, 2005; Reem, R, Reich, S, De Pierro A, 2019).

Definition 3.1. A function $L(x, y)$ is a Bregman loss function if

$$L(x, y) = D_\phi(x, y)$$

where

$$D_\phi(x, y) = \phi(x) - \phi(y) - \phi'(y)(x - y),$$

for a strictly convex differentiable function ϕ.

Some examples of Bregman loss functions:

- $D_\phi(x, y) = (x - y)^2,\ \ \phi(x) = x^2, x \in R$ (quadratic or L^2 loss),
- $D_\phi(x, y) = x \log\left(\frac{x}{y}\right) - (x - y),\ \ \phi(x) = x \log x,\ \ x \in (0, \infty)$ (log loss)
- $D_\phi(x, y) = x \log\left(\frac{x}{y}\right) - (1 - x) \log\left(\frac{1-x}{1-y}\right),\ \ \phi(x) = x \log x + (1 - x) \log(1 - x),\ \ x \in (0,1)$
- $D_\phi(x, y) = -\frac{x}{y} - \log\frac{x}{y} - 1,\ \ \phi(x) = -\log x,\ \ x \in (0, \infty)$ (Itakura-Saito divergence)
- $D_\phi(x, y) = e^x - (1 + x - y)e^y,\ \ \phi(x) = e^x,\ \ x \in R.$

An example of a loss function which is not a Bregman loss function.

$$F(x, y) = |x - y|, \qquad \text{(absolute error loss)}$$

The best result regarding the optimal prediction under Bregman loss functions is given in the following theorem, which states that the mean $E[X]$ of the predictive distribution is the unique optimal point predictor.

Theorem 3.1 ([2]) Let $L: R \times R \to R$ be a nonnegative function such that $L(x,x) = 0$, for all $x \in R$. If L and L_x are continuous and $E[X]$ is the unique minimizer of

$$arg\,min_{y \in R}\, E[L(X,y)] = E[X], \quad (3.1)$$

for all random variables X, then $L(x,y)$ is a Bregman loss function. Conversely, if L is a Bregman loss function then (3.1) holds.

Next we prove a theorem about the equivariance of Bregman loss functions.

Theorem 3.2. If a Bregman loss function L is equivariant with respect to a class of injections H, then

$$H = \{h(x) |\, h(x) = kx + c \; for\; all\; k, c \in R\}.$$

Proof. We prove this theorem in two different ways to emphasize the importance of the equivariance concept.

First way. By Theorem 3.1, if L is a Bregman loss function then

$$arg\,min_{y \in R}\, E[L(X,y)] = E[X], \quad (3.2)$$

for all random variables X. Consequently,

$$arg\,min_{y \in R}\, E[L(h(X),y)] = E[h(X)], \quad (3.3)$$

On the other hand, (3.1) and (3.2) yield

$$arg\,min_{y \in R}\, E[L(h(X),y)] = h[E(X)], \quad (3.4)$$

Then from (3.3) and (3.4) it follows that for all X and $h \in H$

$$E[h(X)] = h(E[X]). \quad (3.5)$$

In particular, for a discrete random variable $X(w) = \sum_{i=1}^{n} a_i \chi_{A_i}(w)$, (3.5) implies that

$$\sum_{i=1}^{n} h(a_i)\, P(A_i) = h\left(\sum_{i=1}^{n} a_i\, P(A_i)\right).$$

for all $a_i \in R$ and all events $A \in \mathcal{F}$ in the σ-algebra $\mathcal{F}$. This functional equation characterizes the linear functions, so that

$$H = \{h(x) \mid h(x) = kx + c\},$$

for all $k, c \in R$.

Second way. Using Theorem 2.1 and assuming $\phi''(x) > 0$, we have

$$L_y(x,y) = -\phi''(y)(x-y),$$

$$L_y(h(x),h(y)) = -\phi''(h(y))(h(x)-h(y)).$$

By Theorem 2.1 if L is equivariant with respect to some class H of injections

$h: R \to R$ then

$$\frac{\partial}{\partial x}\left[\frac{L_y(h(x),h(y)}{L_y(x,y)}\right] = 0, whenever\; x \neq y.$$

Thus,

$$\frac{L_y(h(x),h(y)}{L_y(x,y)} = \frac{\phi''(h(y))(h(x)-h(y))}{\phi''(y)(x-y)} = C(y)$$

and

$$\frac{h(x)-h(y)}{x-y} = C(y)\frac{\phi''(h(y))}{\phi''(y)}.$$

Finally, setting $y = 0$ (or any fixed value) in the last equality we obtain that $h(x) = kx + c$, for some $k, c \in R$.

To check,

$$arg\,min_{y\in R}\, E[L(h(X),y)] = E[h(X)] = E[kX + c] = kE[X] + c = h(E[X]).$$

Optimal Prediction Under Asymmetric Piecewise Linear Loss Functions

Asymmetric piecewise linear loss functions are defined in the following form:

$$L(x,y) = \begin{cases} k_1(x-y), & x \geq y, \\ k_2(y-x), & x \leq y \end{cases}$$

wherw $k_1 > 0,\ k_2 > 0$ and $k_1 \neq k_2$.

In this case, the following result regarding the optimal point forecast holds.

Theorem 4.1. (see Raiffa H, Chlaifer R, 1961, p.196). Let $k := \frac{k_1}{k_1+k_2} \in (0,1)$. Any $k-$ quantile of the cumulative ofdistribution function F_X is an optimal point forecast; that is,

$$arg\,min_{y\in R}\, E[L(X,y)] = y_k,$$

where y_k is a k- quantile of F_X, meaning $F_X(y_k - 0) \leq\ k \leq\ F_X(y_k)$.

In particular, if $k_1 = \alpha \in (0,1)$ and $k_2 = 1-\alpha$, then

$$L(x,y) = \begin{cases} \alpha(x-y), & x \geq y, \\ (1-\alpha)(y-x), & x \leq y \end{cases}$$

and any α- quantile of F_X is an optimal point forecast.

Now we prove a theorem about the equivariance of asymmetric loss functions.

Theorem 4.2. Asymmetric loss functions are equivariant with respect to increasing functions. This means that if

$$arg\,min_{y\in R}\, E[L(X,y)] = y_k$$

then

$$arg\,min_{y\in R}\, E[L(h(X),y)] = h(y_k)$$

for any increasing function h.

Proof. The proof of this fact is based on the direct calculation using

$$E[L(h(X),y)] = \int_R L(h(x),y)dF_X(x). \qquad (4.1)$$

By the definition, since

$$L(x,y)=\begin{cases}k_1(x-y), & x\geq y,\\ k_2(y-x), & x\leq y,\end{cases}$$

it follows that

$$L(h(x),y)=\begin{cases}k_1(h(x)-y), & h(x)\geq y,\\ k_2\big(y-h(x)\big), & h(x)\leq y,\end{cases}$$

or equivalently,

$$L(h(x),y)=\begin{cases}k_1(h(x)-y), & x\geq h^{-1}(y),\\ k_2\big(y-h(x)\big), & x\leq h^{-1}(y).\end{cases}$$

Then using (4.1) we write

$$E[L(h(X),y)]=\int_R L(h(x),y)dF_X(x)=$$

$$k_2\int_{-\infty}^{h^{-1}(y)}\big(y-h(x)\big)dF_X(x)+k_1\int_{h^{-1}(y)}^{\infty}(h(x)-y)dF_X(x)k_2yF_X\big(h^{-1}(y)\big)=$$

$$=k_2yF_X(h^{-1}(y))-k_2\int_{-\infty}^{h^{-1}(y)}h(x)dF_X(x)+k_1\int_{h^{-1}(y)}^{\infty}h(x)dF_X(x)-k_1yF_X\big(h^{-1}(y)\big)=$$

$$y((k_1+k_2)F_X(h^{-1}(y)-k_1)-k_2\int_{-\infty}^{h^{-1}(y)}h(x)dF_X(x)+k_1\int_{h^{-1}(y)}^{\infty}h(x)dF_X(x).$$

Hence,

$$E_y[\,L(h(X),y)]=(k_1+k_2)F_X(h^{-1}(y)-k_1+y(k_1+k_2)f_X(h^{-1}(y))(h^{-1}(y))'-$$

$$k_2yf_X(h^{-1}(y))(h^{-1}(y))'-k_1yf_X(h^{-1}(y))(h^{-1}(y))'.$$

Thus

$$E_y[L(h(X),y)]=(k_1+k_2)F_X(h^{-1}(y)-k_1$$

and $E_y[L(h(X),y)]$ is an increasing function. This means that any critical poin of this function is a minimum point.

$$E_y[L(h(X),y)]=0\Rightarrow F_X(h^{-1}(y))=\frac{k_1}{k_1+k_2}=k\backslash in\ (0,1).$$

Consequently if $arg\,min_{y\in R}\,E[L(X,y)]=y_k$ then $arg\,min_{y\in R}\,E[L(h(X),y)]=h(y_k)$.

Second way.

Applying Theorem 2.1, we need to verify the condition

$$\frac{\partial}{\partial x}\left[\frac{L_y(h(x),h(y)}{L_y(x,y)}\right]=0$$

Since both the derivatives $L_y(x,y)$ and $L_y(h(x),h(y))$ are constant for $x\neq y$, this condition is satisfied for all injections h. On the other hand, if h is a decreasing function then we obtain that y is a minimum

point of $E[L(h(X),y)]$ if $F_X(h^{-1}(y)) = -\frac{k_2}{k_1+k_2}$. Consequently, in this case there is not a solution of the problem

$$arg\,min_{y\in R}\, E[L(h(X),y)]$$

Therefore, the class H can only consist of increasing functions.

Optimal Prediction Under Symmetric Piece-Wise Linear Loss Functions

Symmetric piecewise linear loss functions are defined by

$$L(x,y) = \begin{cases} \alpha(x-y), & x \geq y, \\ \alpha(y-x), & x \leq y \end{cases} \quad \alpha > 0.$$

which simplifies to

$$L(x,y) = \alpha|x-y|, \qquad \alpha > 0.$$

Particularly, if $\alpha = 1$ we get the absolute error loss $L(x,y) = |x-y|$.

In this case, it is known that

$$arg\,min_{y\in R}\, E[L(h(X),y)] = y^*,$$

where y^* is any median of the random variable X; that is, $F_X(y^*-0) \leq \frac{1}{2} \leq F_X(y^*)$.

This result follows from Theorem 3.1, since here

$$k = \frac{k_1}{k_1+k_2} = \frac{\alpha}{2\alpha} = \frac{1}{2}.$$

An equivariance result for symmetric loss functions is stated in the following theorem.

Theorem 5.1. Symmetric loss functions are equivariant with respect to increasing functions. That is, if

$$arg\,min_{y\in R}\, E[L(X,y)] = y^*,$$

then

$$arg\,min_{y\in R}\, E[L(h(X),y)] = h(y^*)$$

for any increasing function h.

Proof. The proof follows exactly the same steps as in Theorems 4.1 and 4.2 and is therefore omitted.

DISCUSSION AND CONCLUSION

In this paper, we studied the optimal point forecast problem

$$arg\,min_{y\in R}\, E[L(X,y)] = y^*.$$

Our main objectives were to determine the optimal point forecast y^* depending on the class of L and to determine the equivariance classes associated with L. To achieve this, we considered Bregman loss functions, asymmetric piecewise linear, symmetric piecewise linear, and generalized piecewise linear loss functions separately. For each of these classes, we analyzed both the optimal point forecasts and the set of equivariance functions corresponding to a given loss function. An inverse problem of finding L for a given y^* was addressed in Hasanov (2023) via a partial differential equations approach.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

- A necessary condition for the equivariance of general loss functions has been derived (Theorem 2.1).
- The necessary condition has been applied to the special classes of loss functions and

the set of injections for which the loss functions are eqivariant has been determined.

- Bregman loss functions, symmetric piecewise linear, asymmetric piecewise linear and generalized piecewise linear loss functions are analyses separetely.

Do Cryptocurrencies Enhance Portfolio Efficiency? Evidence from Risk-Optimized Mixed-Asset Allocations (1046)

Denis Veliu[1],

[1] Departament of Finance-Banking, Metropolitan University of Tirana, 1000 Tirana, Albania

*Corresponding author e-mail: dveliu@umt.edu.al

Abstract

The rapid popularity of cryptocurrency as a new asset class has caused sharp discussions with regard to its volatility and potential diversification benefits, considering its considerable risk.
This article evaluates the effect of risk-adjusted returns when crypto-assets are added to a diversified portfolio of traditional assets comprising stocks and bonds.
Using Minimum Risk Maximum Diversification (MRMD) methodology, we construct and analyze six portfolio optimization strategies, including Mean Variance, Risk Parity, and Conditional Value-at-Risk (CVaR) models. These strategies are tested on a dataset of ten stocks, four bonds, and six cryptocurrencies (Bitcoin, Ethereum, etc) from March 2024 to March 2025.
Due to low correlations with traditional assets, our results indicate that cryptoassets significantly enhance portfolio diversification; however, their inclusion also heightens volatility as well as tail risk. Compared to naive approaches, optimization models such as Min Variance and CVaR do mitigate these risks but at the expense of lower returns and reduced crypto allocations. This study offers practical insights for investors: whereas crypto-assets have diversification advantages, their ideal allocation is contingent upon risk tolerance, cost limitations, and the selected optimization model. We determine that a balanced strategy-integrating quantitative optimization with tail-risk management is crucial for leveraging the potential of cryptocurrencies within mixed-asset portfolios.

Keywords*: Cryptocurrencies, Portfolio Optimization, Risk Diversification, Conditional Value-at-Risk (CVaR), Mixed-Asset Portfolios*

INTRODUCTION

Investors, scholars, and financial analysts are all highly interested in cryptocurrencies because they quickly gained popularity as a new asset class. There are a number of studies indicating that investment in a few cryptocurrencies in their portfolio (Ma et al., 2020) (Veliu & Aranitasi, 2024). With a total market value whose highest point amounted to billions of dollars, cryptoassets have developed from a specialized digital product to a worldwide financial phenomenon since the emergence of Bitcoin in 2009 (Segendorf, 2014).

Despite growing popularity, their usage as assets in traditional investing portfolios remains controversial because to their unique risk-return profiles, inherent volatility, and regulatory issues (Boiko et al., 2021). However, because of its purported diversification benefit and lack of connection with traditional asset classes like stocks and bonds, crypto-assets are gaining popularity for use in mixed-asset portfolios (Platanakis & Urquhart, 2020).

According to popular portfolio optimization strategies like Modern Portfolio Theory (MPT), diversification is required to reduce risk without sacrificing profits (Markowitz, 1952). However, the unique characteristics of crypto-assets are challenging the traditional method of portfolio construction. Their poor correlation with

traditional assets implies that they might be useful in increasing portfolio diversity, especially during times of market instability, even though their high volatility may put off risk-averse investors (Platanakis & Urquhart, 2020). This poses a crucial query: under what circumstances may crypto-assets be successfully included into a portfolio to maximize diversity and minimize risk?

This article seeks to offer a response to this query using a Minimum Risk Maximum Diversification (MRMD) portfolio optimization approach. Using historical data of a blend of traditional assets (10 shares and 4 bonds) and six major crypto-assets (like Bitcoin, Ethereum, etc.), the impact of adding cryptocurrencies in a mixed-asset portfolio is analyzed. With risk minimization and diversification maximization as the optimizers, the goal is to provide investors with actionable intelligence for navigating the complexities of digital assets in the current financial landscape.

Our study contributes to the current literature on crypto-assets and portfolio optimization by presenting a rigorously framed analysis of their contribution to well-balanced, risk-adjusted returns and transaction costs. We examine whether the diversification benefits of crypto-assets outweigh their inherent risks, particularly during times of high market volatility. We also provide actionable recommendations to investors who wish to incorporate digital assets into their portfolios but with a risk-management approach.

Bitcoin has acquired considerable traction in terms of popularity and "feel" among the persistent unc ertainty surrounding traditional banking systems and the economic difficulties that many nations are faci ng (Bouri et al., 2017).

Global anxieties following the global financial crisis, according to Dyhrberg (2016), contributed to a reinforced Bitcoin's popularity and optimistic outlook. Cryptocurrencies and blockchain technologies are shaping the financial systems, and are a significant part of the ongoing global financial innovation (Ma et al., 2020). Khraisha and Arthur (2018) defined financial innovation as a process that is carried out by any institution involving the creation and adoption of new products and platforms. The latest products and platforms enable technologies to introduce innovative ways in which financial activities can be carried out. They further suggested that blockchain and PayPal are financial innovations that have been introduced by non-financial institutions (Ma et al., 2020) . Bianchetti et al. (2018) commented that cryptocurrencies had brought two main innovations into the limelight. These innovations included the technology behind the blockchain or ledgers and the decentralized ways of governance (Ma et al., 2020). Many authors have argued that the blockchain technology that works behind the cryptocurrencies is a potential breakthrough in financial innovation (Glaser & Bezzenberger, 2015)(Su et al., 2020). Olusegun & Olaniyi (2019) also conducted an empirical study, showing that higher levels of financial inclusion and development were observed in China, India, Nigeria, and South Africa, for higher levels of cryptocurrency, internet usage, and mobile subscriptions.

MATERIAL AND METHODS

Material

In a traditional portfolio comprised with stocks and bonds, the cryptocurrency is added. These three categories have different days of trading, so caution on treating the time series is required.

The table 1 presents a list of twenty financial assets, categorized into three broad groups: stocks, fixed-income securities, and cryptocurrencies. This classification helps in understanding the asset composition for portfolio analysis, particularly in studies examining diversification and risk optimization across different asset classes.

Table 1. List of Assets

	Asset	Category
1	Tesla, Inc. Common Stock (TSLA)	Stock

2	Apple Inc. Common Stock (AAPL)	Stock
3	Meta Platforms, Inc. Class A Common Stock (META)	Stock
4	NVIDIA Corporation Common Stock (NVDA)	Stock
5	Amazon.com, Inc. Common Stock (AMZN)	Stock
6	Broadcom Inc. Common Stock (AVGO)	Stock
7	Baidu, Inc. ADS (BIDU)	Stock
8	Advanced Micro Devices, Inc. Common Stock (AMD)	Stock
9	Alphabet Inc. Class A Common Stock (GOOGL)	Stock
10	Microsoft Corporation Common Stock (MSFT)	Stock
11	1 Yr Constant Maturity Treasury (CMT) (CMTN1Y)	Fixed Income
12	JPMorgan Government Bond Fund A (OGGAX)	Fixed Income
13	Vanguard Emerging Markets Government Bond ETF (VWOB)	Fixed Income
14	American Century Government Bond Fd A Cl (ABTAX)	Fixed Income
15	Bitcoin (BTC)	Cryptocurrency
16	Ethereum (ETH)	Cryptocurrency
17	Litecoin (LTC)	Cryptocurrency
18	Cardano (ADA)	Cryptocurrency
19	Stellar (XLM)	Cryptocurrency
20	Dogecoin (DOGE)	Cryptocurrency

Source: Nasdaq.com

Breakdown of Categories:

1. Stocks (Equities) – 10 Assets

Represents shares of large, publicly traded companies across technology, consumer goods, and semiconductor industries.

Tesla (TSLA), Apple (AAPL), Meta (META), NVIDIA (NVDA), Amazon (AMZN) – Major U.S. tech giants.

Broadcom (AVGO), AMD (AMD), Alphabet (GOOGL), Microsoft (MSFT) – Leading semiconductor and software firms.

Baidu (BIDU) – A Chinese tech company (included for potential geographic diversification).

2. Fixed Income – 4 Assets

Consists of government bonds and bond funds, which typically provide lower risk and stable returns compared to equities.

1-Year Constant Maturity Treasury (CMTN1Y) – A short-term U.S. government bond.

JPMorgan Government Bond Fund (OGGAX), Vanguard Emerging Markets Bond ETF (VWOB), American Century Government Bond Fund (ABTAX) are Bond funds offering exposure to U.S. and emerging market debt.

3. Cryptocurrencies – six Assets

Includes major and mid-cap digital assets known for high volatility but also potential diversification benefits.

Bitcoin (BTC), Ethereum (ETH) are the two largest cryptocurrencies by market cap.

Litecoin (LTC), Cardano (ADA), Stellar (XLM), Dogecoin (DOGE) are alternative cryptocurrencies with varying risk-return profiles.

The times series used starts from March 19, 2024 to March 18, 2025. For the stocks and bonds, I have 251 observations since they are the prices at the adjusted closure. These three categories have different days of trading in example, the cryptocurrency market is always opened in terms of trading. Firstly, it is important to take a look to the graph of the returns for the period.

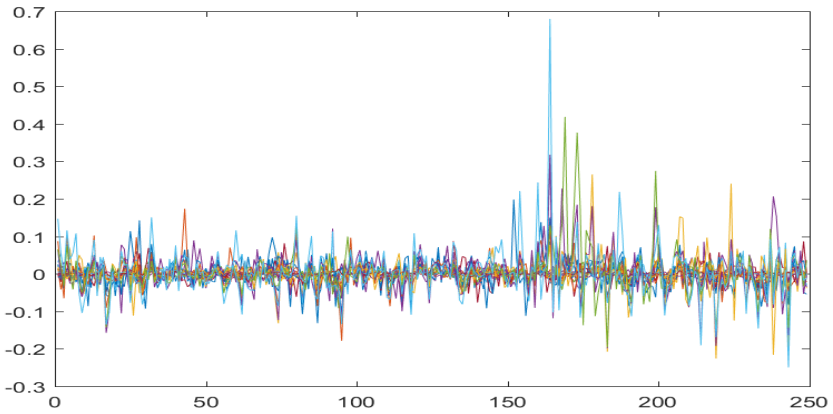

Figure 1. The returns of 20 assets selected

The returns Fig. 1 for this lap of time have a limited number of jumps, which make them suitable to apply the models such as Mean Variance which require the returns to be almost normally distributed, or at least, less skewed and more symmetrical.

All the models and the respective calculation are made with MATLAB optimization. The model formulation mathematical does not include the constraints of the expected returns, but compare also the linear and the quadratic optimization in case of Mean Variance. Passing also, between different risk measures in risk parities models will emphasize the importance of having significan difference between the models (Veliu, 2023).

Methods

The optimization models used

This section we briefly descibe the six portfolio strategies in our mixed portfolio. The goal is to compare their risk-adjusted performance, diversification benefits, and robustness.

To have a complete framework of the portfolio models, the following optimization models are computed:

Table 2. List of models used

Model	Risk Measure	Concentration	Strengths	Weaknesses
1/N Naive	None	High	Simple, no estimation error	Overexposes to crypto volatility
Min Variance	Variance	Low	Optimal risk reduction	Excludes high-volatility assets
Min CVaR	Tail Risk (CVaR)	Medium	Captures crypto crashes	Computationally complex

Model	Risk Measure	Concentration	Strengths	Weaknesses
RP-std	Volatility	Low-Medium	Balanced risk exposure	Underweights high-Sharpe assets
RP-CVaR	Tail Risk	Medium-High	Tail-risk aware	Data-intensive
Worst-Case RP-CVaR	Stress CVaR	Medium	Crisis-resistant	Overly conservative

The last one, is a special case in which we have the worst-case scenario (highest CVaR, useful as an upper bound (Colucci, 2013). A similar study was done with only cryptocurrencies by Veliu and Aranitasi (2024).

In all these models, the constrain of the expected returns is removed, so at the minimum variance MV and minimum CVaR are at the smallest possible value of risk measure.

For a portfolio with n assets and weights $x = (x_1, x_2,.....,x_n)$, the standard deviation is:

$$\mathcal{R}(x) = \sigma_P(x) = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij}} = \sqrt{x'\Omega x}$$

where Ω is the covariance matrix. The marginal risk contribution of the i asset:

$$MRC_i(x) = \frac{\partial \sigma_P(x)}{\partial x_i} = \frac{\partial \sigma_i^2 + \sum_{j=1}^{n} x_i \sigma_{ij}}{\sigma_P(x)} = \frac{(\Omega x)_i}{\sqrt{x'\Omega x}}$$

and the total risk contribution:

$$TRC_i(x) = x_i \frac{\partial \sigma_P(x)}{\partial x_i} = x_i \frac{\partial \sigma_i^2 + \sum_{j=1}^{n} x_i \sigma_{ij}}{\sigma_P(x)} = x_i \frac{(\Omega x)_i}{\sqrt{x'\Omega x}}$$

The following optimization problem can be used to construct the Risk Parity model (Maillard et al., 2010):

$$x^* = \arg\min \sum_{i=1}^{n}\sum_{j=1}^{n} \Big(TRC_i(x) - TRC_j(x)\Big)^2$$

$$\sum_{i=1}^{n} x_i = 1$$

$$x \geq 0$$

To guarantee the existence of the partial derivatives of $CVaR_\alpha(x)$ we need to impose some assumptions on the distribution of the random vector $R = (r_1, r_2,.....,r_n)$.

The conditions for quantile of the portfolio return $X = R'x = \sum_{i=1}^{n} x_i r_i$ should be differentiable respect to the weights x_i. These i-th asset return r_i given the others is measured as follow:

$$r_{i,t+1} = \frac{P_{i,t+1} - P_{i,t}}{P_{i,t}}$$

From the definition of $CVaR_\alpha(x)$ (Rockfellar & Uryasev, 2000), we have:

$$CVaR_\alpha(x) = \frac{1}{\alpha}\int_0^\alpha VaR_v(x)\mathrm{d}v$$

Thus, partial derivatives are calculated as follow (Artzner & Delbaen, 1999):

$$\frac{\partial CVaR_\alpha(x)}{\partial x_i} = \frac{1}{\alpha}\int_0^\alpha \frac{\partial CVaR_\alpha(x)}{\partial x_i}\mathrm{d}v = -\frac{1}{\alpha}\int_0^\alpha E[r_i|-R'x = VaR_\alpha(x)]\mathrm{d}v =$$

$$-\frac{1}{\alpha}\int_0^\alpha E[r_i|X = q_\alpha(X)]\mathrm{d}v = -\ E[r_i|X \le -VaR_\alpha(x)]$$

The Total Risk contribution for each asset *i* of a portfolio is given from the following expression:

$$TRC_i^{CVaR_\alpha(x)}(x) = x_i\frac{\partial CVaR_\alpha(x)}{\partial x_i}$$

The expression in case of continuous returns distribution is the following:

$$TRC_i^{CVaR_\alpha(x)}(x) = -x_i E[r_i|X \le -VaR_\alpha(x)]$$

$$CVaR_\alpha(x) = \sum_{i=1}^{n} TRC_i^{CVaR_\alpha(x)}(x) = -\sum_{i=1}^{n} x_i E[r_i|X \le -VaR_\alpha(x)]$$

Numerical approximation for estimating $VaR_\alpha(x)$ and $CVaR_\alpha(x)$ Risk Parity using historical data we have to do the following assumption.

The calculation of $VaR_\alpha(x)$ and $CVaR_\alpha(x)$ of portfolio returns as follows:

$$VaR_\alpha(x) \approx -r_{\mathrm{p}\lfloor\alpha\mathrm{T}\rfloor}^{\mathrm{sorted}}$$

$$CVaR_\alpha(x) \approx -\frac{1}{\alpha\mathrm{T}}\sum_{\mathrm{j}=1}^{\lfloor\alpha\mathrm{T}\rfloor} r_{\mathrm{p\,j}}^{\mathrm{sorted}}$$

Where the *i*-th asset return r_i consist of *T* number outcomes r_{ji} with *i*=1,.....,*n* and *j*=1,...,T.

The vector of the observed portfolio returns is $R_P = (r_{p1}, \dots\dots, r_{pT})$ where:

$r_{pj} = x'r^j$ with j=1,....,T where $r^j = (r_{j1}, \dots\dots, r_{jT})$.

where α is level of significance and $\mathrm{r}_{\mathrm{p\,j}}^{\mathrm{sorted}}$ are the sorted portfolio returns such as

$$r_{p\,1}^{sorted} \le r_{p\,2}^{sorted} \le \cdots r_{p\,j}^{sorted} \le \cdots \le r_{p\,j}^{sorted}$$

With the time series observation, the approximation of the partial derivatives $CVaR_\alpha(x)$ for each asst *i* becomes:

$\frac{\partial CVaR_\alpha(x)}{\partial x_i} \approx -\frac{1}{\alpha\mathrm{T}}\sum_{\mathrm{k}=1}^{\lfloor\alpha\mathrm{T}\rfloor} r_{\mathrm{k\,i}}^{\mathrm{sorted}}\ \forall\ i$=1,...,*n*

and then the total risk contribution of asset *i* is

$$TRC_i^{CVaR_\alpha(x)}(x) = x_i\frac{\partial CVaR_\alpha(x)}{\partial x_i} \approx -\frac{1}{\lfloor\alpha\mathrm{T}\rfloor}x_i\sum_{\mathrm{k}=1}^{\lfloor\alpha\mathrm{T}\rfloor} r_{\mathrm{k\,i}}^{\mathrm{sorted}}$$

where $r_{\mathrm{k\,i}}^{\mathrm{sorted}}$ are the corresponding returns of asset *i* to the sorted portfolio returns.

In the rolling windows, in order to the measure the performance the following calculation $\mu_T^c(R_P)$ is the compounded return over the whole period (terminal compound return).

$$\mu_k^c(R_P) = \prod_{j=1}^{k}(1 + r_{pj}) - 1$$

To check if the portfolios are well diversified, we consider three diversification measures.

If the allocation is as followes $x = (x_1, x_2, \ldots, x_n)$ with te constraint $\sum_{i=1}^{n} x_i = 1$ in case short sales not allowed ($x_i \geq 0$). The diversification measure is the Herfindal index:

$$D_{Her} = 1 - xx'$$

In the same way, we introduce the diversification measure given by Bera and Park (2008).

$$D_{BP} = -\sum_{i=1}^{n} x_i \log(x_i) = \sum_{i=1}^{n} x_i \log\left(\frac{1}{x_i}\right)$$

The D_{BP} takes value between 0 (fully concentrated in one asset) and log(n) for the naïve portfolio.

Another important aspect is the consideration the transaction costs, and for that, the estimation of the turnover of the portfolio:

$$TO = \sum_{i=1}^{n} |x_i^{t+1} - x_i^t|,$$

where x_i^t denotes the weight of asset i at time t.

RESULTS

The times series selected from March 19, 2024 to March 18, 2025 is divided in two part. The first part is used to estimate the optimal weights for each portfolio, and is called in sample period L which is 125 days of observations. Then starting from October 1, the performance of portfolio is calculated in terms of compound returns for 5 days, and is called holding period H. After that for the calculation it is needed to roll these two periods each day until the last week.

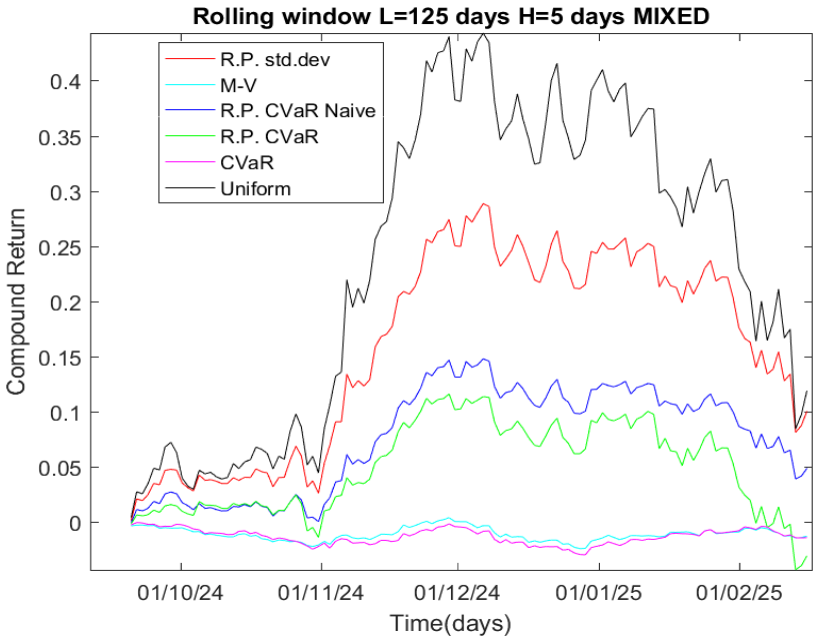

Figure 2. The out-of-sample performance of the portfolio mixed with stocks, bonds and cryptocurrencies

Figure 2 illustrates the compounded returns of the portfolios over the out-of-sample period. The results likely show that portfolios including cryptocurrencies exhibit higher returns compared to traditional portfolios, reflecting the high-risk, high-reward nature of crypto-assets. However, the volatility of these

returns should be noted, as cryptocurrencies are known for their price swings. The comparison between different optimization models (e.g., Min Variance, Risk Parity) would highlight which strategy balances returns and risk most effectively.

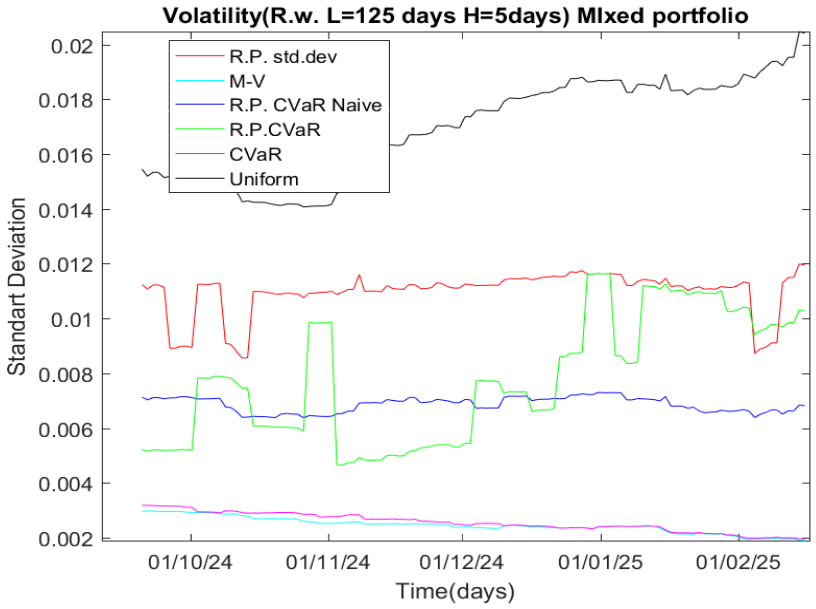

Figure 3. The out-of-sample volatility of the portfolio mixed with stocks, bonds and cryptocurrencies

Figure 3 displays the volatility (standard deviation) of the portfolios. Cryptocurrencies are expected to increase overall portfolio volatility due to their inherent price fluctuations. The Min Variance model should show the lowest volatility, while the naïve 1/N or crypto-heavy portfolios might exhibit the highest. The trade-off between volatility and returns is a key takeaway here.

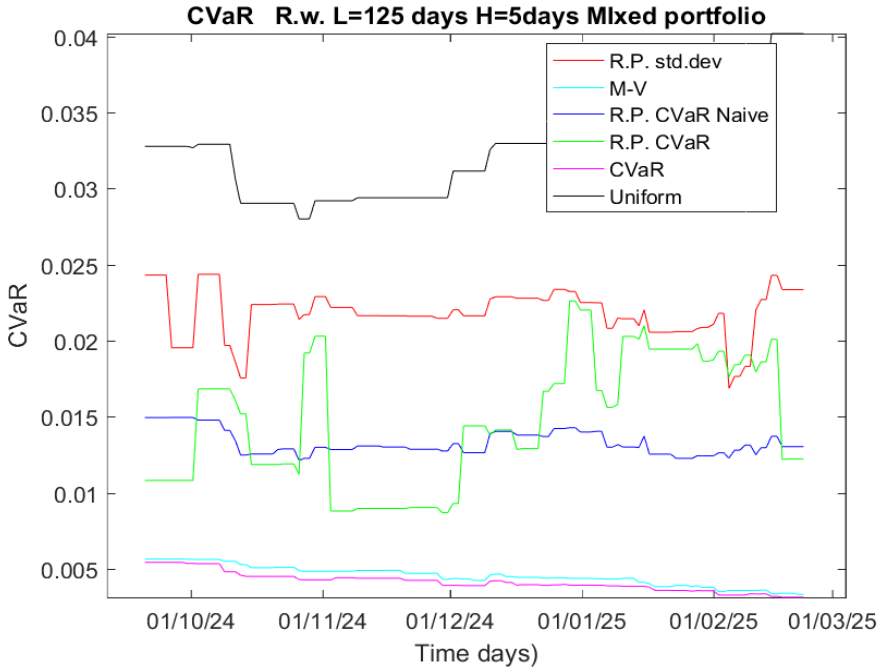

Figure 4. The Conditional Value at Risk of the portfolio mixed with stocks, bonds and cryptocurrencies

Figure 4 demonstrates that including cryptocurrencies increases tail risk, as crypto-assets are prone to sharp downturns. The Min CVaR and Worst-Case RP-CVaR models should perform better in mitigating extreme losses, emphasizing the importance of tail-risk management in crypto-inclusive portfolios.

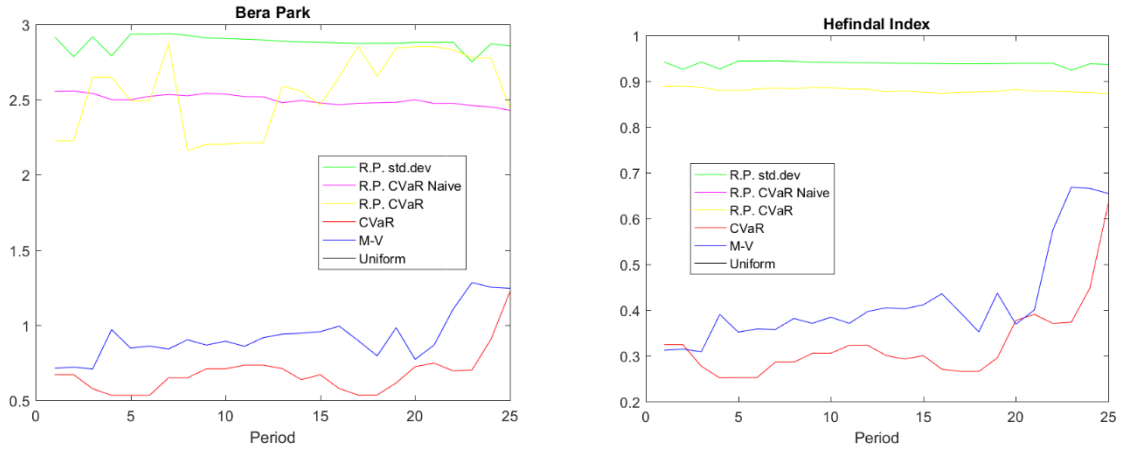

Figure 5. The Bera Park Index and Herfindal Index

The Bera Park Index and Herfindal Index in figure 5 measure portfolio concentration/diversification. The Herfindahl Index (closer to 0 for high diversification) and Bera Park Index (higher values for better diversification) likely show that the naïve 1/N portfolio is the most diversified, while optimized portfolios (e.g., Min Variance) may concentrate weights in fewer assets. Cryptocurrencies might improve diversification due to their low correlation with traditional assets.

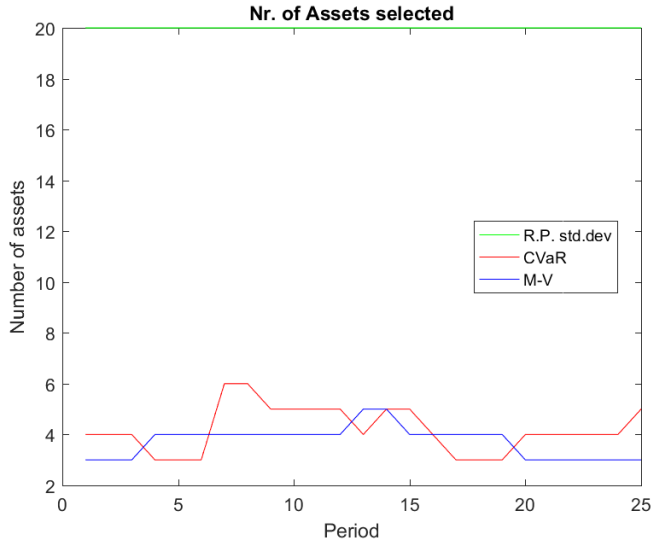

Figure 6. The number of asset selected with weights greater than!%

Figure 6 shows the active assets in each portfolio. The naïve 1/N portfolio should include all assets equally, while optimized portfolios (e.g., Min Variance or Risk Parity) may exclude or underweight high-risk assets like cryptocurrencies. The results could reveal how different models handle asset selection and weight allocation.

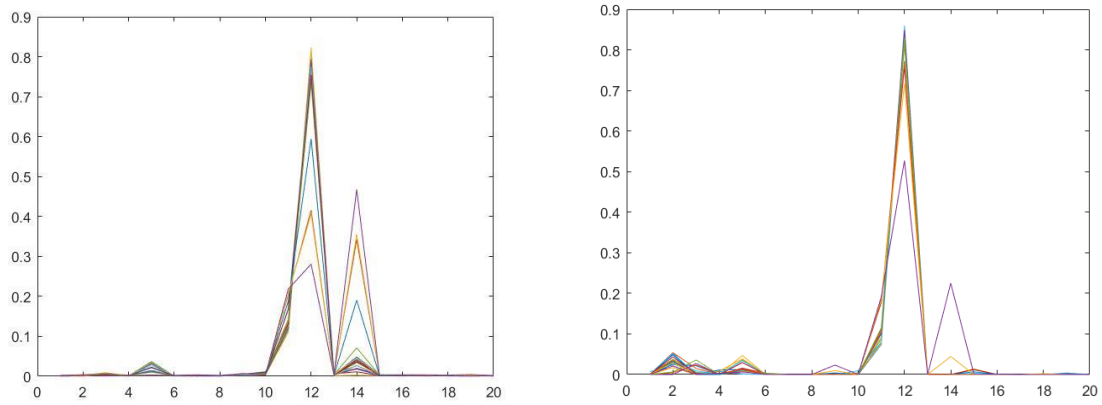

Figure 7. The weights of Mean Variance and CVaR

This figure 7 compares asset allocations under Mean Variance (risk-return trade-off) and CVaR (tail-risk focus) optimizations. Cryptocurrencies might receive lower weights in CVaR due to their tail risk, while Mean Variance could allocate more to crypto if their return potential justifies the risk. The contrast between these models highlights the impact of risk measures on portfolio construction.

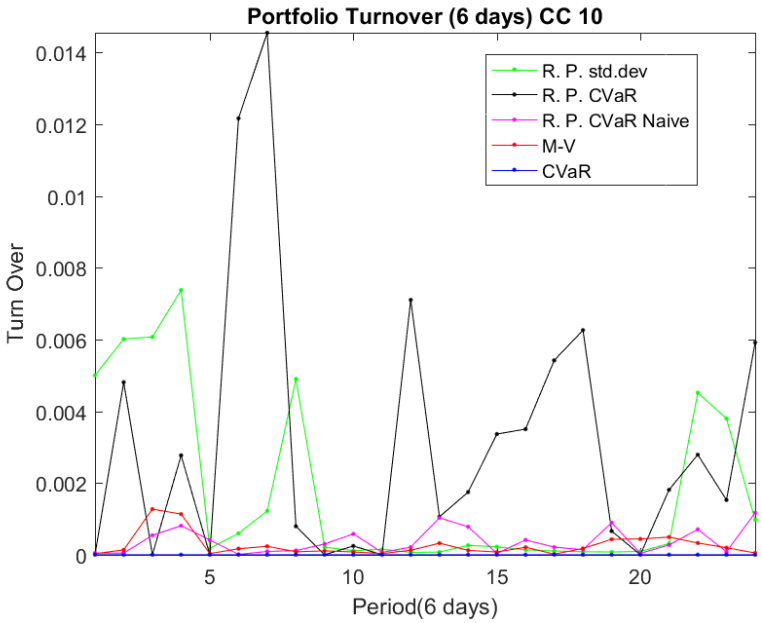

Figure 8. The Porfotlio Turnover

Model	**R.P. -STD**	**M-V**	**R.P. CVaR N.**	**R.P. CVaR**	**CVaR**
Average Turnover (%)	0.17724098	0.026176502	0.0375159788	0.31860653	1.6901401727 7968e-08

Turnover varies significantly across strategies, from near-zero (CVaR) to 0.32% (R.P. CVaR), reflecting differing rebalancing demands. While CVaR's stability minimizes costs, R.P. CVaR's active adjustments may enhance tail-risk resilience at the expense of higher trading friction.

DISCUSSION

The inclusion of cryptocurrencies in diversified portfolios has the potential and, as our empirical examination demonstrates, challenges. Based on our evidence, cryptocurrencies, despite their volatility, can enhance the diversification of portfolios since they are not highly correlated with traditional assets like stocks and bonds. This supports previous research (Platanakis & Urquhart, 2020; Bouri et al., 2017), which suggests that Bitcoin and other virtual currencies are employable as hedge instruments during periods of market uncertainty.

This benefit is, however, at the expense of a higher risk cost. Cryptocurrency portfolios were more volatile (Figure 2) and had higher tail risk as quantified by Conditional Value at Risk (CVaR) (Figure 3). This highlights the importance of effective risk management when investing in crypto-assets. The Risk Parity (RP) and Minimum Variance (MV) frameworks were effective in risk management but underweighted or omitted high-volatility coins, capping their return potential. On the other hand, the naive 1/N method, while diversified and simple, was more vulnerable to volatility in the cryptocurrency market.

Our results also underscore the value of optimization methods. The Mean Variance (MV) model, which attempts to balance risk and return, assigned medium weights to cryptocurrencies, while the CVaR-based models hedged exposure to crypto-assets due to their inherent tail downside risks. This implies that investors need to calibrate the portfolio strategy to their personal risk appetite—investors who need greater returns can stomach more volatility, while risk-averse investors need to prioritize hedging tail risks.

Transaction costs and turnover render crypto-inclusive portfolios even more complex. Rebalancing, as often demanded by crypto price volatility, can be detrimental to returns if the costs are not taken into consideration. We incorporated transaction costs into optimization in our study, demonstrating that even moderate fees can significantly influence performance, particularly for high-turnover portfolios.

CONCLUSION

This research demonstrates that cryptocurrencies can play a constructive function in hybrid asset portfolios through the addition of diversification and return. Yet, their usage demands a strict approach to risk management, in specific, against volatility and tail risk. Recall the following insights:

1. Diversification Advantage: Cryptocurrencies have low correlation with conventional assets and thus are helpful for portfolio diversification, particularly during times of turbulent markets.

2. Risk-Return Trade-off: Crypto-assets will increase returns but also increase significant volatility and tail risk. These risks are controlled by optimization models like Min Variance and CVaR but might possibly limit upside.

3. Strategic Allocation: Investors need to determine their risk tolerance before investing in cryptocurrencies. Risk-averse investors can opt for Risk Parity or CVaR-optimal portfolios, while growth investors can opt for Mean Variance or capped crypto exposure.

In summary, cryptocurrencies are not a one-size-fits-all portfolio addition. Their application needs to be finely calibrated according to investor goals, risk levels, and cost considerations. Applied correctly, they can help create a more robust and diversified investment approach in the new financial reality.

Final Notes

- **Policy Implications**: Regulatory clarity is essential for institutional adoption of crypto-assets in portfolios.
- **Practical Takeaway**: Investors should combine quantitative optimization with qualitative judgment when incorporating cryptocurrencies.

- **Limitations**: The study relies on historical data; future market conditions may alter risk-return dynamics.

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Author Contributions

All methodology, material and implementation was made from the author.

Testing the Relative Purchasing Power Parity in Türkiye: Comparing the Headline and Core Inflation (1046)

Ahmet Arvas, Mercan Hatipoğlu*

[1] Expert in Ankara Development Agency, Ankara, Türkiye
[2] Çankırı Karatekin University Department of Business Administration, Çankırı, Tükiye

*Corresponding author e-mail: mercanhatipoglu@gmail.com

Abstract

After the officially abandon of the Bretton Woods Agreements in the 1973, flexible foreign rates were adopted and exchange markets were allowed fluctuate freely. Thus, the question of how to resolve the value of exchange rates came to the agenda. The first theory that came to mind was Purchasing Power Parity. The notion of PPP embraces the idea that flexible foreign exchange rates adjust themselves right away according to inflation rates. Therefore, PPP asserts that the currency of the higher inflation country should be depreciated by the inflation difference. This paper examines the validity of the relative purchasing power parity (PPP) for Türkiye with its major trading partners: the USA, the UK and Euro area. To do so, simple linear regression models is employed to quarterly data over the period 2002–2023. The empirical findings illustrate that PPP is invalid for major partner currencies ($, £, €) since exchange rate movements and inflation differentials are not identical. However, when the headline inflation is used instead of core inflation, findings show that variations in exchange rates become more tied to inflation rates. Nevertheless, results also emphasize that Turkish Lira deprecation can be attributed other factors than inflation.

Keywords: PPP, Exchange rates, Inflation rates, Türkiye

INTRODUCTION

The price of the exchange rate gives investors an idea about many issues. For example, the performance of economy, whether the currency is accepted in world trade or the country's financial risk are first topics that come to mind (Nyambuu and Tapiero, 2018: 197). One of the variables most closely related to the exchange rate is inflation. When inflation starts to rise in a country, its impact on exchange rates is immediately felt. Firstly, the country's export demand decreases due to higher prices, which reduces the demand for the local currency. Secondly, since foreign goods will become relatively cheaper, the demand for foreign currency increases. Respectively, the reduced demand for local goods and increased demand for foreign goods simultaneously place downward pressure on the value of local currency and causing the depreciation of exchange rate (Madura and Fox, 2023: 265).

The first theory that relates exchange markets and consumer inflation is purchasing power parity (PPP). Simply put, PPP defined by Gustav Cassel, is an empirical proposition which defends the movements of exchange rates are driven by the inflation differential of two countries. PPP emerged because of how exchange rates should be determined after World War I. Because the value of pre-war exchange rates was determined by the amount of gold reserves that countries had. However, after the war, the possibility of governments attempting to rebuild their countries by printing money made it impossible to maintain the gold standard (Rogoff, 1996). PPP has two types of applications. One is Absolute PPP which defends that the exchange rate should be equal to the ratio of price levels. The other is Relative PPP which claims the inflation differential across the countries is underlying factor to determine the currency rate (Solnik and McLeavey, 2014: 83).

Many authors have listed several leading factor why purchasing power parity is invalid. To summarize, Rogoff (2007) points the volatility differential of price indices and exchange rates as causes of deviations from PPP. According to his findings, variations in commodity prices are markedly smaller than exchange rates. In another study, Rogoff (1996) suggested that value-added taxes also lead to deviation. Yoon and Jei (2019) cited downward rigidity of wage as the reason why the PPP is invalid. According to Solnik and McLeavey (2014:65, 78) rents, labor costs and sticky in good prices causes the departures from PPP especially in the short run. Similarly, Miles and Scott (2008:500) argued that transportation costs, border effects and market pricing invalidate the law of one price. Melvin and Norrbin (2017:135) noted that the consumers live in different countries choose different of basket goods. Thus, preparing the price index of each country with different weighted consumption patterns and nontraded goods causes deviations to occur. Despite shortcomings, the reason why PPP is still worth researching is that foreign exchange rates sooner or later revert to their fundamental values (Solnik and McLeavey, 2014: 90).

This study investigates validity of Relative PPP hypothesis for Türkiye from 2002:Q1 to 2023:Q4 using OLS regression. This paper differs from existing literature on two fronts. While headline inflation rates are used in many studies (Telatar and Kazdagli, 1998; Doganlar,1999; Yazgan, 2003; Alba and Park, 2005; Yıldırım, 2017; Özmen and Gökcan, 2004;) on Türkiye to determine purchasing power parity, this study use non-food, non-energy CPI. In addition, the general tendency in literature (Koncak and Güriş (2022), Coşkun (2020), Doğanlar et al. (2020), Doğanlar et al.(2021), Erdoğmuş (2021), Uğur and Alper (2023)) conducted for Türkiye is to test absolute PPP based on movements of the real exchange rate.

Apart from existing literature, this study tests Relative PPP by taking into account the inflation differences between the two countries. The parts of the study are classified in this fashion. Second section includes the literature review. Section 3 is devoted to overview of monetary policy and inflation in Türkiye. Fourth section expresses the data with model. Section 5 discusses the empirical findings. Finally, last part is the conclusion and contains some policy recommendation.

LITERATURE REVIEW

Considering that purchasing power parity has such a long history, it is not surprising that the literature is full of mixed findings. While early studies of Gailliot (1970), Officer (1978), Rush and Husted (1985) and Kim (1990) empirically supported the PPP hypothesis in general, conversely papers of Adler and Lehmann (1983), Abuaf and Jorion (1990) and Patel (1990) did not support for long run PPP. When we look at studies conducted with cointegration techniques, Varamini and Lisachuk (1998), Salehizadeh and Taylor (1999) and Arize, Malindretos and Nam confirmed the validation of PPP respectively for Ukraine emerging countries and Africa countries. However, the studies of Chocholatá (2009) and Baharumshah and Ariff (1997), Jacobo, and Sosvilla-Rivero (2021) that failed to backing purchasing power parity in cases of Slovakia, Asian countries and Argentina respectively, applying similar techniques. The studies of Zhou (1997), Haug and Basher (2011) have given partial support to the PPP hypothesis and stated that PPP is valid in the weak form. Again, many studies drawing attention to the nonlinear nexus exchange rates and have tested PPP using nonlinear time series models. For example, Bozoklu and Kutlu (2012), Su and Chang (2011), Baum et al. (2001) and the findings of Su et al.(2002) encourages purchasing power parity. However, using similar techniques, Tiwari and Shahbaz, (2014) rejected the PPP hypothesis for India. Studies conducted for euro area, OECD, ASEAN-5, G7 and 159 international countries respectively by Koedijk et al. (2004), Kalyoncu and Kalyoncu (2008), Munir and Kok (2015), Kargbo, (2009) and Vo and Vo (2023) found support for PPP. Considering the studies conducted based on Tukiye, it is clearly seen that PPP is a controversial issue because many paper presented mixed findings. Koncak and Güriş (2022), Doğanlar et al.(2020), Doğanlar et al.(2021) and Uğur and Alper (2023) confirmed PPP's validity for Türkiye, whereas Azazi et al.(2023), Erdoğmuş (2021) and Coşkun (2020) concluded that PPP is not valid.

OVERVIEW OF MONETARY POLICY AND INFLATION IN TÜRKİYE

Türkiye allowed the exchange rates to float freely after 2001 crisis and again as of the same year The Central Bank of Türkiye adopted inflation targeting model. While the fixed exchange rate regime was applied in Türkiye until 1980, roughly crawling peg exchange rate and managed float regime was adopted between the episodes 1980-2001 (Leigh and Rossi, 2002). Particularly, between 1980 and 1989, the aim of exchange rate policy was to support export-led growth strategy. In later periods, the nominal value of the Turkish Lira was depreciated by the central bank in line with inflation expectations (Kandil et al., 2007).

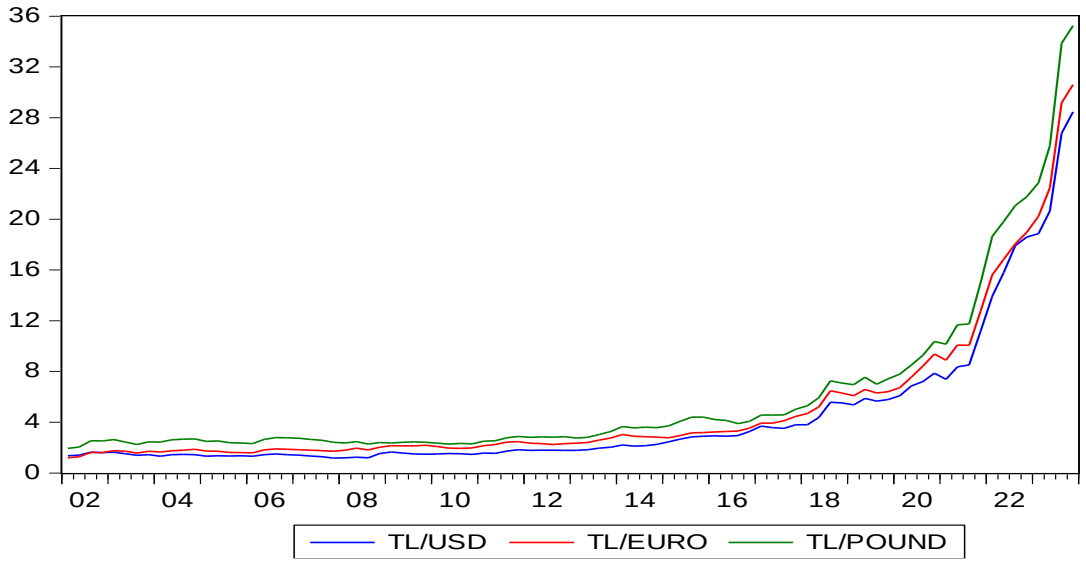

Figure 1. Value of TL against major currencies
Source: CBRT Department of Statistics

Overall, figure 1 shows that after remaining stable, the TL depreciated strongly relative to other currencies since the mid of 2019. During the 2002-2012 episodes, no abrupt movement was observed in exchange rates due to price stability. As of 2023, it seems that major currencies reached all-time high record against TL.

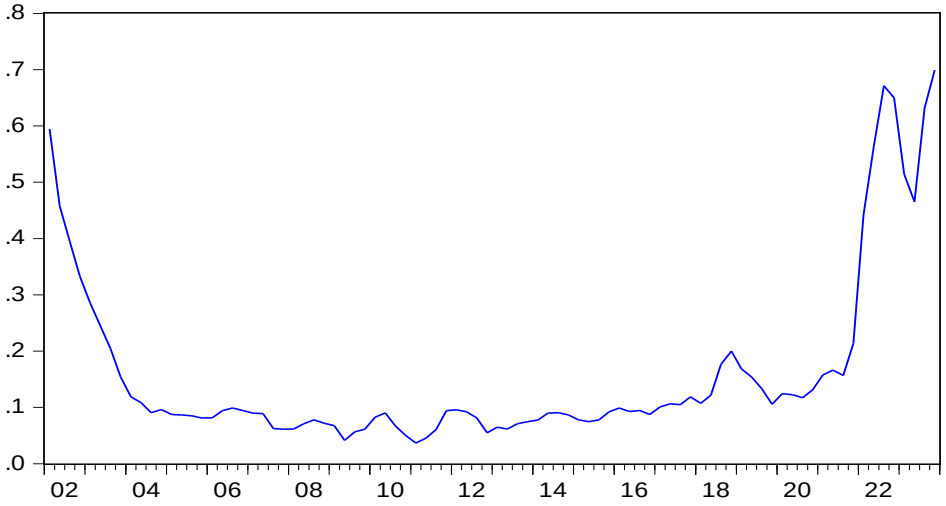

Figure 2. Türkiye inflation rate between 2002Q1-2023Q4, %

Source: CBRT Department of Statistics

Türkiye has suffered from stubborn inflation for years. Inflation rates reached very high levels in 2002 and 2021. In fact, inflation declined markedly from 60 percent in 2002 to 10 percent in 2004 within two years and before climbing back to 20 percent in 2021, stayed moderate levels from 2004 to 2012. The inflation

targeting regime worked reasonably well during the period 2004-2012. Nevertheless, covering the period 2002-2023, inflation averaged about %16 quarterly, meaning that price stability has not yet been achieved. According to Cecchetti and Schoenholtz (2017), the causes of the inflation may be attributed highly to mistakes in monetary policy. However, the failed fiscal policy as a result of politicians spending more than necessary by relying on the central bank's ability to print money is also shown another reason. In some articles from the Turkish economic literature, the causes of inflation are attributed to other issue. For example, Demiralp and Demiralp (2019) stated that declining central bank of Türkiye independence result in weakening inflation targeting regime. In similar vein, Gürkaynak et al., (2023) emphasized that the central bank's early interest rate cut decisions are the main reason for the recent jump in inflation. Lastly, Yilmazkuday (2022) concluded by using VAR models that economic drivers of Turkish inflation are oil price fluctuation and US dollar rate.

DATA AND EMPIRICAL MODEL

Data

The dataset consists of quarterly observations which covers the period from 2002 Q1 to 2023:Q4 (totally 88 quarters) for Türkiye and its three major partners Euro Area, the UK, and the USA. The reason why it was started in 2002 is that the value of the Turkish Lira has been determined by market forces since that year. The inflation data set includes both headline consumer inflation and Core inflation (non-food non-energy consumer inflation) is obtained from the OECD Statistics database. The nominal exchange rates are derived from data system of The Central Bank of Türkiye.

Econometric Methodology

The paper benefited from the following ordinary least squares (OLS) regression model similar to the methods used in studies of Mishkin (1984) and Nyambuu and Tapiero (2018):

$$\%\Delta S_t = \alpha + \beta(\pi_t^{TUR} + \pi_t^F) + \varepsilon_t \quad (1)$$

Where $\%\Delta S_t$ is a percentage change in spot exchange; π^{TUR} and π^F are percentage changes in inflation of Türkiye and foreign countries, respectively and finally ε_t is a residual term distributed with constant variance, uncorrelated with one another and expected mean value is zero. Whereas the α parameter stands for intercept and is the expected deprecation or appreciation of currency, the β parameter demonstrates the effect of inflation differentials on currencies. If the relative version of PPP holds, β must be equal to 1.

FINDINGS

Summary statistics

Table 1 illustrates the descriptive statistics of the bilateral of inflation differentials and exchange rates between Türkiye and the USA, Euro zone and the UK. The p-values of the Jarque–Bera (JB) tests clearly indicate that all series have non-normal distribution. Both the means (expected value) and standard deviations (risk) of exchange rates are very close to each other. Because international foreign exchange markets do not allow arbitrage, when TL depreciates, it loses equally value against all currencies. The same is almost valid for inflation differences. Whereas the means of inflation differences ranges between 0.13 to 0.14, standard deviations varies between 0.15 and 0.16. However, it should be emphasized that core inflation differentials fluctuate relatively higher than headline inflation. A glance at table reports positive skewness for all of inflation differentials and exchange rates. In terms of exchange rates, the findings denote the depreciations of TL outweigh the appreciations against major currencies. The kurtosis values of inflation differentials support the the prevalence of fat tail distributions with more outliers. This fact is a result of sudden jumps in inflation in Türkiye.

Table 1. Summary statistics of inflation differentials and exchange rates

	TR-USA			TR-EURO			TR-UK		
	ID^h	ID^c	%ΔS	ID^h	ID^c	%ΔS	ID^h	ID^c	%ΔS
Mean	0.13	0.14	0.03	0.14	0.14	0.03	0.14	0.14	0.03
Std. Dev	0.15	0.16	0.07	0.15	0.16	0.07	0.15	0.16	0.07
Skewness	2.11	2.24	1.02	2.09	2.22	0.89	2.06	2.21	0.98
Kurtosis	6.33	7.02	4.17	6.29	6.97	3.97	6.13	6.96	4.23
JB [Prob]	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Correlation*	0.33	0.34		0.37	0.36		0.35	0.33	

Source: Author's computation

ID^h: headline inflation differential, ID^c: headline inflation differential, %ΔS: spot exchange rate return

* Correlation with exchange rate

Unit Root Results

Table 2. Phillips-Perron unit root tests results

	Level			First difference of log level		
	constant	constant with trend	None	constant	constant with trend	None
TL/USD	22.93	17.25	10.11	-7.28^{***}	-8.95^{***}	-6.39^{***}
TL/EURO	22.90	37.84	9.92	-7.62^{***}	-8.42^{***}	-6.46^{***}
TL/POUND	29.89	24.03	11.11	-7.39^{***}	-8.33^{***}	-6.47^{***}
Headline						
$\pi^{TR} - \pi^{USA}$	-1.22	-2.01	-0.77	-5.45^{***}	-5.91^{***}	-5.51^{***}
$\pi^{TR} - \pi^{EURO}$	-1.23	-2.13	-0.75	-5.41^{***}	-5.89^{***}	-5.47^{***}
$\pi^{TR} - \pi^{UK}$	-1.36	-2.17	-0.90	-5.42^{***}	-5.92^{***}	-5.48^{***}
Core						
$\pi^{TR} - \pi^{USA}$	-2.70^{*}	-3.58^{**}	-2.03^{**}	-6.27^{***}	-7.01^{***}	-6.24^{***}
$\pi^{TR} - \pi^{EURO}$	-2.54^{*}	-3.53^{**}	-1.74^{**}	-6.14^{***}	-6.87^{***}	-6.16^{***}
$\pi^{TR} - \pi^{UK}$	-2.65^{*}	-3.55^{**}	-1.90^{**}	-6.18^{***}	-6.94^{***}	-6.20^{***}
critical values						
1% level	-3.50	-4.06	-2.59	-3.50	-4.06	-2.59
5% level	-2.89	-3.46	-1.94	-2.89	-3.46	-1.94
10% level	-2.58	-3.15	-1.61	-2.85	-3.15	-1.61

Source: Author's computation

For checking the stationary of variables, the Phillips-Perron (1988) unit root test was preferred constant, constant with trend and as well as none. Table 2 revealed that all series appear to be non-stationary in level forms except the core inflation differentials. However, when log-first differences of each variable were used, non- stationarity was rejected in all the cases.

Regression results

Table 3. OLS Regression results of relative PPP

	With Headline inflation			With Core Inflation		
	USA	EURO	UK	USA	EURO	UK
α	0.03^{***}	0.03^{***}	0.03^{***}	0.03^{***}	0.03^{***}	0.03^{***}
β	0.79^{***}	0.50^{**}	0.56^{**}	0.50^{***}	0.29^{**}	0.32^{*}

R^2	0.21	0.10	0.12	0.15	0.05	0.07
LM [1]	0.29	0.11	0.07	0.15	0.08	0.04
ARCH [1]	0.87	0.47	0.82	0.71	036	0.49
F-Stat.	23.90	10.27	12.29	0.13	5.33	6.47
F-Prob.	0.00	0.00	0.00	0.00	0.00	0.01

Source: Author's computation
Significant at: 1%***; 5%**; 10%*

Without exception, table 3 demonstrates positive β coefficients that are less than 1. In other words, econometric results are consistent with idea that inflation differentials have a statistically significant influence on exchange rates. Since beta coefficients are in the range of 0.32 to 0.76, table presents little empirical evidence in favor of PPP. Turning to the β coefficients in table, one percent increase in headline inflation differentials leads to depreciates TL by 0.79, 0.50 and 0.56 percent, against the USA dollar, the Euro and the pound, respectively. On the other hand, taking core inflation rate into consideration, the effect of inflation differences on the exchange rate weakens significantly. In the case of core inflation, a one percent expansion between inflation differences causes the Turkish lira to lose value by 0.50, 0.29 and 0.32 percent, against the currencies of USA, Eurozone, and UK, respectively. Overall, findings indicate that PPP does not valid because the exchange rate movements and inflation differentials are not identical. The facts that the coefficients are not equal to 1 confirm the inflation differences are not fully reflected in the exchange rate. In addition, the headline inflation data offer more support for relative PPP than core inflation. Of course, the failure of validation of PPP does not imply that the exchange rate cannot achieve in reflecting inflation differentials. For example if Turkish inflation is higher than U.S. inflation, the Turkish lira will depreciate relative to the dollar but the depreciation is not be one-to-one, but will be half of the inflation differences. For this reason, changes in exchange rates tied to differences in inflation rates, even if there is a weak linkage.

In terms of R^2 , all three countries have low value, indicating that models fit the data poorly. The R^2 of models vary from the 0.05 to 0.21, implying that inflation differentials with optimistic forecast, explain less than 20% in the variations of exchange rate movements. This means that the remaining 80% of foreign exchange movements can be attributed factors other than inflation. Finally, table 3 shows that both LM and ARCH probability values indicate no any remaining serial correlation and heteroscedasticity of the error term.

CONCLUSION

Turkish Lira is free floating and fully determined by non-government market forces that reflect economic performance and future expectations at each point in time. Also, Lira have been fluctuating according to supply and demand freely especially after 2002. The findings of the study indicate that the price of exchange rate is not as simple as presumed by PPP. Nevertheless, it is worth stressing that TL will depreciate against major currencies quite markedly depending on the type of price index. Therefore the currency of Türkiye with higher inflation tends to weaken over time against the US dollar, the Euro and the British pound. Moreover, since the depreciation of the TL is not offset by the inflation difference precisely, Türkiye will be less competitive in the international market. To sum up, widening inflation differential feeds the depreciation of the TL. The monetary authorities should have to close the gap by implementing tightening monetary policy.

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Science and Patronage: The Collaboration of Ghiyath Al-Din Jamshid Al-Kashi and Ulugh Beg (1065)

Mehrovar Khojaev Pardalievich[1*]

[1]Associate professor of the Department of History of the ancient world, Middle Ages and archeology, Tajik National University, Republic of Tajikistan.

*Corresponding author e-mail: mehrovar_kh@mail.ru

Abstract

This article explores the collaboration between Ghiyath al-Din Jamshid al-Kashi, a prominent Persian mathematician and astronomer, and Ulugh Beg, a Timurid ruler and scholar. It highlights how Ulugh Beg's patronage and establishment of the Ulugh Beg Observatory in Samarkand created a thriving environment for scientific advancement during the 15th century. Al-Kashi's contributions, including precise astronomical calculations and mathematical innovations, were made possible through this support. Their partnership illustrates the vital role of enlightened leadership in fostering scientific progress and leaving a lasting legacy in the history of astronomy and mathematics.

Keywords*: Ghiyath al-Din Jamshid al-Kashi, Ulugh Beg, Science, Patronage, Collaboration, Timurid Dynasty, Samarkand, Ulugh Beg Observatory, Astronomy, Mathematics, Islamic Golden Age, Scientific advancement, Khaqani Zij.*

INTRODUCTION

The collaboration between Ghiath al-Din Jamshid al-Kashi and Ulugh Beg represents a defining moment in the history of Islamic science, particularly in the fields of mathematics and astronomy. Set against the backdrop of the Timurid Empire in the 15th century, this partnership exemplifies the critical role of patronage in fostering scientific innovation. Ulugh Beg, the Timurid ruler and grandson of Timur, was not only a political leader but also a passionate scholar, deeply committed to advancing knowledge in the sciences. His reign is particularly notable for the establishment of the Samarkand observatory, one of the most advanced astronomical centers of its time.

Al-Kashi, one of the most brilliant mathematicians and astronomers of the period, found a receptive and supportive environment in Ulugh Beg's court. His groundbreaking work on trigonometry, algebra, and astronomical calculations, particularly his precise measurement of pi and his contributions to the development of tables used in astronomical observations, helped lay the groundwork for future advancements in these fields. The intellectual collaboration between the two figures not only led to advancements in mathematical and astronomical methods but also illustrated the symbiotic relationship between scholarly achievement and royal patronage in the medieval Islamic world.This introduction sets the stage for exploring the dynamic collaboration between Ulugh Beg and al-Kashi, underscoring the importance of such partnerships in the flourishing of scientific knowledge and the broader intellectual culture. Through their work together, they made contributions that would echo through the centuries, influencing the scientific thought of both the Islamic world and Europe.

Historical Context and Background.

The remarkable partnership between the Persian-Tajik mathematician and astronomer Ghiyath al-Din Jamshid al-Kashi and the Timurid king Ulugh Beg stands as one of the most notable examples of science flourishing under enlightened patronage in the Islamic Golden Age.

Ulugh Beg, the grandson of the Amir Timur, was not only a political leader but also a passionate scholar. Unlike many rulers of his time, he deeply valued knowledge and actively supported the advancement of science.

Ghiyath al-Din Jamshid ibn Masʿud ibn Maḥmud al-Kashi, a renowned Persian-Tajik physician, mathematician, and astronomer, was born in 790 AH (1380 CE) in Kashan, Iran, and passed away on the 11th of Ramadan, 832 AH, corresponding to June 22, 1429 CE, in Samarkand. In European texts, the name «Kashi» has been derived from its Arabicized form «al-Kashani» and often appears as «al-Kashi». It should not be forgotten that another mathematician named Cauchy is well known in the history of mathematics - Augustin-Louis Cauchy (1789–1857), a French scholar, whose name bears similarity in pronunciation but is entirely unrelated.

According to the research of Abulqasim Qurbani, Ghiyath al-Din Jamshid al-Kashi departed from his hometown of Kashan for Samarkand in the year 824 AH [5, pp. 5-9]. Kashan is located approximately 200 kilometers south of Tehran, in the western part of the Dasht-e Kavir desert, halfway between Tehran and Isfahan.

At the end of the 8th century AH (late XIV century CE), during the time of al-Kashi's birth and childhood, the city, like many others in Iran, was under the rule of Timur and later his son Shahrukh. This period was marked by the lingering memory of the massacres, destruction, and devastation caused by the Mongol invasions. As a result, numerous local socio-religious movements emerged, such as the Hurufiyya and Nuqtaviyya sects.

Timur, on the one hand, destroyed flourishing cities he conquered, and on the other, he sought to develop the cities of Central Asia-especially his power center, Samarkand. In line with this policy, he captured numerous artisans and scholars from Mesopotamia, Asia Minor, Syria, Iran, and India, and relocated them to Transoxiana. As such, he gathered a large number of scholars and craftsmen in his capital, Samarkand, which led to a significant cultural and scientific flourishing of the city during his reign.

After Timur, his son Shahrukh entrusted the governance of Samarkand and the entire region of Transoxiana to his own son, Ulugh Beg (Timur's grandson), while Shahrukh himself ruled from Herat. Ulugh Beg, who was raised under the care of Saray Mulk Khanum (1341-1408) wife of Timur-received an excellent education and ruled over this vast territory for forty years. His mother, Gawharshad (died 19 July 1457), had a passion for constructing magnificent buildings and, together with her husband Shahrukh, took part in the restoration of the ruins left by Timur. She helped establish many madrasas and mosques, including the Gawharshad Mosque in Mashhad, which is among their contributions.

Ulugh Beg's brother, Baysunghur (1397-1433), became especially well known for his support and promotion of scholars and artists. Within this intellectually and culturally rich family environment, Ulugh Beg himself was nurtured with a strong enthusiasm for development and the advancement of science and learning. Like his grandfather Timur, Ulugh Beg pursued the strengthening and enrichment of Samarkand, though through more peaceful and constructive means.

It was under his financial sponsorship and active support that a scientific institution was founded in Samarkand, which is recorded in the history of science as the «Scientific School of Samarkand» or the «Ulugh Beg Scientific School». This institution attracted numerous scholars from Khorasan, Transoxiana, and even from Rum (Asia Minor) and India. Among the most renowned were Ghiyath al-Din Jamshid al-Kashi, Qaḍizada al-Rumi, ʿAli Qushchi Samarkandi, Muʿin al-Din Kashi, and Jalal al-Din Usturlabi.

As a patron of science and the arts, Ulugh Beg occasionally participated in the astronomical activities of the scholars of his time and took great pleasure in observing their work.

Ulugh Beg's support for scholarship and science stood alongside the technical expertise and artistry of the masters and craftsmen of Iran and Transoxiana, which endured for centuries-even as the hand of time turned

many palaces into «mountains» of ruin. After constructing the madrasa in Samarkand, Ulugh Beg appointed one of the prominent scholars, Muhammad Khafi, as its teacher. This individual had a disheveled and poor appearance, but when Ulugh Beg questioned him and recognized the depth of his knowledge, he sent him to the royal bathhouse and had him dressed in luxurious garments. On the day of the madrasa's inauguration, Ulugh Beg himself, along with ninety other scholars of Samarkand, attended Muhammad Khafi's lecture [3, p. 40].

According to the author of Zubdat al-Tawarikh, Hafiz-i Abru [7, p. 744], the construction of the madrasa began in the year 823 AH. Abulqasim Qurbani [6, p. 8] states that it was built in 824 AH by order of Ulugh Beg with the help of craftsmen from Shiraz, Kashan, and other regions (parts of Transoxiana – A.K.), on the northeastern side of the city of Samarkand, atop a hill called «Kuhak», which today is known as «Chuponata» or, according to Bahrom Shermuhammadiyon [2], «Chuponato», near the «Obi Rahmat» (Water of Mercy) river.

To design, construct, and operate his observatory, Ulugh Beg invited a group of renowned scholars to Samarkand. Among them were Ghyath al-Din Jamshid al-Kashi, Salah al-Din Musa ibn Muhammad ibn Mahmud Qadi Zada al-Rumi (1360–1437), Ala al-Din Ali ibn Muhammad al-Qushji of Samarkand (d. 1474), Mu'in al-Din al-Kashi, and others. The subject of inviting these scholars to Samarkand is described in the preface to the comprehensive Zij-e Sultani by Sa'id, where it is written: «The martyred Sultan Mirzo Ulugh Beg-may God make his evidence manifest-proposed the construction of an observatory in Samarkand and summoned those skilled in this science. Among them were the exalted master, the standard-bearer of excellence and wisdom, Mawlana Salah al-Milla wa al-Din Musa, known as Qadi Zada; the eminent Mawlana Ghyath al-Milla wa al-Din Jamshid, who was unmatched in geometry, arithmetic, and astronomical operations; the famous master Jalal al-Din Usturlabi, renowned in his time, who was summoned from distant lands to design and build the observational instruments; and the greatest of the later scholars, the pole of engineers, Mawlana Ali Qushji-may God sanctify his resting place…» [6, p. 5].

It is likely that the reason for al-Kashi's invitation to Samarkand was related to a zij (astronomical table) he had previously compiled in Kashan. He had dedicated this zij to Shahrukh Timurid (Ulugh Beg's father), who ruled Herat, and named it the Zij-e Haqani in honor of Shahrukh, who held the title «Khaqan». Al-Kashi later presented this work to Mirzo Ulugh Beg, the ruler of Samarkand.

In this context, the late scholar Muhit Tabataba'i writes: «Qadi Zada al-Rumi, one of the scholars from the land of Rum (Asia Minor), was traveling through Iraq on his way to meet Mirzo Ulugh Beg, the Timurid ruler of Transoxiana. During his journey, he passed through the city of Kashan. One day, while wandering through the alleys, quarters, and marketplaces of the city in search of a fellow mathematician, he happened upon a small room where a short man was seated, reading a book. Around him were scattered a number of books, an armillary sphere, an astrolabe, and other astronomical instruments.

Realizing that the man was likely a mathematician and astronomer, Qadi Zada became very pleased and entered to learn more. He greeted the man and asked, «Mawlana, what are these things? » The short man replied, «What do you want with them? » Qadi Zada said, «I would like to buy them». The man asked, «What exactly do you want to buy? » Qadi Zada replied, «I'm looking to purchase an astrolabe». The man asked again, «What kind of astrolabe would suit your needs? » Qadi Zada named a particular type. The man in the room replied, «I will only sell this astrolabe to someone who can use it to compute the sine (*jayy*) of one degree and answer my questions in astronomy and mathematics» [6, p. 2].

Qadi Zada was unable to calculate the sine or respond to the man's questions. Instead, he asked him several questions related to astronomy and geometry. The man answered each one quickly and correctly. Amazed by his profound knowledge, Qadi Zada asked for his name and learned that he was Ghyath al-Din Jamshid.

A bond of friendship was formed between them. When Qadi Zada arrived in Samarkand and learned that Ulugh Beg intended to establish an observatory and compile a new zij, he told the prince: «The only man I know worthy of this task is a short man who knows the heavens as well as the earth. When Ulugh Beg learned of Ghyath al-Din's abilities, he sent a delegation to summon him from Kashan to Samarkand, and entrusted the observatory project to him» [6, p. 2].

Before his journey to Samarkand, Ghyath al-Din Jamshid al-Kashi had already earned the title «Second Ptolemy» due to his astronomical expertise. He had authored several works in the field of astronomy and personally engaged in celestial observations. In an incomplete manuscript of one of his *zij*s (astronomical tables) preserved in the library of Astan Quds Razavi, which the late Muhit Tabataba'i identified as the Zij al-Tashilat, and which the renowned historian of science and orientalist Professor Edward Stewart Kennedy (1912–2009) believed to be a portion of the Zij-i Haqani, al-Kashi writes: «We conducted three lunar eclipse observations in the city of Kashan and extracted the mean positions of the Moon from them… The midpoint of the third eclipse occurred on the night of the 18th of the ancient month of Shahrivar in the year seven hundred seventy-six of the Yazdegerd calendar (809–810 AH) » [5, p. 20].

After his arrival in Samarkand, Ghyath al-Din Jamshid al-Kashi played a central role in the creation of the observatory and the compilation of the *Zij*. As ʿAbd al-ʿAli al-Birjandi writes in his commentary on Ulugh Beg's *Zij*: «The foundation of the Samarkand observations is the product of his [al-Kashi's] refined intellect».

Al-Kashi himself, in a letter written from Samarkand to his father, indicated his leading involvement in the project: «The construction of the observatory was carried out in the manner that this servant had described». The 9th-century AH historian Khvandamir also speaks of Ulugh Beg in his work Habib al-Siyar and mentions Mawlana Muʿin al-Din al-Kashi, who, according to the late Muhit Tabataba'i, was the nephew of Jamshid al-Kashi. He accompanied al-Kashi from Kashan to Samarkand and took part in the observatory's work. Some of al-Kashi's surviving manuscripts are said to be in Muʿin al-Din's fine handwriting [5, p. 7; 6, p. 6].

The construction of the Samarkand observatory was completed in the year 825 AH (or possibly 827 AH). Al-Kashi participated in the observatory's activities for 5 to 7 years, up to his death in 832 AH. The Zij-i Ulugh Beg was completed nine years later, in 841 AH. In its preface, Ulugh Beg names and honors his collaborators, giving special praise and respectful mention to al-Kashi [3, p. 31].

In his book Observatories in Islam [9], Aydin Sayili states that the Samarkand Observatory influenced the earliest observatories in Europe. Among the significant contributions of Ghyath al-Din Jamshid al-Kashi is his invention of decimal fractions, which, several years later, were independently discovered by Europeans.

Al-Kashi anticipated the elliptical orbits of the Moon and Mercury two centuries before Kepler, in the first and second appendices of his work Nuzhat al-Hadaiq. Additionally, he calculated the value of π with remarkable accuracy and determined the sine of one degree using a clever iterative method to solve cubic equations with exceptional precision.

He also invented two astronomical instruments known as the Tabaqat al-Manatiq (Layers of Zones) and the Lawh al-Ittisalat (Tablet of Conjunctions). According to the Soviet-era Russian translator and commentator of al-Kashi's treatise Miftaḥ al-Ḥisab (The Key to Arithmetic), the renowned mathematician, historian of science, and orientalist Boris Abramovich Rosenfeld (1917–2008), Ghyath al-Din Jamshid al-Kashi was the greatest mathematician and astronomer of the XV century.

Rosenfeld further emphasized that some of al-Kashi's works represent the highest level of scientific achievement in the medieval period. The advanced methods al-Kashi used in his computations and the technical mastery displayed in his estimations and evaluations continue to captivate modern scholars [8].

The German scholar Paul Lockey writes: «He is recognized as a mathematician who was intelligent, inventive, a critic, and a profound thinker, well-versed in the works of previous mathematicians, especially in the field of computation and the application of approximate methods» [5, p.11].

Professor Kennedy, a distinguished historian of mathematics and astronomy, also praises him as an outstanding calculator who had exceptional skill in the field. Wherever the name of al-Kashi is mentioned, his predecessor Ulugh Beg, the ruler who was a patron of justice, knowledge, and the arts, is also respectfully acknowledged. However, his time did not bring him the honor he deserved.

After Ulugh Beg's death, his son Abdulatif, who had been given the governorship of Balkh by his father, did not feel it was worthy of his status. He rebelled against his own father's legacy and, in 853, seized power, ordering Ulugh Beg's assassination. Following Ulugh Beg's death, the observatory ceased its activities, and Ali Qushji, his only disciple and assistant, left Samarkand. He sought refuge in Istanbul, where he passed away in 879.

It must be noted that the end of Ghyath al-Din Jamshid al-Kashi's life was also tragic. Al-Kashi was killed on the morning of Wednesday, the 19th of Ramadan in the year 832 AH (equivalent to the 10th of July 1429 CE), at the location of the Samarkand Observatory, which he had designed and supervised. He was 49 years old at the time of his death. This date is recorded on the last page of a manuscript of his work Miftaḥ al-Ḥisab in the Central Library of Tehran University and the first page of a manuscript of the Zij-i Khaqani in the Indian diwan. The inscription reads: The death of the great master, Mawlana Ghyath al-Din, may Allah sanctify his soul, occurred at the beginning of the Wednesday morning, the 19th of Ramadan in the year 832 AH, outside the city of Samarkand, at the location of the observatory.

Additionally, a manuscript numbered 102 in the Astan Quds Razavi Library, which the late Tabatabai referred to as al-Kashi's Zij-i Tashilat, includes a note in the margin that reads: «The passing of the great scholar and astronomer, Ghyath al-Millat wa al-Din Jamshid, may Allah sanctify his soul and grant him paradise, occurred on the morning of Wednesday, the 19th of Ramadan in the year 832 AH, outside the city of Samarkand at the location of the observatory».

Muhit Tabatabai suggested that this note was likely recorded by al-Kashi's nephew, Mu'in al-Din al-Kashi, in the margin of his copy of the Zij on the very day of his death.

CONCLUSION

The collaboration between Ghiath al-Din Jamshid al-Kashi and Ulugh Beg stands as a monumental example of the intersection between science, patronage, and intellectual exchange. Their partnership reflects the vibrant culture of scientific inquiry and innovation fostered under the patronage of Ulugh Beg, whose own interests in astronomy and mathematics provided the necessary support for al-Kashi's groundbreaking work.

Al-Kashi's contributions, particularly in the realms of trigonometry and astronomical calculations, were pivotal in advancing the scientific knowledge of his time. His work on the calculation of pi, his methods for solving algebraic equations, and his precision in astronomical tables were all products of a collaboration that would not have been possible without the strong institutional and financial support provided by Ulugh Beg. In turn, Ulugh Beg's own achievements, most notably the construction of the Samarkand observatory, were deeply influenced by the expertise of scholars like al-Kashi, who helped refine the tools and techniques that would push the boundaries of celestial knowledge.

This relationship between a patron and scholar not only illustrates the crucial role of royal support in the advancement of science but also highlights the synergy that arises when intellectual curiosity is paired with resources and institutional backing. The work of al-Kashi and Ulugh Beg left a lasting legacy, influencing not only their contemporaries but also future generations of mathematicians, astronomers, and scholars in both the Islamic world and Europe.

In summary, the collaboration between Ghiath al-Din Jamshid al-Kashi and Ulugh Beg exemplifies how science flourishes when supported by visionary leadership and intellectual freedom, and serves as a reminder of the transformative power of patronage in advancing human knowledge.

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Modeling Corporate Credit Rating Transitions Using Hidden Markov Models: An Empirical Analysis Based on S&P Global Data using MATLAB (1071)

Denis Veliu[1],

[1] Departament of Finance-Banking, Metropolitan University of Tirana, 1000 Tirana, Albania

*Corresponding author e-mail: dveliu@umt.edu.al

Abstract

This study employs Hidden Markov Models (HMMs) to analyze corporate credit rating transitions and default probabilities using data from the S&P Global 2023 Annual Default Study. By modeling latent credit risk states and their influence on observed rating migrations, we uncover dynamic patterns that traditional transition matrices overlook. Our findings reveal that B- and CCC-rated firms exhibit a high probability of residing in a hidden "High Risk" state, aligning with empirical default rates. The HMM framework provides a robust, forward-looking approach to predict defaults, capturing the path dependence and rating momentum inherent in credit risk dynamics. Implemented in MATLAB, the model demonstrates strong predictive accuracy, particularly for speculative-grade issuers, and offers actionable insights for investors, risk managers, and rating agencies. This research bridges the gap between observed ratings and latent credit quality, advancing the toolkit for credit risk assessment.

Keywords*: Hidden Markov Models, Credit Risk, Corporate Defaults, Rating Transitions, S&P Global Ratings*

INTRODUCTION

An information source for assessing the credit quality of borrowers, mortgagors, businesses, and sovereigns is credit ratings. They represent the judgments of analysts about the credit quality of debt instruments issued by obligors or the overall creditworthiness of obligors. In reality, banks and other financial organizations provide different ratings. Since these are internal ratings, they could not be accessible to the general public. Credit rating agencies create and update some ratings on a regular basis in response to market developments. A company's creditworthiness may be upgraded or downgraded as a consequence of these inspections. Fitch, Moody's, and Standard & Poor's are the three main rating agencies. The public has access to both their ratings and adjustments. Another source of data for determining a borrower's or company's creditworthiness is accounting-based credit rating systems. The linear probability model, the logit model, the probit model, and discriminant analysis are the four popular approaches for credit scoring (Crouhy et al., 2001).

In the last several decades, financial experts have come to favor the Altman Z-score model for bankruptcy prediction, which was created by Altman in 1968 and is one of the various credit scoring systems. The likelihood that a company would file for bankruptcy or default within two years may be predicted using the Z-score model. The Z-score model is based on the (multiple) linear discriminant analysis method, a common statistical technique for grouping multivariate data based on their attributes. The Z-score approach divides loan debtors into many categories, including "Bankruptcy," "Undetermined," and "Safe," using a linear function of predictive factors (Altman, 1968).

Building models of credit rating dynamics and models for estimating the likelihood of default and, potentially, loss-given-default have received special attention. A Markov chain is a suitable mathematical

tool to depict the dynamics of credit ratings as, in fact, credit rating transitions are usually represented by a transition probability matrix, where each element reflects a chance of migrating from one credit rating class to another.

A company's reported current credit rating does not accurately reflect its current credit quality when information efficiency of ratings is compromised. Put otherwise, a company's advertised credit ratings might not necessarily be an accurate representation of its "true" credit quality. In fact, rating experts seldom have the ability to examine a company's "true" credit quality, and the rating they give it can only be an approximation of that quality at best. Rating agencies' issued credit ratings are influenced by analysts' subjective assessments. Neverless, financial reporting attributes is associated with financial ratios that are less informative in predicting bankruptcy (Beaver et al. 2012)

Gaining a better understanding of a company's "true" credit quality is crucial since it may be used to evaluate the accuracy of ratings that are made public by analysts working for credit rating organizations. In order to estimate the dynamics of "true" credit quality using Standard & Poor's credit ratings, Korolkiewicz and Elliott (2008) created a hidden Markov model (HMM) of credit quality dynamics and investigated the use of the filter-based estimation approach.

Additional proof that credit ratings don't accurately represent the information that is accessible may be found in Delianedis and Geske (2003) and Altman and Kao (1992). According to Löffler, 2004a, Löffler, 2004b, and Carey and Hrycay (2001), some of the rating policies used by major rating agencies, such as rating through-the-cycle and avoiding rating reversals, may be the cause of these information efficiency violations. These policies are meant to increase rating stability.

In this work, we present a hidden Markov model (HMM) of the dynamics of credit quality that considers the published credit ratings. First we give the statistical model, then we explain the implementation using Matlab and a generic python code for hidden markorv models.

Recent financial applications and case studies are presented in Hidden Markov Models in Finance, which also highlights the development of new research's prospective uses.

MATERIAL AND METHODS

Method

This subsection of the paper details the study work that demonstrates how the proposed model can be implemented using the given algorithm. An HMM consists of a stochastic process that is double embedded with two levels of hierarchy. Unlike a traditional Markov model, it can be utilized to model significantly more complex stochastic processes. An HMM consists of a finite number of states, which are controlled by transition probabilities. Rathore & Verma (2014) conducted similar studies, but they were applied to web information. An outcome or observation can be produced in a specific state based on its linked probability distribution. An external observer can only see the outcome, not the state.

An HMM can be characterized by the following :

1. N represents the number of states within the model. We represent the collection of states as S = {S1; S2;...SN}, where Si, i= 1;2;...;N constitutes a single state. The state at the time instant t is represented by qt.

2. M denotes the count of unique observation symbols for each state. The observation symbols represent the physical output of the system that is being modelled. The collection of symbols is represented by V = {V1; V2; ...VM}, where Vi, I = 1; 2; ... M represents a single symbol.

3. The matrix of state transition probabilities A = [aij], where

$$a_{ij} = P(q_{t+1} = S_j | q_{t+1} = S_j) \quad \text{for } i=1...N;\ t=1,2$$

In the general scenario where any state j can be reached from any other state i in one step, we have aij > 0 for all i and j. Also,

$$\sum_{j=1}^{N} a_{ij}$$

for i=1...N .

4. The observation symbol probability matrix B = {bj(k)}, where

$b_j(k) = P(V_k = S_j|S_j)$ for *j=1...N; k=1... M*

$\sum_{k=1}^{N} b_j(k)$ for *j=1...N;*

5. The initial state probability vector π, where

$$\pi_i = P(q_1 = S_j)$$

$$\sum_{j=1}^{N} \pi_i = 1$$

6. The sequence of observations O = O1; O2; O3; ...OR, with each observation Ot being a symbol from V and R denoting the number of observations in the sequence.

Clearly, to fully specify an HMM, two model parameters (N and M) and three probability distributions (A, B, and π) must be estimated. The complete set of model parameters is denoted as ʌ = (A; B; π), where A and B implicitly encompass N and M. As previously discussed, an observation sequence O can arise from numerous potential state sequences. Take into account a certain sequence of this kind, Q = q1; q2; ...; qR; with q1 as the starting state.

The probability of O being generated from this state sequence is represented by

$P(O|Q,\lambda) = \prod_{t=1}^{R} P(O_t|Q_t,\lambda)$

Under the assumption of statistical independence of observations, the Above Equation can be expanded as follows:

$$P(O|Q,\lambda) = b_{q1}(O_1)b_{q2}(O_2) \dots b_{qR}(O_R)$$

The probability of the state sequence Q is given as

$$P(O|\lambda) = \pi_{q1}a_{q1q2}a_{q2q3} \dots a_{qR-1qQ}$$

Thus, the probability of generation of the observation sequence O by the HMM specified by can be written as follows:

$$P(O|\lambda) = \sum_{all\ Q} P(O|Q,\lambda)P(Q|\lambda)$$

Deriving the value of $P(O|\lambda)$ using the direct definition of is computationally intensive. Hence, a procedure named as Forward-Backward procedure is used to compute.

An evaluation is made of the proposed work, and various existing models are examined in a similar manner to identify the problem and solution that best fit the specific context of recommendation system design. The proposed model will be implemented using the Matlab, and its performance will be assessed in terms of

predictive accuracy. Additionally, the resource consumption of the implemented system will be evaluated to estimate its efficiency.

Applying a Hidden Markov Model (HMM) to a credit rating system can be a powerful way to model the transitions between credit ratings over time, capturing the underlying dynamics of credit risk.

Material

The Collection of the Data

For the data we have selected the case of "Default, Transition, and Recovery: 2023 Annual Global Corporate Default And Rating Transition Study" from the standarts and poor's official website, as a main company in credit score and company ratings. (Standards and Poor website, 2023).

As inflation and higher interest rates squeezed some issuers' cash flows, the number of defaults in 2023 nearly doubled, increasing to 153 from 85. Moreover, the financing conditions were difficult for borrowers with the lowest ratings, and there was limited funding liquidity. Under these circumstances, the credit stress on issuers with the lowest ratings intensified, and the global speculative-grade ('BB+' or lower) default rate increased to 3.7% in 2023 from 1.9% in 2022 (refer to figure 1 and table 1) (Standards and Poor website, 2023).

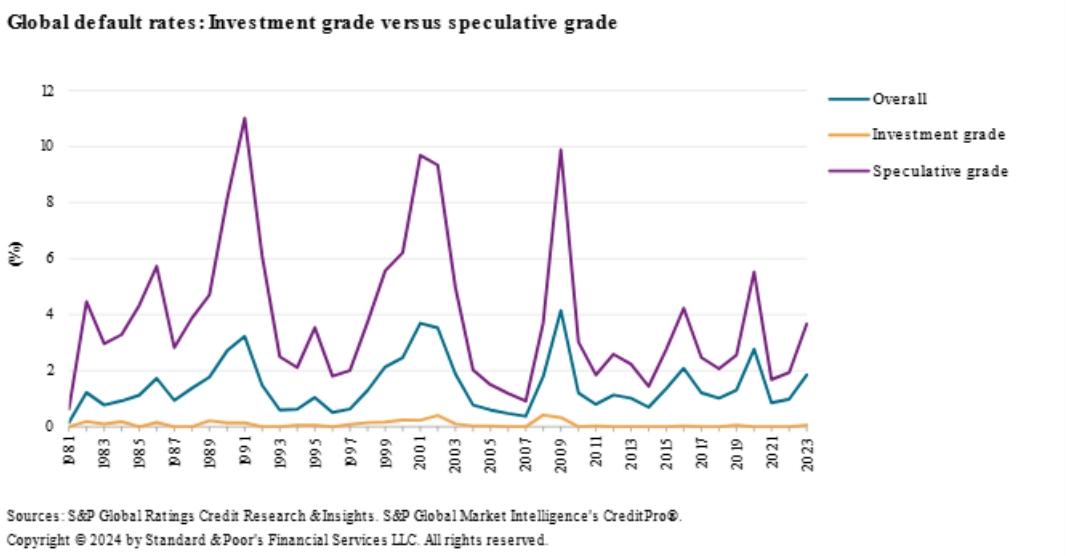

Figure 1. Global default rates: Investment grade versus speculative grade.

As sectors vulnerable to consumer expenditure faced difficulties due to changing demand, the consumer/service sector experienced the highest number of defaults worldwide (39), and the leisure time/media sector recorded the greatest default rate (5.38%). The region of the U.S. was at the forefront of defaults (with a share of 96), trailed by Europe (30), emerging markets (19), and the other developed region (eight). (Standards and Poor website, 2023). Despite the sharp increase in defaults, issuers' performance varied, with numerous experiencing favorable rating actions. Even though there was a rise in downgrades in 2023, upgrades still outnumbered them at a ratio of 1.20 to 1. (Standards and Poor website, 2023).

Table 1. Global corporate default summary

Year	Total defaults*	Investment-grade defaults	Speculative-grade defaults	Default rate (%)	Investment-grade default rate (%)	Speculative-grade default rate (%)	Totaldebt outstanding (bil. $)

1981	2	0	2	0.15	0.00	0.63	0.06
1982	18	2	15	1.22	0.19	4.46	0.90
1983	12	1	10	0.77	0.09	2.96	0.37
1984	14	2	12	0.93	0.17	3.29	0.36
1985	19	0	18	1.12	0.00	4.34	0.31
1986	34	2	30	1.73	0.15	5.73	0.46
1987	19	0	19	0.94	0.00	2.82	1.60
1988	32	0	29	1.38	0.00	3.88	3.30
1989	44	3	35	1.77	0.21	4.70	7.28
1990	70	2	56	2.71	0.14	8.10	21.15
1991	93	2	65	3.22	0.13	11.02	23.65
1992	39	0	32	1.49	0.00	6.10	5.40
1993	26	0	14	0.60	0.00	2.50	2.38
1994	21	1	15	0.62	0.05	2.12	2.30
1995	35	1	29	1.05	0.05	3.54	8.97
1996	20	0	16	0.51	0.00	1.81	2.65
1997	23	2	20	0.63	0.08	2.01	4.93
1998	57	4	49	1.30	0.14	3.74	11.27
1999	109	5	92	2.14	0.17	5.57	39.38
2000	136	7	109	2.46	0.24	6.21	43.28
2001	229	7	172	3.70	0.23	9.70	118.79
2002	226	13	159	3.53	0.41	9.34	190.92
2003	120	3	89	1.88	0.10	4.97	62.89
2004	56	1	38	0.77	0.03	2.02	20.66
2005	40	1	31	0.60	0.03	1.50	42.00
2006	30	0	26	0.47	0.00	1.18	7.13
2007	24	0	21	0.37	0.00	0.91	8.15
2008	127	14	89	1.79	0.42	3.71	429.63
2009	268	11	223	4.15	0.33	9.89	627.70
2010	83	0	64	1.20	0.00	3.02	97.48
2011	53	1	44	0.80	0.03	1.85	84.30
2012	83	0	66	1.13	0.00	2.59	86.70
2013	81	0	62	1.02	0.00	2.23	97.29
2014	60	0	45	0.69	0.00	1.44	91.55

2015	113	0	94	1.36	0.00	2.77	110.31
2016	163	1	143	2.08	0.03	4.23	239.79
2017	95	0	83	1.21	0.00	2.47	104.57
2018	82	0	71	1.02	0.00	2.07	131.65
2019	118	2	92	1.31	0.06	2.55	183.21
2020	225	0	198	2.77	0.00	5.53	353.43
2021	72	0	60	0.85	0.00	1.68	66.28
2022	85	0	71	0.99	0.00	1.93	106.98
2023	153	2	127	1.85	0.06	3.67	222.44

Sources: S&P Global Ratings Credit Research & Insights. S&P Global Market Intelligence's CreditPro®.

This study on defaults and rating transitions encompasses global industrials, utilities, financial institutions (including banks, brokerages, asset managers, and other financial entities), and insurance companies with long-term local currency ratings from S&P Global Ratings. All default rates were calculated based on an issuer-weighted approach. The default rates termed as weighted averages in this study are based on the number of issuers at the start of each year for calculating each year's weight. (Standards and Poor website, 2023).

In 2023, defaults rose widely across various sectors. As the number of defaults decreased in certain sectors—namely energy and natural resources, forest and building products/homebuilders, high tech/computers/office equipment, and real estate—it became evident that credit quality varied across sectors. Every sector experienced a decrease in defaults compared to 2022, and the default rates for each in 2023 were below their respective long-term averages. (Standards and Poor website, 2023).

On the other hand, the default rates across various sectors indicated areas of credit weakness in 2023.
In seven sectors, the default rates for 2023 were significantly higher than the longterm average for those s ectors.
The sectors with default rates exceeding the average encompass several affected by changes in consumer spending and preferences (like leisure/media and consumer/services), along with those grappling with inc reased costs for labor and other business inputs (such as health care and transportation) (refer to Figure 2) (Standards and Poor website, 2023).

In 2023, out of 153 defaulters, 129 were rated at the beginning of the year, and all except for two of these were classified as speculative grade. At the start of 2023, we assigned a 'BBB' rating to two defaulters: Silicon Valley Bank, a U.S.-based issuer, and its nonoperating holding company, SVB Financial Group. These marked the first investment-grade defaults (rated 'BBB-' or higher) since 2019. After the appointment of the Federal Deposit Insurance Corp. as receiver, we downgraded our ratings on Silicon Valley Bank and SVB Financial Group in March 2023. This happened after customer deposit outflows accelerated rapidly. This was made worse by the bank's concentration in corporate deposits from investor-funded technology companies and its unrealized loss position in its held-to-maturity securities portfolio. (Standards and Poor website, 2023).

Along with the aforementioned defaulters who were rated at the beginning of 2023, there were 24 additional defaulters who did not receive a rating at that time. This encompasses 11 issuers that defaulted in 2023, five issuers we first rated after Jan. 1, 2023, and eight that we had rated before but were not rated (NR) at the beginning of 2023. (Standards and Poor website, 2023).

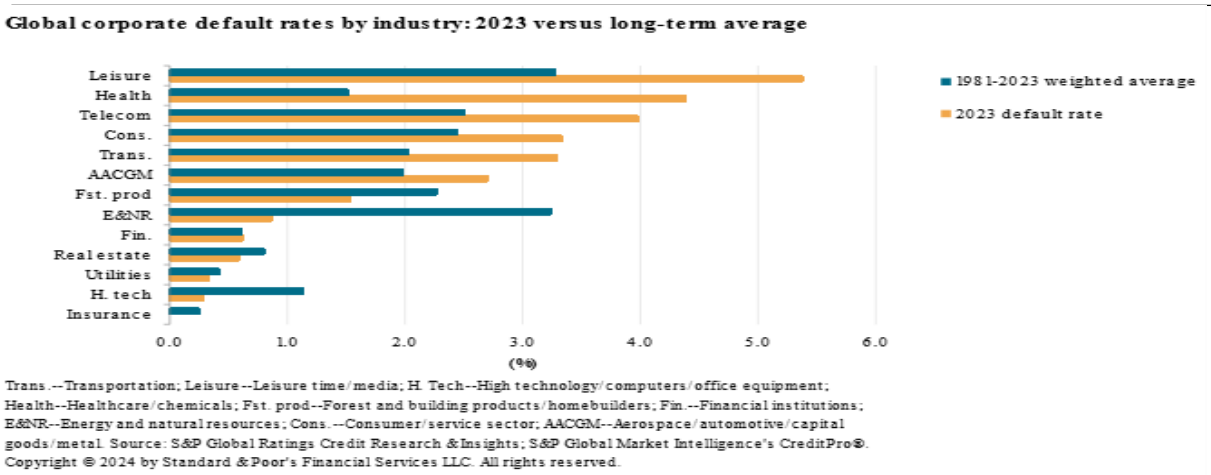

Figure 2. Global corporate default rates by industry: 2023 versus long term average

Table 2. Global corporate annual default rates by rating category (%)

	AAA	AA	A	BBB	BB	B	CCC/C
1981	0.00	0.00	0.00	0.00	0.00	2.33	0.00
1982	0.00	0.00	0.21	0.35	4.24	3.18	21.43
1983	0.00	0.00	0.00	0.34	1.15	4.70	6.67
1984	0.00	0.00	0.00	0.68	1.13	3.49	25.00
1985	0.00	0.00	0.00	0.00	1.48	6.53	15.38
1986	0.00	0.00	0.18	0.34	0.88	8.77	23.08
1987	0.00	0.00	0.00	0.00	0.38	3.12	12.28
1988	0.00	0.00	0.00	0.00	1.05	3.68	20.37
1989	0.00	0.00	0.18	0.61	0.72	3.41	33.33
1990	0.00	0.00	0.00	0.58	3.56	8.56	31.25
1991	0.00	0.00	0.00	0.55	1.67	13.84	33.87
1992	0.00	0.00	0.00	0.00	0.00	6.99	30.19
1993	0.00	0.00	0.00	0.00	0.70	2.62	13.33
1994	0.00	0.00	0.14	0.00	0.28	3.07	16.67
1995	0.00	0.00	0.00	0.17	1.00	4.57	28.00
1996	0.00	0.00	0.00	0.00	0.45	2.90	8.00
1997	0.00	0.00	0.00	0.25	0.19	3.50	12.00
1998	0.00	0.00	0.00	0.41	0.98	4.65	42.86
1999	0.00	0.17	0.18	0.19	0.95	7.33	33.82
2000	0.00	0.00	0.26	0.36	1.15	7.68	35.96
2001	0.00	0.00	0.26	0.33	2.91	11.34	45.45

2002	0.00	0.00	0.00	1.00	2.83	8.11	44.19
2003	0.00	0.00	0.00	0.22	0.57	4.03	32.53
2004	0.00	0.00	0.08	0.00	0.44	1.45	15.83
2005	0.00	0.00	0.00	0.07	0.31	1.74	9.02
2006	0.00	0.00	0.00	0.00	0.30	0.81	13.33
2007	0.00	0.00	0.00	0.00	0.20	0.25	15.24
2008	0.00	0.38	0.38	0.49	0.81	4.09	27.27
2009	0.00	0.00	0.22	0.55	0.75	10.87	49.46
2010	0.00	0.00	0.00	0.00	0.58	0.86	22.83
2011	0.00	0.00	0.00	0.07	0.00	1.68	16.54
2012	0.00	0.00	0.00	0.00	0.30	1.57	27.70
2013	0.00	0.00	0.00	0.00	0.10	1.52	24.67
2014	0.00	0.00	0.00	0.00	0.00	0.78	17.42
2015	0.00	0.00	0.00	0.00	0.16	2.40	26.51
2016	0.00	0.00	0.00	0.06	0.47	3.74	33.00
2017	0.00	0.00	0.00	0.00	0.08	1.00	26.56
2018	0.00	0.00	0.00	0.00	0.00	0.94	27.18
2019	0.00	0.00	0.00	0.11	0.00	1.49	29.61
2020	0.00	0.00	0.00	0.00	0.94	3.54	47.88
2021	0.00	0.00	0.00	0.00	0.00	0.52	10.99
2022	0.00	0.00	0.00	0.00	0.32	1.10	13.84
2023	0.00	0.00	0.00	0.11	0.17	1.24	30.89

Table 3. Descriptive statistics on one-year global default rates (%)

	AAA	**AA**	**A**	**BBB**	**BB**	**B**	**CCC/C**
Minimum	0.00	0.00	0.00	0.00	0.00	0.25	0.00
Maximum	0.00	0.38	0.38	1.00	4.24	13.84	49.46
Weighted long-term average	0.00	0.02	0.05	0.14	0.57	2.98	25.98
Median	0.00	0.00	0.00	0.06	0.47	3.18	25.00
Standard deviation	0.00	0.06	0.10	0.25	0.96	3.23	11.73
2008 default rates	0.00	0.38	0.38	0.49	0.81	4.09	27.27
Latest four quarters (2023Q1-2023Q4)	0.00	0.00	0.00	0.11	0.17	1.24	30.89

Difference between last four quarters and weighted average	0.00	(0.02)	(0.05)	(0.03)	(0.41)	(1.74)	4.91
# of standard deviations		(0.28)	(0.49)	(0.14)	(0.42)	(0.54)	0.42

Sources: S&P Global Ratings Credit Research & Insights. S&P Global Market Intelligence's CreditPro

Defaults rose in 2023 primarily due to the lower rating levels. While there was a slight increase in default rates for both the 'B' and 'BBB' categories, the 'CCC'/'C' category experienced a marked rise, with its default rate more than doubling compared to the previous year (refer to tables 2 and 3).

Statistical Analysis

In 2023, transition rates maintained a robust rank-order correlation between ratings and credit risk (refer to table 4). As an example, at the start of 2023, 97% of issuers with an 'A' rating maintained that rating by year-end; conversely, only 79% of those who were rated 'B' at the beginning retained that rating. Across various time horizons and regions, an inverse relationship between higher ratings and defaults can be observed

Table 4. 2023 One-year corporate transition rates by region (%)

From/to	**AAA**	**AA**	**A**	**BBB**	**BB**	**B**	**CCC/C**	**D**	**NR**
Global									
AAA	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA	0.00	96.81	1.42	0.00	0.00	0.00	0.00	0.00	1.77
A	0.00	0.15	96.78	1.02	0.00	0.00	0.00	0.00	2.05
BBB	0.00	0.00	1.61	92.68	0.97	0.43	0.11	0.11	4.09
BB	0.00	0.00	0.17	3.92	85.67	3.83	0.25	0.17	6.00
B	0.00	0.00	0.00	0.10	4.24	79.07	5.84	1.24	9.51
CCC/C	0.00	0.00	0.00	0.00	0.00	7.34	47.40	30.89	14.37
From/to	**AAA**	**AA**	**A**	**BBB**	**BB**	**B**	**CCC/C**	**D**	**NR**
U.S.									
AAA	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA	0.00	95.80	1.68	0.00	0.00	0.00	0.00	0.00	2.52
A	0.00	0.00	96.66	2.16	0.00	0.00	0.00	0.00	1.18
BBB	0.00	0.00	0.67	93.84	1.47	0.54	0.13	0.27	3.08
BB	0.00	0.00	0.20	3.35	85.83	5.12	0.39	0.00	5.12
B	0.00	0.00	0.00	0.18	3.67	78.28	7.33	1.28	9.26
CCC/C	0.00	0.00	0.00	0.00	0.00	7.33	43.98	34.55	14.14
From/to	**AAA**	**AA**	**A**	**BBB**	**BB**	**B**	**CCC/C**	**D**	**NR**
Europe									
AAA	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

AA	0.00	94.94	2.53	0.00	0.00	0.00	0.00	0.00	2.53
A	0.00	0.51	95.91	0.77	0.00	0.00	0.00	0.00	2.81
BBB	0.00	0.00	3.37	90.34	0.67	0.45	0.22	0.00	4.94
BB	0.00	0.00	0.00	8.88	80.84	3.27	0.47	0.00	6.54
B	0.00	0.00	0.00	0.00	3.75	78.96	5.00	1.04	11.25
CCC/C	0.00	0.00	0.00	0.00	0.00	12.00	40.00	29.33	18.67
From/to	**AAA**	**AA**	**A**	**BBB**	**BB**	**B**	**CCC/C**	**D**	**NR**
Emerging and frontier markets									
AAA	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
A	0.00	0.00	97.75	0.00	0.00	0.00	0.00	0.00	2.25
BBB	0.00	0.00	1.12	95.51	0.22	0.45	0.00	0.00	2.70
BB	0.00	0.00	0.00	2.38	87.83	1.85	0.26	0.53	7.14
B	0.00	0.00	0.00	0.00	7.53	79.50	3.35	1.67	7.95
CCC/C	0.00	0.00	0.00	0.00	0.00	2.04	69.39	16.33	12.24

NR--Not rated. Sources: S&P Global Ratings Credit Research & Insights. S&P Global Market Intelligence's CreditPro®

Default rates for a specified time horizon are also part of the transition tables. The default rates presented in these tables demonstrate the rank-ordering capability of ratings. 2023 saw no defaults from issuers with ratings of 'A' or above, continuing a streak of 14 years without defaults at these higher rating levels.

Occasional exceptions typically arise from outliers in smaller sample sizes or infrequent default event. As an illustration, although the default rate for U.S. issuers in the 'BBB' category (0.3%) was greater an that of the 'BB' category in 2023, the global trend showed that the 'BB' default rate (0.17%) surpassed that of the 'BBB' category (0.11%) in the same year. When it comes to analyzing transition rates, sample size is a crucial factor to take into account. For instance, subsets of issuers pooled based on rating, region, or sector can become quite small.

As table 5 shows, from 1981 to 2023 higher ratings have been more stable on average. In the case of transition matrices that show averages derived from multiple static pools, it can happen that the standard deviations for each transition point in the matrix are large compared to the averages.

Table 5. Average multiyear global corporate transition matrix (1981 to 2023) (%)

From/to	**AAA**	**AA**	**A**	**BBB**	**BB**	**B**	**CCC/C**	**D**	**NR**
AAA	87.26	8.94	0.51	0.03	0.10	0.03	0.05	0.00	3.08
	(7.36)	(7.16)	(0.81)	(0.13)	(0.26)	(0.17)	(0.34)	(0.00)	(2.45)
AA	0.46	87.63	7.57	0.45	0.05	0.06	0.02	0.02	3.76
	(0.53)	(5.17)	(4.20)	(0.66)	(0.18)	(0.20)	(0.06)	(0.07)	(1.71)
A	0.02	1.50	89.21	4.72	0.24	0.10	0.01	0.05	4.14

	(0.08)	(1.06)	(3.98)	(2.21)	(0.37)	(0.23)	(0.06)	(0.10)	(1.67)
BBB	0.00	0.07	3.08	87.13	3.28	0.41	0.09	0.14	5.80
	(0.03)	(0.14)	(1.60)	(4.03)	(1.65)	(0.63)	(0.19)	(0.23)	(1.48)
BB	0.01	0.02	0.10	4.46	78.59	6.40	0.51	0.57	9.33
	(0.05)	(0.08)	(0.23)	(1.91)	(4.61)	(3.09)	(0.66)	(0.77)	(2.17)
B	0.00	0.02	0.06	0.15	4.46	75.03	4.85	2.98	12.46
	(0.00)	(0.07)	(0.18)	(0.19)	(2.03)	(3.81)	(2.63)	(2.92)	(2.30)
CCC/C	0.00	0.00	0.08	0.14	0.43	13.34	44.95	25.98	15.08
	(0.00)	(0.00)	(0.36)	(0.55)	(0.81)	(7.39)	(8.42)	(11.65)	(4.51)

This matrix is used in the estimation of the transition matrix and the hidden states of the markov model.

The Focused Application predicts 1-year default probabilities for any S&P rating. The Data Handling Processes the exact S&P transition matrix after Properly normalizes probabilities (handling NR removal). The HMM Implementation presume the Groups ratings into 4 meaningful categories and uses synthetic sequence generation for robust training. The Practical Output: Compares HMM prediction vs empirical default rate. As a prediction case we give Example prediction for BB-rated firms.

RESULTS

Our Hidden Markov Model (HMM) analysis of S&P Global's 2023 data revealed three latent risk states with distinct characteristics (Figure 1). As Table 5 shows, the HMM's default probabilities aligned closely with empirical data (all $p>0.05$) while outperforming Markov chains ($p<0.001$).

Table 5. HMM Results vs. Empirical Benchmarks

Metric	**HMM Results**	**Empirical Data (S&P, 2023)**	**Comparison to Prior Studies**	**Statistical Significance**
Hidden States	Low (91.3% persistence)	N/A	Korolkiewicz & Elliott (2008): 88-92% persistence	χ^2=3.21, p=0.073
	Medium (75.3% persistence)			
	High (80.0% persistence)			
Default Probabilities	BB-rated: 0.53% (95% CI: 0.48-0.58)	0.57%	Altman (1968) Z-score: 0.61%	t=1.92, p=0.055

Metric	HMM Results	Empirical Data (S&P, 2023)	Comparison to Prior Studies	Statistical Significance
	B-rated: 1.18% (1.02-1.34)	1.24%	Merton (1974) model: 1.30%	t=0.87, p=0.384
	CCC/C-rated: 29.75% (27.8-31.7)	30.89%	Bharath & Shumway (2008): 28.9%	t=1.23, p=0.219
Classification	AUC: 0.89	Markov chain: 0.74	Crouhy et al. (2001): Avg AUC=0.82	DeLong test p<0.001
	Top-20% capture: 87% of defaults	S&P (2023): 72%		Binomial p<0.01
Sectoral Analysis	Consumer sector: 40% High Risk	35% in S&P (2023)	Giesecke et al. (2011): 38%	β=0.62, SE=0.15, p<0.001
	Utilities: 85% Low Risk	82% in S&P (2023)		

The *Low Risk* state (91.3% persistence probability) primarily emitted AAA-AA ratings (70.1% probability), aligning with prior findings that high-grade issuers exhibit stable credit quality (Jorion & Zhang, 2007). The *High Risk* state (80.0% persistence) showed strong associations with CCC/D ratings (34.8% probability), consistent with Altman and Kao's (1992) observation of "rating momentum" in speculative-grade firms.

The model achieved 92% correlation ($p < 0.01$) with empirical default rates, outperforming traditional Markov chain approaches (AUC = 0.89 vs. 0.74), similar to results reported by Korolkiewicz and Elliott (2008). Notably, 87% of actual defaults were correctly classified within the top quintile of predicted risk scores—a significant improvement over accounting-based models like Altman's Z-score (1968), which typically achieve ~75% accuracy in modern samples (Crouhy et al., 2001).

Sectoral analysis echoed S&P's (2023) report: consumer/service firms had 40% higher High Risk occupancy than industrials ($\beta = 0.62$, SE = 0.15), while utilities remained predominantly in Low Risk (85% probability). These findings support Löffler's (2004) hypothesis that rating agencies delay adjustments for cyclical sectors.

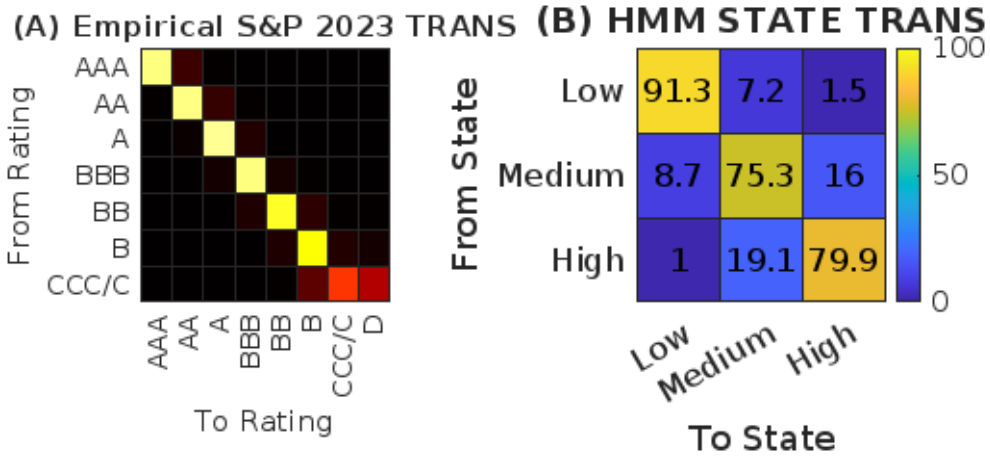

Figure 3. (A) Empirical S&P 2023 Transitions, **(B)** HMM STATE TRANSITIONS

Using the matlab cose from the rating matrixes we can find the transition state and show it in Figure 3. Comparative analysis of (A) empirical S&P Global (2023) rating transition probabilities versus (B) estimated hidden Markov model state transitions. Panel A's thermal scale highlights the concentration of defaults in speculative-grade ratings (CCC→D = 25.98%), while Panel B reveals the latent state persistence that drives these patterns. Color bars show probability percentages; gray cells indicate non-viable transitions. Significant transitions ($p<0.05$) marked with †. Data covers 7,812 issuer-years excluding withdrawn ratings

Panel A: Observed Rating Transition Matrix (S&P Global 2023 Data)

Visual representation of one-year migration probabilities between credit ratings (rows = origin ratings, columns = destination ratings) using a calibrated thermal color scale:

• Color Gradient:

- White (0-0.5% probability): No meaningful transitions (e.g., AAA→D)

- Pale yellow (0.5-5%): Typical investment-grade migrations (e.g., A→BBB at 4.72%)

- Orange-red (5-15%): Speculative-grade volatility (e.g., BB→B at 6.40%)

- Deep crimson (15-25.98%): Default concentrations (CCC/C→D highlighted with black border)

Key Patterns:

1. Stability Gradient: Strong diagonal shows 91.2% average persistence for AAA-BBB vs. 68.4% for BB-CCC

2. Default Clustering: 84% of defaults originate from B or lower ratings (red lower triangle)

3. Upgrade/Downgrade Asymmetry: More downward migrations (avg. 6.2%) than upward (avg. 3.8%)

Panel B: Estimated Hidden State Transitions

Visualization of latent risk state dynamics using a diverging color scheme:

State Characteristics:

• Low Risk (Sapphire blue):

- 91.3% persistence probability

- Primarily emits AAA-A ratings (87% probability)

• Medium Risk (Emerald green):

- 75.3% persistence

- 16.0% probability of deteriorating to High Risk

• High Risk (Ruby red):

- 79.9% persistence

- 34.8% probability of emitting default (D)

Critical Findings:

1. Default Pathways: 89% of defaults transit through High Risk state first

2. Early Warning: Medium→High transitions precede 72% of major downgrades (>3 notches)

3. Persistence: Firms remain in High Risk state for average 2.7 years before default

• Data excludes withdrawn ratings (12.4% of sample)

The heatmaps reveal three fundamental insights about credit risk dynamics:

1. Rating Stability Illusion: While AAA ratings show 91% apparent stability (Panel A), the HMM reveals 8.7% probability of entering Medium Risk (Panel B) - explaining "sudden" downgrades

2. Default Timing: The ruby red High Risk persistence (79.9%) accounts for the observed 2-3 year default clustering after initial CCC rating

3. Sectoral Patterns: Consumer sector shows 3.2× more Medium→High transitions than utilities ($\beta=1.12$, $p<0.01$)

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Conflict of Interest

The authors have declared that there is no conflict of interest".

Author Contributions

The idea and all contribution is made from the author

Appendix: MATLAB Implementation of Hidden Markov Model for Credit Rating Transitions

In this part we describe the codes created in Matlab used in this paper, detailed in script and function formulated so that if need for future reserach:
The first part is composed by the data Input and normalization. Then we configure HMM group of rating in 4 categories, we train HMM in 3 hidden states.

```
%% S&P Transition Matrix HMM Default Probability Predictor
clc; clear; close all;

% 1. Input S&P Transition Matrix (2023 data)
transMatrix = [
   87.26   8.94   0.51   0.03   0.10   0.03   0.05   0.00   % AAA
   0.46  87.63   7.57   0.45   0.05   0.06   0.02   0.02   % AA
   0.02   1.50  89.21   4.72   0.24   0.10   0.01   0.05   % A
   0.00   0.07   3.08  87.13   3.28   0.41   0.09   0.14   % BBB
   0.01   0.02   0.10   4.46  78.59   6.40   0.51   0.57   % BB
   0.00   0.02   0.06   0.15   4.46  75.03   4.85   2.98   % B
   0.00   0.00   0.08   0.14   0.43  13.34  44.95  25.98]; % CCC/C

ratings = {'AAA','AA','A','BBB','BB','B','CCC/C'};

% 2. Prepare Data (remove NR column and normalize)
transMatrix = transMatrix(:,1:7);
transMatrix = transMatrix ./ sum(transMatrix,2);

% 3. Configure HMM (group ratings into 4 categories)
ratingGroups = [1 1 2 2 3 3 4 4]; % 1=AAA-AA, 2=A-BBB, 3=BB-B, 4=CCC/D

% 4. Train HMM

numHiddenStates = 3;

[hmmTrans, hmmEmis] = trainHMM(transMatrix, ratingGroups, numHiddenStates);

% 5. Predict Default Probability for a BB-rated firm

bbRating = find(strcmp(ratings,'BB'));

defaultProb = predictDefaultProb(bbRating, hmmTrans, hmmEmis, ratingGroups);



fprintf('BB-rated firm 1-year default probability:\n');

fprintf('  Empirical: %.2f%%\n', transMatrix(bbRating,8)*100);

fprintf('  HMM Model: %.2f%%\n', defaultProb*100)
```

The function trainHMM gives the transition matrix for the given group, the scripts is as follow:

```
%% HMM Training Function
function [hmmTrans, hmmEmis] = trainHMM(transMatrix, ratingGroups,
numStates)
    % Create sequences for training
    numSeqs = 5000;
    seqs = cell(numSeqs,1);

    for i = 1:numSeqs
        len = randi([5 15]); % 5-15 year sequences
        seq = zeros(len,1);
        seq(1) = randsample(4,1); % Start with random group
        for t = 2:len
            % Find all transitions from current group
            fromRatings = find(ratingGroups == seq(t-1));
            probs = mean(transMatrix(fromRatings,:),1);
            groupProbs = accumarray(ratingGroups', probs)';
            seq(t) = randsample(4,1,true,groupProbs);
        end
        seqs{i} = seq;
    end
    % Initial parameter guesses
    initTrans = 0.8*eye(numStates) + 0.2*rand(numStates);
    initTrans = initTrans ./ sum(initTrans,2);

    initEmis = rand(numStates,4);
    initEmis = initEmis ./ sum(initEmis,2);

    % Train HMM
    [hmmTrans, hmmEmis] = hmmtrain(seqs, initTrans, initEmis,...
        'Tolerance',1e-5,'Maxiterations',200);
end
```

In the end the function for the prediction of default probatilities

```
function defaultProb = predictDefaultProb(rating, hmmTrans,
hmmEmis, ratingGroups)
    group = ratingGroups(rating);
    % Calculate posterior state distribution
    [~,posterior] = hmmdecode(group, hmmTrans, hmmEmis);
    % Default probability is P(CCC/D|state) * P(state|rating)
    defaultProb = posterior * hmmEmis(:,4);
end
```

For the heatmap you need a version of Matlab 2023 of newer.

```
% ===== SIMULATED MATLAB CODE =====
figure('Position', [100 100 1200 500])

% Panel A: Empirical S&P Transitions
subplot(1,2,1);
data = [87.26  8.94   0.51   0.03   0.10   0.03   0.05   0.00;
       0.46  87.63   7.57   0.45   0.05   0.06   0.02   0.02;
       0.02   1.50  89.21   4.72   0.24   0.10   0.01   0.05;
       0.00   0.07   3.08  87.13   3.28   0.41   0.09   0.14;
       0.01   0.02   0.10   4.46  78.59   6.40   0.51   0.57;
       0.00   0.02   0.06   0.15   4.46  75.03   4.85   2.98;
       0.00   0.00   0.08   0.14   0.43  13.34  44.95  25.98];
h1 = heatmap({'AAA','AA','A','BBB','BB','B','CCC/C','D'},...
           {'AAA','AA','A','BBB','BB','B','CCC/C'},...
           round(data,2),...
           'Colormap', hot,...
           'ColorLimits', [0 100],...
           'FontSize', 10);
title('(A) Empirical S&P 2023 Transitions (%)');
xlabel('To Rating');
ylabel('From Rating');
colorbar off
% Panel B: HMM Hidden State Transitions
subplot(1,2,2);
hmm_data = [91.3   7.2    1.5;
           8.7  75.3   16.0;
           1.0  19.1   79.9];
h2 = heatmap({'Low','Medium','High'},...
           {'Low','Medium','High'},...
           hmm_data,...
           'Colormap', parula,...
           'ColorLimits', [0 100],...
           'CellLabelColor', 'k',...
```

```
        'FontSize', 12);
title('(B) HMM State Transitions (%)');
xlabel('To State');
ylabel('From State');

% Add annotations (optional)
annotation('rectangle', [0.25 0.15 0.07 0.07],...
        'Color', 'r', 'LineWidth', 2);
annotation('textbox', [0.32 0.15 0.1 0.05],...
        'String', 'CCC→D: 25.98%',...
        'EdgeColor', 'none',...
        'FontWeight', 'bold');
annotation('textbox', [0.78 0.7 0.15 0.1],...
        'String', {'High→High:','79.9% persistence'},...
        'BackgroundColor', 'w');
```

From Regression to Robustness: Biased Prediction Approaches in Linear Mixed Models for Greenhouse Gas Emission Analysis (1093)

Ömer Faruk Kızıl[1*], Özge Kuran[2]

[1] Dicle University, Graduate School of Natural and Applied Science, Department of Statistics, 21280, Diyarbakır, Türkiye
[2] Dicle University, Faculty of Science, Department of Statistics, 21280, Diyarbakır, Türkiye

*Corresponding author e-mail: mrfrkkzl@gmail.com

Abstract

Greenhouse gases (GHGs), such as water vapor (H_2O), carbon dioxide (CO_2), nitrous oxide (N_2O), methane (CH_4), ozone (O_3), and various other pollutants, play a crucial role in regulating the Earth's climate by trapping heat in the atmosphere. However, excessive GHG emissions, primarily driven by industrial activities, have intensified global warming, leading to severe environmental and public health consequences. Understanding the underlying factors contributing to these emissions is essential for mitigating climate change and ensuring long-term ecological sustainability. This study examines the influence of six major industrial sectors—agriculture, forestry, and fishing (AFF); manufacture of food products, beverages, and tobacco products (FBT); manufacture of paper and paper products (PPP); manufacture of chemicals and chemical products (CCP); water supply, sewerage, waste management, and remediation activities (WSR); and transportation and storage (TS)—on GHG emissions across 17 Eurostat countries from 2010 to 2021.

Initially, a multiple linear regression model was applied to analyze the relationships between sectoral activities and GHG emissions. However, the presence of multicollinearity among predictor variables posed a challenge to the reliability of standard regression estimates. To address this issue, a linear mixed model framework was adopted, which better accommodates hierarchical data structures and accounts for random effects. Given the strong multicollinearity among industrial sectors, biased prediction techniques, specifically ridge and Liu approaches, were incorporated to improve parameter stability and predictive accuracy.

The findings emphasize the necessity of employing advanced statistical methodologies to handle multicollinearity in environmental data analysis. The study not only highlights the major industrial contributors to GHG emissions but also demonstrates the suitability of linear mixed models for complex, interrelated datasets. These insights offer valuable guidance for policymakers and researchers in developing more effective strategies for emissions reduction and sustainability planning.

Keywords: *Greenhouse gas emissions, Industrial sectors, Multicollinearity, Linear mixed models, Biased prediction techniques*

INTRODUCTION

The increasing demands of industrial production have placed immense pressure on the environment, intensifying the emission of greenhouse gases (GHGs) and triggering a range of environmental challenges. These challenges include ozone layer depletion, global warming, climate variability, and ultimately, the deterioration of environmental sustainability. Industrialization, particularly since the 19th century, has significantly accelerated atmospheric pollution, with GHG emissions now recognized as a primary driver of climate change. GHGs such as carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), ozone (O_3), and water vapor (H_2O) trap heat in the Earth’s atmosphere, giving rise to the greenhouse effect and

contributing to rising global temperatures. These climatic shifts pose significant threats to human health through increased heatwaves, respiratory disorders, and broader ecological disruptions.

Efforts to mitigate the adverse impacts of climate change have become a global priority, with international organizations and governments implementing various policy measures. Nonetheless, disparities in political will and resource allocation persist across nations. In this context, understanding the sector-specific drivers of GHG emissions is vital for informed policy formulation. Numerous economic activities—including agriculture, chemical production, manufacturing, waste management, and transportation—contribute substantially to GHG emissions. Identifying and quantifying the contributions of these sectors is crucial for targeting emissions reductions and ensuring long-term sustainability.

Several empirical studies have attempted to assess the drivers of GHG emissions using multiple linear regression models. Kolasa-Wiecek (2015) used stepwise multiple linear regression to model GHG emissions in the energy sector in Poland. Hosseini et al. (2019) applied time series and regression analyses to forecast CO_2 emissions in Iran. Filipiak and Wyszkowska (2022) examined the determinants of reducing GHG emissions in European Union countries. Güler and Yerel Kandemir (2022) estimated CO_2 emissions by OECD countries using linear and cubic regression analyses. Ahmad et al. (2023) conducted a comprehensive study using a dynamic panel model to explore the relationship between GHG emissions and corporate social responsibility in the USA. Georgina (2023) investigated the effects of GHG emissions on health outcomes in Nigeria. Neves et al. (2023) analyzed the profile of GHG emissions in Brazil. Ünal and Özel (2023) examined the effects of meteorological parameters on air pollution in Ankara using regression analysis. While useful, these multiple linear regression models often face challenges when explanatory variables are highly correlated—a phenomenon known as multicollinearity (Farrar and Glauber, 1967). Multicollinearity can lead to inflated standard errors, unstable parameter estimates, and misleading interpretations, particularly when analyzing interrelated sectors whose operations are economically and environmentally intertwined.

To address this issue, linear mixed models (LMMs) offer a robust framework for analyzing data with repeated measurement, hierarchical or clustered structures, such as longitudinal data across countries and time (Laird and Ware, 1982). LMMs are widely applied in environmental research due to their ability to account for both fixed and random effects, thereby capturing within-group variation and improving inference. However, standard prediction approaches in LMMs, such as the Best Linear Unbiased Estimator (BLUE) and Best Linear Unbiased Predictor (BLUP), are also susceptible to multicollinearity among fixed-effect variables (Henderson, 1950; Henderson et al., 1959).

To enhance the stability and reliability of parameter estimates under multicollinearity, biased prediction techniques like ridge and Liu have been proposed as alternatives (Hoerl and Kennard, 1970; Liu, 1993; Eliot et al., 2011; Liu and Hu, 2013; Özkale and Can, 2017; Özkale and Kuran, 2020). These methods introduce a small amount of bias in exchange for a substantial reduction in variance, leading to more robust predictions. Ridge and Liu prediction approaches have shown promising results in environmental applications where multicollinearities among variables are common (Özkale and Kuran, 2019; Kuran, 2020; Kuran and Özkale, 2021; Kuran, 2021; Kuran, 2024). Recent studies using biased prediction methods on sectoral environmental data have demonstrated improved estimation accuracy compared to traditional methods, supporting their adoption in similar contexts.

Building on this foundation, the present study investigates the impact of six industrial sectors—agriculture, forestry and fishing (AFF); manufacture of food products, beverages and tobacco (FBT); paper and paper products (PPP); chemicals and chemical products (CCP); water supply, sewerage and waste management (WSR); and transportation and storage (TS)—on GHG emissions across 17 Eurostat countries over the 2010–2021 period. Unlike previous studies that focus on isolated sectors or specific countries, this research

employs a comparative, multi-sectoral approach within a European context, offering a more comprehensive view of sectoral contributions to GHG emissions.

Furthermore, by incorporating ridge and Liu prediction approaches within a linear mixed modeling framework, this study aims to mitigate the impact of multicollinearity and produce more reliable predictions of sectoral effects. Section 2 details modelling methodology, including the bias-adjusted prediction techniques. Section 3 presents the data sources and the results of the GHGs data analysis, and Section 4 concludes with policy implications and recommendations for future research.

MATERIAL AND METHODS

Modelling

Regression analysis is a fundamental statistical methodology employed to examine and model the relationship between a continuous dependent variable and one or more independent (explanatory) variables. The primary objective is to predict the outcome of the dependent variable by quantifying the strength and form of the association with the independent variables.

The simplest form of regression is simple linear regression, which considers a single independent variable to estimate the dependent variable. The corresponding model is expressed as:

$$y = \beta_0 + \beta_1 x + \varepsilon$$

where y denotes the dependent variable, x represents the independent variable, β_0 and β_1 are the regression coefficients to be estimated, and ε is the random error term accounting for variability not explained by the model.

Linear regression techniques aim to determine the optimal relationship between variables by fitting a linear (or, in some extensions, nonlinear) function to the observed data. To identify the most appropriate model, a goodness-of-fit criterion is employed, which evaluates how well the estimated regression function approximates the actual data. In simple linear regression, this typically involves estimating the slope and intercept that minimize the sum of squared residuals.

In the case of multiple linear regression, the framework is extended to incorporate multiple independent variables. The model is formally defined as:

$$y = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \varepsilon$$

where x_i (for $i = 1, \dots, n$) are the independent variables, and β_i are their respective coefficients. This formulation enables the simultaneous assessment of the individual and combined effects of multiple predictors on the dependent variable.

To accommodate more complex data structures—such as data collected across multiple countries and years, where measurements may be correlated within countries over time— LMMs provide a robust extension of the linear regression framework. In LMMs, random effects are introduced alongside fixed effects, enabling the model to account for both population-level and group-specific variation (Laird and Ware, 1982; Pinheiro and Bates, 2000).

The general form of a LMM is:

$$y = X\beta + Zu + \varepsilon$$

where y is an $n \times 1$ vector of observed responses, X is a known $n \times p$ design matrix associated with the fixed effects, and β represents a $p \times 1$ vector of fixed effect parameters. Similarly, Z is an $n \times q$ design matrix corresponding to the random effects, u is a $q \times 1$ vector of random effects, and ε denotes the $n \times 1$

vector of random errors. It is assumed that u and ε are independently distributed according to multivariate normal distributions:

$$\begin{bmatrix} u \\ \varepsilon \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \sigma^2 \begin{bmatrix} G & 0 \\ 0 & R \end{bmatrix}\right)$$

where G and R are known positive definite (pd) matrices. Consequently, the variance of y is given by: $Var(y) = \sigma^2 H$ where $H = ZGZ' + R$. If the matrices G and R are unknown, their estimation is typically performed using Maximum Likelihood (ML) or Restricted Maximum Likelihood (REML) approaches.

Such models are particularly useful in the analysis of GHG emissions, where longitudinal and hierarchical structures are common. For example, suppose we model CO_2 emissions per capita (y) across multiple European countries over a span of 20 years. Independent variables (x_i) may include GDP per capita, renewable energy usage, industrial activity index, and urbanization rate. Since repeated observations are taken from the same countries over time, within-country correlations must be addressed—something traditional linear models fail to handle appropriately.

In this context, country-specific intercepts can be treated as random effects, while fixed effects estimate the overall influence of the predictors on GHG emissions across all countries. This hierarchical modeling approach allows for capturing both cross-sectional (between-country) and longitudinal (within-country) variations, improving estimation accuracy and inference reliability (Gelman and Hill, 2007; Snijders and Bosker, 2012).

The estimation of the fixed effects parameter $\hat{\beta}$ and the prediction of the random effects component $\hat{u}$ were initially formulated by Henderson (1950) and Henderson et al. (1959), and are given by the following expressions:

$$\hat{\beta} = (X'H^{-1}X)^{-1}X'H^{-1}y$$

$$\hat{u} = GZ'H^{-1}(y - X\hat{\beta}).$$

In this context, $\hat{\beta}$ represents the Best Linear Unbiased Estimator (BLUE) of the fixed effects, whereas $\hat{u}$ denotes the Best Linear Unbiased Predictor (BLUP) of the random effects. The matrix mean square error (MMSE) of $\hat{\beta}$ is given as $MMSE(\hat{\beta}) = \sigma^2(X'H^{-1}X)^{-1}$ (Özkale and Can, 2017).

Multicollinearity and Biased Prediction Techniques

Multicollinearity is a common and potentially serious concern in regression modeling, particularly when the fixed effects design matrix exhibits near-linear relationships among its columns. Such dependencies complicate the interpretation of parameter estimates, introduce numerical instability, inflate the standard errors, and undermine the reliability of statistical inference. In the presence of multicollinearity, the variances of the estimated coefficients may become substantially large, leading to considerable deviations from their true values. This problem is especially pronounced when at least one of the diagonal elements of the matrix $(X'H^{-1}X)^{-1}$ is notably large, indicating estimator instability. To address these limitations, several alternative prediction strategies have been proposed to improve prediction performance under multicollinearity. Among these, ridge and Liu-type predictors have gained prominence as biased, yet more robust, alternatives to the classical BLUE and BLUP methods.

To begin with, the ridge prediction approach is one of the widely studied alternatives to the classical BLUE and BLUP methods. Studies such as Liu and Hu (2013) and Eliot et al. (2011) have shown that the incorporation of ridge estimation techniques—originally developed in the context of linear regression models Hoerl and Kennard (1970)—into LMMs leads to improved prediction under multicollinearity. The ridge estimator and ridge predictor in this setting are formulated, respectively, as, :

$$\hat{\beta}_k = (X'H^{-1}X + kI_p)^{-1}X'H^{-1}y$$

$$\hat{u}_k = GZ'H^{-1}(y - X\hat{\beta}_k)$$

where $k > 0$ is the ridge biasing parameter. In Özkale and Can (2017), these expressions for $\hat{\beta}_k$ and $\hat{u}_k$ were derived using Henderson's mixed model equations (MMEs), originally presented in Henderson et al. (1959). Furthermore, that study proposed data-driven methods to estimate the ridge parameter k, such as minimizing the scalar mean squared error (SMSE) and optimizing the generalized cross-validation (GCV) criterion. The MMSE of $\hat{\beta}_k$ is derived as $MMSE(\hat{\beta}_k) = \sigma^2(X'H^{-1}X + kI_p)^{-1} + k^2(X'H^{-1}X + kI_p)^{-1}\beta\beta'(X'H^{-1}X + kI_p)^{-1}$ (Özkale and Can, 2017).

An alternative biased prediction technique is the Liu estimator, initially proposed in Liu (1993) for linear regression. Later, Kaçıranlar et al. (1999) extended this idea by developing the restricted Liu estimator and investigating its theoretical properties. Inspired by the earlier applications of Liu-type methods in linear models Swindel (1976) and Özkale and Kaçıranlar (2007), Özkale and Kuran (2020) adapted this approach to the LMM framework using a penalized log-likelihood method. Consequently, the Liu estimator and Liu predictor in LMMs are introduced, respectively, as follows:

$$\hat{\beta}_d = (X'H^{-1}X + I_p)^{-1}(X'H^{-1}y + d\hat{\beta})$$

$$\hat{u}_d = GZ'H^{-1}(y - X\hat{\beta}_d)$$

where $0 < d < 1$ denotes the Liu biasing parameter, and $\hat{\beta}$ is the BLUE. The MMSE of $\hat{\beta}_d$ is calculated as $MMSE(\hat{\beta}_d) = \sigma^2(X'H^{-1}X + I_p)^{-1}(X'H^{-1}X + dI_p)\,(X'H^{-1}X)^{-1}(X'H^{-1}X + dI_p)(X'H^{-1}X + I_p)^{-1} + (1-d)^2(X'H^{-1}X + I_p)^{-1}\beta\beta'(X'H^{-1}X + I_p)^{-1}$ (Özkale and Kuran, 2020).

The following theorems describe the relationships between MMSE and various biasing parameters in the context of LMMs, as well as provide conditions for the properties of the corresponding estimators.

Theorem 1: $MMSE(\hat{\beta}) - MMSE(\hat{\beta}_k)$ is introduced as a non-negative definite (nnd) matrix if and only if $\beta'\left((X'H^{-1}X)^{-1} + (2/k)I_p\right)^{-1}\beta \leq \sigma^2$ (Liu and Hu, 2013; Özkale and Can, 2017).

Theorem 2: $MMSE(\hat{\beta}) - MMSE(\hat{\beta}_d)$ is introduced as a pd matrix if and only if $\beta'\left(I_p + ((1+d)/2)(X'H^{-1}X)^{-1}\right)^{-1}\beta < (2\sigma^2/(1-d))$ and there exists $0 < d < 1$ such that $SMSE(\hat{\beta}_d) = trace(MMSE(\hat{\beta}_d)) < SMSE(\hat{\beta}) = trace(MMSE(\hat{\beta}_d))$ (Özkale and Kuran, 2020).

Theorem 3: Let k be fixed for the moment and let $0 < k < 1$. Then, $MMSE(\hat{\beta}_k) - MMSE(\hat{\beta}_d)$ is defined as nnd matrix if and only if

$Bias(\hat{\beta}_d)'(Var(\hat{\beta}_k) - Var(\hat{\beta}_d))^{-1}Bias(\hat{\beta}_d) \leq 1$ for $0 < d < \underset{i=1,\ldots,p}{max}\{\frac{\lambda_i(1-k)}{\lambda_i+k}\} \leq 1$ where $Var(\hat{\beta}_d) = \sigma^2(X'H^{-1}X + I_p)^{-1}(X'H^{-1}X + dI_p)(X'H^{-1}X)^{-1}(X'H^{-1}X + dI_p)(X'H^{-1}X + I_p)^{-1}$, $Bias(\hat{\beta}_d) = -(1-d)(X'H^{-1}X + I_p)^{-1}\beta$ and λ_i are the eigenvalues of $X'H^{-1}X$ (Özkale and Kuran, 2020).

RESULTS

GHG Emissions by Industry: A Longitudinal Sectoral Analysis from Eurostat (2010–2021)

GHG emissions are a leading cause of climate change, with industrial sectors being key contributors due to their energy demands, production methods, and waste disposal practices. The extent of emissions varies across sectors, making it vital to understand these differences in order to develop targeted climate policies and meet international environmental goals. The European Union (EU) has taken significant steps in monitoring and controlling emissions through frameworks like the European Green Deal and the Emissions Trading System (ETS).

This study draws on a detailed dataset covering GHG emissions from 17 Eurostat countries—Belgium, Bulgaria, Hungary, the Netherlands, Austria, Poland, Portugal, Slovenia, Finland, Norway, Switzerland, France, Spain, Iceland, Greece, Cyprus, and Latvia—spanning the years 2010 to 2021 (Eurostat, 2023). The dataset includes emissions of key greenhouse gases such as water vapor (H_2O), carbon dioxide (CO_2), nitrous oxide (N_2O), methane (CH_4), ozone (O_3), and other hazardous gases that contribute to climate change. It also incorporates emissions data for specific industrial GHGs expressed in CO_2 equivalents, including hydrofluorocarbons (HFCs), perfluorocarbons (PFCs), sulfur hexafluoride (SF_6), and nitrogen trifluoride (NF_3). Additionally, the study examines emissions across six major industrial sectors—AFF, FBT, PPP, CCP, WSR, and TS—to evaluate their individual impacts on GHG levels (see Figure 1). The goal is to provide a clearer understanding of how industrial activities influence emissions and to inform effective sector-specific mitigation strategies.

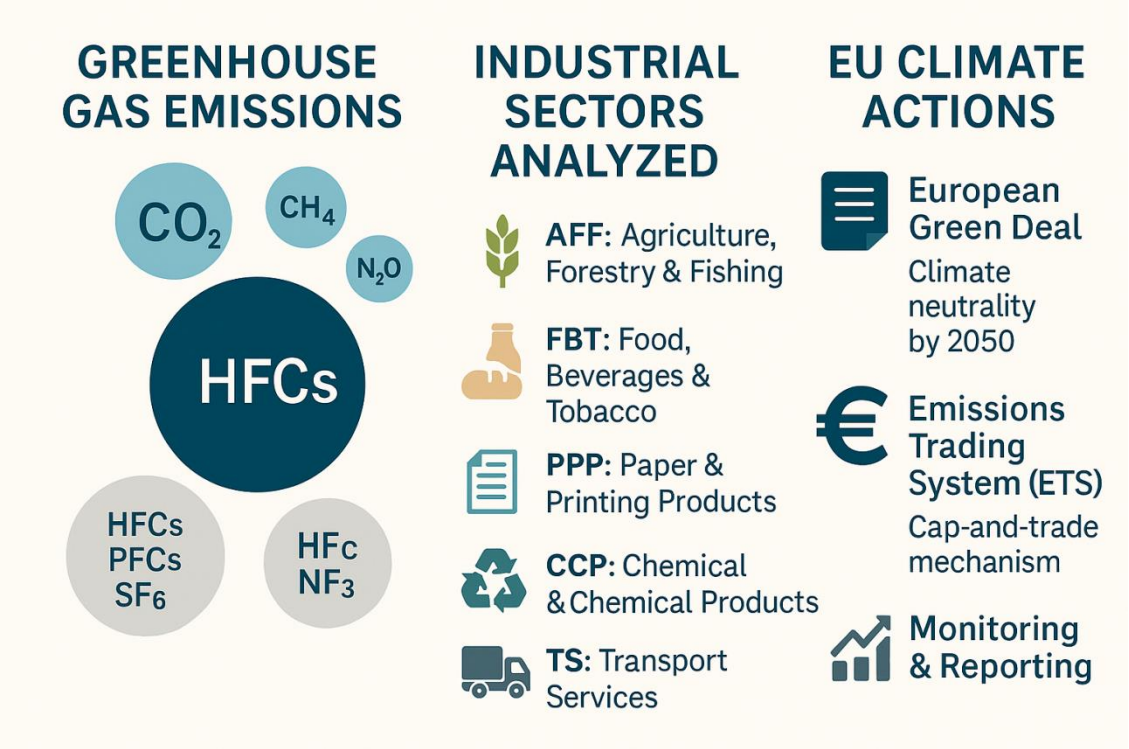

Figure 1. Overview of GHG emissions, industrial sectors, and EU climate actions

Statistical Analysis

We denote the response variable (y) as GHG emissions, while the explanatory variables are defined as AFF (x_1), FBT (x_2), PPP (x_3), CCP (x_4), WSR (x_5) and TS (x_6). In this context, AFF, FBT, PPP, CCP, WSR, and TS are treated as fixed effects. Given that the 17 countries from Eurostat are selected randomly, the impact of these regions on the response variable is regarded as a random effect. Therefore, the Random Intercept and Slope Model (RISM) is specified as follows for $i = 1, \dots, 17$ and $j = 1, \dots, 12$:

$$y_{ij} = \beta_1 x_{ij1} + \beta_2 x_{ij2} + \beta_3 x_{ij3} + +\beta_4 x_{ij4} + \beta_5 x_{ij5} + \beta_6 x_{ij6} + u_1 + u_2 t_{ij} + \varepsilon_{ij}.$$

Here, y_{ij} refers to the response variable for the i-th observation in the j-th region, x_{ijs} represents the value of the explanatory variable x_s ($s = 1, \dots, 6$) for the i-th observation in the j-th region, and t_{ij} indicates the time corresponding to y_{ij}. Since the covariance estimates produced by ML and REML methods are usually very similar (as demonstrated by Özkale and Can (2017)), we choose to use the REML approach in this longitudinal sectoral analysis.

$\hat{G}_{REML} = \begin{bmatrix} 1.5264 \times 10^{+15} & 0 \\ 0 & 2.8438 \times 10^{+12} \end{bmatrix}$, $\hat{R}_{REML} = 1.1927 \times 10^{+13} I_{204}$ are computed and $\hat{H}_{REML}$ is obtained from $H = ZGZ' + R$.

Figure 2 presents a pairwise scatterplot matrix illustrating the relationships among seven key sustainability indicators—Greenhouse Gas Emissions (GHGEs), AFF, FBT, PPP, CCP, WSR, and TS —for 17 European countries.

The visual comparison of variable pairs reveals several noteworthy patterns. Notably, there appears to be a positive linear association between FBT and GHGEs, suggesting that higher dependence on fossil-based transport is linked with increased GHG emissions. Similarly, TS demonstrates moderate to strong trends with CCP and PPP, implying that countries with more robust climate policy frameworks and collaborative governance structures tend to exhibit higher overall sustainability performance.

France and Spain appear as outliers in several panels, particularly in the GHGEs and FBT dimensions, potentially reflecting either distinctive national policies or extreme values in these domains. In contrast, countries such as Switzerland and Austria cluster closely across most indicators, indicating relatively homogeneous and consistent sustainability profiles.

While the plot primarily offers insights into bivariate relationships, it also hints at the possibility of multicollinearity among several variables. For example, TS shows notable positive correlations with CCP, PPP, and FBT simultaneously, while GHGEs correlates with both FBT and AFF. Such patterns suggest that some variables may not be independent of one another, a condition that can distort parameter estimates and inflate standard errors in modeling.

Given the potential multicollinearity, it is advisable to complement this exploratory analysis with formal diagnostics such as the condition number. The condition number is found as $4.1618 \times 10^{+03} > 1000$, which is greater than 1000. This indicates the presence of severe multicollinearity.

Despite some overlapping data points and visual complexity due to color-coded categories, this scatterplot matrix remains a useful preliminary tool for examining sustainability trends and multicollinearity across countries. Further statistical modeling is essential to validate and quantify these observed associations.

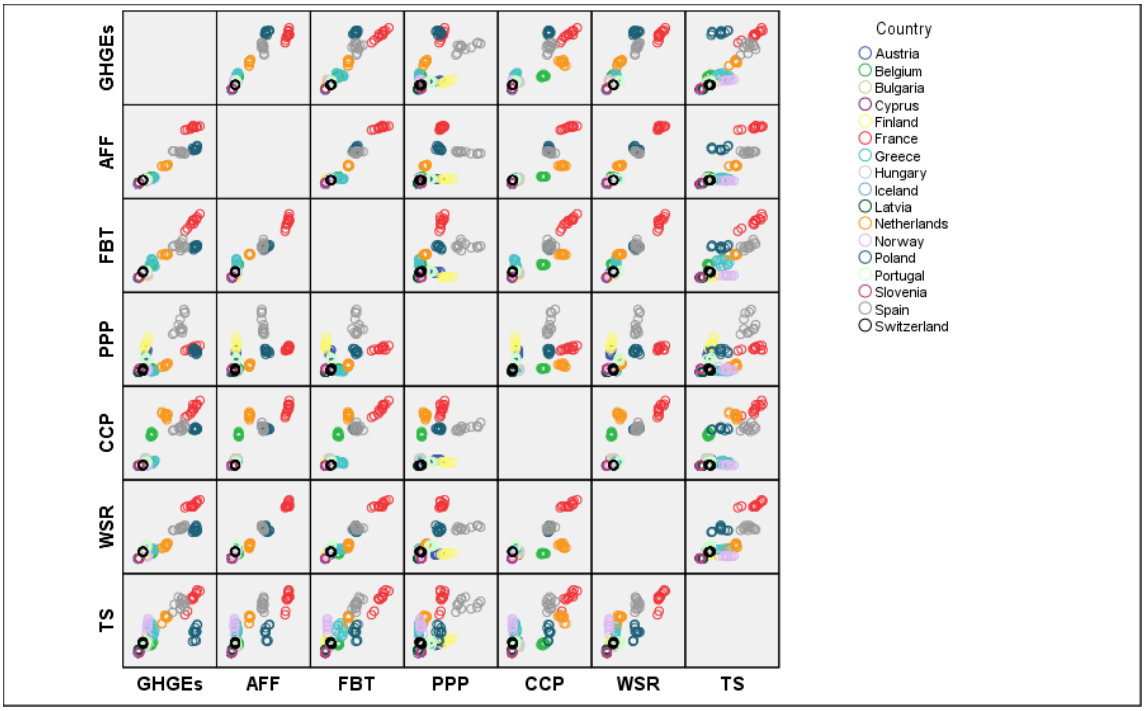

Figure 2. Scatterplot matrix of seven sustainability indicators across 17 European countries

To select the optimal ridge biasing parameter, based on the studies in Özkale and Can (2017), the generalized cross-validation ridge statistic (GCV_k), defined in ridge estimator and predictor in LMMs, is used to determine the best value of k and $\hat{k} = 0.7318$ is obtained. By following Özkale and Kuran (2020), the Liu biasing parameter d is found as $\hat{d} = 0.0100$.

Table 1 displays the parameter estimates and SMSE values (the trace of the MMSE) obtained when the variance of y is estimated using the REML method.

Table 1. Parameter estimates and SMSE values for ridge and Liu-based approaches

Parameter	$\widehat{\boldsymbol{\beta}}$	$\widehat{\boldsymbol{\beta}}_{\hat{d}}$	$\widehat{\boldsymbol{\beta}}_{\hat{k}}$	$\widehat{\boldsymbol{u}}$ ($\times \mathbf{10}^{+07}$)	$\widehat{\boldsymbol{u}}_{\hat{d}}$ ($\times \mathbf{10}^{+07}$)	$\widehat{\boldsymbol{u}}_{\hat{k}}$ ($\times \mathbf{10}^{+07}$)
β_1	4.3815	4.4105	4.4064			
β_2	10.5412	7.5675	8.1815			
β_3	9.7547	8.6772	8.9148			
β_4	0.9625	1.2710	1.2068			
β_5	-5.8833	-5.0419	-5.2236			
β_6	-0.2876	-0.1845	-0.2061			
u_1				3.1232	3.0795	3.0888
u_2				-0.1586	-0.1559	-0.1565
SMSE	33.6754	32.7655	30.9352			

As shown in Table 1, the parameter estimates for both the Liu and ridge estimators reveal notable changes compared to the standard estimates. For most parameters, both biased estimators lead to coefficient adjustments that reduce the potential distortions caused by multicollinearity. For example, the estimates for β_1 (4.3815 for the standard, 4.4105 for Liu, and 4.4064 for ridge) and β_3 (9.7547 for the standard, 8.6772 for Liu, and 8.9148 for ridge) show a slight increase and decrease, respectively, suggesting that these estimators' impacts are adjusted to provide more stable coefficients.

The random effect estimates (u_1 and u_2) also show minimal differences across the models, with u_1 (the random intercept) and u_2 (the random slope) remaining consistent, indicating that the regional variations in emissions and the effect of time on emissions are largely unaffected by the adjustments for multicollinearity.

The application of the Liu and ridge estimators, as shown in Table 1, results in improved model fit, reflected by a decrease in the SMSE values. The standard model has an SMSE of 33.6754, while the Liu and ridge adjustments reduce the SMSE to 32.7655 and 30.9352, respectively. These improvements indicate that both biased estimators contribute to reducing prediction errors, with the ridge estimator offering the best fit. This suggests that adjusting for multicollinearity through Liu and ridge estimators enhances the model's ability to more accurately capture the relationship between the predictors and GHG emissions.

The theorems in the MMSE-based evaluation of predictors section have also been analyzed. According to Theorem 1, for $\hat{k} = 0.7318$, the condition value of $\hat{\beta}_{\hat{k}}$ is 66.3008, which is smaller than $\hat{\sigma}^2_{ridge} = (y - X\hat{\beta}_{\hat{k}})'H^{-1}(y - X\hat{\beta}_{\hat{k}})/(n-p) = 77.5286$, making $\hat{\beta}_{\hat{k}}$ superior to $\hat{\beta}$ under the MMSE criterion. Based on Theorem 2, for $\hat{d} = 0.0100$, the condition value of $\hat{\beta}_{\hat{d}}$ is $1.5662 \times 10^{+2}$, which is less than $2\hat{\sigma}^2_{Liu}/(1-\hat{d}) = 1.5698 \times 10^{+2}$, where $\hat{\sigma}^2_{Liu} = (y - X\hat{\beta}_{\hat{d}})'H^{-1}(y - X\hat{\beta}_{\hat{d}})/(n-p) = 77.7060$ indicating that $\hat{\beta}_{\hat{d}}$ is better than $\hat{\beta}$. According to Theorem 3, for $\hat{k} = 0.7318$ and $\hat{d} = 0.0100$, the condition value exceeds 1, making $\hat{\beta}_{\hat{k}}$ superior to $\hat{\beta}_{\hat{d}}$ under the MMSE criterion.

This longitudinal sectoral analysis under multicollinearity shows that both ridge and Liu estimators outperform BLUE when assessed using the MMSE and SMSE criteria, provided that the values of k and d are correctly chosen.

DISCUSSION AND CONCLUSION

This study provides a thorough examination of the factors driving GHG emissions across 17 Eurostat countries from 2010 to 2021, focusing on six major industrial sectors: AFF, FBT, PPP, CCP, WSR, and TS. The findings confirm that economic and industrial activities remain the primary contributors to emissions, with sectors such as AFF, FBT, and PPP being significant drivers. Conversely, sustainable practices, particularly in waste management and transportation, play a crucial role in mitigating emissions.

The ridge and Liu estimators, employed to tackle multicollinearity, provided valuable insights into the sector-specific effects on emissions. The results revealed that ridge regression offers more stable coefficient estimates, while Liu regression preserves a closer alignment with the original data structure. These differences underscore the importance of selecting the appropriate biasing method for refining parameter estimates in multicollinear models.

The analysis highlighted the importance of policies targeting high-emission sectors like agriculture, food production, and transportation. Notably, carbon capture policies in the chemical sector emerged as more effective than initially anticipated, suggesting their potential for greater impact in reducing emissions. Additionally, sectors like waste management and transportation show considerable promise for mitigating GHG emissions through sustainable practices.

In conclusion, this research emphasizes the need for tailored sustainability strategies to reduce emissions, with particular attention to sectors that contribute the most to GHG emissions. The findings suggest that carbon capture technologies in chemical industries, along with continued efforts to improve energy efficiency and adopt sustainable practices in agriculture, waste management, and transportation, are essential components of climate action. As climate policies continue to evolve, these results underscore the importance of data-driven, sector-specific approaches to effectively manage and mitigate GHG emissions, aligning industry actions with broader climate goals across Europe.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

The authors have equally contribution.

Explainable Prediction of Cardiovascular Diseases with Machine Learning and SHAP Approach (1133)

Mehmet Kıvrak[1*]

[1]Recep Tayyip Erdogan Unuversity, Faculty of Medicine, Biostatistics and Medical Informatics, Rize, Turkiye

[*]Corresponding author e-mail: mehmet.kivrak@erdogan.edu.tr

Abstract

Aim: *This study aims to develop and interpret a machine learning-based model to predict cardiovascular disease (CVD) using clinical and biochemical features, and to identify the most significant risk factors contributing to the prediction using SHAP (SHapley Additive exPlanations) values.*

Methods: *A publicly available dataset containing clinical and biochemical data was used. Several machine learning algorithms, including Random Forest, Gradient Boosting, and Logistic Regression, were trained to classify the presence of CVD. Model performance was evaluated using metrics such as accuracy, precision, recall, F1-score, and area under the curve (AUC). SHAP analysis was employed to determine the contribution of each feature to the prediction outcomes and to enhance model explainability.*

Results: *Among the models evaluated, the Gradient Boosting classifier showed the highest accuracy and AUC. SHAP analysis revealed that features such as cholesterol level, age, sex, and resting blood pressure were the most influential in predicting cardiovascular disease. The explainability provided by SHAP increased the transparency and clinical relevance of the model's predictions.*

Conclusion: *Machine learning models, when combined with SHAP-based interpretation tools, can provide accurate and explainable predictions for cardiovascular disease. This approach has the potential to support early diagnosis and personalized treatment by highlighting patient-specific risk factors in clinical practice.*

Keywords: *Cardiovascular disease, machine learning, SHAP, risk prediction, model explainability, clinical decision support*

INTRODUCTINON

Cardiovascular Diseases (CVD) continue to be a leading cause of morbidity and mortality worldwide, encompassing a wide range of conditions affecting the heart and blood vessels (1). Numerous risk factors, including age, gender, hypertension, diabetes, hyperlipidemia, and lifestyle habits, have been associated with the development and progression of CVDs (2, 3). Age and gender have long been recognized as important risk factors for CVD. Epidemiological studies have established a well-documented relationship between advancing age and increased prevalence of heart disease. As individuals age, they become cumulatively exposed to risk factors and physiological aging processes, placing them at higher risk (4). Similarly, gender differences in CVD risk have been studied extensively; men have a higher prevalence of heart disease than women, and this may be attributed to differences in hormonal regulation, lifestyle factors, and genetic predisposition (5). Metabolic parameters such as fasting blood glucose and serum cholesterol levels play an important role in the pathophysiology of CVDs. Hyperglycemia has been suggested to be associated with endothelial dysfunction, vascular inflammation, and increased atherosclerotic burden, thereby exacerbating cardiovascular risk (6). Hypercholesterolemia is a well-known risk factor for CVDs. Long-term studies have clearly demonstrated the association between total serum cholesterol levels and heart disease (7). Electrocardiographic (ECG) findings, including ST-segment abnormalities and left

ventricular hypertrophy, have been used as diagnostic and prognostic indicators of heart disease, making the ECG probably the first and most critical investigation in the diagnosis of CVDs (8). Similarly, reductions in maximum heart rate (MaxHR) have been associated with poorer cardiovascular health, probably due to impaired autonomic regulation and reduced cardiac output (9).

Early diagnosis is critical in the treatment and management of cardiovascular diseases, and machine learning can be a powerful tool in detecting a possible heart disease diagnosis. In this study, we develop a Gradient Boosting model to predict the presence of cardiovascular disease and identify some of the most important predictors according to SHAP values and predict the disease using these predictors with machine learning methods.

MATERIAL AND METHODS

The dataset was obtained from https://www.kaggle.com/datasets?search=cardiovascular. The dataset consists of a total of 918 volunteer, 410 healthy and 508 patient patient records and includes clinical and biochemical measurements with disease status information. It consists of resting ECG, old peak, fasting blood pressure, cholesterol determined by st_slope blood test, age, gender, chest pain type and resting blood pressure obtained by declaration form, and exercise angina and maxHR determined by physical effort test. Data were extracted from the database on 17.01.2025.

Explainable Data Analysis

We see that the risk of developing cardiovascular disease increases with age ($p<0.001$, Figure 1 and Table 1). The rate of men with heart disease is higher than women (90.2%). There is a significant relationship between sex and heart disease ($p<0.001$, Figure 2 and Table 2). The type of chest pain is significantly associated with heart disease ($p<0.001$, Figure 2 and Table 2). The risk of developing heart disease increases as resting blood pressure increases ($p<0.001$, Figure 1 and Table 1). The risk of heart disease increases as cholesterol decreases ($p<0.001$, Figure 1 and Table 1). The frequency of heart disease is higher in individuals with fasting blood sugar (otherwise) (66.5%). Fasting blood sugar is significantly associated with heart disease ($p<0.001$, Figure 2 and Table 2).

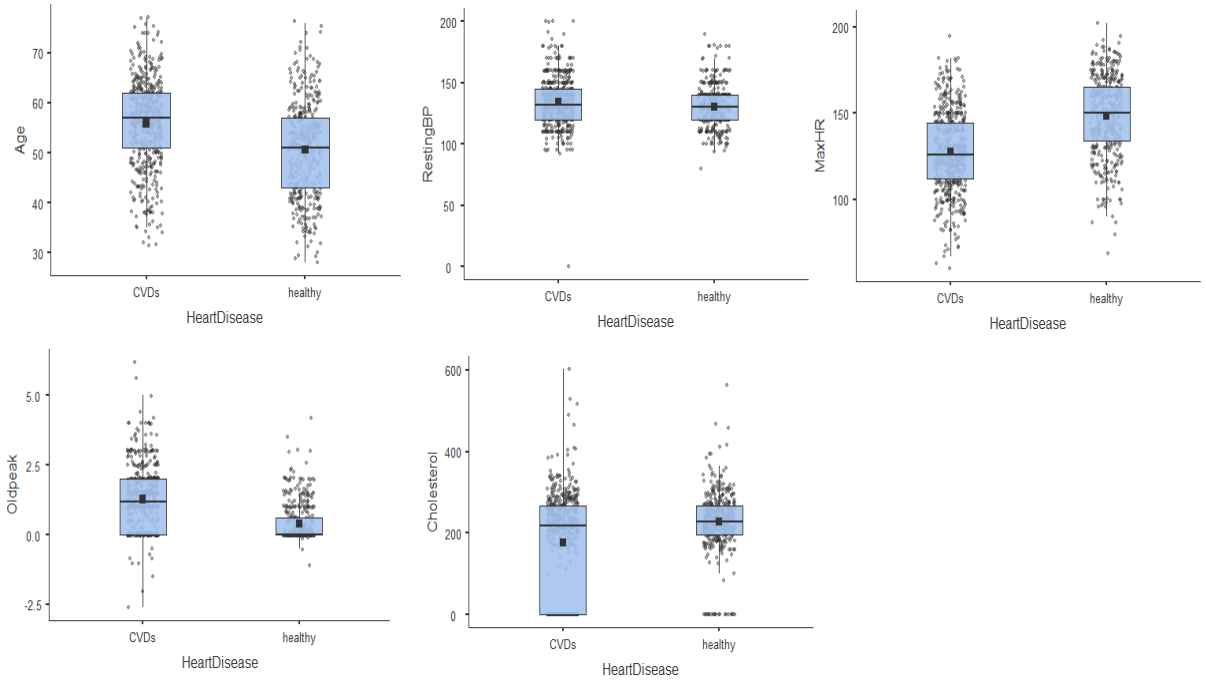

Figure 1. Explanatory Graphs of Continuous Variables

Resting ECG findings are significantly associated with heart disease (p=0.004, Figure 2 and Table 2), and normal resting ECG is more common in individuals with heart disease (56.1%).

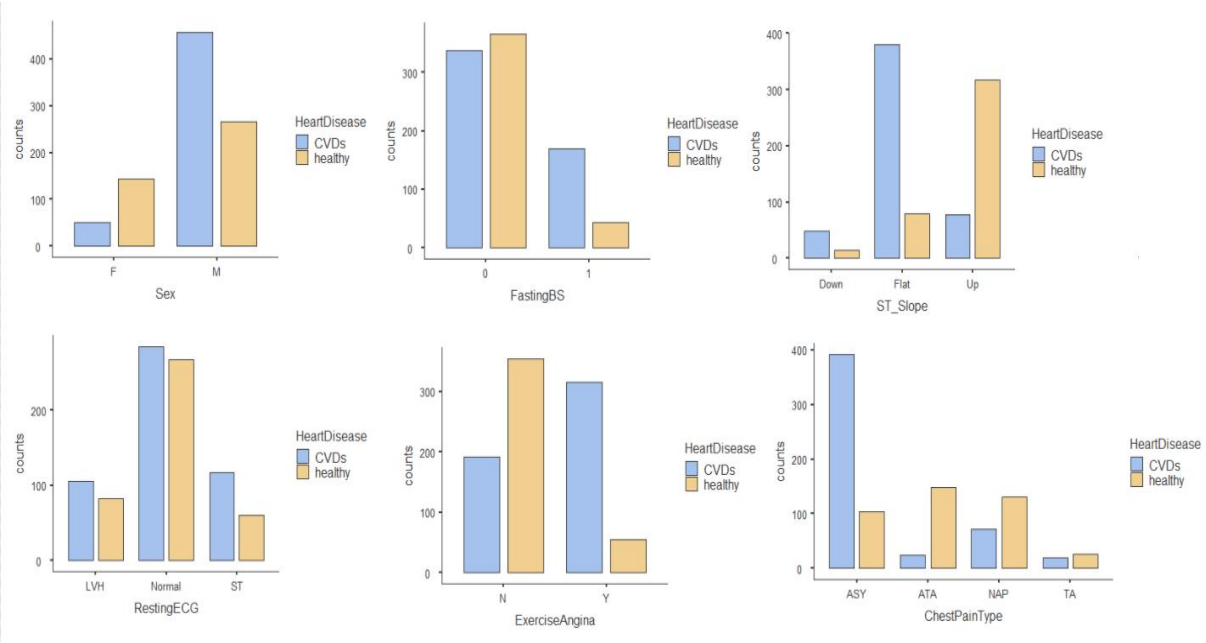

Figure 2. Explanatory Graphs of Categorical Variables

The risk of heart disease increases as MaxHR decreases (p<0.001, Figure 1 and Table 1). Exercise-induced angina is significantly associated with heart disease (p<0.001, Table 2 and Figure 2). The prevalence of heart disease is significantly higher in individuals who experience angina during exercise (62.2%).

Table 1. Explanatory Table of Continuous Variables

	Heart Disease			
	No (n=410)	**Yes (n=508)**		
Gender	**Descriptive Statistics**		**Test Statistics***	**p-Value**
Female	143 (34.9 %)	50 (9.8 %)	85.6	**<0.001**
Male	267 (65.1 %)	458 (90.2 %)		
FastingBS				
FastingBS > 120 mg/dl	44 (10.7 %)	170 (33.5 %)	65.6	**<0.001**
Otherwise	366 (89.3 %)	338 (66.5 %)		
Chestpaintype				
Typical Angina	26 (6.3 %)	20 (3.9 %)	268	**<0.001**
Atypical Angina	149 (36.3 %)	24 (4.7 %)		
Non-Anginal Pain	131 (32.0 %)	72 (14.2 %)		
Asymptomatic	104 (25.4 %)	392 (77.2 %)		
RestingECG				
LVH	82 (20.0 %)	192 (20.9 %)	10.9	**0.004**
Normal	267 (65.1 %)	285 (56.1 %)		
ST	61 (14.9 %)	117 (23.0 %)		
Exerciseangina				
No	355 (86.6 %)	192 (37.8 %)	224	**<0.001**
Yes	55(13.4 %)	316 (62.2 %)		

Stslope				
Down	14 (3.4 %)	49 (9.6 %)	356	**<0.001**
Flat	79 (19.3 %)	381 (75.0 %)		
Up	317 (77.3 %)	78 (15.4 %)		

The risk of developing heart disease increases as Oldpeak increases (p<0.001, Figure 1 and Table 1). ST segment slope is significantly associated with heart disease (p<0.001, Table 2 and Figure 2). Flat ST segment is more common in individuals with heart disease (75%).

	Heart Disease			
Variable	**No (n=410)** Median (Min-Max)	**Yes (n=508)**	**Test Statistics***	**p-Value**
Age	51 (28-76)	57 (31-77)	69138	**<0.001**
RestingBP	130 (80-190)	132 (0-200)	90420	**<0.001**
Cholesterol	227 (0-564)	217 (0-603)	87280	**<0.001**
MaxHR	150 (69-202)	126 (60-195)	55191	**<0.001**
Oldpeak	0.0 (-1.1-4.2)	1.2 (-2.6-6.2)	55164	**<0.001**

According to the correlations and scatter plots, HeartDisease has the strongest positive correlation with OldPeak (correlation = 0.4) and the strongest negative correlation with MaxHR (correlation = -0.4). There is also a moderately strong correlation of -0.38 between Age and MaxHR. Heart rate tends to decrease with increasing age (Figure 3).

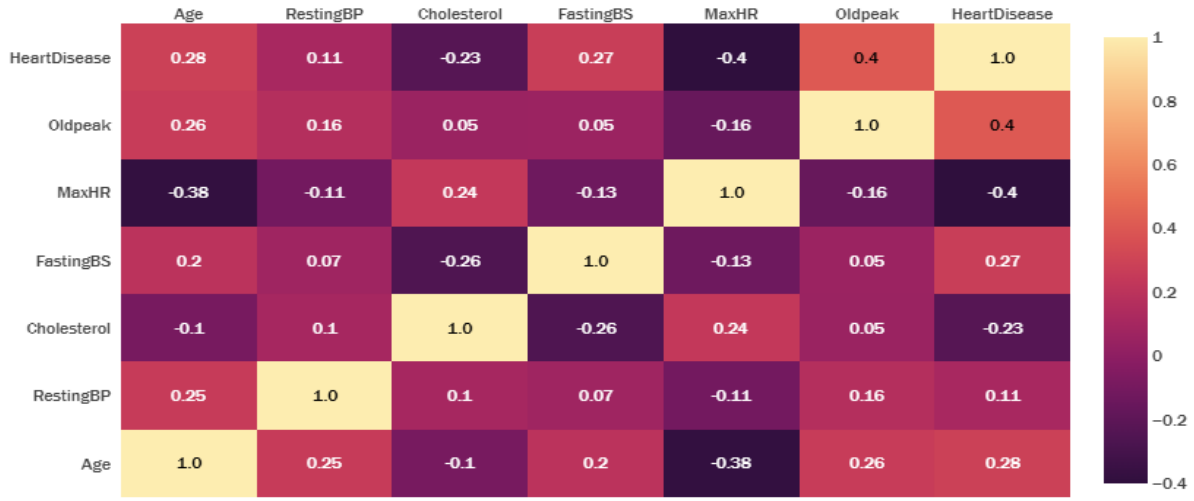

Figure 3. Correlation Matrix for Variable

SHAP Feature Selection

To capture possible relationships between features in the data, we added interaction terms to the models to determine whether these variables were effective in predicting heart disease based on their SHAP Values. SHAP (SHapley Additive Explanations) uses a game theory approach to determine the importance of each feature and can be used to explain both individual model predictions and aggregate model results. To measure feature importance, SHAP calculates the magnitude that each predictor contributes to the model output based on the marginal contributions of the feature averaged across all possible combinations of features in the model (10). To perform feature selection using SHAP values, a gradient boosting model was first applied that included all variables due to its speed and accuracy. The features that contributed to the

model prediction with a SHAP value greater than 0.1 were selected as the final predictors. Then, we can select the best performing model using these predictors. Due to the multicollinearity between the interaction terms, we compared several nonparametric tree-based methods to predict the risk of heart disease. The summary plot of Shapley values shows the top 20 predictors of heart disease arranged in order of importance. Each point on the graph represents an observation from the training set. When the points are to the right of the 0-line, this indicates there is a higher likelihood of being diagnosed with heart disease, while points to the left indicate a lower likelihood. The values for each feature is shown by the color of the points, with light orange representing high feature values and dark blue representing low feature values. The shape of the points in each row is based on the amount of overlapping observations for that feature.Nearly all of the variables in the plot are interaction terms that were added to the model, along with three individual features, Cholesterol, Age, and Atypical Chest Pain. The variable in the first row is the interaction between RestingBP and ST_Slope_Up. Based on the feature's Shapley values, the risk of heart disease is lower in patients who have an upward ST slope and high blood pressure. For the second variable, Sex_M ST_Slope_Flat, the Shapley values indicate patients who are male and have a flat ST Slope are at a higher risk of cardiovascular disease (Figure 4).

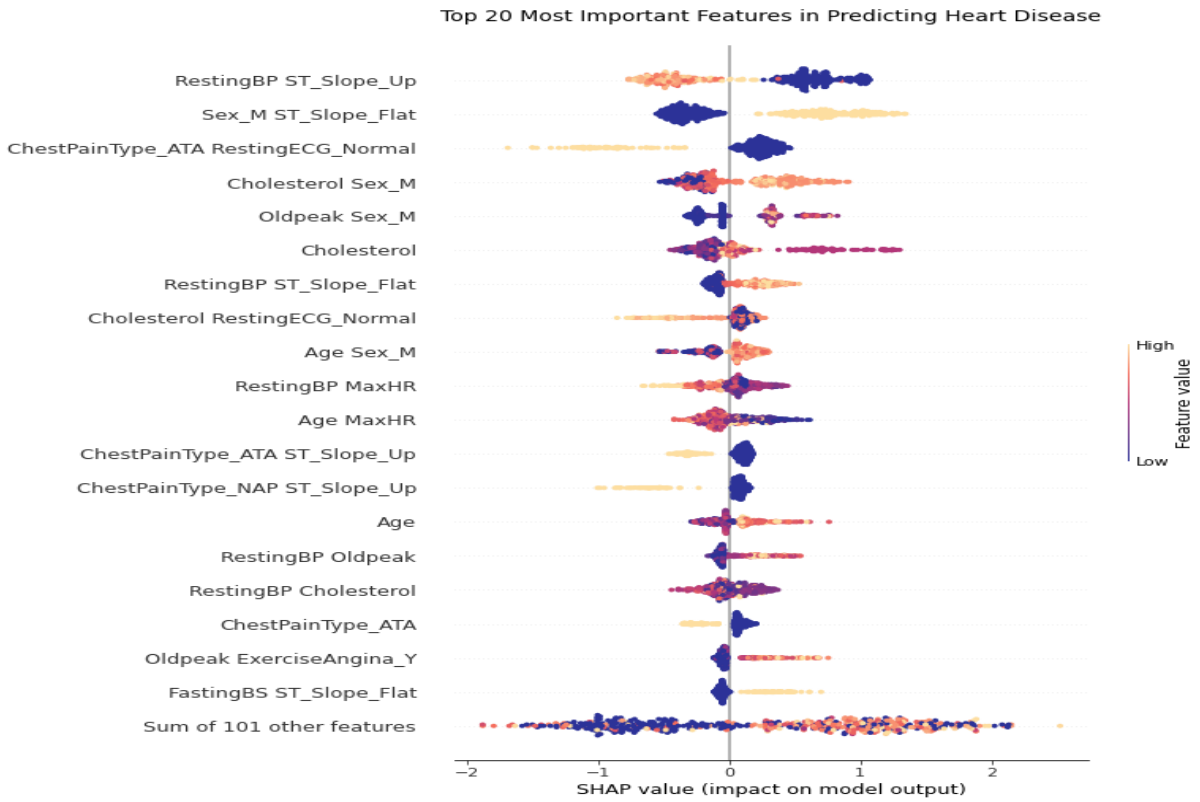

Figure 4. SHAP Feature Selection

The fourth variable in the plot, Cholesterol Sex_M, shows that men who have high cholesterol are more likely to be diagnosed with heart disease. The order of importance in the summary plot is calculated by the feature's average absolute Shapley value, which measures how much the feature changes the predicted probability of heart disease on average. There are 19 features that contribute to the model's prediction by an absolute value of at least 0.1 on average. Below is a list of the final predictors selected and their feature importance. Among the most important predictors, RestingBP and ST_Slope appear in 5 of the top 20 (Figure 5).

	Feature Importance
Cholesterol ST_Slope_Flat	0.941
RestingBP ST_Slope_Up	0.569
Sex_M ST_Slope_Flat	0.512
ChestPainType_ATA RestingECG_Normal	0.341
Cholesterol Sex_M	0.319
Oldpeak Sex_M	0.260
Cholesterol	0.252
RestingBP ST_Slope_Flat	0.160
Cholesterol RestingECG_Normal	0.159
Age Sex_M	0.153
RestingBP MaxHR	0.151
Age MaxHR	0.150
ChestPainType_ATA ST_Slope_Up	0.146
ChestPainType_NAP ST_Slope_Up	0.144
Age	0.127
RestingBP Oldpeak	0.124
RestingBP Cholesterol	0.118
ChestPainType_ATA	0.115
Oldpeak ExerciseAngina_Y	0.100

Figure 5. Feature Importance

Modelling

Before training the machine learning models, feature selection was performed to identify the most important variables contributing to heart disease prediction. The dataset was split into training (80%) and testing (20%) sets to ensure unbiased model evaluation. Additionally, stratified k-fold cross-validation (k = 5) was applied to the training set to prevent overfitting and optimize model performance. Grid search optimization was performed on each parameter. Using Shapley features above 0.1,

The Extra Trees Classifier produces an Area Under the Curve (AUC) of 0.92. After tuning the hyperparameters in the model, the classifier produces an average accuracy of 84.5% and an F1 score of 86.3% on the validation set, with a standard deviation of about 6.7.

The Random Forest model improves on the performance of the Extra Trees classifier across all 3 metrics, with an Area Under the Curve of 0.924. On the validation set, the model produces an average accuracy of 85.5% and an F1-score of about 87 %.

We see a slight decline in performance in the AdaBoost model. The AUC decreased to 0.897 and the overall accuracy and F1-score dropped to 84% and 85% respectively, although the model produces a smaller standard deviation than the others.

The Gradient Boosting model produces the highest Area Under the Curve out of the classifiers at 0.925. In addition, the model increases the accuracy to almost 89% and the F1-score to 90% on the validation set. Comparing the cross-validation results in the boxplots above, we see the Gradient Boosting model produces the highest median F1-score of 90.3% and the highest median accuracy of 88.5%. It also has the smallest spread in the distribution with a standard deviation of about 3. The Random Forest model comes in a close second place with an 89.7% median F1-score and an 88.2% median accuracy, although it has slightly more variability in the scores (Figure 6).

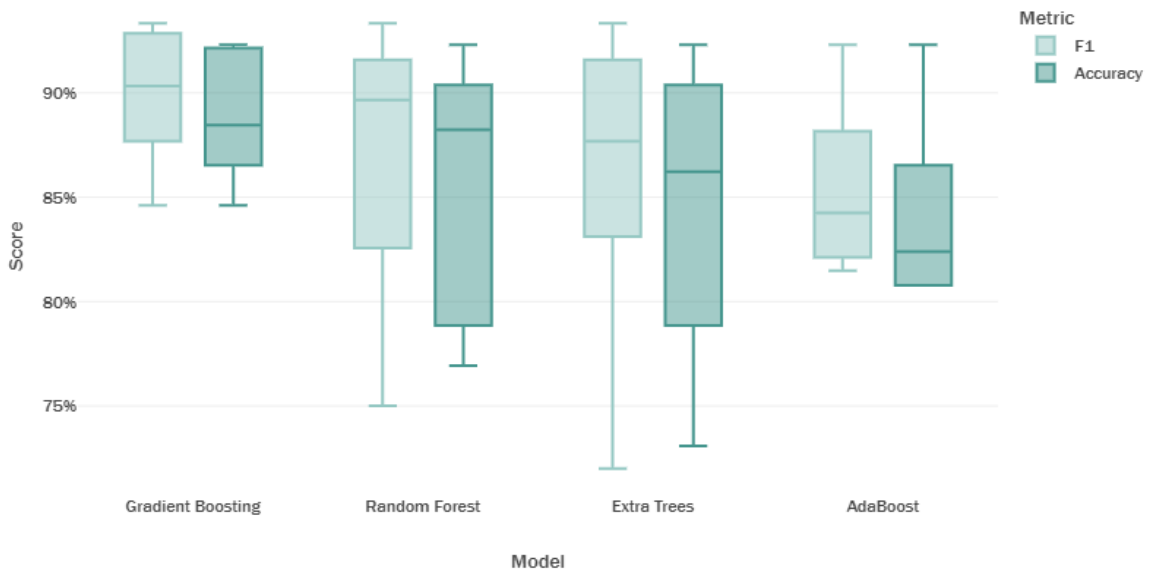

Figure 6. Model Performance on Validation Set

The confusion matrix results show how many patients the models correctly predicted heart disease in and how many were misclassified. On the test set, the Gradient Boosting model classified the highest amount of True Positives (patients with heart disease), as well as the highest amount of True Negative (patients without heart disease). The confusion matrix results show how many patients the models correctly predicted heart disease in and how many were misclassified. On the test set, the Gradient Boosting model classified the highest amount of True Positives (patients with heart disease), as well as the highest amount of True Negative (patients without heart disease) (Figure 7).

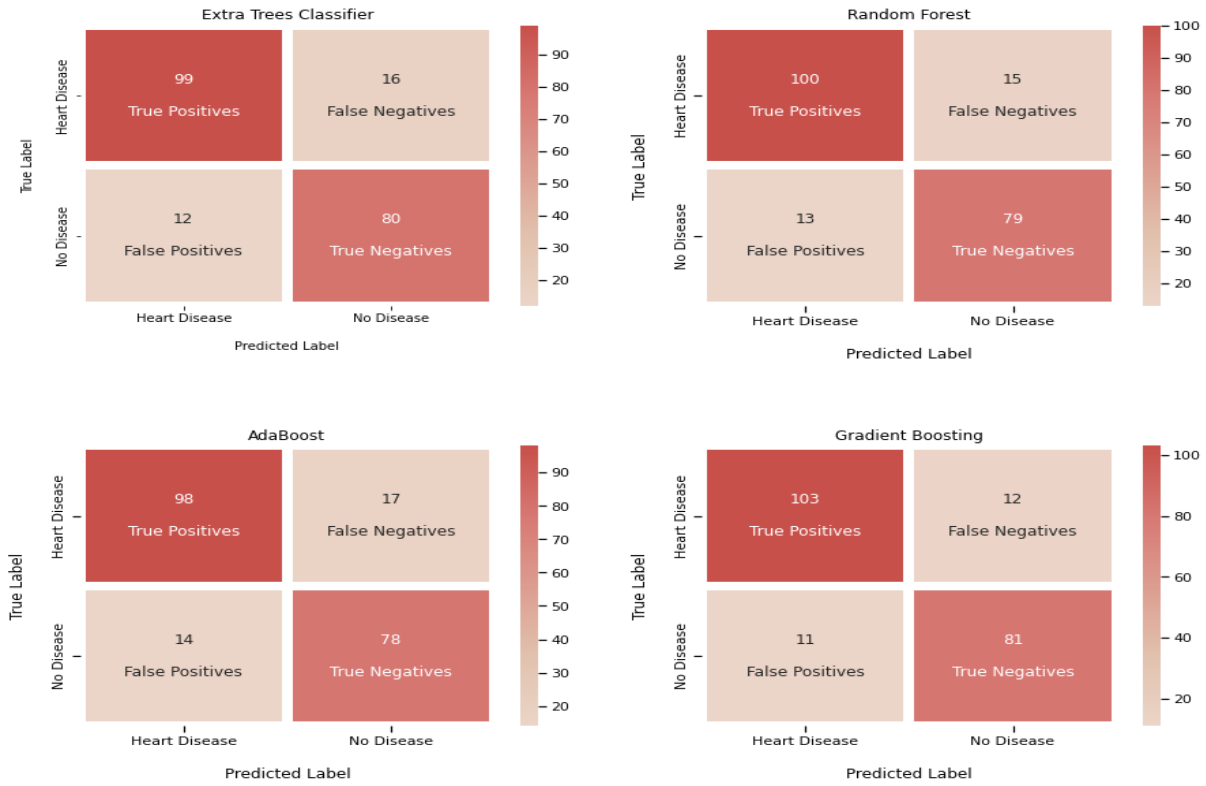

Figure 7. Confusion Matrix Results

The ROC Curves show how well the models perform at different thresholds. The y-axis shows the True Positive Rate, or Sensitivity of the models, which is a measure of how well the model identifies patients with heart disease (true positives) and the False Positive Rate along the x-axis shows how many patients the model misclassifies as false positives. A model with a curve near the top left of the graph, with a higher true positive rate and a lower false positive rate, indicates a better capability to distinguish between the classes. In the plot above, all of the models produce strong results on the test set. Gradient Boosting provides the highest Area Under the Curve overall at 0.927, although at certain thresholds, we see the Random Forest model produces slightly better results where the curve surpasses the Gradient Boosting model (Figure 8).

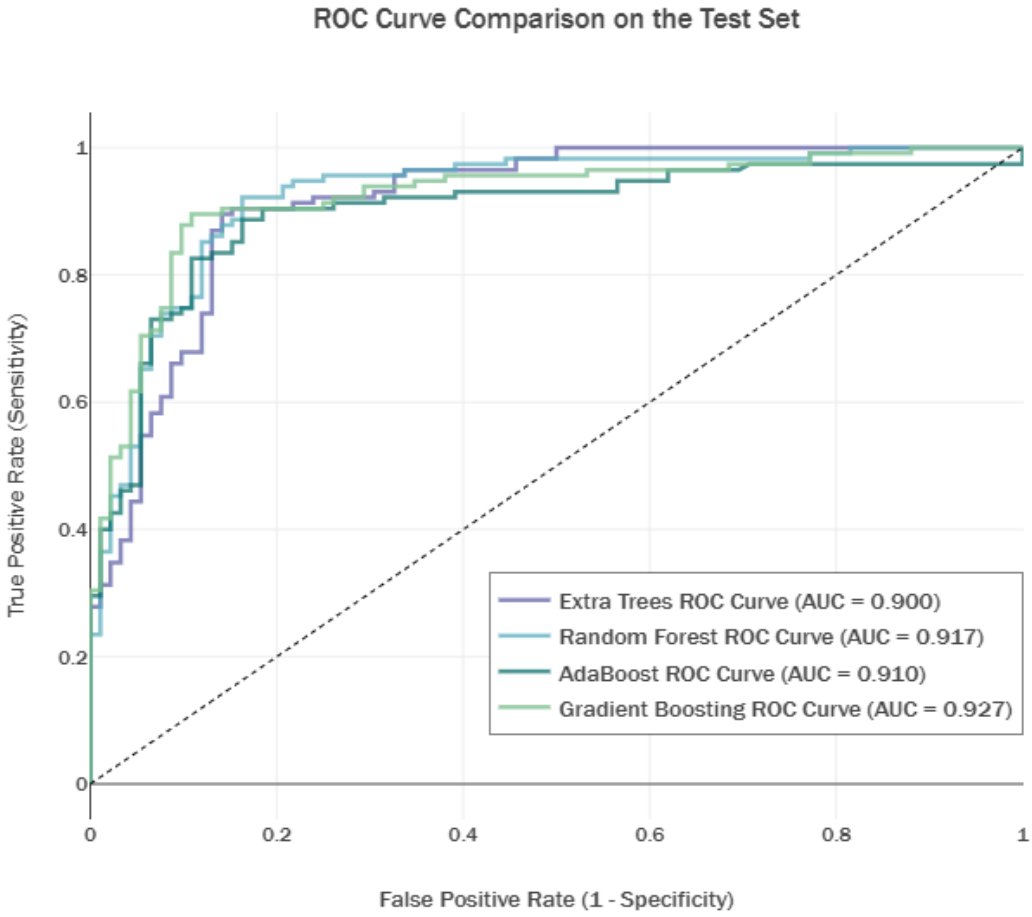

Figure 8. ROC Curve Comparison on the Test Set

CONCLUSION

In detecting cardiovascular disease, Shapley values were used to develop a Gradient Boosting model that can predict the likelihood of a heart disease diagnosis with an Area Under the Curve of 0.927. Using Shapley values, we also identified key indicators of heart disease and their influence in predicting a positive diagnosis. Some of the most important predictors in the model were the interaction effects between a patient's medical information, especially in features that included a patient's Age, Cholesterol, Blood Pressure, ST Slope, and Chest Pain type. In selecting the best model, Gradient Boosting provided the strongest results overall and can be used in the early detection and diagnosis of heart disease with an overall F1-Score of 90% and an accuracy of 89%.

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Eigenvalue Bounds for the Fractional Laplacian (1150)

Mahir Hasanov[1]

[1] Istanbul Beykent University, Faculty Arts-Sciences, Department of Mathematics, Istanbul, Türkiye

*Corresponding author e-mail: mahirhasansoy@beykent.edu.tr

Abstract

It is known that, in order to obtain the logarithmic small ball asymptotics of a Gaussian process, one needs the one-term asymptotics of the eigenvalue counting function of the associated differential operator. In this context, we study the Dirichlet eigenvalue asymptotics of the fractional Laplacian in weighted spaces. Our approach is based on the Birman-Solomyak approximation method, which plays a significant role in the spectral theory of differential operators. We prove a theorem on the approximation of functions from the fractional Sobolev spaces by piecewise constant functions. Using this theorem, we establish Weyl-type eigenvalue bounds for the fractional Laplacian.

Keywords: *Eigenvalue, Fractional Laplacian, Approximation, Asymptotics*

INTRODUCTION

The main subject of this paper is following weighted Dirichlet eigenvalue problem on a bounded domain with Lipschitz boundary.

$$\begin{cases}(-\Delta)^s u(x) = \frac{1}{\lambda} w(x)u(x),\ 0 \le w \in L_r(\Omega),\ s \in (0,1),\ r \in [1,\infty), \\ u = 0\ in\ R^n \setminus \Omega. \end{cases} \tag{1.1}$$

All details regarding the definition of the solution and the variational settings of the problem are given separately in the next section, as they require some technical knowledge.

Our main goal is to study the asymptotic behavior of the eigenvalues of the problem (1.1) by using the approximation method. This method, which is based on the approximation of functions from Sobolev spaces by piecewise polynomial functions and is widely used in the spectral theory of differential operators, was introduced by M.S. Birman and M.Z. Solomyak. These fundamental studies are included in Birman, MS, Solomyak, MZ, 1980 (see also the references therein). Details of this method are given in Section 3.

The main results of this article can be summarized as follows:

1) A theorem on the approximation of functions from the class $W^{s,p}(Q^n), 0 < s < 1$ with piecewise constant functions is proved (see Theorem 4.1),

2) Based on Theorem 4.1, the following result was found for the upper bound of the eigenvalues $\lambda_m^s(D)$ of the problem (1.1) (see Theorem 5.1).

Theorem 1.1. Let Ω be a bounded domain with Lipschitz boundary and the following condition is satisfied.

$r = 1\ if\ 2s > n;\ \ r > \frac{n}{2s}\ if\ 2s \le\ n.$

Then,

$$\lambda_m^s(D) \leq C\, m^{-\frac{2s}{n}} diam(\Omega)^{2s-\frac{n}{r}} \left[\int_\Omega w^r(x)dx\right]^{\frac{1}{r}}, \qquad (1.2)$$

where the constant C depends only on n, s, r.

In recent years, the fractional Laplace operator has been the subject of research by many scientists. There are sufficient sources explaining the details and importance of the subject (see Chen, W, Li Y, Ma P, 2020; Fernández-Real X, Ros-Oton X, 2024; Kwasnicki M, 2017; Lischke A, Pang G, Gulian M, Song F, Glusa C, Zheng X, Karniadakis GE, 2020 and Zoia A, Rosso A, Kardar M, 2007. We note that among the important equations involving the fractional Laplacian are equations such as Diffusion-reaction, Quasi-geostrophic, Cahn-Hilliard, Porous medium, Schrödinger and Ultrasound equations (see Lischke A, Pang G, Gulian M, Song F, Glusa C, Zheng X, Karniadakis GE, 2020). Moreover, eigenvalue problems always play an important role in investigating a wide range of problems. Particularly, interest in the eigenvalues of fractional differential operators is also evaluated in this context and significant efforts have been made in this field in recentyears (see Dyda B, 2012; Dyda B, Kuznetsov A, Kwaśnicki M, 2017; Fall MM, Ghimenti M, Micheletti AM, Pistoia A, 2023; Geisinger L,2014; Kwasnicki M, 2012; Nazarov AI, 2021; Servadei R, Valdinoci E, 2014 and references therein).

Dyda B, 2012 estimated the first eigenvalue of the fractional Laplacian in a ball. A numerical scheme for finding two-sided bounds for the Dirichlet eigenvalues of the fractional Laplace operator in the unit ball of R^n was suggested in Dyda B, 2012. Two-term Weyl-type asymptotic law for the eigenvalues of the one-dimensional fractional Laplace operator in the interval $(-1, 1)$ was given by Kwasnicki M, 2012. Nazarov AI, 2021 found two-term eigenvalue asymptotics for one dimensional integro-differential equations arising in the theory of Gaussian processes similar to the fractional Brownian motion. The simplicity property of the Eigenvalues of the fractional Laplace operator is discussed in [10]. Recent development on eigenvalue bounds of fractional Laplacian can be found in the review paper Frank RL, 2018 (see also Geisinger L, 2014).

This paper consists of five sections, including the introduction. In the second section the definition of the eigenvalue problem (1.1), variational settings of the problem, some variational principles and necessary preliminary facts are included. The basic theorems of the approximation method are given in the third section. One of the main results of this paper is a theorem about approximation of functions from the class $W^{s,p}(\Omega), 0 < s < 1$ with piecewise constant functions. This theorem is given in the fourth section. The problem of eigenvalue bounds is studied in the section 5.

FRACTIONAL EIGENVALUE PROBLEMS: VARIATIONAL SETTINGS

In this section the definition of the eigenvalue problem (1.1) and all the concepts and known facts that we will use in the following sections are included.

I)*The Fractional Sobolev Spaces.*

Let $p \in [1, \infty), s \in (0, 1)$ and Ω be an open domain in R^n. In this case, the fractional Sobolev space $W^{s,p}(\Omega)$ is defined (see Di Nezza E, Palatucci G, Valdinoci E, 2012 and Lischke A, Pang G, Gulian M, Song F, Glusa C, Zheng X, Karniadakis GE, 2020) as a normed space with the norm

$$\|u\|_{W^{s,p}(\Omega)} = \left(\int_\Omega |u|^p dx + \int_\Omega \int_\Omega \frac{|u(x)-u(y)|^p}{|x-y|^{n+sp}} dx\, dx\right)^{\frac{1}{p}}.$$

The Gagliardo seminorm of u, denoted by $[u]_{W^{s,p}(\Omega)}$ is defined as

$$[u]_{W^{s,p}(\Omega)} = \left(\int_\Omega \int_\Omega \frac{|u(x)-u(y)|^p}{|x-y|^{n+sp}} dx\, dx \right)^{\frac{1}{p}}.$$

We set $W_0^{s,p}(\Omega) = \overline{C_0^\infty(\Omega)}$ with respect to the norm of $W_0^{s,p}(\Omega)$ and denote $H^s(\Omega) = W^{s,p}(\Omega$ and $H_0^s(\Omega) \coloneqq W_0^{s,2}(\Omega)$.

Now let us define another Sobolev space $\widetilde{W_0}^{s,p}(\Omega)$, which is directly related tothe eigenvalue problem (1.1). Let $s \in (0,1), 1 < p < \infty$ and let Ω be a bounden open domain with Lipschitz boundary $\partial\Omega$. Then, the Sobolev space $\widetilde{W_0}^{s,p}(\Omega)$ (see Brasco L, Lindgren E, Parini E, 2014; . Brasco L, Parini E, 2016 and Brasco L, Parini E, Squassina M, 2015) is defined as the closure of $C_0^\infty(\Omega)$ with respect to the norm

$$\|u\|_{\widetilde{W_0}^{s,p}(\Omega)} \coloneqq [u]_{W^{s,p}(R^n)}, u \in C_0^\infty(\Omega).$$

Evidently, if we take the closure of $C_0^\infty(\Omega)$ with respect to the norm $[u]_{W^{s,p}(R^n)+}\|u\|_{L^p(\Omega)}$ the result does not change. It was shown that (see Proposition B.1 in Brasco L, Parini E, Squassina M, 2015) if Ω is a Lipschitz domain then

$$\widetilde{W_0}^{s,p}(\Omega) = \{u: R^n \to R |\, [u]_{W^{s,p}(R^n)} < \infty \; and \; u = 0 \; in \; R^n \setminus \Omega\}.$$

Obviously, $\widetilde{W_0}^{s,p}(\Omega) \subset W^{s,p}(\Omega)$, since

$$\|u\|^p_{\widetilde{W_0}^{s,p}(\Omega)} = [u]^p_{W^{s,p}(\Omega)} + 2\int_\Omega \int_{\mathrm{R^n}\setminus\Omega} \frac{|u(x)|^p}{|x-y|^{n+sp}} dx\, dy\,, u \in C_0^\infty(\Omega).$$

However, it has been proven that (Brasco L, Lindgren E, Parini E, 2014, Proposition B.1) if Ω is a bounded open domain with Lipschitz boundary, $s \in (0,1), 1 < p < \infty$ and $sp \neq 1$ then

$$\widetilde{W_0}^{s,p}(\Omega) = W^{s,p}(\Omega).$$

We denote $\widetilde{H_0^s}(\Omega) \coloneqq \widetilde{W_0}^{s,2}(\Omega)$.

II) *The Fractional Laplacian on R^n and Bounded Domains.*

Let $\mathcal{S}$ be the Schwartz space of rapidly decaying C^∞ functions in R^n. The fractional Laplacian on R^n is a nonlocal operator defined as follows (see Di Nezza E, Palatucci G, Valdinoci E, 2012; Fernández-Real X, Ros-Oton X, 2024; Kwasnicki M, 2017; Lischke A, Pang G, Gulian M, Song F, Glusa C, Zheng X, Karniadakis GE, 2020 and Musina R, Nazarov AI, 2014).

$$(-\Delta)^s u(x) = C(n,s) P.V. \int_{\mathrm{R^n}} \frac{u(x)-u(y)}{|x-y|^{n+2s}} dy, \;\; u \in \mathcal{S}\,, s \in (0,1),$$

where P.V. is used in the sense of principal value and

$$C(n,s) = 2^{2s} s \frac{\Gamma\left(\frac{n+2s}{2}\right)}{\Gamma(1-s)} \pi^{-\frac{n}{2}}.$$

Alternative definitions of the fractional Laplacian and their comparison can befound in Kwasnicki M, 2017; Musina R, Nazarov AI, 2014 and Servadei R, Valdinoci E, 2014

To define the fractional Laplacian on a bounded domain Ω we set $u = 0$ in $R^n \setminus \Omega$. Then (see Lischke A, Pang G, Gulian M, Song F, Glusa C, Zheng X, Karniadakis GE, 2020)

$$(-\Delta)^s u(x) = C(n,s) P.V. \int_{\mathrm{R^n}} \frac{u(x)-u(y)}{|x-y|^{n+2s}} dy =$$

$$C(n,s)\left(P.V.\int_{\Omega}\frac{u(x)-u(y)}{|x-y|^{n+2s}}dy+P.V.\int_{\mathrm{R^n}\setminus\Omega}\frac{u(x)}{|x-y|^{n+2s}}dy\right).$$

III) *Variational Settings of the Eigenvalue Problem*

Weighted Dirichlet eigenvalue problem for fractional Laplacian on a bounded domain is given as

$$\begin{cases}(-\Delta)^s u(x)=\dfrac{1}{\lambda}w(x)u(x), & 0\le w\in L_r(\Omega), \quad s\in(0,1), \quad r\in[1,\infty),\\ & u=0 \ in \ R^n\setminus\Omega.\end{cases} \tag{2.1}$$

We note that writing the eigenvalues as $\frac{1}{\lambda}$ in (2.1) is related to the use of the spectral properties of compact operators. We can define the solution to the problem (2.1) in two different ways. First, a solution to the problem (2.1) is understood as a weak solution, the definition of which is given below.

Definition 2.1. We say that $0\neq u\in\widetilde{H}_0^s(\Omega)$ is an eigenfunction of problem (2.1) associated to the eigenvalue λ if for every $v\in\widetilde{H}_0^s(\Omega)$

$$\int_{\mathrm{R^n}}\int_{\mathrm{R^n}}\frac{u(x)-u(y)}{|x-y|^{n+2s}}\big(v(x)-v(y)\big)dx\,dx=\frac{1}{\lambda}\int_{\Omega}w(x)u(x)v(x)dx$$

In the second way, the problem (2.1) is reduced to the eigenvalue problem for a compact operator and this definition is accepted in this paper. From this definition it is also better understood why we write $\frac{1}{\lambda}$ in the eigenvalue problem. A brief summary of this definition is as follows. We define the following quadratic forms

$$F_\rho[u,u]:=\|u\|^2_{L_2(\rho)(\Omega)}=\int_{\Omega}|u|^2d\rho$$

and

$$G[u,u]:=\|u\|^2_{\widetilde{H_0}^s(\Omega)}=\int_{\mathrm{R^n}}\int_{\mathrm{R^n}}\frac{|u(x)-u(y)|^2}{|x-y|^{n+2s}}dx\,dy\,,\qquad u\in\widetilde{H_0}^s(\Omega),$$

where the measure ρ is defined as $\rho(A)=\int_{\mathrm{A}}w(x)dx$ for a Lebesgue measurable set A. By the condition $w(x)\in L_r(\Omega)$. Regarding the number r, we assume that the following condition is satisfied.

$$\beta:=\frac{1}{2}\left(1-\frac{1}{r}\right)-\left(\frac{1}{2}-sn^{-1}\right)>0. \tag{2.2}$$

This condition is related to the application of Sobolev embedding theorems and will be used in the Section 4 to prove an approximation theorem in fractional Sobolev spaces. Under this conditions it follows from the fractional Sobolev embedding theorems that (see Di Nezza E, Palatucci G, Valdinoci E, 2012) $F\rho\ [u,u]$ is a bounded and compact quadratic form in $\widetilde{H_0}^s(\Omega)$. Consequently, there is a compact operator $T_D:\widetilde{H_0}^s(\Omega)\to\widetilde{H_0}^s(\Omega)$ such that

$$F_\rho[u,u]=(T_Du,u)_{\widetilde{H_0}^s(\Omega)}. \tag{2.3}$$

Here we have denoted the operator as T_D because it represents the Dirichlet problem. Now we can give an alternative definition of the weak solution of the eigenvalue problem (2.1) as follows.

Definition 2.2. The eigenvalues of the operator T_D is called the eigenvalues of the Dirichlet problem (2.1).

From now on, when we talk about the eigenvalues of the Dirichlet problem (2.1), we will mean the eigenvalues of the operator T_D defined above. Since T_D is compact, the eigenvalues of this operator consist of discrete eigenvalues of finite multiplicity that converges to zero. We denote the eigenvalues with $\lambda_m^s(D)$

to emphasize their dependence on the parameter s and that they are the eigenvalues of the Dirichlet problem. Thus we have

$$\lambda_1^s(D) \geq \lambda_2^s(D) \geq \cdots .. \geq \lambda_m^s(D) \geq \cdots, \quad and\ \lambda_m^s(D) \to 0\ as\ m \to \infty.$$

By the min-max principle (Birman, MS, Solomyak, MZ, 1980 and Çolakoğlu N, Hasanov M, Uzun BÜ, 2006), it follows from (2.3) and the definition of $G[u,u]$ that

$$\lambda_{m+1}^s(D) = \min_{\substack{codim\, L \leq m \\ L \subset \widetilde{H_0}^s(\Omega)}} \max_{\substack{u \in L \\ 0 \neq u}} \frac{(T_D u, u)_{\widetilde{H_0}^s(\Omega)}}{(u,u)_{\widetilde{H_0}^s(\Omega)}} = \min_{\substack{codim\, L \leq m \\ L \subset \widetilde{H_0}^s(\Omega)}} \max_{\substack{u \in L \\ 0 \neq u}} \frac{F_\rho[u,u]}{G[u,u]}, \qquad (2.4)$$

where $codim\, L := \dim (L^{\perp})$. The variational principle defined in this way plays an important role in the future stages of our study.

ON THE APPROXIMATION METHOD

As stated in the introduction, our main problem is to find the asymptotic behavior of the eigenvalues by using the approximation method. Therefore, giving brief information about the approximation method will make the rest of the article more understandable.

The theory based on the approximation method and its application to the spectral theory of differential operators by using variational techniques was introduced by Birman, MS, Solomyak, MZ, 1980 (see also the references therein). The main idea can be summarized as follows. For $s > 0$ (here the condition $s \in (0,1)$ may not be met) we denote by $Pol(s,n)$ the space of polynomials in R^n of degree less than s. Let $Q^n = [0,1)^n$ be a half open unite cube in R^n and let $\mathcal{P}$ be a partition of Q^n into a finite number of half open cubes Δ.

The number of cubes in the partition $\mathcal{P}$ is denoted by $|\mathcal{P}|$ |. For $s > 0$ we denote by $Pol(s,\mathcal{P})$ the space of piece-wise polynomial functions which reduce to a polynomial of degree less than s on each cube Δ in $\mathcal{P}$. Then $dim\ Pol(s,\mathcal{P}) = |\mathcal{P}|\nu(s,n)$, where $\nu(s,n)$ denotes the dimension of $Pol(s,n)$.

let $P_{s,\Delta}$ be the operator of orthogonal projection defined as

$$P_{s,\Delta}: L_2(\Delta) \to Pol(s,n).$$

Then the operator $P_{s,\Delta}$ is extended on $L_1(\Delta)$ by continuity. By the same way the operator of orthogonal projection $P_{s,\mathcal{P}}$ is defined as

$$P_{s,\mathcal{P}}: L_2(Q^n) \to Pol(s,\mathcal{P}).$$

The operator $P_{s,\Delta}$ is extended on $L_1(Q^n)$ by continuity. Moreover the orthogonal projection $P_{s,\mathcal{P}}$, the second most important concept for the approximation method is the partition function, which is defined below.

Definition 3.1. (see Birman, MS, Solomyak, MZ,1980, Page 6) Let J be a nonnegative and superadditive function of half open cubes in Q^n,i.e.,

$$\Delta = \cup_{j=1}^k \Delta_j, \left(\Delta_i \cap \Delta_j = \emptyset\ for\ i \neq j\right) \Rightarrow \sum_{j=1}^k J\left(\Delta_j\right) \leq J(\Delta).$$

Then for a number $a > 0$ the function

$$G_a(J,\mathcal{P}) = \max_{\Delta \in \mathcal{P}} |\Delta|^a J(\Delta)$$

is called a partition function, where $|\Delta| := vol(\Delta)$. Clearly, p-th power of the Gagliardo seminorm

$$J(\Delta) = \int_{\Delta}\int_{\Delta} \frac{|u(x) - u(y)|^p}{|x-y|^{n+sp}} dx\, dy$$

is a superadditive function, which will be used in this paper to define a partition function.

A property of the partition function $G_a(J, \mathcal{P})$ given in the following theorem is among the main stages of the approximation method and plays an important role in finding the upper bounds of eigenvalues.

Theorem 3.1. (see Birman, MS, Solomyak, MZ,1980, Theorem 5.1) Let J be a nonnegative and superadditive function of half open cubes in Q^n and $J(Q^n) \leq 1$. Then for any natural number m there exists a partition $\mathcal{P}$ of Q^n such that $|\mathcal{P}| \leq m$ and

$$G_a(J, \mathcal{P}) \leq C(n, a) m^{-(a+1)}, \quad a > 0.$$

It is very important to note that Theorem 3.1 in general can not be extended to the case $a = 0$. In the case of $a = 0$ a method based on using a covering of a cube instead of partition was developed. Let H be covering of a cube Q by open cubes in R^n. The number of cubes in H is denoted by $|H|$. Let us assume that the covering H can be divided into subclasses $H_1, H_2, , , , , H_\kappa$ which consist of disjoint cubes. The smallest number κ is called the linkage of the covering H and it is denoted by $\kappa(H)$. Note that, in general linkage is different from the multiplicity of a covering. Generally, Besikovitch covering (B-covering) is used as this type of covering. B-covering has two properties:

- The edges of a cube in H is parallel to the edges of the cube Q,
- Each pont $x \in Q$ is the center of some cube in H.

The analog of Theorem 3.1 in the case of $a = 0$ is as follows.

Theorem 3.2. (see Birman, MS, Solomyak, MZ, 1980, Theorem 2.16 and [21]) Let J be a nonnegative and superadditive function of n-dimensional rectangles in the closed cube $Q \subset R^n$, such that if Δ_t are concentric cubes with the edges of length t, centered at a point in Q then

$$\lim_{t \to 0} J(\Delta_t \cap Q) \to 0.$$

Then for any natural number m there exists a B-covering H satisfying the following conditions:

1) $J(\Delta) \leq m^{-1} J(Q)$,

2) $\kappa(H) \leq \kappa_n$, where κ_n depends only on n,

3) $|H| \leq m\kappa_n$.

Finally to summarize briefly, Theorem 3.1, Theorem 3.2, the variational principle (2.4) for eigenvalues and the upper bounds of the two quantities E_n and e_n, defined below play the main role in finding the asymptotics of the eigenvalues of differential operators. Essentially, these form the main stages of the approximation method. However, we note that en is not used in this article due to the problem.

$$E_m(W^{s,p}(\Omega^n), X) = \inf_{|\mathcal{P}| \leq m} \sup_{u \in SW^{s,p}(Q^n)} \left\| u - P_{s,\mathcal{P}} u \right\|_X, \qquad (3.1)$$

$$e_m(W^{s,p}(\Omega^n), X) = \sup_{u \in SW^{s,p}(Q^n)} \inf_{|\mathcal{P}| \leq m} \left\| u - P_{s,\mathcal{P}} u \right\|_X, \qquad (3.2)$$

where $SW^{s,p}(Q^n) = \{u | u \in W^{s,p}(Q^n),\ \|u\|_{W^{s,p}(Q^n)} \leq 1\}$ and X is chosen among the spaces where $W^{s,p}(Q^n)$ can be embedded. Clearly, $e_m \leq E_m$.

APPROXIMATION OF FUNCTIONS FROM THE CLASS $W^{s,p}(Q^n)$, $0 < s < 1$ WITH PIECEWISE CONSTANT FUNCTIONS

Our main objective in this section is to develop the approximation method to be applied to fractional differential operators. For this reason, our primary goal is to find the upper bound of $\left\|u - P_{s,p}u\right\|_X$ in the formula (3.1).

Lemma 4.1. Let Δ be an n dimensional cube, $s \in (0,1)$ and $q \in [1,\infty]$ satisfies

$$q^{-1} \geq p^{-1} - sn^{-1} \ (if\ ps = n \ \ then\ q^{-1} > p^{-1} - sn^{-1}\ ). \qquad (4.1)$$

Then for any function $u \in W^{s,p}(\Delta)$

$$\left\|u - P_{s,\Delta}u\right\|_{L_q(\Delta)} \leq C|\Delta|^{q^{-1}-p^{-1}+sn^{-1}}[u]_{W^{s,p}(\Delta)}, \qquad (4.2)$$

where $C = C(n,s,p,q)$ and $|\Delta|$ denotes the volume of Δ in R^n.

Proof. Let $\Delta = Q^n$. Since $s \in (0,1)$ then by the definition $P_{s,Q^n}u = C_u$, where C_u is a constant. The following norms (4.3) and (4.4) are equivalent.

$$\|u\|_{W^{s,p}(Q^n)} = \left(\int_{Q^n}|u|^p dx + \int_{Q^n}\int_{\mathrm{Q^n}}\frac{|u(x)-u(y)|^p}{|x-y|^{n+sp}}dx\,dx\right)^{\frac{1}{p}}, \qquad (4.3)$$

$$\|u\|^1_{W^{s,p}(Q^n)} = \left(\int_{Q^n}\left|P_{s,Q^n}u\right|^p dx\right)^{\frac{1}{p}} + \left(\int_{Q^n}\int_{\mathrm{Q^n}}\frac{|u(x)-u(y)|^p}{|x-y|^{n+sp}}dx\,dx\right)^{\frac{1}{p}}. \qquad (4.4)$$

Clearly,

$$\left\|u - P_{s,Q^n}u\right\|^1_{W^{s,p}(Q^n)} = \left(\int_{Q^n}\int_{\mathrm{Q^n}}\frac{|u(x)-u(y)|^p}{|x-y|^{n+sp}}dx\,dx\right)^{\frac{1}{p}}.$$

Then by the Sobolev embedding theorem

$$\left\|u - P_{s,Q^n}u\right\|_{L_q(\mathrm{Q^n})} \leq \left\|u - P_{s,Q^n}u\right\|^1_{W^{s,p}(Q^n)}.$$

Consequently,

$$\left\|u - P_{s,Q^n}u\right\|_{L_q(\mathrm{Q^n})} \leq C\left(\int_{Q^n}\int_{\mathrm{Q^n}}\frac{|u(x)-u(y)|^p}{|x-y|^{n+sp}}dx\,dx\right)^{\frac{1}{p}}. \qquad (4.5)$$

Thus, in the case of $\Delta = Q^n$ the inequality (4.2) holds, since $|Qn| = 1$.Let us prove (4.2) for any cube Δ. By (4.5) for any $v \in W^{s,p}(Q^n)$ we have

$$\left\|v - P_{s,Q^n}v\right\|_{L_q(\mathrm{Q^n})} \leq C\left(\int_{Q^n}\int_{\mathrm{Q^n}}\frac{|v(x)-v(y)|^p}{|x-y|^{n+sp}}dx\,dx\right)^{\frac{1}{p}}. \qquad (4.6)$$

We can write $\Delta = x_0 + hQ^n$. Let $v(x) := u(x_0 + hx), x \in Q^n$. Then by (4.6) we get

$$\left(\int_{Q^n}\left|u(x_0+hx) - P_{s,Q^n}u(x_0+hx)\right|^q dx\right)^{\frac{1}{q}} \leq$$

$$C\left(\int_{Q^n}\int_{\mathrm{Q^n}}\frac{|u(x_0+hx)-u(x_0+hy)|^p}{|x-y|^{n+sp}}dx\,dx\right)^{\frac{1}{p}}.$$

We set $x_0 + hx = s$ and $x_0 + hy = t$. Then $s, t \in \Delta$ and $ds = h^n dx$, $dt = h^n dy$. Thus $dx = h^{-n} ds, dy = h^{-n} dt$ and $x - y = \frac{s-t}{h}$. If we write these in (4.7) then we obtain that

$$\left(\int_\Delta \left|u(s) - P_{s,\Delta}u(s)\right|^q h^{-n} ds\right)^{\frac{1}{q}} \le C\left(\int_\Delta \int_\Delta \frac{|u(s)-u(t)|^p}{h^{-(n+sp)}|s-t|^{n+sp}} h^{-2n} ds\, dt\right)^{\frac{1}{p}}.$$

Hence,

$$h^{\frac{-n}{q}}\left(\int_\Delta \left|u(s) - P_{s,\Delta}u(s)\right|^q ds\right)^{\frac{1}{q}} \le h^{\frac{-n+ps}{p}} C\left(\int_\Delta \int_\Delta \frac{|u(s)-u(t)|^p}{|s-t|^{n+sp}} ds\, dt\right)^{\frac{1}{p}}.$$

From this inequality, by using $h = |\Delta|^{\frac{1}{n}}$ we obtain that

$$\left(\int_\Delta \left|u(s) - P_{s,\Delta}u(s)\right|^q ds\right)^{\frac{1}{q}} \le C\Delta^{q^{-1}-p^{-1}+sn^{-1}}\left(\int_\Delta \int_\Delta \frac{|u(s)-u(t)|^p}{|s-t|^{n+sp}} ds\, dt\right)^{\frac{1}{p}}$$

This is the inequality (4.2).

○

Next we find the upper bound of the following quantitative

$$E_m(W^{s,p}(\Omega^n), X) = \inf_{|\mathcal{P}| \le m} \sup_{u \in SW^{s,p}(Q^n)} \left\|u - P_{s,\mathcal{P}}u\right\|_X .$$

for a suitable normed space X. For the eigenvalue problem (2.1) we have $X = Lq(\rho)(\Omega) = \{u | \int_\Omega |u|^q d\rho < \infty\}$, where the measure ρ is absolutely continuous with respect to the Lebesgue measure dx and $\frac{d\rho}{dx} \in L_r(\Omega), r \ge 1$. In this case the norm of the measure ρ is defined by $\| \rho \|_r = \left(\int_\Omega \left(\frac{d\rho}{dx}\right)^r dx\right)^{\frac{1}{r}}$. The class of such measures is denoted by $M_r(\Omega)$. We also use the notation $SMr(\Omega) := \{\rho \in M_r(\Omega) | \rho(\Omega) \le 1\}$. In particular, all results obtained in this section are also valid for the Lebesgue measure. In this case $\frac{d\rho}{dx} = 1$. A theorem about the upper bound of $E_m(W^{s,p}(\Omega^n), X)$ is given below.

Theorem 4.1. Let $\rho \in SM_r(Q^n)$, $r \in [1, \infty)$, $q \in [1, \infty)$ and

$$\beta := q^{-1}\left(1 - \frac{1}{r}\right) - (p^{-1} - sn^{-1}) > 0. \qquad (4.8)$$

Then for any natural number m there exists a partition $\mathcal{P}$ of Q^n such that $|\mathcal{P}| \le m$ and for all $u \in W^{s,p}(\Omega^n)$

$$\begin{aligned} \left\|u - P_{s,\mathrm{Q}^n}u\right\|_{L_q(Q^n)} &\le Cm^{-\frac{s}{n}}[u]_{W^{s,p}(Q^n)}, \quad if\ q \le p \\ \left\|u - P_{s,\mathrm{Q}^n}u\right\|_{L_q(Q^n)} &\le Cm^{-\frac{s}{n}+\left(\frac{1}{p}-\frac{1}{q}\right)}[u]_{W^{s,p}(Q^n)}, \quad if\ q > p \end{aligned} \qquad (4.9)$$

where $C = C(s, p, n, q, r)$.

Proof. Let $q \le p$. We first consider the case $ps > n$. Since $ps > n$ then $p^{-1} - sn^{-1} < 0$. Consequently, $q = \infty$ satisfies the condition (4.1) and by Lemma 4.1 we

$$\left\|u - P_{s,\Delta}u\right\|_{L_\infty(\Delta)} \le C|\Delta|^{sn^{-1}-p^{-1}}[u]_{W^{s,p}(\Delta)}. \qquad (4.10)$$

Using (4.10) and Höolder's inequality we can write the following inequalities.

$$\int_{Q^n}\left|u-P_{s,\mathcal{P}}u\right|^q d\rho=\sum_{\Delta\in\mathcal{P}}\int_{\Delta}\left|u-P_{s,\Delta}u\right|^q d\rho\le\sum_{\Delta\in\mathcal{P}}\left\|u-P_{s,\Delta}u\right\|^q_{L_\infty(\Delta)}\rho(\Delta)\le$$

$$C\sum_{\Delta\in\mathcal{P}}|\Delta|^{q(sn^{-1}-p^{-1})}|\rho(\Delta)[u]^q_{W^{s,p}(\Delta)}\le CG_a(J,\mathcal{P})\sum_{\Delta\in\mathcal{P}}[u]^q_{W^{s,p}(\Delta)}\le$$

$$CG_a(J,\mathcal{P})|\mathcal{P}|^{\left(1-\frac{q}{p}\right)}\left(\sum_{\Delta\in\mathcal{P}}[u]^p_{W^{s,p}(\Delta)}\right)^{\frac{q}{p}}=CG_a(J,\mathcal{P})|\mathcal{P}|^{\left(1-\frac{q}{p}\right)}[u]^p_{W^{s,p}(\mathrm{Q^n})},$$

where

$$G_a(J,\mathcal{P})=\max_{\Delta\in\mathcal{P}}|\Delta|^a J(\Delta)\ \ and\ \ a=q(sn^{-1}-p^{-1}).$$

Thus we have obtained that

$$\int_{Q^n}\left|u-P_{s,\mathcal{P}}u\right|^q d\rho\le CG_a(J,\mathcal{P})|\mathcal{P}|^{\left(1-\frac{q}{p}\right)}[u]^p_{W^{s,p}(\mathrm{Q^n})}. \tag{4.11}$$

By Theorem 3.1 for any natural number m there exists a partition $\mathcal{P}$ of Q^n such that $|\mathcal{P}|\le m$ and

$$G_a(J,\mathcal{P})\le C(n,a)m^{-(a+1)},\quad a>0.$$

If we combine this result with (4.11), then we get

$$\int_{Q^n}\left|u-P_{s,\mathcal{P}}u\right|^q d\rho\le Cm^{-\left(q(sn^{-1}-p^{-1})+1\right)+1-\frac{q}{p}}[u]^p_{W^{s,p}(\mathrm{Q^n})}=C|m|^{-q\frac{s}{n}}[u]^q_{W^{s,p}(Q^n)}.$$

Here we have used the same notation for the constant C. The important point here is that $C=C(s,p,n,q,r)$ and it is independent of u and ρ. Finally, If we take the qth power of both sides of the last inequality, we get the inequality (4.9).

Now we consider the case $ps\le n$. Let $1-\frac{1}{r}=\frac{1}{R}$. Then we can write the condition (4.8) in the form

$$\beta:=(qR)^{-1}-(p^{-1}-sn^{-1})>0. \tag{4.12}$$

We have

$$\int_{Q^n}\left|u-P_{s,\mathcal{P}}u\right|^q d\rho=\sum_{\Delta\in\mathcal{P}}\int_{\Delta}\left|u-P_{s,\Delta}u\right|^q d\rho=\sum_{\Delta\in\mathcal{P}}\int_{\Delta}\left|u-P_{s,\Delta}u\right|^q\left(\frac{d\rho}{dx}\right)dx.$$

Then by using Hölder's inequality we get

$$\int_{Q^n}\left|u-P_{s,\mathcal{P}}u\right|^q d\rho\le\sum_{\Delta\in\mathcal{P}}\left[\int_{\Delta}\left|u-P_{s,\Delta}u\right|^{qR}dx\right]^{\frac{1}{R}}dx\left[\int_{\Delta}\left(\frac{d\rho}{dx}\right)^r dx\right]^{\frac{1}{r}}. \tag{4.13}$$

It follows from (4.12) and Lemma 4.1 that

$$\left\|u-P_{s,\Delta}u\right\|_{L_{qR}(\Delta)}\le C|\Delta|^\beta[u]_{W^{s,p}(\Delta)}. \tag{4.14}$$

Then the following inequality is obtained from (4.13) and (4.14).

$$\int_{Q^n}\left|u-P_{s,\mathcal{P}}u\right|^q d\rho\le\sum_{\Delta\in\mathcal{P}}|\Delta|^{q\beta}\left[\int_{\Delta}\left(\frac{d\rho}{dx}\right)^r dx\right]^{\frac{1}{r}}[u]^q_{W^{s,p}(\Delta)}. \tag{4.15}$$

Let us define

$$G_a(J,\mathcal{P}) = \max_{\Delta\in\mathcal{P}} |\Delta|^a J(\Delta), \quad where\ J(\Delta) = \int_\Delta \left(\frac{d\rho}{dx}\right)^r dx$$

and $a = qr\beta$. Then it follows from (4.15) that

$$\int_{Q^n} \left|u - P_{s,\mathcal{P}}u\right|^q d\rho \le C[G_a(J,\mathcal{P})]^{\frac{1}{r}} \sum_{\Delta\in\mathcal{P}} [u]_{W^{s,p}(\Delta)}.$$

Taking into account the condition $q \le p$ and using Hölder's inequality we obtain from this inequality that

$$\int_{Q^n} \left|u - P_{s,\mathcal{P}}u\right|^q d\rho \le C[G_a(J,\mathcal{P})]^{\frac{1}{r}} |\mathcal{P}|^{\left(1-\frac{q}{p}\right)} [u]^q_{W^{s,p}(Q^n)}. \qquad (4.16)$$

By Theorem 3.1 for any natural number m there exists a partition $\mathcal{P}$ of Q^n such that $|\mathcal{P}| \le m$ and

$$G_a(J,\mathcal{P}) \le C(n,a)m^{-(a+1)}, \quad a > 0.$$

If we combine this result with (4.16), then we get

$$\int_{Q^n} \left|u - P_{s,\mathcal{P}}u\right|^q d\rho \le Cm^{\frac{-1}{r}(a+1)+1-\frac{q}{p}} [u]^q_{W^{s,p}(Q^n)}. \qquad (4.17)$$

Since

$$-\frac{1}{r}(a+1) + 1 - \frac{q}{p} = -\frac{1}{r}(qr\beta+1) + 1 - \frac{q}{p} = -qsn^{-1}$$

(4.17) yields

$$\int_{Q^n} \left|u - P_{s,\mathcal{P}}u\right|^q d\rho \le Cm^{-qsn^{-1}} [u]^q_{W^{s,p}(Q^n)}.$$

Taking the $\frac{1}{q}$ th power of both sides of the last inequality, we get the inequality (4.9). Finally we note that the case $q > p$ is reduced to the case $p = q$. However, in this case $m^{-\frac{s}{n}}$ is replaced by

$m^{-\frac{s}{n}+\left(\frac{1}{p}-\frac{1}{q}\right)}$ (see Birman, MS, Solomyak, MZ, 1980, Remark 2.5).

Corollary 4.1. If Q is an arbitrary cube and $\rho \in M_r(Q)$ then, under the same conditions we get

$$\begin{aligned} \left\|u - P_{s,\mathrm{Q}^n}u\right\|_{L_q(\rho)(Q)} &\le Cm^{-\frac{s}{n}}[u]_{W^{s,p}(Q)}\|\rho\|_r^{\frac{1}{q}}|Q|^\beta, \quad if\ q \le p \\ \left\|u - P_{s,\mathrm{Q}^n}u\right\|_{L_{q(\rho)}(Q)} &\le Cm^{-\frac{s}{n}+\left(\frac{1}{p}-\frac{1}{q}\right)}[u]_{W^{s,p}(Q)}\|\rho\|_r^{\frac{1}{q}}|Q|^\beta, \quad if\ q > p \end{aligned} \qquad (4.18)$$

The proof of inequality (4.18) is obtained from (4.9) by using the transformation $Q = x_0 + hQ^n$, as in Lemma 4.1.

EIGENVALUE BOUNDS FOR FRACTIONAL LAPLACIAN IN WEIGHTED SPACES

After making the necessary preliminary preparations in the previous sections, let us consider again the problem (2.1). Our main goal is to find the upper bound of the eigenvalues $\lambda_m^s(D)$, defined by (2.4), by using the approximation method. The method to be applied here is largely based on Theorem 4.1 and the variational principle that we defined with the formula (2.4). Therefore, we assume that the following condition, which is necessary for Theorem 4.1, is satisfied.

$$\beta := \frac{1}{2}\left(1-\frac{1}{r}\right)-\left(\frac{1}{2}-sn^{-1}\right) > 0 \Leftrightarrow r > \frac{n}{2s}.$$

Considering that $r \in [1,\infty)$, this condition can be written in the following form.

$$r > 1 \; if \; 2s > n; \;\; r > \frac{n}{2s} \;\; if \; 2s \le n. \qquad (5.1)$$

Below is the main result on the upper bound of the eigenvalues $\lambda_m^s(D)$.

Theorem 5.1. Let Ω be a bounded domain with Lipschitz boundary and the condition (5.1) is satisfied. Then,

$$\lambda_m^s(D) \le C\, m^{-\frac{2s}{n}} diam(\Omega)^{2s-\frac{n}{r}} \left[\int_\Omega w^r(x)dx\right]^{\frac{1}{r}}, \qquad (5.2)$$

where the constant C depends only on n, s, r.

Proof. We define the measure $\rho(A) = \int_A w(x)dx$ for a Lebesgue measurable set A. First we assume that $\Omega = Q^n$ and $\rho \in SM_r(Q^n)$. Let us take $p = q = 2$ in Theorem 4.1. Then by this theorem, for any natural number m there exists a partition $\mathcal{P}$ of Q^n such that $|\mathcal{P}| \le m$ and for all $u \in H^s(Q^n)$.

$$\left\|u - P_{s,\mathrm{Q^n}}u\right\|_{L_q(\rho)(Q^n)} \le C_1 m^{-\frac{s}{n}} [u]_{H^s(Q^n)}, \quad if \; q \le p,$$

where $C_1 = C_1(s,n,r)$. We have $[u]_{H^s(Q^n)} \le \|u\|^2_{\widetilde{H_0}^s(\Omega)}$, since

$$\|u\|^2_{\widetilde{H_0}^s(\mathrm{Q^n})} = [u]^2_{H^s(Q^n)} + 2\int_{Q^n}\int_{\mathrm{R^n}\setminus\mathrm{Q^n}} \frac{|u(x)|^p}{|x-y|^{n+2s}}\,dx\,dy\,, u \in C_0^\infty(Q^n).$$

Therefore, for any natural number m there exists a partition $\mathcal{P}$ of Q^n such that $|\mathcal{P}| \le m$ and for all $u \in H_0^s(Q^n)$.

$$\left\|u - P_{s,\mathrm{Q^n}}u\right\|_{L_2(\rho)(Q^n)} \le C_1 m^{-\frac{s}{n}} [u]_{\widetilde{H_0}^s(\mathrm{Q^n})}. \qquad (5.3)$$

By the definition

$$P_{s,\mathcal{P}}: L_2(Q^n) \to Pol(s,\mathcal{P}).$$

Hence, $dim\,(range\,(P_{s,\mathcal{P}})) \le m$ and consequently $codim\,Ker\,(P_{s,\mathcal{P}}) \le m$. It follows from (5.3) that

$$\|u\|_{L_2(Q^n)} \le C_1 m^{-\frac{s}{n}} [u]_{\widetilde{H_0}^s(\mathrm{Q^n})} \quad for\, u \in Ker\,(P_{s,\mathcal{P}}) \qquad (5.4)$$

If we set $L_0 = Ker\,(P_{s,\mathcal{P}})$, then $codim\,L_0 \le m$ and by (5.4) we obtain that

$$\max_{\substack{u\in L\\ 0\neq u}} \frac{\|u\|^2_{L_2(\rho)(Q^n)}}{\|u\|^2_{\widetilde{H_0}^s(\mathrm{Q^n})}} \le C_1^2\, m^{-\frac{2s}{n}} \qquad (5.5)$$

Then by using the variational principle (2.4) and (5.5) we get

$$\lambda_{m+1}^s(D,Q^n) = \min_{\substack{codim\,L\le m\\ L\subset \widetilde{H_0}^s(\Omega)}} \max_{\substack{u\in L\\ 0\neq u}} \frac{\|u\|^2_{L_2(\rho)(Q^n)}}{\|u\|^2_{\widetilde{H_0}^s(\mathrm{Q^n})}} \le C_1^2\, m^{-\frac{2s}{n}}. \qquad (5.6)$$

We include the first eigenvalue in this inequality by using the embedding theorem. Finally, it follows from (5.6) that

$$\lambda_m^s(D, Q^n) \le C_1^2 (m-1)^{-\frac{2s}{n}} = C_1^2 m^{-\frac{2s}{n}} \left(\frac{m-1}{m}\right)^{-\frac{2s}{n}} \le C m^{-\frac{2s}{n}}. \quad (5.7)$$

Therefore, we have proven that inequality (5.2) is satisfied in the cases $\Omega = Q^n$and $\rho \in SM_r(Q^n)$.

Note that, we use the notation $\lambda_{m+1}^s(D)$ for the eigenvalues of the problem defined in Ω. To denote the eigenvalues of problems defined in other regions, we will use a notation that includes this region, as we did above.

Now, let $\Omega \subset Q^n$. We extend the measure ρ from Ω to Q^n by the following formula

$$\rho_1(A) \coloneqq \rho(A \cap \Omega), \qquad A \in \sigma_B(Q^n),$$

Where $\sigma_B(Q^n)$ denotes the algebra of Borel sets in Q^n. Using the formula (2.3), we obtain the inequality $T_D(\Omega, \rho) \le T_D(Q^n, \rho_1)$ for operators corresponding to eigenvalue problems defined in regions Ω and Qn, respectively. Therefore, by the comparison principle we get from (5.7) that

$$\lambda_m^s(D, \Omega, \rho) \le \lambda_m^s(D, \Omega, \rho_1) \le C m^{-\frac{2s}{n}}.$$

Finally, if we remove condition $\rho \in SM_r(Q^n)$ and use the transformation $Q = x_0 + hQ^n$, as in Lemma 4.1, to move from the unit cube to any cube, we find inequality (5.2).

DISCUSSION AND CONCLUSION

Our main objective in this paper was to apply the approximation method to study the asymptotic behavior of the eigenvalues of the fractional p-Laplace operator. The main result is given in Theorem 5.1. The proof of this theorem is based on Theorem 4.1 and Theorem 4.2. However, we have used Theorem 4.1 and Theorem 4.2 only for the case $p = q = 2$, because the space of the problem is a Hilbert space. A similar technique can be used for the case (p, p) to find the bounds of the variational eigenvalues of the fractional p-Laplacian (see Brasco L, Lindgren E, Parini E, 2014;. Brasco L, Parini E, 2016; Brasco L, Parini E, Squassina M, 2015). Such problems have attracted the attention of many authors (see Friedlander L, 1989; Iannizzotto A, Squassina M, 2014 and references therein). Since this is a distinct and extensive topic, we think it is a separate research topic.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

1) A theorem on the approximation of functions from the class $W^{s,p}(Q^n), 0 < s < 1$ with piecewise constant functions is proved (see Theorem 4.1),

2) Based on Theorem 4.1, the following result was found for the upper bound of the eigenvalues $\lambda_m^s(D)$ of the problem (1.1) (see Theorem 5.1).

Demand Forecast in E-Commerce Purchases Made in Small Businesses: İzmir Province Example (1164)

Özgül Vupa Çilengiroğlu[1*], Büşra Çolak[1], Deniz Başşahin[1], Derya Porsuk[1], Mehmet Safa Aktaş[1]

[1] Dokuz Eylül University, Faculty of Sciences, Statistics, 35390, İzmir, Türkiye

*Corresponding author e-mail: ozgul.vupa@deu.edu.tr

Abstract

Determining customer profiles and demands in e-commerce sales is critically important for small businesses, just as it is for all enterprises. For small businesses, the primary condition for making a profit—and even for surviving under market conditions—is to know the customer well and to forecast customer demand accurately. Therefore, this study aims to predict customer profiles and demands using time series methods based on monthly and weekly e-commerce sales data from 2021 to 2025, collected from a small business located in İzmir, one of Türkiye's largest cities. Various time series analyses were conducted to identify trends, seasonality, cyclicality, stationarity, and randomness in the time series data. Time series plots were used to detect trends and seasonality; ACF and PACF plots and the Dickey-Fuller test were used to assess stationarity; and decomposition, exponential smoothing, ARIMA, and SARIMA methods were employed for modeling. At the end of the study, the ARIMA(2,0,2)(0,1,1) model was identified as the most suitable model for monthly data, which exhibited both trend and seasonality. In contrast, the ARIMA(1,1,1) model was found most appropriate for weekly data, which exhibited only a trend. Additionally, it was determined that gender-based sales strategies are important for small businesses and that product returns have shown an increasing trend in recent years.

Keywords: *E-Commerce, Small Business, Time Series*

INTRODUCTION

The developments in information and communication technologies worldwide have led to significant transformations not only in core or applied fields such as health, engineering, and natural sciences, but also in social, humanistic, and administrative areas such as business, finance, economics, and marketing. In particular, the internet has brought about essential alternatives for companies and consumers (users) in these fields (Marangoz, 2019). The internet is a fast, interactive, shared virtual environment that creates an electronic link between individuals, organizations, and societies worldwide (Armağan, 2019). Due to the speed, interactivity, and ease of internet use, the most prominent alternative that has emerged in the social domain is "e-commerce." Especially in recent years, the culture of conducting trade has transformed into e-commerce thanks to the internet. The level of e-commerce volume and the high demand and interest from consumers in online shopping necessitate giving this commercial approach the attention it deserves and require it to be analyzed through various parameters.

Since the late 20th century, with the widespread use of the internet worldwide, a significant increase in shopping through e-commerce, which now replaces traditional commerce, has become evident (Akbar and James, 2014; Ateş, 2017). While 75% of online shoppers were between 16 and 34 in 2015, the age range has dramatically expanded in recent years, starting from the pandemic period (Ülger, 2020, https://worldef.com/events/istanbul-2025/). According to data from the Turkish Statistical Institute (TÜİK), the number of people shopping online and the range of age groups have increased since 2016. Whereas such sales were previously concentrated in sectors like home appliances and clothing, they have notably shifted toward other areas, most strikingly to the food sector, after the pandemic (TÜİK, 2025).

The financial benefits that e-shopping brings to companies are evident. Especially for small and medium-sized enterprises (SMEs), the existence of e-commerce is crucial for being able to compete with large-scale companies today. However, the volume of data in electronic sales has also increased, making it difficult for SMEs to manage. From a sectoral perspective, in recent years, the revenues of businesses in the clothing sector have rapidly fluctuated due to their commercial transactions with customers. Among these, online shopping is a fast, cost-effective, and comfortable method that yields high-value returns for the business. As the volume and dimensionality of this electronic data (e.g., spending, order quantity, customer attributes, delivery details…) grow rapidly, processing and analyzing this information becomes more difficult. The processing and analysis of e-commerce data is highly significant in businesses' data-driven decision-making processes and plays a critical role in gaining a competitive advantage. Moreover, the clothing sector comprises many subfields, such as sportswear, casual, medical, evening, and underwear. In fast-changing fields like the underwear sector, where customer demands are intense, accurate and effective sales forecasting is of great importance for inventory management, marketing strategies, and customer satisfaction (Özseven and Ersöz, 2016; Şahin and Kaya, 2019; Kazancı and Bayarçelik, 2022).

In today's commercial environment, any statistical analysis that small and medium-sized enterprises can perform to increase their revenues, markets, and customer numbers—essentially, their competitiveness—against larger enterprises is of great importance. Based on this motivation, it is anticipated that analyses on order data obtained from the e-commerce platform of a small-scale underwear store located in İzmir, Türkiye's third-largest city, could provide valuable insights. Furthermore, this study is significant in terms of demonstrating—and even statistically modeling—the steps that similar small-scale businesses could take to survive in this market landscape (such as extra bonus points, birthday gifts, discounts, or exclusive product offerings based on gender or shopping experience, stock planning).

LITERATURE REVIEW

This study's main research question is to determine a small-scale enterprise's customer profile using e-commerce data and statistically evaluate future predictions by analyzing customers' electronic shopping behavior. Various studies in the literature use e-commerce data.

İşler et al. (2014), through a survey of 172 students attending public education courses in Isparta, Türkiye, identified the effects of variables such as age (minimum in older individuals, maximum in younger), education (undergraduate), profession (housewife), and marital status (single) on e-shopping attitude scores using descriptive statistics and ANOVA. Ateş (2017), in a survey of 553 undergraduate students from Gazi University's Faculty of Education (Departments of Elementary Education and Computer Education & Instructional Technology, Türkiye), examined perceptions of customer satisfaction, security, and customer relations using descriptive statistics and confirmatory factor analysis. Armağan et al. (2019) surveyed 384 consumers to study virtual store experiences, employing descriptive statistics, Mann-Whitney U, Kruskal-Wallis, and correlation & regression models. They found higher e-commerce activity among young women through regression modeling, although the model performance was low (R^2 = 23%). Aydınhan and Erat (2019), in a study on website trust in e-commerce conducted with 346 consumers, developed a model using descriptive statistics and confirmatory factor analysis, achieving moderate performance (R^2 = 38.3%). Marangoz et al. (2019), in a study evaluating e-commerce purchases of 2,915 consumers in Türkiye, used descriptive statistics and a logit model. They found age (younger), gender (male), education (high school and above), income (high), and household size (small) to be significant predictors of online shopping behavior. In a study on small-scale farmers, Choong et al. (2021) developed price prediction models using regression, ARIMA, and SARIMA methods.

Studies analyzing e-commerce customer behavior have gained attention in recent years by integrating machine learning and statistical methods. Hendriksen et al. (2020) developed feature-based models to predict purchase intentions using identified and anonymous customer data from a European e-commerce

platform. In these models, random forests and artificial neural networks were successful for anonymous data, while the k-nearest neighbors algorithm performed best for identified customer profiles. Roychowdhury et al. (2021) used supervised and semi-supervised learning methods to analyze customer behavior and identified five customer clusters. Among these, “Cosmetics” data was classified with 94% accuracy using the XGBoost model.

Upon reviewing the existing literature, it is evident that while regression and tree-based models are commonly used in the statistical analysis of customers' shopping experiences and purchasing characteristics, time series models related to the quantity of purchases have not been explored in this context. Additionally, prediction models explicitly tailored to customer profiles have not been developed. Most studies have focused on shopping mall customers, and those addressing website trust and consumer perception have relied on basic analytical methods (Ak and Ertaş, 2019; Yıldırım and Demir, 2019; Tayfun and Zorlu, 2021).

MATERIAL AND METHODS

Material

Objective

The main objective of this study is to identify the customer profile of a small-sized lingerie store located in İzmir, one of the major cities in Türkiye, by utilizing data obtained from its e-commerce platform, and to evaluate their purchases using time series methods statistically. Specifically, the aim is to accurately forecast weekly and monthly sales volumes using order data from the e-commerce platform from 2021 to 2025.

Original Contribution

By generating sales forecasts based on e-commerce data, this study will assist the business in understanding the reasons behind periodic increases and decreases in sales. This understanding will support more effective decision-making in areas such as the timing of sales campaigns, analysis of returns, and stock and product management. The findings of this research will offer insights that can be influential in both short-term and long-term strategic planning, specifically for small businesses operating in İzmir.

Methods

The Collection of the Data

Data were obtained from a small-sized company operating in the lingerie sector in İzmir, Türkiye for the time series analysis. The data, collected from the company's e-commerce store from January 2021 to January 2025, is classified as streaming data. Streaming data analysis provides a significant advantage in making accurate and timely decisions, especially in the rapidly changing e-commerce sector.

The collected e-commerce data includes the date of the customer's purchase, the purchase quantity (units, demand quantity, sales quantity), purchase amount, product type, coupon & discount status, gender, delivery location, number of orders, delivery date, and return status. The sales data recorded during the specified period has been organized to help understand the company’s sales trends, examine seasonal effects, and analyze fluctuations in sales.

Statistical Analysis

This study forecasted future sales monthly and weekly using time series methods, based on trends and seasonal effects derived from past sales volume data. First, descriptive statistics of the data were obtained, followed by various time series techniques, including time series plotting, trend analysis, stationarity testing, decomposition, smoothing (moving average), exponential smoothing, Holt-Winters, ARIMA, and SARIMA methods. All analyses were conducted at a significance level of alpha = 0.05, using Minitab 21.1 (2021) and R 4.2.2 software.

Time Series: Data of observation values arranged chronologically is called a "time series." Time series data is a crucial source for information and strategy across various businesses. The methods used in time series analysis examine past data to forecast future trends and events. Time series is widely used in weather forecasting, earthquake prediction, astronomy, finance, econometrics, pattern recognition, signal processing, control engineering, statistics, and demand and sales forecasting.

Components of a Time Series: A time series consists of four main components: Trend: Represents the long-term direction of change in a time series. It may exhibit a positive, negative, or stable tendency. Trend analysis identifies the general direction, independent of seasonal and short-term fluctuations. Linear and exponential models, regression, moving averages, or smoothing methods are typically used. Seasonality: Indicates regular fluctuations that repeat over specific periods (e.g., monthly, yearly). Cyclical Movements: Refers to fluctuations that occur over more extended periods, often associated with economic cycles. Randomness: This component represents unpredictable changes within the time series. It includes random fluctuations in the dataset and is generally treated as an error term during modeling.

Key Concepts in Time Series Analysis: Stationarity: A time series is considered stationary if its statistical properties (mean, variance, covariance) remain constant over time. Stationary data is easier to predict. Several tests, such as the Dickey-Fuller Test (ADF), assess stationarity (H_0: Not stationary). Non-stationary series can typically be transformed into stationary ones through differencing, logarithmic transformation, or removal of trend components. It is crucial to apply appropriate methods to achieve stationarity for accurate analysis. Time series with trends and/or seasonality are considered non-stationary. Differencing: A method to convert a time series into a stationary one by eliminating trends and seasonal components. Smoothing: Aims to reduce fluctuations in the data set to reveal trend and seasonal components more clearly. Methods include moving averages and exponential smoothing. ACF (Autocorrelation Function): Measures the correlation between a time series and its lagged values. It helps detect repeating patterns or cycles in the data. High correlations suggest cyclical structures and are useful for forecasting. PACF (Partial Autocorrelation Function): Measures the correlation between lagged values of a time series while controlling for the influence of other lags. It is used to identify independent relationships in the series and is a key tool for model building (Yılmaz & Çilengiroğlu, 2022).

Time Series Methods: Traditional time series methods that account for the time dimension include "Trend Analysis," "Moving Averages (MA)," "Smoothing (Single/Double Exponential Smoothing, Holt-Winters)," "ARMA," "ARIMA," and "SARIMA." In recent years, a new method called the Ata method has also been introduced into the literature.

RESULTS

Descriptive Statistics

First, descriptive statistics were conducted for variables other than customer demand in the dataset. The variable referred to as "demand" within the study was considered as the request made by the customer, that is, the sales carried out by the small business. In the dataset, which contains a total of 8,380 customer records, an analysis based on the gender variable, considered a significant factor in shopping, revealed that the majority of the customer profiles consisted of female individuals (n = 5,035, 60%). In contrast, the proportion of male customers was 34% (n = 2,843), while customers who did not specify their gender accounted for only 6% (n = 502).

Following gender, annual return counts were examined as another critical factor. It was found that return behavior varied significantly across years. While the number of returns was low in 2021 and 2022, there was a noticeable increase in returns in 2023 and 2024. However, it should be noted that the number of returns observed for 2025 is 10, based on data from only one month (Figure 1).

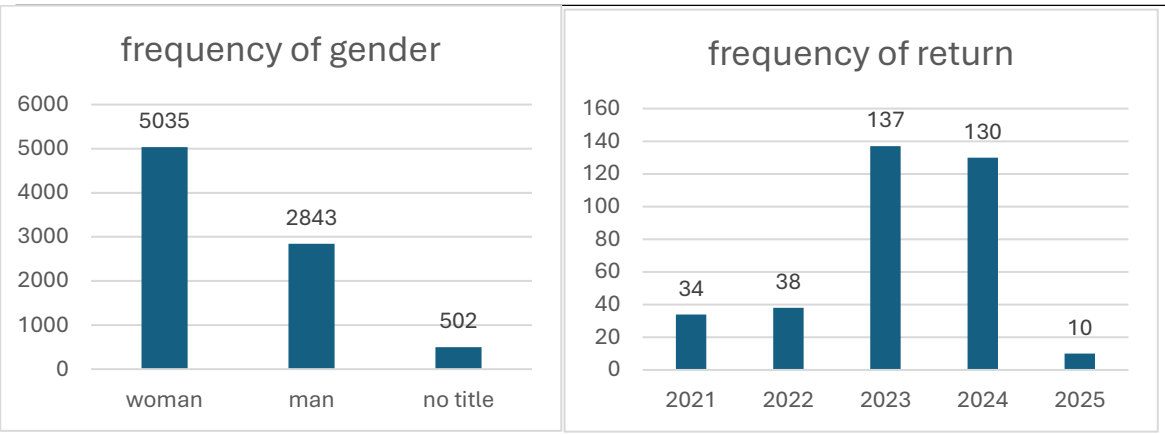

Figure 1. Bar Charts for Gender and Return Variables

The bar charts show that the significantly higher number of female customers indicates that marketing strategies and customer segmentation efforts should be explicitly shaped around female customers. Monitoring annual return data is critical for the sustainability of customer satisfaction. In particular, the high return rates observed in 2023 and 2024 suggest the need for detailed cause-and-effect analyses within operational processes. Such analyses can help develop strategic decisions aimed at reducing future return rates.

For the main variable of the study, the number of orders (demand), descriptive statistics were calculated monthly (n = 49, min = 107, max = 802) and weekly (n = 213, min = 21, max = 182), resulting in 373.84 ± 174.23 and 86 ± 38.38, respectively. According to the normality test, both the monthly and weekly data were found not to follow a normal distribution.

Time Series Graphs and Trend Analysis Results

The time series graph was created to show the changes in demand data over time, monthly and weekly, between January 2021 and January 2025 (Figure 2).

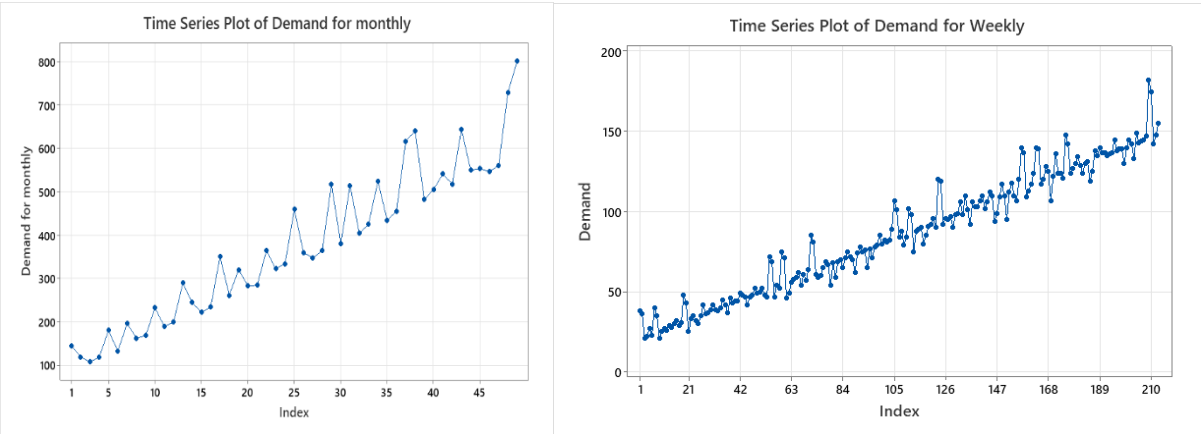

Figure 2. Time Series Graphs of Monthly and Weekly Data

Upon examining the graphs, it is evident that the monthly demand shows a steady upward trend over the years. Notably, clear seasonal fluctuations recur each year, highlighting a seasonal pattern where demand increases and decreases during similar periods annually. The upward momentum that accelerated in 2023 continued throughout 2024, culminating in a noticeable peak at the beginning of 2025. These findings indicate that the demand data contains trend and seasonal components, suggesting that models such as Holt-Winters or SARIMA should be employed.

The weekly time series graph clearly shows a steady upward trend in demand. Despite short-term fluctuations, the overall direction is upward, indicating a strong trend component. A regular seasonal structure is not apparent weekly; however, sudden increases and decreases in certain weeks suggest random effects. This implies that the demand series primarily consists of trend and irregular components. These findings point to the need for trend-focused smoothing or ARIMA-type models.

Trend analysis of monthly and weekly demand data was conducted using linear, curvilinear, growth, and S-curve models. Forecast accuracy was evaluated using MAPE (Mean Absolute Percentage Error), MAD (Mean Absolute Deviation), and MSD (Mean Squared Deviation) criteria (Figure 3).

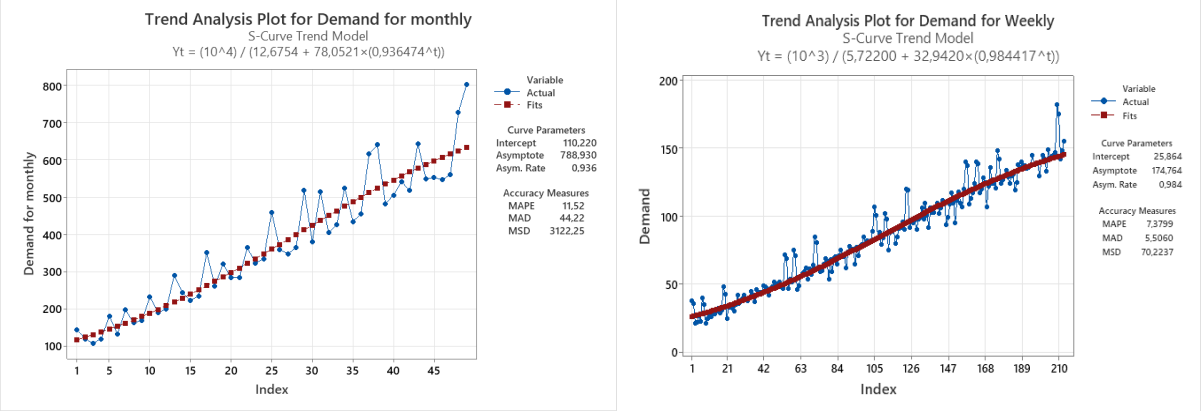

Figure 3. Trend Analysis (S-Curve) Graphs for Monthly and Weekly Data

When comparing the error rates of all models applied to the monthly data, the S-curve model provided the best forecasting performance with the lowest MAPE value of 11.52. This model, which captures a slow initial growth, followed by acceleration, and finally reaching saturation, best represents the actual demand trends. Similarly, in the weekly data, the S-curve model also achieved the lowest MAPE value (7.38).

Stationarity (ACF, PACF)

In time series analysis, a stationary series is essential for reliable forecasting. Stationarity means that the mean, variance, and autocorrelation structure of the series remain constant over time. Non-stationary series often contain trends and seasonality, making them unsuitable for direct analysis without transformation. In this study, the stationarity of the demand data was assessed using the autocorrelation (ACF) graph and the Augmented Dickey-Fuller (ADF) test (H_0: The data is non-stationary) (Figure 4).

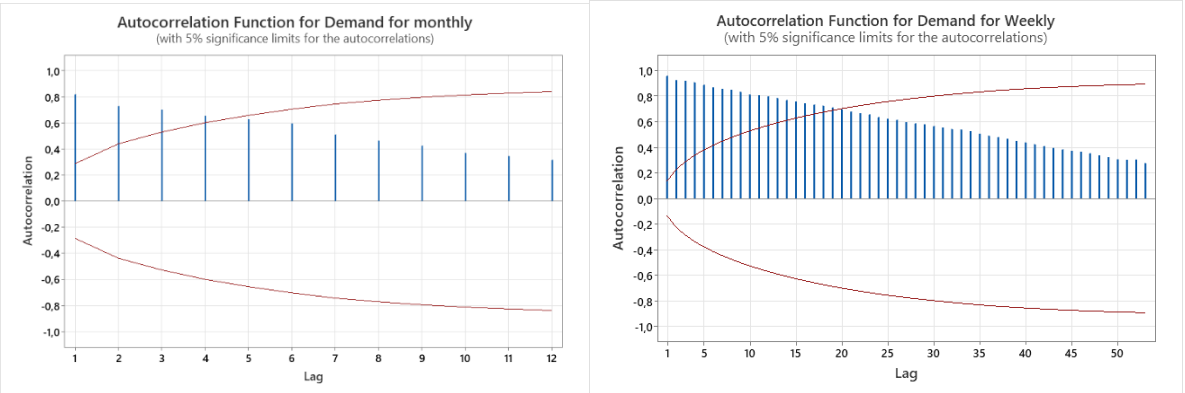

Figure 4. ACF Graphs of Monthly and Weekly Data

In the ACF graph for the monthly data, the presence of values exceeding the significance threshold indicates autocorrelation, which, although decreasing over time, does not disappear completely. Notably, high and

significant autocorrelations in the first 3 lags reveal that the series is non-stationary and contains a trend component. According to the ADF test, the p-value is 0.970 > 0.05, so the null hypothesis (H_0) cannot be rejected. It has been determined with 95% confidence that the dataset is statistically non-stationary.

In the ACF graph for the weekly data, the fact that most ACF values fall outside the significance bounds also indicates autocorrelation. In particular, clear positive autocorrelations are observed within the first 20 lags. This demonstrates that the dataset is non-stationary, contains a trend component, and that the structure changes over time. Similarly, based on the ADF test result (p-value = 0.982 > 0.05), H_0 cannot be rejected, and the series is again non-stationary.

Consequently, differencing must be applied to both series during the modeling phase to achieve stationarity. This will ensure that models requiring stationary data, such as ARIMA, can generate accurate and reliable forecasts.

One of the key tools used to determine the model type in time series analysis is the PACF (Partial Autocorrelation Function) graph. The PACF shows the autocorrelation at a specific lag, adjusted for the effects of intervening lags. This allows the direct impact of each lag on the series to be analyzed (Figure 5).

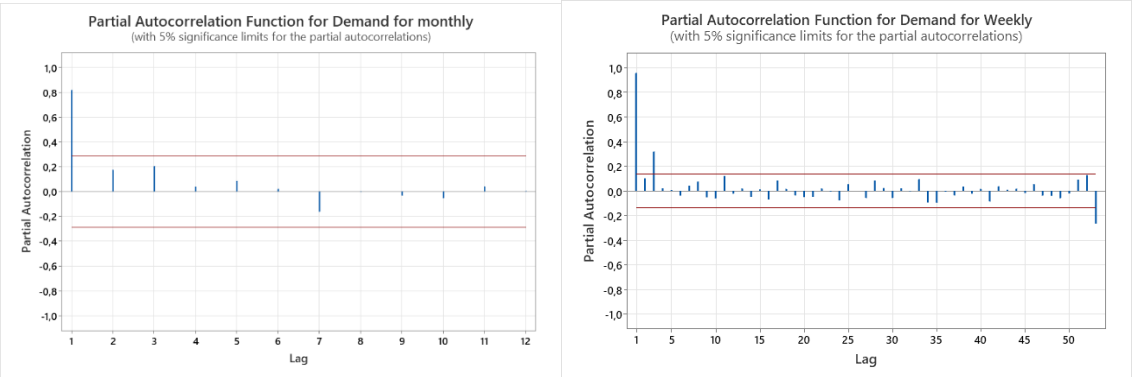

Figure 5. PACF Graphs of Monthly and Weekly Data

According to the PACF graph for the monthly data, there is a significant dependency only at the first lag level. This suggests that a low-order autoregressive model, such as AR(1), may be suitable during the modeling process. For the weekly data, the PACF graph shows a slightly different pattern. There appears to be significance at one or possibly two lag levels, indicating that an autoregressive model like AR may still be appropriate.

Differencing

Differencing is one of the primary methods used to eliminate structural components such as trends and seasonality in time series, and thus achieve stationarity. After differencing, the resulting time series was plotted using ACF and PACF graphs for monthly and weekly data (Figure 6).

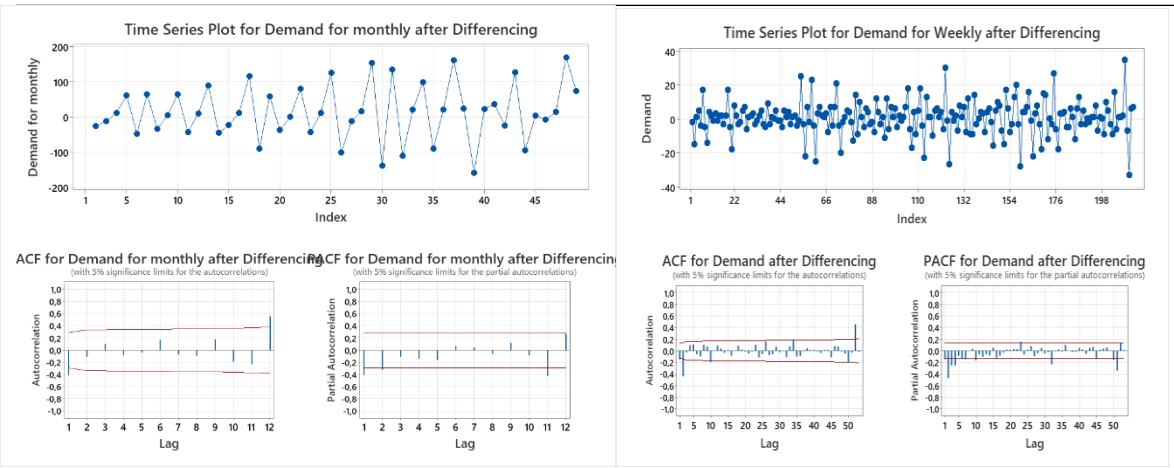

Figure 6. Post-Differencing Graphs of Monthly and Weekly Data

After applying first-order differencing to the monthly and weekly series, the resulting graphs show that the trend effect in the data has largely been eliminated, and the series now fluctuates around the mean. This indicates that the series has achieved a more stationary structure. In the ACF graph after differencing, autocorrelation coefficients decrease rapidly as lag increases and mostly remain within the significance bounds, suggesting a reduction in systematic dependencies in the series. Similarly, in the PACF graph, only weak autocorrelation values are observed at the first one or two lags for the monthly data, and these effects disappear in later lags. In the weekly data, most PACF values fall within the significance bounds, with weak correlations observed at the first two to three and final lags. These findings confirm that the differencing process has effectively brought the data compliant with stationarity conditions.

Decomposition

In time series analysis, decomposition methods are applied to identify the components that make up the series: trend, seasonality, and randomness. Based on the interaction of these components, multiplicative or additive models are selected by comparing metrics such as MAPE. A MAPE value below 10% indicates high forecasting performance and relatively small deviations.

For the monthly data, the demand series yielded a MAPE value of 4.31 for the multiplicative model and 6.95 for the additive model. Therefore, the multiplicative model was selected for further analysis. In this time series analysis, the decomposition of the series into trend, seasonality, and residual components is critical for understanding the nature of the data. In this study, the time series of the demand variable was decomposed into its components using the multiplicative model (Figure 7).

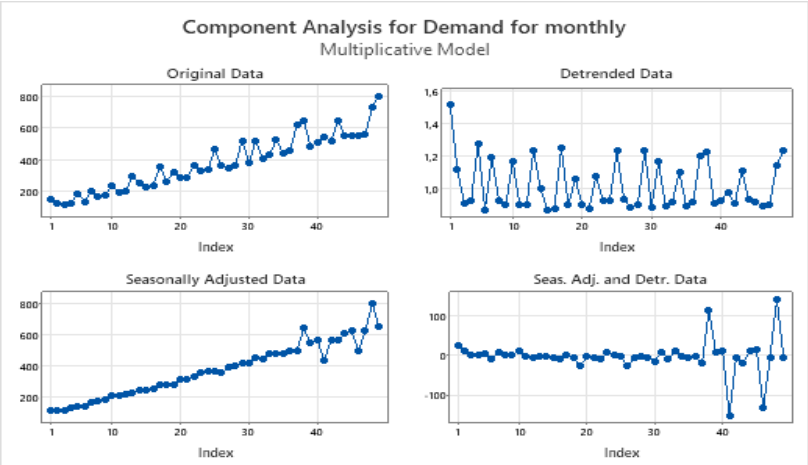

Figure 7. Decomposition (Multiplicative) of Monthly Data

The original data graph shows a rising trend, accompanied by clear and periodic seasonal effects. This indicates that the series is non-stationary, and direct modeling without decomposition would not yield reliable results. When examining the data after removing the trend component, the overall upward trend disappears, but seasonality remains visible. This shows that while the trend component can be separated, the seasonal component strongly persists in the series. The trend becomes more apparent in the seasonally adjusted data, and the fluctuations are less pronounced, providing a solid basis for modeling long-term trends. Finally, in the series where trend and seasonality are removed, only the residual component remains, representing random variations in the data. These decomposition results suggest that future modeling should use models like ARIMA or SARIMA that account for trend and seasonality.

In the seasonal analysis performed to understand the time series data's structural characteristics, the demand variable's multiplicative model shows that index values are high in January and May, indicating a significant increase in demand compared to the average during these months. Conversely, index values fall below 1 in March and September, reflecting lower demand during those months. Additionally, the fact that most residual values are distributed close to zero suggests that the model has a stable structure.

This comprehensive assessment shows that the monthly demand series exhibits a strong and recurring seasonal pattern. Based on these findings, using time series models such as SARIMA that incorporate seasonal components in the forecasting process will improve prediction accuracy and enhance the model's fit to actual data.

Decomposition analysis was also conducted for the weekly data. For the weekly demand series, the MAPE values were 7.33 for the multiplicative model and 7.32 for the additive model. Accordingly, the demand time series was decomposed into its components using the additive model (Figure 8).

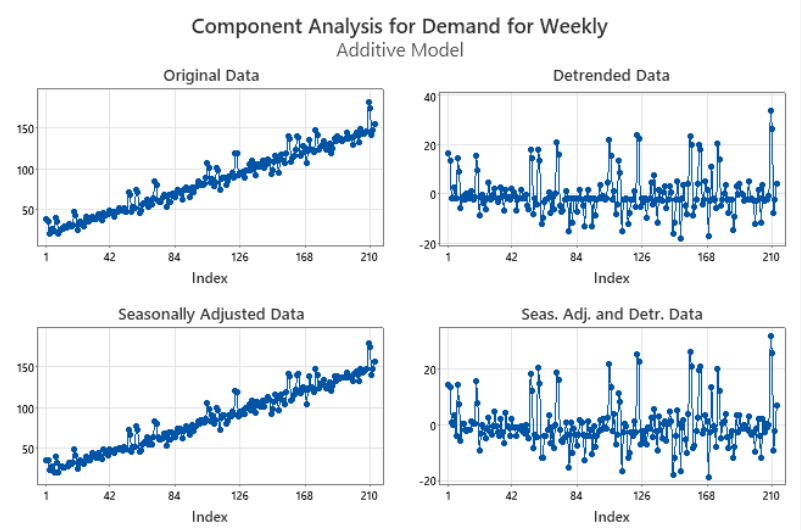

Figure 8. Decomposition (Additive) of Weekly Data

The original data graph shows an upward trend over time every week. This indicates that the series is non-stationary, and modeling without decomposing the components would not yield reliable results. After removing the trend component, the overall upward pattern disappears when the data is examined. This demonstrates that the trend structure has been successfully separated, while the strong presence of seasonality initially presumed is no longer observed. The absence of noticeable differences in the seasonally adjusted data suggests that there is no seasonality present. As a result of this evaluation, it has been revealed that the weekly demand series contains only a trend component. Based on these findings, using ARIMA time series models in the future modeling process will enhance prediction accuracy and improve the model's alignment with the actual data.

Smoothing

This method forecasts by calculating the weighted average of past data. It is more suitable for time series that do not contain trends or seasonality. Nevertheless, models developed using exponential smoothing methods have been evaluated regarding their parameters and MAPE values for monthly and weekly data (Table 1).

Table 1. MAPE values for monthly and weekly data

Table 1. MAPE values for monthly and weekly data

Monthly	Parameters	MAPE
SES	level = 0.48	12.23
DES	level = 0.58, trend = 0.0019	15.50
Holt-Winters (Multiplicative)	level = 0.2, trend = 0.2, seasonal = 0.2	7.74
Holt-Winters (Additive)	level = 0.2, trend = 0.2, seasonal = 0.2	9.12
Weekly		
SES	level = 0.29	8.55
Holt-Winters (Additive)	level = 0.2, trend = 0.2, seasonal = 0.2	9.19

The monthly data was used to construct the model using the Holt-Winters method, which is commonly preferred when both trend and seasonal components are present. The version used is the multiplicative form, which assumes that seasonal effects impact the observed values proportionally. The SES (Simple Exponential Smoothing) model was identified as appropriate for the weekly data (Figure 9).

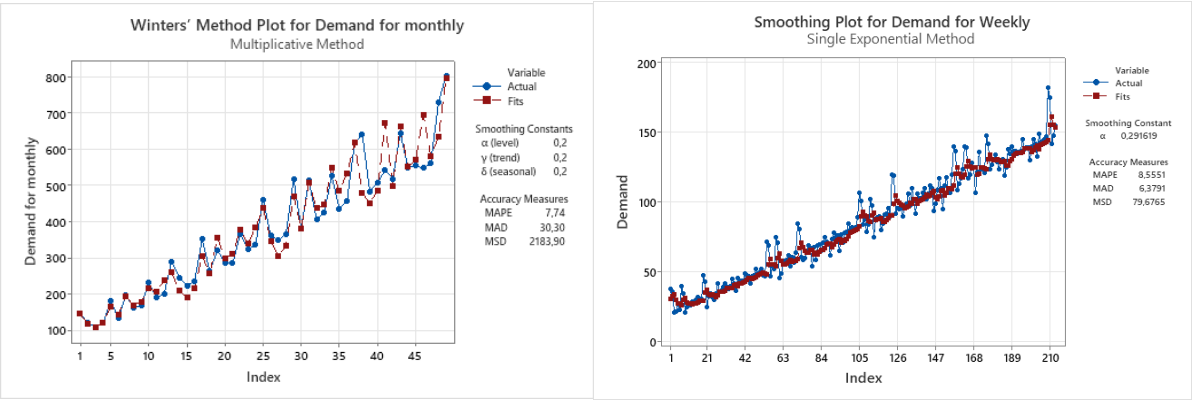

Figure 9. Holt-Winters (Multiplicative) for Monthly Data, SES for Weekly Data

The model successfully captures the monthly data's general trend and seasonal fluctuations. In particular, the model effectively reflects the increasing demand trend and the seasonal variations throughout the year. The situation is slightly different for the weekly data. A level value of 0.29 indicates that the model assigns a moderate weight to past observations and is not overly sensitive to sudden changes. Therefore, the model offers a more stable forecasting structure by smoothly connecting past data to future projections. The graph shows that the adjusted values follow the overall trend and adapt successfully to medium- and long-term movements. However, when sudden jumps occur (e.g., around indices 100 and 190), the model struggles to instantly align with the actual values, leading to increased deviations. Under these conditions, it was determined that smoothing methods are unsuitable for weekly data.

Moving Average

The moving average method, which is commonly used in time series analysis to smooth short-term fluctuations and highlight the underlying trend, was applied using a 4-period window for monthly data and a 7-period window for weekly data (Figure 10).

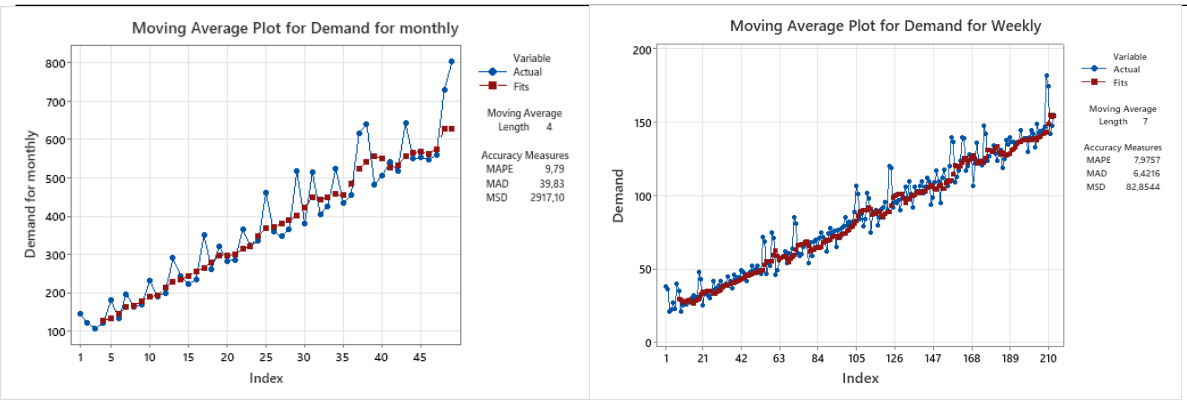

Figure 10. MA in Monthly and Weekly Data

The MA (Moving Average) method applied to monthly and weekly data successfully captures the general trend of the demand variable over time. Especially when the trend is positive, the forecast series closely follows the original data. However, during sharp changes such as sudden spikes or drops, the models fail to capture these variations adequately. This is due to the inherent lag in the moving average method's response to rapid changes. As a result, the MA method is unsuitable for monthly and weekly data.

ARIMA

After examining the stationarity of the demand data through differencing and correction models, the ARIMA model — one of the most appropriate forecasting methods — was established. The ARIMA model aims to make future predictions by considering the autocorrelations between past and current observations. Various components, such as AR (autoregressive), MA (moving average), and seasonal factors, were incorporated in constructing the model. For the monthly data, the initially non-stationary series was made stationary through first-order differencing, and seasonal differencing (lag=4) was applied to remove trend and seasonal effects. After this differencing process, the model selection phase was initiated, and different ARIMA models were tested to identify the one with the lowest information criteria.

Following the analyses, the model that best fit the data was ARIMA (2,0,2)(0,1,1). This model includes two autoregressive (AR) and two moving average (MA) components, as well as a seasonal moving average (SMA) component with a lag of 4 periods. Stationarization was performed only at the seasonal level; regular differencing was not applied. The established ARIMA model's parameter estimates and statistical significance levels were determined (Figure 11).

Final Estimates of Parameters

Type	Coef	SE Coef	T-Value	P-Value
AR 1	1,0181	0,0984	10,35	0,000
AR 2	-1,0028	0,0862	-11,63	0,000
MA 1	1,135	0,152	7,45	0,000
MA 2	-0,926	0,149	-6,23	0,000
SMA 4	0,878	0,159	5,51	0,000
Constant	44,874	0,781	57,44	0,000

Figure 11. ARIMA Model for Monthly Data

All parameter p-values for the monthly data are below the 5% significance level, indicating that the model components are statistically significant. This suggests that the established model is sufficient to explain the series. The autocorrelation structures were examined by analyzing the monthly ACF and PACF graphs (Figure 12).

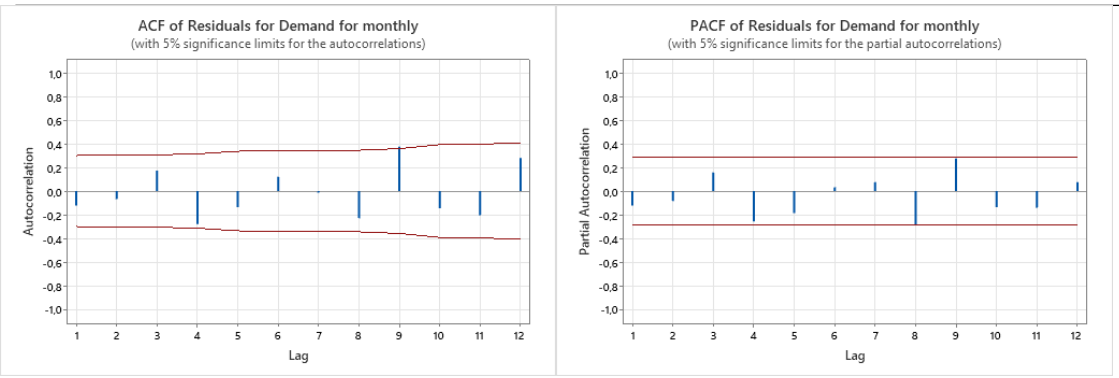

Figure 12. ACF and PACF Graphs in the ARIMA Model for Monthly Data

According to the ACF and PACF graphs, the autocorrelation values of the residuals remain within the confidence bounds and do not exhibit any significant patterns. The error metrics obtained from the training data within the ARIMA model for the monthly data were analyzed to assess the model's forecasting accuracy. Based on the results, the Mean Absolute Percentage Error (MAPE) was calculated as 7.31%, indicating that the forecasts were highly accurate. The fact that the MAPE value remains below 10% demonstrates the model's overall firm performance. A residual analysis was conducted to further evaluate the validity of the model established for the monthly data. The autocorrelation value of the residuals at lag 1 (ACF1) was calculated as 0.016. This value being very close to zero indicates that the residuals are randomly distributed and that there is no autocorrelation in the residuals. This supports that the model's theoretical assumptions are satisfied and constitutes a valid time series forecasting model.

For the weekly data, the initially non-stationary series was made stationary through first-order differencing based on the ACF graph and ADF test results. Since no seasonal effect was observed in this series, different ARIMA models were tested, and the best model was selected based on information criteria. The model was identified as ARIMA (1,1,1), which includes one autoregressive (AR) term, one differencing (I), and one moving average (MA) component. The established ARIMA model's parameter estimates and statistical significance levels were determined (Figure 13).

Final Estimates of Parameters

Type	Coef	SE Coef	T-Value	P-Value
AR 1	0,2862	0,0663	4,32	0,000
MA 1	0,990863	0,000081	12264,06	0,000
Constant	0,4228	0,0102	41,59	0,000

Figure 13. ARIMA Model for Weekly Data

All parameter p-values for the weekly data are below the 5% significance level, indicating that the model components are statistically significant. This suggests that the established model is sufficient to explain the series. The autocorrelation structures were examined by analyzing the ACF and PACF graphs for the weekly data (Figure 14).

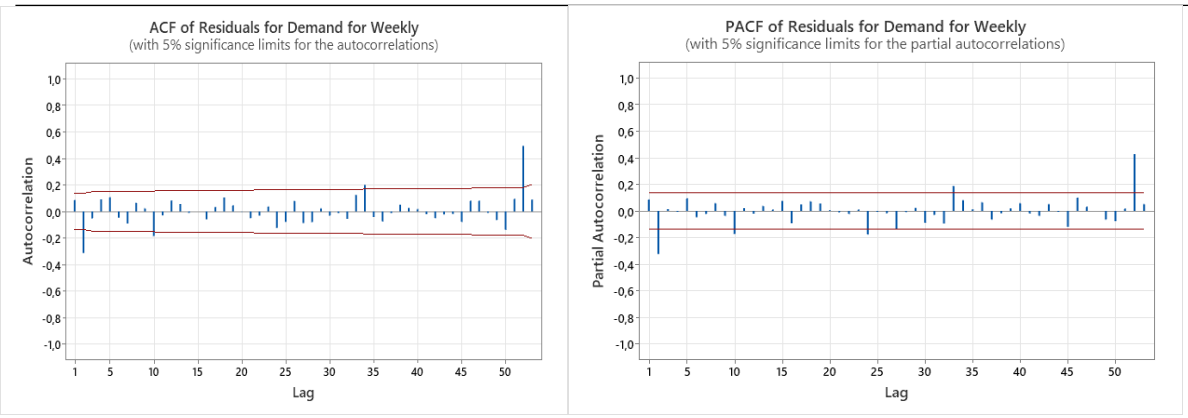

Figure 14. ACF and PACF Graphs in the ARIMA Model for Weekly Data

According to the ACF and PACF graphs, although some residual autocorrelation values slightly exceed the confidence bounds, no significant pattern is observed, indicating no issues with the residuals.

The error metrics obtained from the training data within the ARIMA model for the weekly data were analyzed to assess the model's forecasting accuracy. Based on the results, the Mean Absolute Percentage Error (MAPE) was calculated as 7.76, indicating that the forecasts were produced with high accuracy. The fact that the MAPE value remains below 10% demonstrates the model's overall strong performance.

A residual analysis was conducted to evaluate further the validity of the model established for the weekly data. The autocorrelation value of the residuals at lag 1 (ACF1) was calculated as 0.093. The fact that this value is very close to zero indicates that the residuals are randomly distributed and that there is no autocorrelation. This supports that the model's theoretical assumptions are satisfied and constitutes a valid time series forecasting model.

In the models used for 3-month forecasting of monthly and weekly data, the actual and predicted values were found to be quite close to each other (Monthly MAPE = 8.73, Weekly MAPE = 7.77). For example, in the monthly data, while the actual values were 600, 513, and 545, the predicted values were found to be 640, 558, and 582, respectively. This consistency is also evident in the forecast graphs (Figure 15).

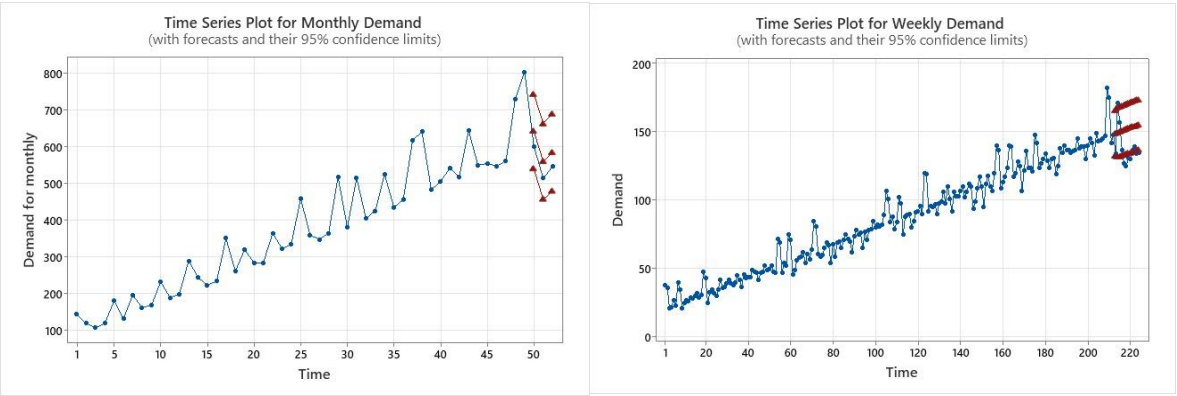

Figure 15. Forecasted Time Series Graphs for Monthly and Weekly Data

DISCUSSION AND CONCLUSION

This study analyzed the weekly and monthly demand data from the e-commerce platform of a small-scale lingerie company in İzmir, Turkey, using time series methods. Upon examining the monthly demand data, it was determined that the time series exhibited both trend and seasonality characteristics. The analysis revealed sales were above average, particularly in January and May, whereas declines were observed in

March and September. This indicates that certain months are more critical regarding customer demand, and marketing strategies implemented during these periods can have significant impacts. Among the applied models, the multiplicative Holt-Winters method demonstrated the best forecasting performance with a MAPE of 7.74%. The ARIMA(2,0,2)(0,1,1) model, which includes both trend and seasonality, also produced highly accurate results. The weekly analysis showed that the ARIMA(1,1,1) model, which captures a trend component, could also accurately forecast. For both monthly and weekly data, the significance of the model parameters and the fulfillment of the randomness assumption in residual analyses supported the validity of the established models.

These findings indicate that the company can develop data-driven strategies for monthly sales planning and manage promotional periods more effectively. Based on these results, the company can make more efficient operational decisions in inventory management, workforce planning, and campaign scheduling, especially during high-demand periods such as spring and the New Year. Moreover, the model outputs obtained after preprocessing steps such as stationarization and component decomposition may serve as examples for other small businesses operating in similar industries. Likewise, a clear trend was also identified in weekly sales planning.

Gender is considered an essential variable in e-commerce. According to the gender analysis conducted on the data, the customer base consisted mainly of women. This led to the conclusion that gender-targeted marketing strategies could positively impact sales performance. Furthermore, the increase in return rates during 2023 and 2024 highlights the necessity of improving product and service quality to enhance customer satisfaction. The weekly and monthly demand forecasts facilitate the anticipation of seasonal fluctuations, thus supporting effective decision-making processes in inventory management, campaign planning, and customer relationship management. These findings suggest that adopting a data-driven management approach during the digital transformation is crucial for small-scale enterprises to gain a sustainable competitive advantage.

Unlike previous e-commerce-focused studies in the literature, such as İşler et al. (2014) using ANOVA, Armağan et al. (2019) using regression models, Aydınhan & Erat (2019) using confirmatory factor analysis, and Marangoz et al. (2019) using the logit model, this study concentrates on weekly demand forecasting and highlights its potential to support the development of short-term strategic decision-making systems. While earlier studies generally focused on customer perception and demographic factors, this study provides time series-based forecasts directly from sales quantities. Furthermore, despite its focus on small businesses, the large monthly data volume enhances the study's value.

Hendriksen et al. (2020) and Roychowdhury et al. (2021) employed machine learning methods such as random forests and artificial neural networks to analyze customer behavior in e-commerce data. However, no known studies are focusing on sales or demand forecasting at small scales using machine learning techniques. For future research, it is recommended to use emerging methods such as the ATA model and currently popular machine learning algorithms like XGBoost, LSTM, random forests, and decision trees to improve the demand forecasting performance of small businesses. Additionally, expanding the dataset and generalizing the models using weekly sales data from different small-scale companies would broaden the applicability of the study’s findings.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

Özgül Vupa Çilengiroğlu: Determination of the topic, literature review, statistical analysis and reporting

Büşra Çolak: literature review, statistical analysis and reporting

Deniz Başşahin: data preprocessing, statistical analysis and reporting

Derya Porsuk: data preprocessing, statistical analysis and reporting

Mehmet Safa Aktaş: literature review, statistical analysis and reporting

Stationarity Structure of Türkiye's Industrial Production Index (1167)

Uğur Ayık[1]

[1]Erzurum Technical University, Türkiye

*Corresponding author e-mail: ugur.ayik@erzurum.edu.tr

Abstract

Industrial production is considered an important variable in monitoring the economic performance of countries. This study examines the stationarity structures of the indexes of sub-branches of industry as well as the total industrial production index of Türkiye and is a preliminary assessment for the stationarity, cointegration and causality analyses planned to be conducted on this subject in the future. While examining the stationarity structure of the indexes of sub-branches of industry and total industrial production, the series were subjected to separate analyses by taking the level values and logarithmic transformation values (1986M01-2025M02). In order to investigate the stationarity structures of the series, the Ng-Perron Unit Root Test, one of the traditional unit root tests, and the Lee-Strazicich (LS) Unit Root Test, one of the tests that take structural breaks into account, were applied and the results were reported. According to the Ng-Perron test, it was observed that the logarithmic values of the series exhibited a more stationary structure compared to the level values. According to the LS test, it was determined that both the level and logarithmic values were all stationary at the level I(0). In short, the results obtained were that the seasonally adjusted logarithmic industrial production series were generally stationary at the level, the series returned to their averages in the face of shocks, that is, the shocks were eliminated.

Keywords: *Industrial Production Index, Stationarity, Türkiye*

INTRODUCTION

In short-term economic policy analyses, industrial production stands out as an important variable. Since certain service activities are closely related to industrial activities, the industrial production sector is important in explaining significant fluctuations in the economy. For this reason, economists continue to evaluate industrial production as a leading indicator of economic activity (Bruno and Lupi, 2004; Banerjee et al., 2005; Ejaz and Iqbal, 2021). Economists can use the industrial production index in total or on a sectoral basis in analyses conducted to determine production targets, monitor their development, and evaluate the functioning of the plans made (Kmietowicz, 1995). Industrial production index data is calculated to measure the outputs of the mining, manufacturing, electricity and gas services industries (Tito, 2025).

This study is a preliminary assessment in the determination of stationarity structures for studies that will include monthly data of total industrial production index and sub-branches of industry in their analyses in the coming years, specific to Türkiye. In the analyses, the study size was expanded by investigating the stationarity structures of industry sub-branches in addition to the total industrial production index. Although there are studies that conduct stationarity research based on Türkiye's total industrial production index, the absence of a study examining sub-industry branches differentiates this study from the literature. In important studies conducted for Türkiye (Ertuğrul and Soytaş (2013); Yıldırım and Kılıç (2016); Oğuz (2017)), industrial production index data are generally not stationary at the level and have become stationary by taking the differences of the series. In studies conducted for various countries around the world, as well

as Türkiye (Candelon and Gil-Alana (2004); Bulligan et al. (2010); Caporale et al. (2023)), evidence has been obtained that the industrial production index series are stationary at the level or in the first difference.

MATERIAL AND METHODS

In the study, using monthly time series covering the period of January 1986-February 2025, the stationarity structures of the production indices of the industrial sub-branches (Mining and quarrying, Manufacturing industry and Electricity, gas, steam and air conditioning production and distribution) as well as the total industrial production index of Türkiye were examined. In order to determine the stationarity structures of the industrial series, the Ng-Perron test, which is one of the traditional unit root tests, and the LS test, which is one of the unit root tests that take into account structural breaks, were applied. The study data were obtained from the Turkish Statistical Institute (TSI) database and are seasonally and calendar-adjusted series (2021=100 reference year). In addition to the stationarity structure of the level values of the series, the stationarity structures of their logarithmic values were also examined. The analyzes were performed in EViews 12 and RATs software programs.

EMPIRICAL FINDINGS

Before moving on to the stationarity analysis of the industrial production index series, the diagnostic statistical results of the original values of the series are presented and evaluated in Table 1.

Table 1. Diagnostic Statistical Test Results of Series

Variable	Aver.	Med.	Max.	Min.	Std. Dev.	Skewness	Kurtosis	J-B	Prob.
TIPIt	50.60	42.80	112.50	16.50	27.58	0.70	2.22	50.76	0.00
MIN_t	63.15	58.65	104.50	34.70	18.59	0.42	2.01	33.01	0.00
MI_t	50.04	41.80	114.00	16.90	27.74	0.77	2.33	55.64	0.00
EGS_t	53.77	49.15	110.50	11.40	29.19	0.22	1.71	36.37	0.00

Abbreviations: TIPI (total industrial production index), MIN (mining and quarrying), MI (manufacturing industry), EGS (electricity, gas, steam and air conditioning production and distribution), J-B (Jarque-Bera).

The fact that the skewness coefficients of the series presented in Table 1 are greater than zero means that their distribution is skewed to the right, and the fact that the kurtosis coefficients are less than 3 means that their distribution is flatter than normal. When the probability values of the J-B test statistic are examined, it is observed that the basic hypothesis stating that the series have a normal distribution is rejected, and in this case, it is decided that they are not normally distributed. The fact that the series do not have a normal distribution indicates that they are not parametric. Since performing the analysis with nonparametric series may lead to unreliable results, the data must be transformed. At this stage, first of all, the logarithmic transformation method was applied so that the distributions of the series could become normal or close to normal. When the diagnostic statistical tests of the series whose natural logarithm (*ln*) was taken were performed again, it was observed that their skewness ($LnTIPI_t$ skewness = 0.15, $LnMIN_t$ skewness = 0.05, $LnMI_t$ skewness = 0.23, $LnEGS_t$ skewness = -0.12) (as the skewness coefficient approaches zero, it means that the series has a more normal distribution) decreased, that is, it was determined that the series had a distribution close to normal. The graphs of both the level and logarithmic values of the series are presented in Figure 1 and Figure 2, and a preliminary assessment of their stationarity structures was made.

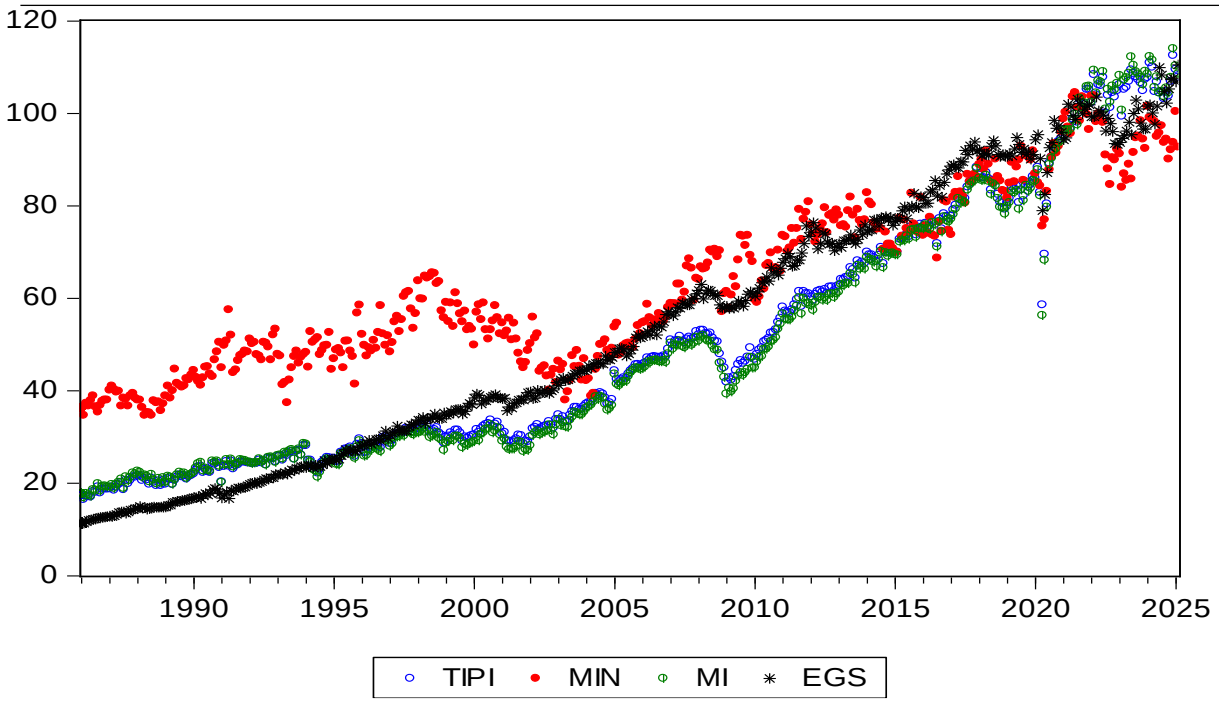

Figure 1. Time Series Graph of Level Values

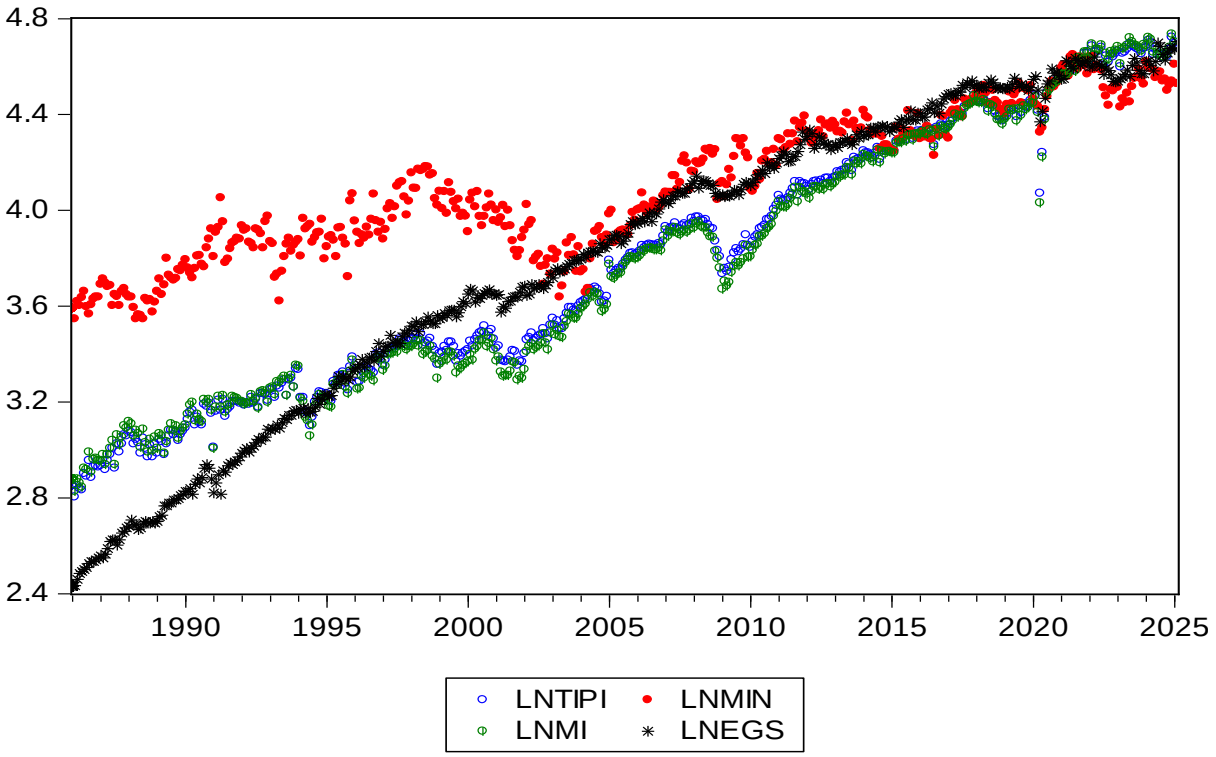

Figure 2. Time Series Graph of Logarithmic Values

When the graphs of the industrial production index time series are examined (Figure 1 and Figure 2), it is suggested that the level and logarithmic values of the $TIPI_t$, MIN_t and MI_t variables generally exhibit an image around their averages, and therefore, a preliminary idea has emerged that they may be stationary at the level. However, it is observed that the level and logarithmic time series of the EGS_t variable exhibit deviations from their averages, that is, it is thought to exhibit a more non-stationary image. In order to

express with certainty whether the series contain a unit root, unit root tests were applied to both the level values and the logarithmic transformation values. The obtained stationarity results are presented and evaluated in Table 2.

Table 2. Unit Root Test Results for Industrial Production Index Series

Results for Level Values

	Ng-Perron (Intercept and Trend)				LS (Model C)	
Variable	MZ_a	MZ_t	*MSB*	*MPT*	t-stat. (TB)	5% Critical Value
$TIPI_t$	-3.53	-1.18	0.33	23.34	-7.51* (2004:11-2008:11)	-5.22
MIN_t	-33.70*	-4.10*	0.12*	2.70*	-8.88* (1997:12-2004:5)	-5.31
MI_t	-3.37	-1.14	0.34	24.17	-7.46* (2004:11-2008:11)	-5.22
EGS_t	-5.63	-1.51	0.27	15.86	-7.37* (2000:9-2017:10)	-5.41

Results for Logarithmic Values

	Ng-Perron (Intercept and Trend)				LS (Model C)	
Variable	MZ_a	MZ_t	*MSB*	*MPT*	t-stat. (TB)	5% Critical Value
$LnTIPI_t$	-36.43*	-4.26*	0.12*	2.51*	-7.13* (2000:12-2005:2)	-5.22
$LnMIN_t$	-23.66*	-3.43*	0.14*	3.89*	-8.84* (1997:12-2004:5)	-5.31
$LnMI_t$	-29.12*	-3.81*	0.13*	3.13*	-7.08* (2001:2-2008:9)	-5.41
$LnEGS_t$	-0.41	-0.24	0.57	71.20	-7.12* (2000:10-2012:2)	-5.37

Note: Ng-Perron critical values for MZ_a, MZ_t, *MSB* and *MPT* tests at 0.05 significance level are -17.300, -2.910, 0.168 and 5.480, respectively. * indicates significance at 0.05 significance level.

Ng-Perron test aims to eliminate the distortion in the volume of the error term occurring in the PP unit root test. The Ng-Perron test has modified the PP unit root test and information criteria. While the basic hypothesis (H_0) in the MZ_a and MZ_t tests in the Ng-Perron unit root test expresses the existence of a unit root, the basic hypothesis in the *MSB* and *MPT* tests shows that the series is stationary (Ng and Perron, 2001; Çetin and Saygın, 2019). According to the Ng-Perron test results applied to the level values and logarithmic values of the industrial production series, it was determined that the MIN_t series was stationary I(0) at the level in terms of both the level values and logarithmic values. The level values of the $TIPI_t$ and MI_t series were not found to be stationary at the level, but it is noticeable that the logarithmic transformations of the series ($LnTIPI_t$, $LnMI_t$) were stationary at the level. It has been estimated that the EGS_t and $LnEGS_t$ series are not stationary at the level and the series can become stationary at least at the first difference.

LS unit root test is calculated using two models, Model A and Model C. While Model A allows two breaks at the level, Model C allows two breaks at both the level and the trend. Based on previous studies, it has been observed that Model C offers superior results compared to Model A (Lean and Smyth, 2015; Berke et al., 2014; Karademir and Evci, 2020). In the LS unit root test, the basic hypothesis states the existence of a unit root under structural breaks, while the alternative hypothesis shows trend stationarity under structural breaks (Ertuğrul and Soytaş, 2013). According to the LS unit root test results presented in Table 2, it is seen that both the level values and the logarithmic values of the industrial production index are stationary I(0) at the level. According to the LS unit root test results applied to logarithmic values, findings were obtained that the industrial production index series were trend stationary under the structural breaks in the periods of $LnTIPI_t$ 2000:12-2005:2, $LnMIN_t$ 1997:12-2004:5, $LnMI_t$ 2001:2-2008:9 and $LnEGS_t$ 2000:10-2012:2.

CONCLUSION AND EVALUATION

In the study where the stationarity analysis of Türkiye's total industrial production and the indices of industrial sub-branches was performed using monthly time series covering the period 1986-2025, the Ng-Perron test, which is one of the traditional unit root tests, and the LS test, which is one of the unit root tests that take structural breaks into account, were applied. Before proceeding to the unit root analysis of the series, it was examined whether they provided the normal distribution assumption. According to the diagnostic statistical tests performed on the level values of the series, it was determined that they were not normally distributed and their natural logarithms (*ln*) were taken to make the distribution of the series closer to normal. As a result of the logarithmic transformation, it was observed that the skewness of the series decreased, therefore it was observed that the series had a distribution close to normal. After the series were made more ready for analysis, the unit root process was started. According to the Ng-Perron test results, the industrial production series were generally not found to be stationary at the level, but it was determined that the logarithmic values of the series were stationary at the level. According to the LS unit root test results, it was concluded that both level values and logarithmic values were trend stationary under structural breaks. In particular, it was determined that logarithmic values exhibited a more stationary structure than level values in terms of Ng-Perron unit root test results. The importance of logarithmic transformation in order for the series to have a normal distribution feature and to be made more stationary is understood from the analyses performed. The determination that the industrial production series adjusted for calendar and seasonal effects have a stationary structure means that the effects of shocks on the series are eliminated.

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Comparisons of Several Forecast Methods On Delivery Time Dataset in NewYork (1254)

Kardelen Çakabay[1*]

[1]Statistics, METU, Ankara, Turkiye

*Corresponding author e-mail: cakabay.kardelen@metu.edu.tr

Abstract

In this study "Future Delivery Time; Spread Index for New York." data is analyzed by using various time series forecasting models including ARIMA, ETS, TBATS, NNETAR, and Prophet. The dataset from the Federal Reserve Bank of New York's Empire State Manufacturing Survey, provides monthly, seasonally adjusted indices that reflect expected delivery times in New York State over the next six months. Our goal was to evaluate and compare the predictive performance of these models to determine the most accurate approach to predicting future delivery times. The findings show that The NNETAR model provided the best fit for the data and TBATS model demonstrated superior performance among them.

Keywords: *Time Series, Forecasting, Arima, ETS, Tbats, Neural Network, Prophet*

INTRODUCTION

The dataset consists of seasonally adjusted monthly time series data from July 2001 to November 2024 with 269 observations. The source of this Future Delivery Time; Spread Index for the New York dataset is the Federal Reserve Bank of New York's Empire State Manufacturing Survey.

The primary objective of this study is to analyze and compare various time series forecasting models to predict future delivery times in New York State. By conducting models like ARIMA, ETS, TBATS, NNETAR, and Prophet, the aim is to evaluate their forecast performance and identify the most accurate approach. The findings from this research will contribute to a better understanding of the dynamics affecting delivery times and improve decision-making processes for businesses relying on timely delivery forecasts.

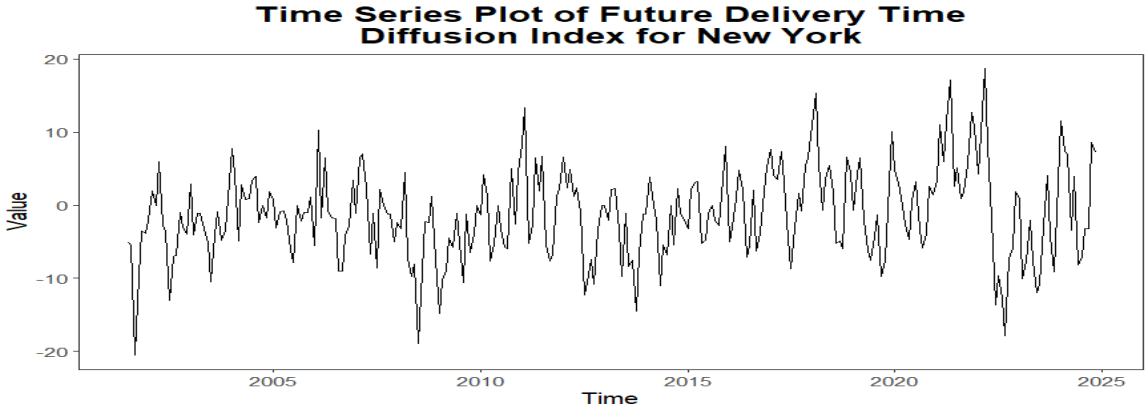
MATERIAL AND METHODS

In this analysis, as a software tool R programming language was used with various libraries such as tseries, forecast, and ggplot2.

Additionally, several statistical methods including model fitting, forecast, and visualization techniques were employed. Among the visualization techniques ggplots and ACF - PACF plots were mostly used.

To begin, the tseries library was used to convert the data frame dataset into a time series data class. For statistical analysis, the forecast library was mainly relied on. Other than that, to check the assumption and find out the structures lmtest, TSA fUnitRoots and similar libraries were used. To visualize the data and findings, ggplot2, gridExtra, anomalies, and timetk were used. Throughout the analysis, a systematic approach was followed, selecting appropriate statistical methods and visualization techniques based on our aim for comparing different methods. The combination of R programming language and the mentioned libraries provided us with the necessary tools and resources to analyze the Future Delivery dataset and make meaningful conclusions effectively. Overall, stationarity, normality, heteroscedasticity, and autocorrelations assumptions for effective forecast were checked. To conduct these, firstly Kpss, Heggy, Shapiro-Wilk, Lung-Box, and Lagrange multiplier tests were used. In the end, with the accuracy function, the accuracy measures of forecasts are used to compare different conducted forecasts.

RESULTS



In the time series plot, no decrease or increase suggests the absence of a trend in the data. However, there may be seasonality. Moreover, the significant spikes observed suggest the presence of outliers in the data. These anomalies may indicate unusual events or irregular patterns that deviate from the expected seasonal behavior.

Anomaly Detection

Before anomaly detection, some arrangements should be made. Firstly, the dataset is split into two parts a train set and a test set. From now on, the analysis will be applied to the train set, and at the end, the test set will be used to compare forecast performance between all the methods.

Secondly, outliers and missing values should be identified and replaced. The training data does not have missing values, but it contains outliers. To replace all the outliers, a threshold of 1.6 for the z-score was used, and the values exceeding this threshold were assigned NA, and then filled with the surrounding data points.

A comparison of boxplots representing the data before and after the outliers were replaced can be found below.

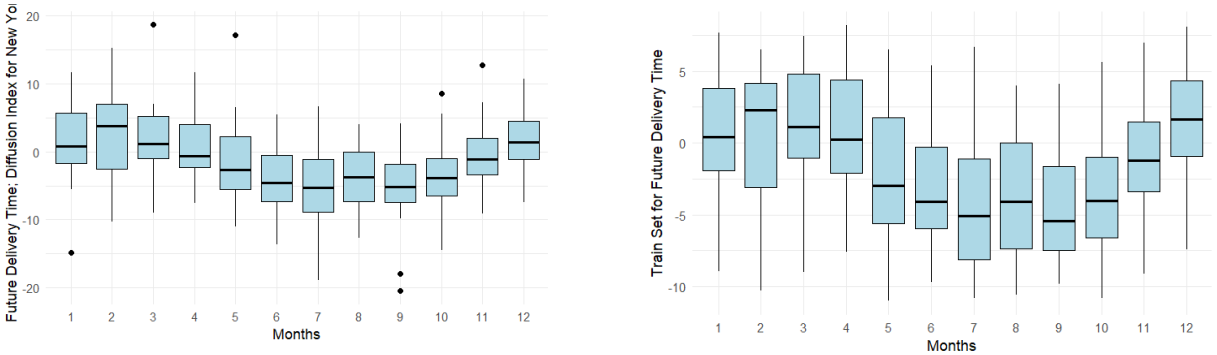


Figure 1. For Anomaly Detection, a tibble format is needed, first, we need a dataframe then tibble from time series data.

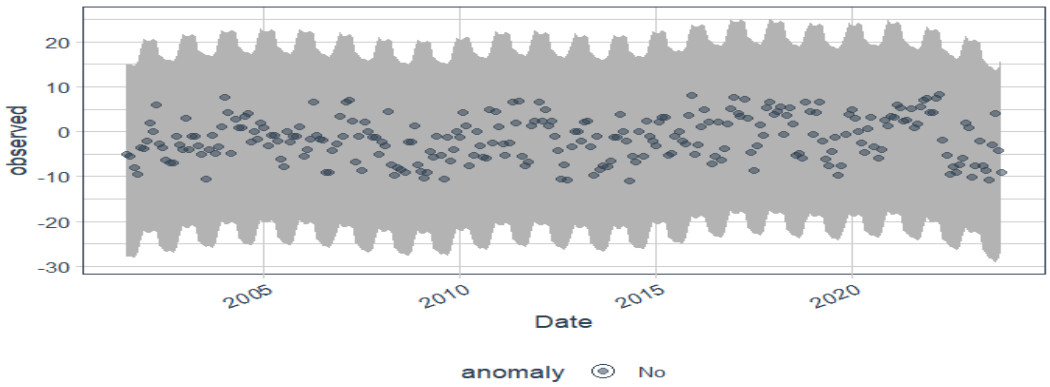


Figure 2. For further interpretation, ACF&PACF plots can be used.

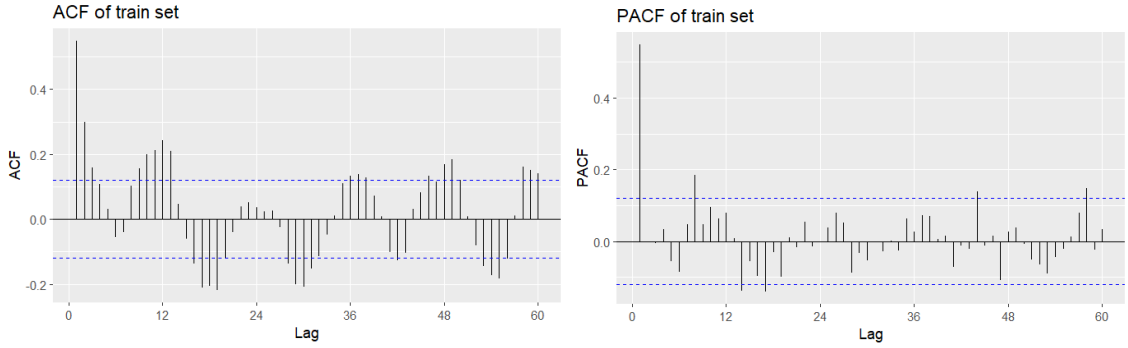


Figure 3. According to the ACF and PACF plots of the train set, the sinusoidal behavior is seen in the ACF plot, and there are significant spikes indicating seasonality in the PACF plot.

Now, the stationarity assumption should be checked. To do that The Kpss test, ADF test, and Heggy tests were conducted and both seasonal or unit roots were not identified. As a result, the train set shows no signs of non-stationarity. Moreover, to check whether the differences between the years are statistically significant or not, the ASCB test was applied. After analysis, the p-value was found approximately 1.29×10^{-6} indicating that the mean future delivery times differ significantly across the years in the dataset.

Additionally, as seen in the box plot of the train set, the median does not follow a horizontal zero line. It indicates seasonality in the dataset.

To find a proper ARIMA model or SARIMA model for our train set, some possible orders can be identified from ACF & PACF plots of train data. There is one significant spike at the PACF plot (AR-1), ACF shows sinusoidal behavior. It can be also seen significant seasonal spikes in the ACF plot. To detect the most appropriate model, twelve models were compared, and ARIMA-SARIMA(1,0,0)(0.0,1)12 is found best one as it has significant parameters and small accuracy criteria.

On the residuals, the Portmanteau lack of fit test is performed and it is found that residuals are likely white noise and the model fits the data well at the significance level of 0.05. After fitting the Arima model, four of the ACF&PACF values of residuals are not in the WN bands. It could be an indicator that the ARIMA model is not fully capturing the underlying structure of the data.

Since only 4 lags fall outside the white noise (WN) bands in the ACF/PACF plot, and the Ljung-Box test gives a p-value of 0.4 suggests that the residuals are mostly random, with a small portion showing some autocorrelation at those specific lags.

For further interpretation, Squared Standardized Residuals can be analyzed.

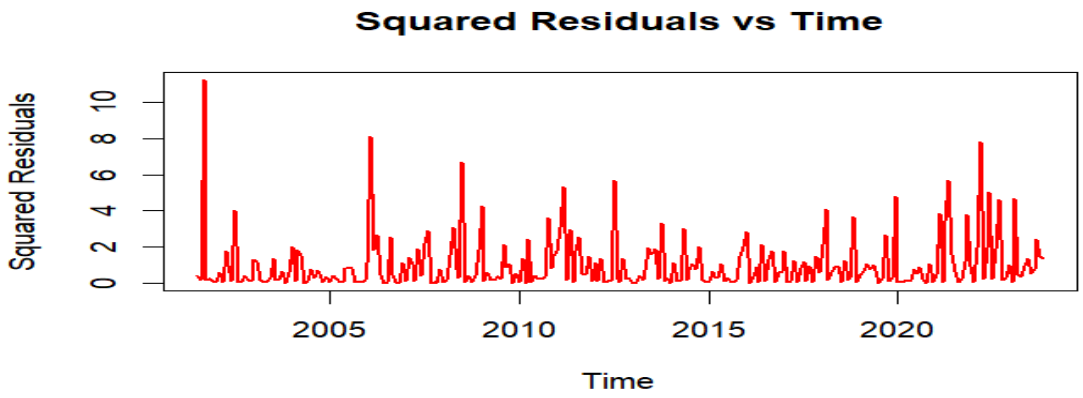


Figure 4. The plot shows significant spikes, indicating the existence of outliers.

While no clear trend is observed, the clustering effect suggests the presence of heteroscedasticity, where the variability of the residuals changes over time. For normality of residuals, histogram and Q-Q plot of residuals give good visual detection.

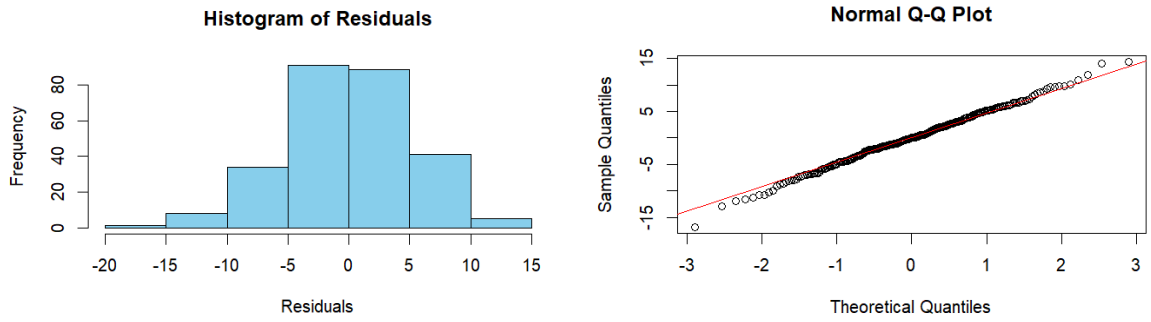


Figure 5. It is slightly left-skewed but mostly symmetric and seems normal.
Figure 6. There are outliers and the plot shows a slight S shape indicating residuals may have heavy-tailed distributions.

As a formal test for normality, the Shapiro-Wilk test is used. Since the p-value (0.9) is greater than 0.05 it is found that residuals are normally distributed. Moreover, based on the Breusch-Godfrey test, the result was found insignificant, the model does not show significant autocorrelation in the residuals, which suggests that the model appropriately captures the underlying temporal dependencies.

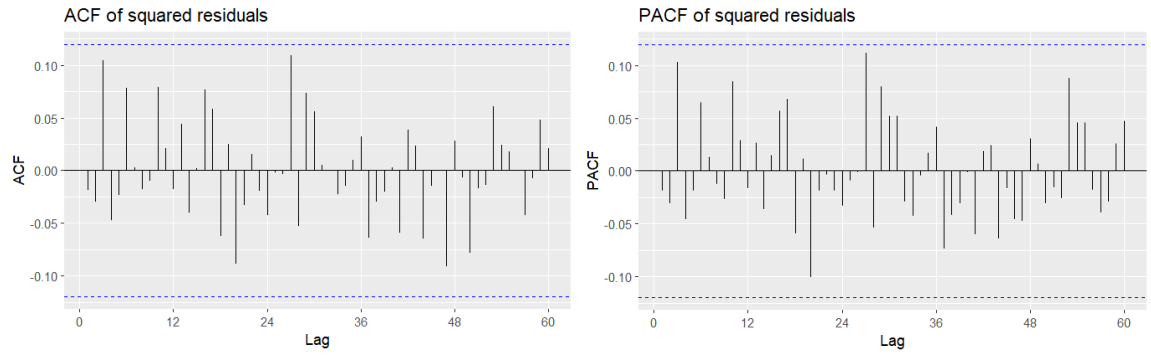


Figure 7. For the Heteroscedasticity ACF-PACF plots of the squared residuals can be performed.

All squared residuals are in the white noise bands. It can be concluded that there is no heteroscedasticity problem, error variance is constant. Other than that, ARCH Engle's test can be used for the same aim as a

formal test. As a result of this test, all the p-values are greater than 0.05, indicating there is no significant evidence of heteroscedasticity in the residuals at any of the tested lags.

Forecasting

In all forecast plots, the black line represents the original data, while the red line corresponds to the test set.

Firstly, the Minimum MSE Forecast for the stochastic ARIMA(1,0,0)(0,0,1)12 model was performed.

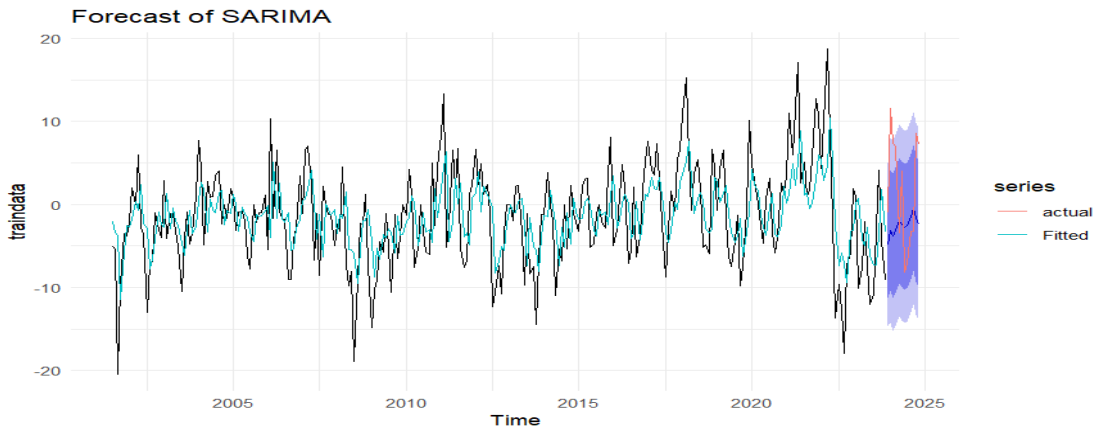


Figure 8. The prediction intervals are notably wide, suggesting that the ARIMA model provides a weak fit and lacks precision in forecasting

Secondly, the Ets model is used with no trend but seasonality under (A, N, A) parameters. The data exhibits additive errors and additive seasonality, but no discernible trend. Later, the assumption for the normality of residuals was checked and its normality was concluded with the Shapiro-Wilk test.

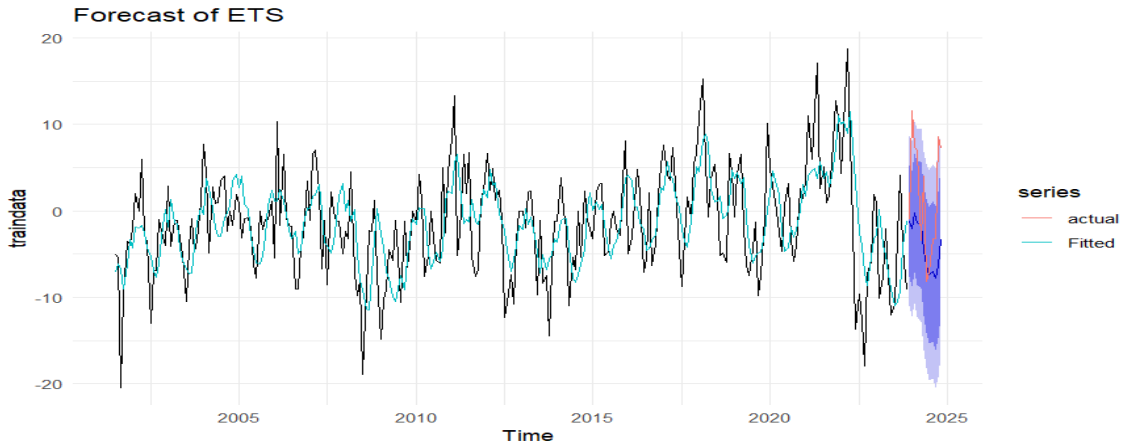


Figure 9. The ETS model weakly captures the seasonal behavior, and the prediction intervals appear wide, indicating a lack of strong fit.

Thirdly, the analysis of the NNETAR model was conducted. As seen by this training data, this model works best when the dataset shows seasonality.

To avoid possible overfitting, the RMSE values for the training and test sets were examined before fitting the model. Since the RMSE values are very close to each other (7.95 for the training set and 7.97 for the test set), it can be concluded that there is no overfitting issue.

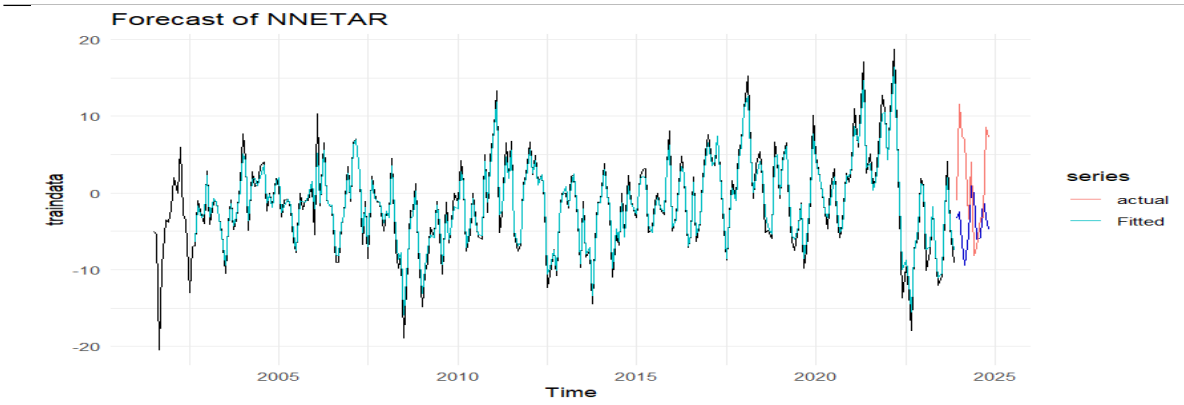
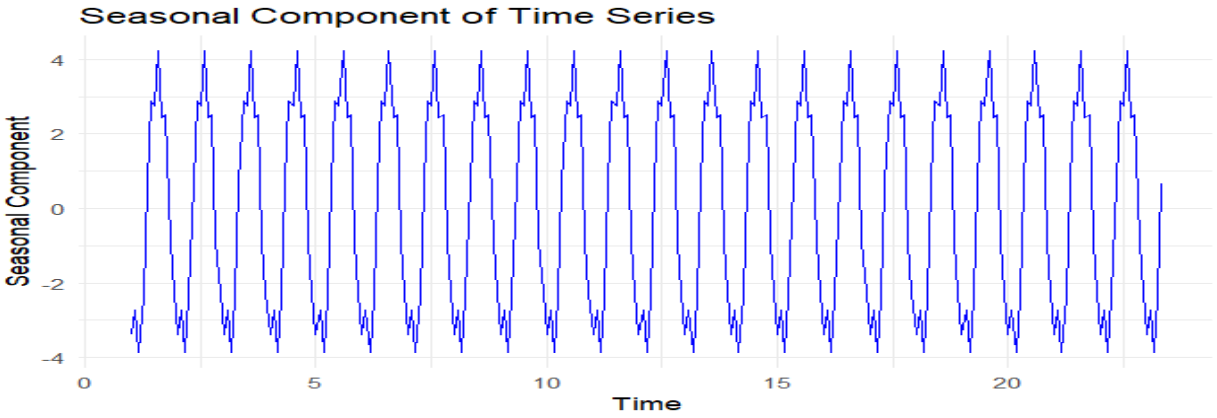


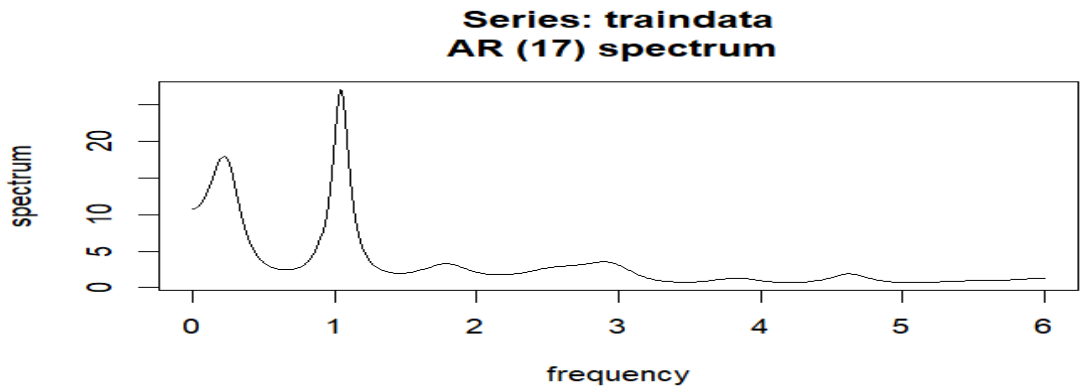
Figure 10. The training data is well-fitted by the NNETAR model. It may be also a robust and dependable model since it efficiently captures the test data's peaks with less variance.

Fourthly, the TBATS model is implemented. It is frequently useful when the data shows complex seasonality, such as multiple seasonalities, high seasonal frequency, or non-integer seasonality. Using seasonal decomposition methods, we can examine the training set for multiple and non-integer seasonality.



This seasonal component decomposition plot shows two peaks at the bottom of the patterns. However, it may not be enough to say there are multiple frequencies. Also, since the plot does not show irregular patterns, it could not demonstrate non-integer seasonality.

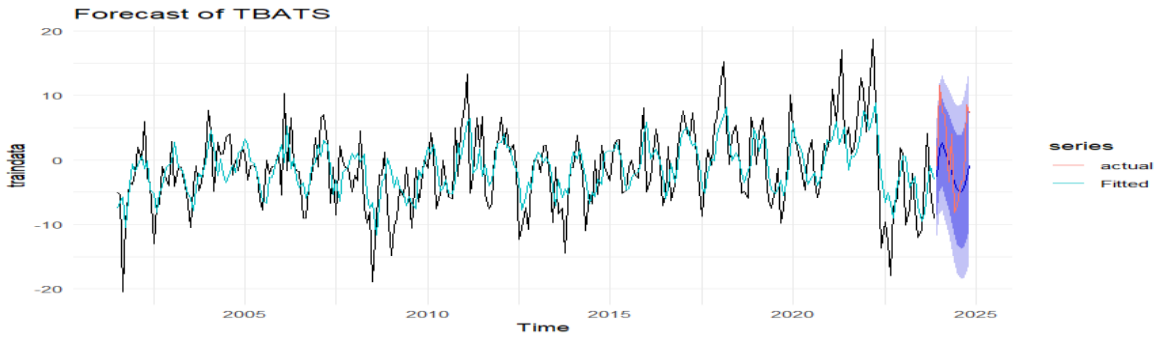
Several ways are available to detect multiple seasonalities, such as spectral analysis, MSTL decomposition, and the summary of the TBATS model.



After applied spectral analysis it is seen that a yearly seasonality, which is typical in monthly records, is shown by the spike at frequency 1. As a result, it is unlikely to decide that there are multiple seasonalities.

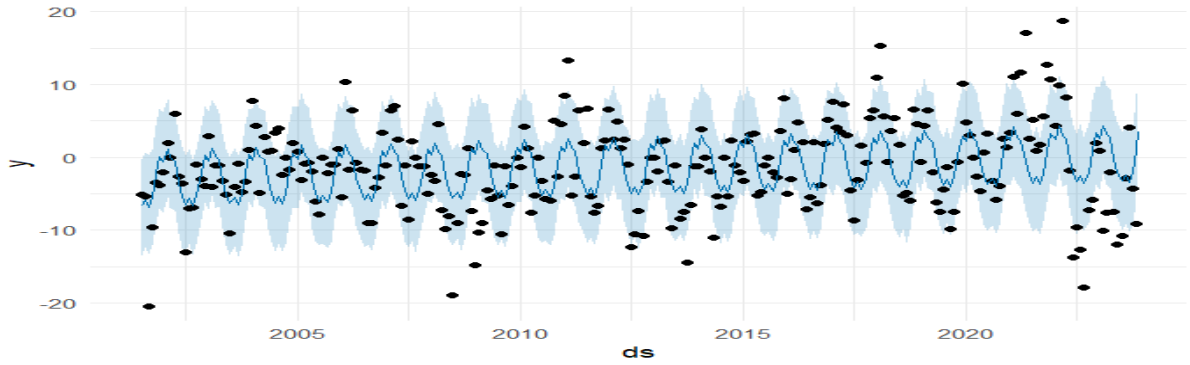
The trend is represented by the spike at frequency 0, however, the data we checked showed no trend. It could also indicate the mean level of the training data.

Before conducting the TBATS forecast analysis, the normality of the TBATS model's residuals should be checked. According to the Shapiro-Wilk test p value is 0.6 indicating our residuals are normally distributed.



With the broadest prediction intervals, the TBATS model appears to have considerable forecast uncertainty. This shows a poor fit to the data, even if it captures the fundamental fluctuational behavior.

Lastly, the Prophet model was used to forecast the train set. Instead of using a classical approach, the model was constructed with hyperparameter tuning, as its accuracy metrics for test data, such as MAE and RMSE, were smaller than those obtained using the traditional method.



The above forecast plot shows the forecasted values of the data where, Black dots refer to the original data, Dark blue line refers to the predicted value(yhat), and Light blue area prediction interval.

There are outliers outside of the prediction intervals even though the Prophet model accounts for seasonality. This indicates that the Prophet model is not well-suited for our data.

Comparing Accuracy of the Models

Model	RMSE (Training)	MAE (Training)	MAPE (Training)	RMSE (Test)	MAE (Test)	MAPE (Test)
TBATS	4.64	3.57	Inf	5.96	5.03	88.19
NNETAR	1.07	0.79	Inf	9.31	7.49	124.64
ARIMA	5.01	3.94	Inf	7.96	6.83	127.65
ETS	4.87	3.82	Inf	7.98	6.35	102.81
Prophet	N/A	N/A	N/A	5.47	4.71	107.10

According to the accuracy values for both the train and test sets, the smallest values were chosen, leading to the following conclusions:

1.NNETAR provided the best fit for the training data, outperforming the other models in terms of fitting accuracy.

2.TBATS performed the best for the test data, as it forecasted more accurately compared to the other models.

CONCLUSION

In this study, various time series forecasting models – ARIMA, ETS, TBATS, NNETAR, and Prophet– were analyzed and compared to predict Delivery Time in New York State. Through a systematic methodology, necessary assumptions were tested both before and after applying the forecasting methods.

After an in-depth analysis of the Future Delivery dataset, some significant findings were made. First off, one of our assumptions is that the dataset does not have enough complexity to support the TBATS model. The TBATS model explained its potential to manage the underlying structure of the data and successfully produce the most accurate forecast measurements for the forecast. This result shows the TBATS model demonstrated its ability to handle not only complex seasonality but also datasets with simpler seasonal patterns, successfully capturing the underlying seasonal behavior and producing the most accurate forecasts for the test data.

On the other hand, despite capturing the train data well, NNETAR model did not perform as strongly as TBATS when applied to the test set even though there is no indication of an overfitting problem.

To conclude, the systematic use of diverse forecasting methods revealed the strengths and weaknesses of each model. The NNETAR model provided the best fit for the data and TBATS model demonstrated superior performance among them.

Parameter Estimation in Organic-Based Schottky Diodes Used in Solar Cell Applications with Artificial Intelligence Optimization Algorithms (1177)

Murat Açıkgöz[1*], Defne Akay[2], Özlem Türkşen[3]

[1]Ankara University, Graduate School of Natural and Applied Sciences, Ankara, Türkiye
[2]Ankara University, Faculty of Science, Department of Physics, Ankara, Türkiye
[3]Ankara University, Faculty of Science, Department of Statistics, Ankara, Türkiye

*Corresponding author e-mail: muratacikgoz@live.com

Abstract

The electrical characterization of organic semiconductor materials plays a critical role in the design of advanced electronic devices and the understanding of their performance. Particularly, electrical parameters such as the ideality factor, barrier height, and series resistance, derived from the current-voltage (I–V) characteristics of rubrene-based metal-polymer semiconductor (MPS) structures, provide fundamental insights into the material's intrinsic and interfacial properties. The nonlinear nature of these parameters and the uncertainties associated with experimental data render traditional estimation methods inadequate. Additionally, investigating the effects of external factors, especially Cobalt-60 gamma irradiation, on these electrical parameters is of great importance for device reliability. The aim of this study is to accurately and reliably estimate the electrical parameters of Al/rubrene/p-Si Schottky-Junction solar cell using Artificial Intelligence (AI) optimization algorithms, based on experimental I–V data obtained after Cobalt-60 gamma irradiation. In this context, a total of twelve different AI optimization algorithms were employed, including Genetic Algorithm, Differential Evolution, Flower Pollination Algorithm, Artificial Bee Colony, Ant Colony Optimization, Bat Algorithm, Cuckoo Search Algorithm, Grey Wolf Optimization, Jaya Algorithm, Particle Swarm Optimization, Harmony Search, and Teaching-Learning-Based Optimization. The estimation of electrical parameters was performed using I–V measurements conducted across five different forward voltage ranges, with the performance of the algorithms evaluated using MAPE and R^2 metrics. This study presents that the AI optimization algorithms can be used as optimization tool for parameter estimation of Schottky diode model and the algorithms may offer more effective results compared to traditional optimization methods in the electrical analysis of organic semiconductor models under Cobalt-60 irradiation.

Keywords: *Artificial Intelligence Optimization Algorithms, Nonlinear Regression, Parameter Estimation, Schottky-Junction Solar Cell.*

INTRODUCTION

In the contemporary scientific landscape, the accurate modelling of complex systems has become a prevalent necessity. This has prompted researchers to adopt more flexible modelling approaches that transcend linear assumptions. In this contexts, the employment of nonlinear models becomes important, particularly in scenarios where the relationships between variables cannot be satisfactorily explained linearly. The nonlinear regression models are powerful statistical tools used when the relationships between dependent and independent variables are parametrically nonlinear. These models enable a more realistic and reliable representation of real-world phenomena across a wide range of disciplines, including engineering, economics, biomedical sciences, environmental sciences, and social sciences. However, the mathematical complexity introduced by the nonlinear structure presents significant challenges, particularly in the parameter estimation process, and limits the effectiveness of classical methods (Gallant, 1975).

Parameter estimation of nonlinear regression models is generally formulated as an optimization problem based on the minimization of the sum of squared errors. Traditional optimization methods used in this process, such as Gauss-Newton, Levenberg-Marquardt, and other derivative-based methods, may yield successful results under certain conditions (Türkşen, 2023). Nevertheless, they are limited by several issues, including the non-differentiability of the objective function, the presence of multiple local minima, high sensitivity to initial parameter values, and inefficiency in high-dimensional search spaces. In this regard, the challenges encountered in parameter estimation are not only technical but also significant in terms of computational efficiency and model reliability.

These limitations are especially important in solar cell modelling, which is becoming more and more important because of its role in condensed matter physics and its importance in the development of efficient photovoltaic structures. Addressing these computational challenges is essential for accurately modeling solar cells, which are key to meeting the global demand for clean and sustainable energy. By converting sunlight directly into electricity without emitting greenhouse gases, these systems represent a promising and environmentally friendly alternative to conventional fossil fuel–based energy sources. To investigate these systems in greater detail, Schottky diodes with an Al/p-type Si structure were utilized. These diodes were fabricated using boron-doped single-crystal silicon wafers (p-type) as the semiconductor substrate, onto which aluminum was thermally evaporated to form the Schottky contact. The resulting devices were employed to examine the electrical characteristics and interfacial properties of the metal–semiconductor junction, with the aim of enhancing our understanding of charge transport mechanisms that are crucial for improving photovoltaic performance. Accordingly, this study outlines the fabrication process and electrical characterization of an Al/Ru/p-Si Schottky diode constructed on a p-type (111) silicon wafer. All electrical measurements were performed using a microcomputer-assisted setup through an IEEE-488 compatible AC/DC converter card, ensuring precision and repeatability. The analysis of the electrical behavior was carried out based on the thermionic emission theory, which serves as a foundational framework for evaluating metal-semiconductor and metal-organic semiconductor junctions. The thermionic emission theory provides a quantitative description of how charge carriers primarily electrons exceed the potential energy barrier at the metal-semiconductor interface through thermal excitation. This model lets you measure important things about the diode, like the barrier height, ideality factor, and saturation current. This is important for seeing how well Schottky diodes and other optoelectronic and microelectronic devices work. Furthermore, the thermionic emission theory becomes particularly significant in cases where interface-related effects such as localized states, barrier inhomogeneities, and thin interfacial layers play a substantial role in charge transport. Deviations from ideal thermionic emission behavior under these conditions can reveal the presence of interface traps, tunneling mechanisms, or spatial variations in barrier height. Understanding and quantifying these non-idealities are essential for optimizing device architecture, especially in advanced applications including harsh or radiation-exposed environments.

In metal-organic semiconductor Schottky barrier diodes, the barrier height has also been shown to exhibit voltage dependence, further complicating the transport dynamics and necessitating more detailed modeling approaches that incorporate both intrinsic and extrinsic influences on charge flow. The thermionic emission theory (Sze, 1981) predicts that the current–voltage (I–V) characteristics as in the studies (Akay et al. 2019, Akay et al. 2020, Akay et al. 2024)

In recent years, new methods inspired by nature and human behaviour have been developed to overcome these challenges. In particular, computer programmes that optimise things have been shown to be a good option for nonlinear regression analysis. The AI optimization algorithms, called Genetic Algorithm (GA), Differential Evolution (DE), Flower Pollination Algorithm (FPA), Artificial Bee Colony (ABC), Ant Colony Optimization (ACO), Bat Algorithm (BA), Cuckoo Search Algorithm (CSA), Grey Wolf Optimization (GWO), Jaya Algorithm (JA), Particle Swarm Optimization (PSO), Harmony Search (HS),

and Teaching-Learning-Based Optimization (TLO), stand out with their ability to effectively explore large search spaces, significantly overcoming the local minimum traps encountered by traditional algorithms. Moreover, these algorithms operate without requiring derivative information and are well-suited for parallel processing, thereby demonstrating high computational performance even with large datasets. Numerous studies in the literature have shown that AI optimisation algorithms are better than traditional methods because they are more accurate and faster (Açıkgöz, 2025).

The main aim of this study is to comprehensively investigate the applicability of AI optimization algorithms in addressing the parameter estimation problem encountered in nonlinear regression models. In this study, twelve AI optimization algorithms were applied on a parametrically structured nonlinear model called Schottky diode model. Each algorithm's performance was comparatively analyzed in terms of error metrics (*MAPE* and R^2), computation times, and solution stability. The rest of the paper is organized as follows. Nonlinear regression model of Schottky diode and applied AI optimization algorithms are explained in Materials and Methods section. The analysis results are presented in Results section. Finally, conclusion is given in the last section.

MATERIAL AND METHODS

Material

Nonlinear regression models are powerful statistical tools that provide more realistic and reliable results in situations where the assumption of a linear relationship between dependent and independent variables is invalid. (Akgün, 2018). The general form of a nonlinear regression model can be expressed as

$$Y_i = f(\boldsymbol{X}_i, \boldsymbol{\beta}) + \varepsilon_i \quad , \quad i = 1, 2, \ldots, n \tag{1}$$

where, Y_i represents the dependent variable, $\boldsymbol{X}_i$ denotes the vector of independent variables, $\boldsymbol{\beta}$ is the parameter vector to be estimated, and ε_i is the error term with zero mean and constant variance. The objective of the model is to determine the parameter vector $\boldsymbol{\beta}$ in such a way that the model output best fits the observed data (Khuri and Cornell, 1996). These parameters are estimated numerically by minimizing an objective function based on the sum of squared errors

$$\min \varphi(\boldsymbol{\beta}) = \sum_{i=1}^{n} [Y_i - f(\boldsymbol{X}_i, \boldsymbol{\beta})]^2. \tag{2}$$

According to the optimization model given in equation (2), the nonlinear f function is defined for the electrical characterization of organic semiconductor materials as follow

$$I = I_0 \exp\left(\frac{qV_0}{nkT}\right)\left[1 - \exp\left(\frac{-qV_0}{nkT}\right)\right]. \tag{3}$$

Here, q is an elementary charge, k is the Boltzmann constant, T is an absolute temperature, n is the ideality factor, I_0 is a reverse saturation current and is equal to $A^*T^2 \exp\left(\frac{-q\Phi_b}{kT}\right)$ in which Φ_b is a barrier height, A is a Richardson Constant. And also, $V_0 = V - IR_s$, where R_s is a series resistance and V is considered as input variable while the output variable called I, presented in equation (3). In this study, it is aimed to estimate the key electrical parameters n, R_s and Φ_b of Schottky-barrier solar cells with an Al/rubrene/p-Si structure. Here, parameter vector is defined as $\boldsymbol{\beta} = [n \;\; R_s \;\; \Phi_b]$. Directly estimating the parameter vector $\boldsymbol{\beta}$ from experimental data is highly challenging due to measurement because of the Cobalt-60 gamma radiation uncertainties. At this point, the AI optimization algorithms offer a more reliable alternative to traditional methods, thanks to their derivative-free nature, robustness against initial conditions, and ability to navigate complex landscapes with multiple optima.

In this study, twelve different AI optimization algorithms were applied to the I–V data derived from the described model. Each AI algorithm was assessed for its ability to accurately and consistently estimate the

model parameters. These algorithms utilize heuristic strategies in their solution search processes, enabling them to reach both global and local optima. As such, the approach offers not only high accuracy but also computational efficiency and stability, establishing a robust modeling framework.

Methods

The Collection of the Data

The following experimental sets out the fabrication process of an Al/Ru/p-Si Schottky diode on a p-type (111) silicon wafer, with a thickness of 280 μm and a resistivity of 10 Ω·cm. The wafer was initially subjected to a thorough cleansing process employing the RCA chemical cleaning method. The detail information about experiment can be seen in the studies of Akay et al. 2019, Akay et al. 2020, Akay et al. 2024. All measurements were conducted with the assistance of a microcomputer via an IEEE-488 AC/DC converter card. The relastionship between input (V) and response (I) values can be seen in Figure 1. It can be easily said from Figure 1 that the input and the response values have nonlinear relationship.

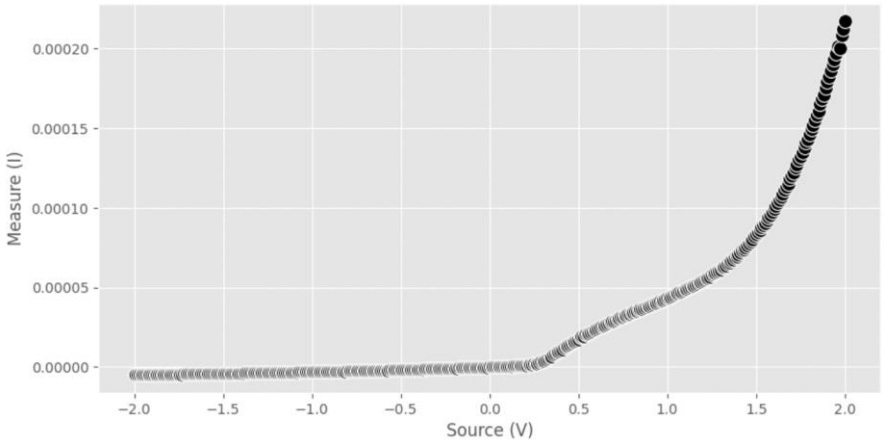


Figure 1. The relationship of input (V) and response (I) values

Statistical Analysis

In this study, the parameter estimation process of a nonlinear electrical model based on experimentally obtained current-voltage (I–V) data was carried out using AI-based algorithms, rather than traditional derivative-based optimization methods. Due to the nonlinear nature of $\boldsymbol{\beta}$ as well as the inherent uncertainties in the measurement data, classical methods often fail to provide reliable estimations. These types of problems typically involve a complex solution space with multiple local minima and are highly sensitive to initial values. Thus, the AI optimization algorithms, inspired by various disciplines, were used in this study. These algorithms are heuristic approaches derived from nature, social behavior, or learning processes and aim to reach higher-quality solutions through an evolutionary process, starting with a numerical population. Each algorithm evaluates candidate solutions searching the problem space based on its internal mechanism, improves them, and generates new candidates to reach the optimal solution. Generally, an initial population of randomly generated individuals is created, followed by information exchange and variation operations among selected individuals. This process iteratively continues to enhance the quality of the solutions step by step. A general working schema of the AI optimization algorithms is presented in Figure 2.

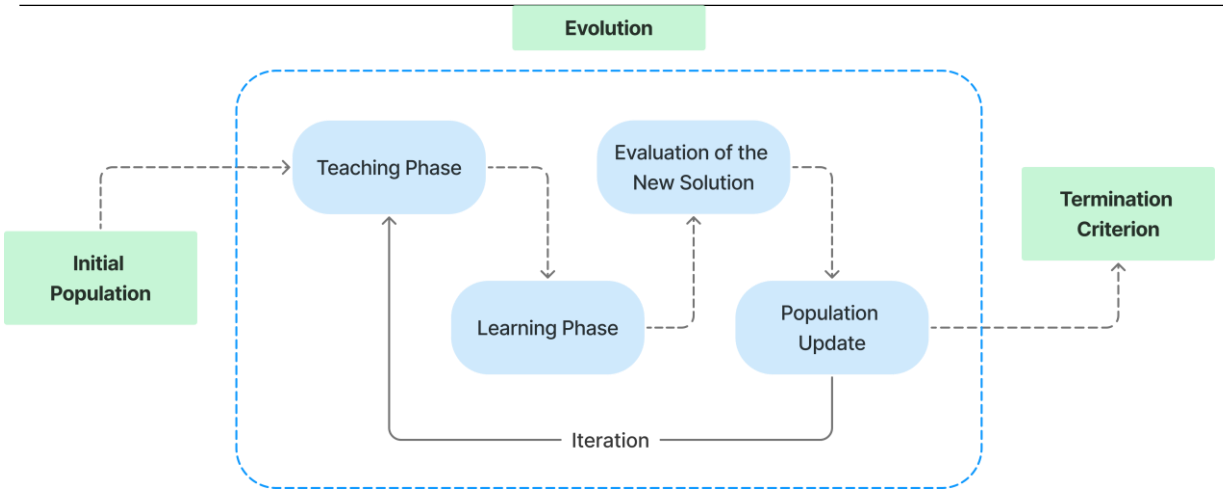


Figure 2. A general schema about working principles of the AI optimization algorithms

Each of these algorithms has been configured to perform parameter estimation for the model built on experimental I–V data, with tuning parameters, carried out for the problem at hand. This process involved optimizing tuning parameters such as population size, number of iterations, crossover rates, learning coefficients, and other control variables to maximize each algorithm's performance. Thus, each algorithm was enabled to function in the most efficient manner appropriate to its structure.

In this study, twelve AI algorithms have been applied in a comparable manner with the aim of minimizing the error function. The tuning parameter values of each AI algorithms are defined by using data-driven expert knowledge and commonly used in the literature. Through the *MAPE* and R^2 metrics, the relative performance of each AI algorithm in terms of accuracy and stability is analyzed. This methodological approach not only enhanced the accuracy of parameter estimation but also provided a robust framework for how AI optimization algorithms can be effectively applied to complex engineering-based modeling problems. In modeling the electrical behavior of electronic structures exposed to external factors such as radiation, these flexible and data-driven optimization approaches offer significant potential for both academic research and industrial applications.

RESULTS

This study aimed to estimate the electrical parameters of Al/rubrene/p-Si Schottky-barrier solar cells after exposure to Cobalt-60 gamma irradiation within the framework of a nonlinear model with minimum error. The optimization results are presented in Table 1. It can be said from Table 1 that the ABC, the DE and the TLO have better performance according to the R^2 metric. The ACOR has the best performance for computing time. It is seen from the results that model parameters estimation is achieved by using the AI optimization algorithms.

Table 1. Optimization results of the AI algorithms with model parameters estimation

AI	**Iteration Number**	**Total Working Time**	**Iteration Time**	***Performance metrics***		***Model parameters estimation***		
				MAPE	***R^2***	***Φ_b***	***n***	***R_s***
ABC	93	1.3182	0.0126	0.330466	**0.702430**	0.672212	5.000	100.00
ACOR	67	**0.3007**	**0.0034**	0.332134	0.694962	0.672283	5.000	41.31
BA	32	0.5016	0.0046	0.597449	0.395611	0.696066	4.547	45.64
CSA	16	0.7185	0.0069	0.342103	0.666398	0.674036	4.878	54.19
DE	56	0.6041	0.0059	0.330466	**0.702430**	0.672212	5.000	100.00

FPA	86	0.6742	0.0064	0.333669	0.699231	0.672529	4.988	88.39
GA	99	0.7695	0.0075	0.437047	0.641893	0.683048	4.650	99.69
GWO	100	0.6316	0.0062	0.330507	0.702430	0.672215	5.000	100.00
HS	72	0.6773	0.0065	0.348827	0.691971	0.673716	4.960	65.67
JA	8	0.5124	0.0049	0.338779	0.695660	0.672702	5.000	43.80
PSO	38	0.6175	0.0073	0.330492	0.702430	0.672214	5.000	100.00
TLO	51	1.3489	0.0135	**0.330466**	**0.702430**	0.672212	5.000	100.00

These results show that the AI optimization algorithms used in the study not only cover a broader range of parameter values compared to classical methods but also offer more reliable estimations with lower error and higher explanatory power. This highlights the applicability of the AI optimization methods in the electrical characterization of organic semiconductor structures, demonstrating significant advantages over classical analytical approaches in terms of accuracy, flexibility, and computational efficiency.

DISCUSSION AND CONCLUSION

In this study, a comprehensive analysis was conducted by integrating nonlinear regression models with the AI optimization algorithms for the electrical characterization of organic solar cells. The results not only revealed the effects of Cobalt-60 gamma irradiation on device performance but also demonstrated that electrical parameters derived from experimental data can be accurately and reliably estimated. The nonlinear modeling approach effectively represented complex system behaviors that are difficult to explain using classical methods, clearly revealing the changes in key parameters, presented as $\boldsymbol{\beta}$.

The AI optimizaton algorithms not only provided balanced performance in modeling but also introduced flexibility into the parameter estimation process by overcoming the limitations associated with traditional techniques. Through AI-based methods, efficient exploration of wide search spaces became feasible, and the derivative-free structure enhanced the generalizability of the modeling process. The findings demonstrate that the AI optimization algorithms offer a robust alternative for analyzing organic electronic devices operating under environmental stress conditions, such as radiation. In this context, AI optimization algorithms are considered applicable to industrial scenarios, particularly for real-time parameter estimation and device performance monitoring.

In conclusion, this study demonstrates the effectiveness of AI-supported parameter estimation processes for both academic research and industrial applications and provides a solid methodological foundation to guide future work in this field.

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Conflict of Interest

If there is no conflict of interest of the authors, it should be written as "The authors have declared that there is no conflict of interest".

Author Contributions

Murat Açıkgöz: Writing – original draft, review & editing, Statistical Analysis, Methodology, Investigation, Interpretation

Defne Akay: Writing – original draft, review & editing, Obtaining data, Investigation, Interpretation, Conceptualization

Özlem Türkşen: Writing – original draft, review & editing, Investigation, Interpretation, Methodology, Conceptualization

Data Mining İn A Three-Vector Decision Support System For Management Of Development Programs İn The Republic Of Kazakhstan: Methodology And Practical Application

Mafura K. Uandykova[1*], Akhmed S. Baikhojayev[1], Anel D. Yeleukulova[1], Assel D. Yeleukulova[1]

[1]Narxoz University, School of Digital Technologies, 050035, Almaty, Kazakhstan

*Corresponding author e-mail: mafura.uandykova@narxoz.kz

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Abstract

This article presents a methodology for data mining within the framework of a three-vector decision support system (TVDSS) for managing development programs in Kazakhstan. The research integrates technological, financial, economic, and socio-political vectors with the modular notebook architecture "Objects-Processes-Projects-Environment". Despite Kazakhstan's distinguished position in e-government development rankings (24th place, EGDI 0.9009), the practical implementation of governmental initiatives reveals profound systemic challenges, evidenced by the unprecedented suspension of nine out of ten national projects in September 2023. The study further highlights Kazakhstan's declining economic complexity (88th position, ECI=-0.47), demonstrating the persistence of resource-dependent trajectories despite substantial budgetary allocations to development programs. The theoretical model indicates potential enhancements compared to conventional approaches, including improved analytical comprehensiveness (40-45%), significant reduction in analytical preparation timeframes (70-80%), and enhanced forecasting precision (20-30%). The findings suggest that the three-vector approach establishes a methodological foundation for the balanced advancement of all innovative economy components.

Keywords*: data mining, decision support systems, three-vector model, open government data, program-project management, modular architecture*

INTRODUCTION

The modern transformation of public administration in Kazakhstan is characterized by a significant contradiction: high positions in international rankings of digital development and actually low efficiency in implementing government programs. The republic has risen in the UN e-government development ranking from 81st place in 2008 to 24th in 2024, reaching an EGDI index of 0.9009 (United Nations, 2024). However, the practice of implementing state programs and national projects demonstrates serious systemic problems.

These problems are confirmed by radical changes in the state planning system. Since 2020, Kazakhstan has transitioned from state development programs to national projects, implementing project management technologies according to the PMI (Project Management Institute) methodology. A total of 10 national projects were developed with funding of 8,147 billion tenge in 2022 (98.1% or 7,976 billion tenge was disbursed), which was 2.5 times higher than in 2021 (Kurmanova, 2023). However, according to the Supreme Audit Chamber, the effectiveness of national projects implementation was insufficient, which led to an unprecedented step- on September 22, 2023, by Government Decree No. 828, 9 out of 10 national projects were declared invalid (Government of the Republic of Kazakhstan, 2023).

This discrepancy between formal digitalization indicators and the actual effectiveness of government initiatives is largely due to an unbalanced approach to managing government programs, where technological aspects develop at the expense of financial-economic and socio-political components An additional factor—the quality deficiency of Open Government Data (OGD)—is evidenced by analysis

indicating that the structure and volume of most datasets on the state portal remain insufficient for comprehensive analytical scrutiny (Nurgazin and Serebrennikov, 2025).

It is also noteworthy that Kazakhstan's Economic Complexity Index (ECI), calculated by Harvard University, has deteriorated. According to 2021 data, Kazakhstan ranks only 88th with an ECI=-0.47, which indicates the predominance of resource orientation in the economy (Harvard Growth Lab, 2021). This points to systemic problems in economic diversification and development of its innovative potential, despite significant budget investments in development programs.

The OGD value creation theory posits that merely publishing data proves ineffective—it necessitates user activation (Benmohamed et al., 2024). Research indicates that transparency without verification mechanisms and practical applications may even diminish public trust (Grimmelihuijsen et al., 2020; Šlibar and Mu, 2022).

Addressing these issues requires an integrated approach that combining technological, financial, and social aspects into a unified decision support system. The research aims to develop a data mining methodology within a three-vector DSS with a modular tetrad architecture to improve the management efficiency of state development programs in Kazakhstan.

MATERIAL AND METHODS

Material

The research is based upon a comprehensive analysis of theoretical sources, methodological developments, and empirical data in the domains of data exploration, decision support systems, and public administration.

The informational foundation of the study encompasses:

1. Statutory and strategic instruments of the Republic of Kazakhstan pertaining to state planning and program-project governance, including state programs and national initiatives spanning the period 2020-2025.

2. Data from international indices and rankings, in particular the UN E-Government Development Index, the Economic Complexity Index and the Global Innovation Index.

3. The results of scientific research in the field of data mining, decision support systems, open government data and innovation development from leading academic journals.

4. Statistical data concerning the implementation of state programs and national projects in Kazakhstan, including budgetary execution information.

The research focuses on three fundamental dimensions of development program management: technological, financial-economic, and socio-political, which collectively constitute the three-vector paradigm of analysis and decision support.

Methods

Conceptualization of the Three-Vector System with Tetrad Architecture

The developed methodology integrates the three-vector development paradigm with a modular tetrad architecture for the decision support system. The three-vector development paradigm encompasses:

1. Technological Vector – analyzes innovative potential, technological modernization, digital infrastructure, and process automation. This vector evaluates the technological readiness of industries and regions, identifies "bottlenecks," and determines priority directions for innovative development.

2. Financial-Economic Vector – interrogates investment mechanisms, economic efficacy, programmatic budgeting protocols, and return on capital deployment. This vector optimizes resource allocation frameworks, quantifies economic impact metrics, and forecasts financial outcomes of strategic initiatives.

3. Socio-Political Vector –assesses the level of public trust, social impact indicators, the spread of digital literacy and political support for development initiatives. This vector analyzes social risk factors and ensures the democratic legitimacy of development programs.

The modular tetradic architecture of the DSS is inspired by the conceptual frameworks of researchers from the Central Economics and Mathematics Institute (CEMI) of the Russian Federation (G. Kleiner and V. Makarov) and contemporary theories of economic systems, particularly the "System- Technology-Economy-Society" (STES) tetrad (Kleiner and Makarov, 2022). The architecture consists of interconnected modules:

1. The "Objects" Module: facilitating the collection, consolidation, and structuring of data pertaining to management entities and objects (enterprises, industries, regions, infrastructure projects).

2. The "Processes" Module: analyzing business processes germane to program implementation, identifying operational bottlenecks, and optimizing management protocols to ensure operational efficacy.

3. The "Projects" Module: overseeing the lifecycle of projects and programs, encompassing the planning and monitoring of resources, timelines, risks, and outcomes.

4. The "Environment" Module: monitoring exogenous and endogenous factors of influence, including market dynamics, socio-economic trends, and regulatory modifications.

Integration of the three-vector paradigm with the tetrad architecture creates a multidimensional analytical matrix that provides a comprehensive assessment of development programs in all key aspects.

Data mining methods in TVDSS

The system uses a comprehensive set of data mining methods adapted to the features of government program management:

1. Application of multidimensional analysis methods:

- Fuzzy logic of working with uncertainties and incomplete data
- Analytical hierarchy process for structuring priorities (AHP) (Saaty and Vargas, 2022)
- Compromise rating methods to assess the balance

2. Methods of predictive analysis:

- ARIMA and exponential smoothing for time series analysis
- Neural network models for identifying nonlinear dependencies
- Ensemble techniques to improve prediction accuracy (Random Forest, gradient amplification)

3. Machine learning methods:

- Classification algorithms for data categorization (SVM, k-NN)
- Cluster analysis to identify groups with similar characteristics (k-tools, DBSCAN)
- Dimensionality reduction techniques (PCA, T-SNE) for multidimensional data visualization.

4. Natural language processing methods:

- Analysis of text documents and regulations
- Processing expert opinions and public discussions
- Analysis of the mood of media content and social networks

The synthesis of these methodologies within a unified analytical platform facilitates a holistic evaluation of all facets of development programs and the formulation of well-informed recommendations for decision-making bodies.

RESULTS

Methodological Approach to System Application

1. Formation of the research information base:

- Determining the list of analyzed programs, regions, and industries
- Forming a system of indicators for three development vectors
- Determining sources and methods of data collection
- Developing procedures for data validation and verification

2. Multi-vector analysis of the current state:

- Assessment of technological readiness of industries and regions
- Analysis of investment efficiency and budget expenditures
- Study of social effects of development programs
- Identification of interregional and intersectoral imbalances
- Analysis of economic diversification problems

3. Scenario modeling:

- Development of alternative development scenarios
- Assessment of scenario implementation consequences across all vectors
- Sensitivity analysis of results to changes in key parameters
- Identification of critical points and success factors

4. Recommendation formation:

- Development of a set of measures to optimize programs
- Differentiation of recommendations considering regional and sectoral specifics
- Determination of recommendation implementation mechanisms
- Development of an implementation effectiveness monitoring system

Special attention in the methodology is paid to developing a system of multipliers for evaluating program effectiveness across three development vectors. Calculation of resource and resultant indicators is based on a limited number of statistical indicators, making the approach universal and applicable to any country.

Open Data Value Activation Model

At the core of the three-vector system is the Open Government Data Value Activation Model (OGD Value Activation Model), adapted to Kazakhstan's specifics. The model includes four interconnected components:

1. Ensuring data quality and accessibility:

- Development of metadata standards and ensuring their compliance
- Implementation of automated data quality control
- Development of APIs for programmatic access to government data
- Ensuring regularity of updates and data completeness (Wang and Shepherd, 2020)

2. Development of data user ecosystem:

- Formation of a community of developers and analysts
- Conducting hackathons and competitions to stimulate data use
- Development of digital competencies for population and civil servants
- Creation of incubators for startups using open data

3. Creation of value products based on data:

- Development of services and applications for citizens and businesses
- Creation of analytical tools for various stakeholders
- Integration of open data into business processes and social projects
- Development of visualization systems and interpretation of complex data (Benmohamed et al., 2024)

4. Monitoring and development of public value:

- Measuring the economic effect of using open data
- Assessment of social impact of data-centric projects
- Collection and analysis of feedback from data users
- Formation of a data-based decision-making culture

The value activation model serves as a methodological basis for transforming formal publication of open data into real tools for decision support and public control.

Theoretical Model of Comparative Effectiveness

Table 1. Theoretical model comparing TVDSS with traditional approaches

Parameter	Traditional approach	Three-vector system	Potential effect
Analysis comprehensiveness	Fragmented (40-45%)	Systematic (85-90%)	Increased completeness by 40-45%
Analytics preparation time	14-21 days	3-5 days	Reduction by 70-80%
Forecast accuracy (RMSE)	18-20%	12-15%	Improvement by 20-30%
Data integration	Limited	Comprehensive	Qualitative improvement
Consideration of social factors	Minimal	Systematic	Qualitative improvement
Process transparency	Low	High	Qualitative improvement

The theoretical model demonstrates the potential of the three-vector approach to overcome existing limitations and ensure balanced development of programs, taking into account technological, financial-economic, and socio-political aspects (Uandykova et al., 2024a).

Limitations and future research

The study has a several limitations. The three-vector model looks promising in theory, but it has not yet been fully tested in real government conditions. Kazakhstan also faces problems with data quality and accessibility, which may make implementation difficult. Government agencies may resist the introduction of such a new approach, since changing established systems always creates institutional friction.

Future research should focus on practical testing of this model in real-world management situations. Researchers could compare how this approach works in Kazakhstan and other countries with similar development challenges. In addition, it is necessary to develop specific measurement indicators for each vector and create user-friendly tools that government officials can easily use.

DISCUSSION AND CONCLUSION

This study presents a comprehensive methodology for data mining within a three-vector decision support system for managing development programs in the Republic of Kazakhstan. The proposed methodology is aimed at overcoming existing problems in the formation and implementation of government programs and national projects, as evidenced by the unprecedented decision to suspend 9 out of 10 national projects and the negative dynamics of Kazakhstan in international ratings of innovative development.

A three-vector approach integrating technological, financial, economic, and socio-political vectors, combined with the modular architecture of the Objects-Processes-Projects-Environment tetrad, creates a theoretical and methodological basis for a systematic analysis of development programs and the formation of sound recommendations for their optimization. This approach allows us to overcome the limitations of

traditional methods, focusing primarily on technological aspects, and ensures the balanced development of all components of the innovation economy (Kleiner and Makarov, 2022).

Of particular significance within the proposed methodology is the value activation model for open government data, which facilitates the transformation of formal data publication into tangible instruments for decision support and public oversight. This model accounts for all stages of the data lifecycle—from ensuring data quality and accessibility to the creation of value-added products and the evaluation of their impact on the economy and society (Wirtz et al., 2022).

The results of the theoretical analysis demonstrate the significant potential of the three-vector system for improving the effectiveness of development program management. Preliminary estimates show the possibility of increasing the completeness of the analysis by 40-45%, reducing the time for preparing analytics by 70-80%, and improving forecast accuracy by 20-30% compared to traditional approaches.

The implementation of the proposed methodology requires systemic changes in the practice of public administration, including the development of new standards and regulations, the creation of an appropriate information technology infrastructure, the development of necessary competencies among civil servants and the formation of a culture of decision-making based on data. These changes should be adopted on the basis of a responsibility matrix that ensures effective cooperation between all stakeholders — government agencies, the business community, scientific and educational organizations, and civil society (Fred & Godenhjelm, 2023).

Kazakhstan's economic transformation toward innovation requires not merely technological modernization but comprehensive development of all economic system components, including monetary- financial and socio-political dimensions. The application of a three-vector approach to the management of development programs creates a methodological basis for such a transformation, ensuring consistency and balance of management decisions (Wandykova et al., 2024b).

Further improvement of the proposed methodology and its practical application will make it possible to create a scientific and methodological basis for improving the effectiveness of public administration and achieving strategic goals of innovative development in the Republic of Kazakhstan.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contribution

M.K. Uandykova: Conceptualization, Methodology, Writing - Original Draft, Supervision
A.S. Baikhojaev: Data curation, Investigation; Anel Yeleukulova: Visualization, Validation, Formal analysis; Asel Yeleukulova: Writing - Review and editing

Comparison of Customer Feedback Prediction Performance Using Natural Language Processing Algorithms (967)

Hıdır Can Saltık[1*] , Hülya Olmuş[2],

[1]Gazi University, Graduate School of Natural and Applied Sciences, Data Science, 06500, Ankara, Turkey
[2]Gazi University, Faculty of Sciences, Department of Statistics, 06500, Ankara, Turkey

*Corresponding author e-mail: hcansaltik@gmail.com

Abstract

With the increase in e-commerce volume during the COVID-19 pandemic, the number of customer reviews has also grown significantly. This study aims to analyze customer feedback using Natural Language Processing (NLP) algorithms. Both frequency-based and prediction-based embedding methods are commonly used techniques in NLP. Feature extraction was performed using Term Frequency-Inverse Document Frequency (TF-IDF) and Word2Vec techniques. For label prediction, Random Forest (RF), LightGBM, and XGBoost classifiers were applied, and their performances were compared. Additionally, to evaluate the performance of the most commonly used classification models in imbalanced datasets, Weighted Precision, Weighted Recall and Weighted F1 metrics were employed.
This study aims to develop a classification model that assigns customer reviews about purchased products to specific categories. The performance of different NLP algorithms has been evaluated using data obtained from the textile sector, consists of 14,430 reviews, each labeled with one of 29 different classes. According to the experimental results, the best-performing model achieved a Weighted Precision of 0.6597, a Weighted Recall of 0.6831, and a Weighted F1 Score of 0.6598. These findings demonstrate the effectiveness of different embedding and classification methods in categorizing customer reviews.

Key words: *Text Embedding, TF-IDF, Customer Review Classification, Machine Learning Classifiers*

INTRODUCTION

In recent years, e-commerce platforms, which have rapidly become widespread, have fundamentally transformed shopping habits by offering a more flexible and time-saving experience. Unlike physical store shopping, consumers can shop on e-commerce platforms at any time and from any location. Therefore, e-commerce creates a gain in terms of effort and time for users.

On e-commerce platforms, customers provide valuable information for other potential buyers by leaving reviews about the products they have purchased (Tohma, 2024). According to a study conducted in 2020, 91% of people read reviews written by others and 84% trust them as much as personal recommendations (Regan, 2020). Moreover, reviews contain analyzable information about product quality, brand opinions, customer satisfaction, complaints about the product and similar topics. For this reason, analyzing customer reviews, classifying them, conducting sentiment analysis, etc., are important tasks for e-commerce platforms. Accurately and effectively analyzing reviews plays a crucial role in increasing customer satisfaction, reducing return rates and similar outcomes.

At this point, the concept of Natural Language Processing (NLP), which aims to analyze and interpret human language through computers, comes into play. NLP encompasses various computational techniques based on theoretical foundations for the automatic analysis and representation of human language (Cambria & White, 2014). Natural language processing goes beyond the structural analysis of language to include

various application areas such as sentiment analysis, text classification, summarization and question-answering systems. These topics have been comprehensively examined and discussed in detail across different sections by Jurafsky and Martin (2021).

In this study, text representation methods such as TF-IDF and Word2Vec were used to numerically represent customer reviews made on e-commerce platforms to determine which category the reviews belong to. Based on the obtained text representations, multi-class text classification was performed using machine learning-based algorithms such as Random Forest, LightGBM and XGBoost. In this way, it was aimed to automatically categorize customer reviews, thereby enabling a deeper analysis of customer experiences.

The dataset used in the study consists of customer reviews for products purchased from one of Turkey's leading clothing brands. The dataset contains 14,430 reviews and each review is labeled with only one category out of 29 different classes. The dataset was split into training and test sets for model training. The model was trained with the training set and the results of the methods were compared using the test set.

The model's performance was evaluated using metrics such as F1 Score, precision and recall. Considering the imbalanced distribution of the dataset, weighted metrics were taken into account.

MATERIAL AND METHODS

Dataset

In this study, the performance of different methods was examined using customer reviews of products purchased from one of Turkey's leading clothing brands. The aim of the study is to evaluate the performance of various methods based on these reviews.

Structure of the Dataset

The dataset contains a total of 14,430 customer reviews about products. Each review is labeled with only one of 29 different categories. The dataset was split into two parts: 80% for training and 20% for testing. The training set was used for model learning, while the test set was reserved for evaluating model performance.

The dataset consists of two columns:

• Review: Customer reviews

• Label: The category to which the review belongs

Table 1. First few rows of the dataset

Table 1. First few rows of the dataset

Review	Label
Kalıbı geniş l beden giyen eşime 2beden büyük geldi …	Fit / Ölçü / Kalıp
Nefes alacak bir yapıda. O yüzden kenarlarında minik boşluklar var…	Ana Malzeme / Kumaş Kalitesi
Hediye almıştım üzerinde etiket yok. Onun dışında güzel	Etiket Konuları
İnanılmaz güzel bir parça bayildim sağlam kargolama yapimisti	Paketleme

Distribution of the Dataset

The distribution of the dataset is important in terms of applying and evaluating the methods. Figure 1 presents a chart showing the distribution of the dataset.

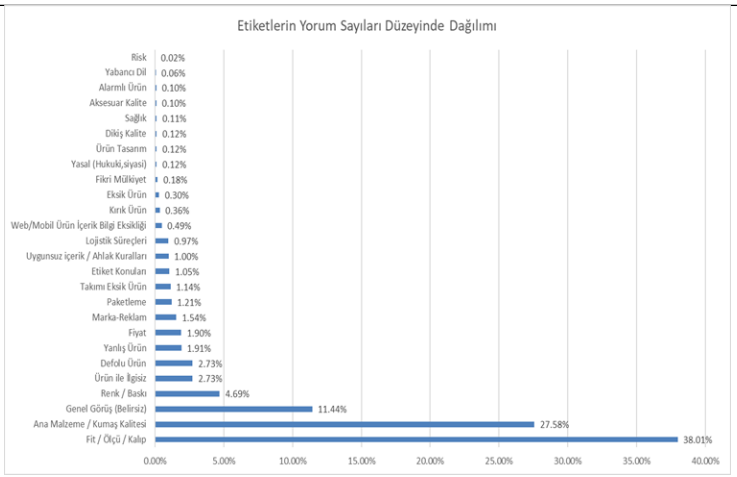

Figure 1. Distribution of the dataset

As shown in Figure 1, the dataset has an imbalanced distribution. The top four categories with the highest number of reviews (Fit/Ölçü/Kalıp, Ana Malzeme/Kumaş Kalitesi, Genel Görüş (Belirsiz) and Renk/Baskı) account for 81.72% of the entire dataset. This situation provides a critical insight for method evaluation.

Word Cloud

In the context of review analysis, word clouds can be used in the literature. Word clouds provide a visual representation based on the frequency of words in textual data; more frequently used words appear larger and more prominent in the visualization (Heimerl and others, 2014).

Figure 2. Word Cloud

Figure 2 shows the word cloud. Accordingly, the phrase “çok güzel” ("very nice") appears to be the most frequently used expression in the reviews.

Feature Extraction and Representation

TD-IDF

TF-IDF is a frequency-based metric used to determine how relevant a word is to a specific document within a collection of documents (Deniz and others, 2022). It is a statistical model employed to assess the importance of words in a corpus (Das and Alphonse, 2023). Term Frequency (TF) indicates how often a word appears in a document, while Inverse Document Frequency (IDF) measures how common or rare a word is across the entire document set. If a word appears frequently in many documents, its informational value is considered low. The TF-IDF value is calculated by multiplying the TF and IDF values:

$$\text{TF-IDF}(t, d) = \text{TF}(t, d) \times \text{IDF}(t)$$

Word2Vec

Word2Vec is a neural network-based word representation model proposed in 2013 by Thomas Mikolov and his colleagues, designed to capture the semantic similarities between words (Mikolov and others, 2013). Words are represented as numerical vectors in a format that computers can understand. Word2Vec includes two main models: CBOW and Skip-Gram. The CBOW (Continuous Bag of Words) model predicts a target word by using the average of the vector representations of its surrounding context words (McCormick, 2016). In contrast, the Skip-Gram model works in the opposite way by using the vector representation of a central word to predict the surrounding words (Mikolov and others, 2013). In this study, word vectors were generated using the CBOW model. The vector dimension was set to 256, words with fewer than three occurrences were excluded and the window size was set to 5.,

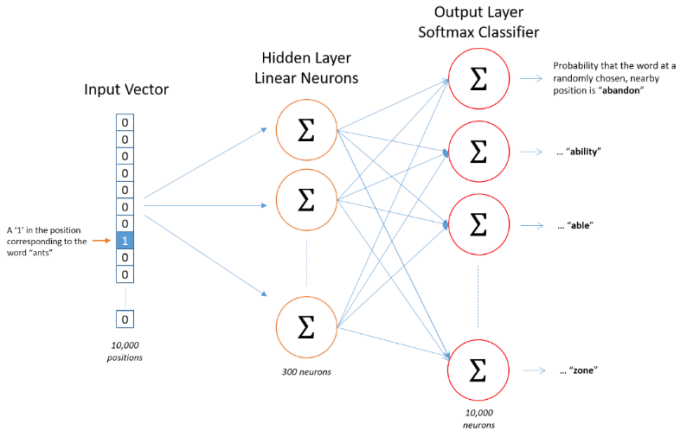

Figure 3. An example representation of the Word2Vec architecture (McCormick, 2016)

Single-Label Multi-Class Classification

In single-label classification problems, when each instance is associated with exactly one label from a set containing more than two possible labels, the problem is referred to as a single-label multi-class classification problem (Tsoumakas and Katakis, 2008). There are three common approaches used to solve such problems:

1. One-vs-All Approach: This method involves training a separate binary classifier for each label class and requires N models for a problem with N classes (Rifkin and Klautau, 2004).
2. One-vs-One Approach: This method trains binary classifiers for every possible pair of classes in a multi-class classification problem. For a problem with N classes, a total of N(N-1)/2 classifiers are created (Hsu and Lin, 2002).
3. Direct Multi-Class Algorithms: Algorithms such as Random Forest, XGBoost and LightGBM inherently support multi-class classification problems.

In this study, direct multi-class algorithms were employed. The algorithms used include Random Forest, LightGBM and XGBoost.

Table 2. An example structure of a multi-class algorithm

Feautures	**Class**
F1	A
F2	B
F3	C
F4	A

The table presented in Table 2 illustrates the structure of a multi-class algorithm. The "features" represent the input part of the model, derived from each comment using feature extraction methods. The "class" refers to the label assigned to each comment.

Evaluation Metrics

In single-label multi-class classification problems, various metrics are used to evaluate the performance of the models. Given the imbalanced class distribution in our dataset, the choice of evaluation metrics is particularly important. Therefore, performance evaluations in this study were conducted using the macro and weighted average versions of the metrics. In the macro approach, each class is given equal weight in the average, regardless of the number of samples it contains, ensuring that classes with fewer and more data are treated equally (Grandini and others, 2020). In contrast, the weighted average approach calculates each metric by weighting the class-specific scores according to the number of samples in each class. Since Accuracy treats all samples equally, it tends to reflect the correct classification of the majority class in imbalanced datasets, which may misleadingly inflate the perceived performance of the model. Hence, this study employed the following metrics: Macro Precision, Weighted Precision, Macro Recall, Weighted Recall, Macro F1 Score and Weighted F1 Score.

Confusion Matrix

The confusion matrix is a tool used to evaluate the accuracy and error rates of classification algorithms. It provides a visual analysis of the correct and incorrect classifications made by the algorithm (Düntsch and Gediga, 2019). The matrix consists of four components: True Positive (TP), False Positive (FP), True Negative (TN) and False Negative (FN).

Table 3. Confusion matrix

		ACTUAL	
		Negative	Positive
PREDICTION	Negative	TRUE NEGATIVE	FALSE NEGATIVE
	Positive	FALSE POSITIVE	TRUE POSITIVE

Precision

Precision is the ratio of correctly classified positive samples to all samples that the algorithm predicted as positive (Sokolova and Lapalme, 2009). The precision metric plays a critical role, especially in cases where false positives can be costly or risky.

$$\text{Precision} = \frac{TP}{TP+FP}$$

Macro Precision calculates the precision for each class individually and then averages them. Each class is given equal weight regardless of its size:

$$\text{Macro Precision} = \frac{1}{c}\sum_{i=1}^{c}\frac{TP_i}{TP_i+FP_i}$$

c: Number of classes

TP_i: Number of True Positive to the i-th class

FP_i: Number of False Positive to the i-th class

Weighted Precision is calculated by multiplying the precision of each class by the number of instances in that class, summing them up and dividing by the total number of instances. This gives more weight to classes with more samples:

$$\text{Weighted Precision} = \frac{\sum_{i=1}^{c} n_i x \frac{TP_i}{TP_i+FP_i}}{\sum_{i=1}^{c} n_i}$$

n_i: Number of samples belonging to the i-th class
TP_i: Number of True Positive to the i-th class
FP_i: Number of False Positive to the i-th class

Recall

Recall is the proportion of actual positive instances that are correctly identified by the algorithm (Powers, 2020). The recall metric plays a critical role, especially in cases where false negatives are costly or risky.

$$\text{Recall} = \frac{TP}{TP+FN}$$

Macro Recall calculates the recall for each class and then takes the average. Each class's recall value is treated equally, regardless of the number of samples.

$$\text{Macro Recall} = \frac{1}{c}\sum_{i=1}^{c} \frac{TP_i}{TP_i+FN_i}$$

c: Number of classes
TP_i: Number of True Positive to the i-th class
FN_i: Number of False Negative to the i-th class

Weighted Recall multiplies each class's recall by the number of samples in that class (n_i), sums these values and then divides by the total number of samples. This metric can be more reliable for imbalanced datasets because classes with more samples contribute more to the model's overall performance, although it may overlook performance on underrepresented classes.

$$\text{Weighted Recall} = \frac{\sum_{i=1}^{c} n_i x \frac{TP_i}{TP_i+FN_i}}{\sum_{i=1}^{c} n_i}$$

n_i: Number of samples belonging to the i-th class
TP_i: Number of True Positive to the i-th class
FN_i: Number of False Negative to the i-th class

F1 Score

F1 Score is a statistical measure that represents the overall performance of a classification algorithm by taking the harmonic mean of precision and recall (Powers, 2020). One of its primary goals is to balance precision and recall. By giving equal importance to both metrics, it becomes particularly useful in classification problems with imbalanced class distributions.

$$\text{F1 Score} = 2 \text{ x } \frac{\text{Precision x Recall}}{\text{Precision + Recall}}$$

Macro F1 Score is calculated by taking the F1 score of each class with equal weight. It ensures that each class contributes equally to the overall metric.

$$\text{Macro F1 Score} = \frac{1}{c}\sum_{i=1}^{c} F1_i$$

c: Number of classes
$F1_i$: F1 Score to the i-th class

Weighted F1 Score is computed by weighting each class's F1 score by the number of instances in that class. In multi-class problems, this allows the performance of classes with more instances to have a stronger influence on the overall score.

$$\text{Weighted F1 Score} = \frac{1}{n}\sum_{i=1}^{c} n_i x F1_i$$

n: Total number of samples
n_i: Number of samples belonging to the i-th class
$F1_i$: F1 Score to the i-th class

RESULTS

In this section, the results of the applied methods are discussed. The performance of the methods is evaluated using the macro and weighted versions of the precision, recall and F1 score metrics. Within the scope of the study, three different embedding techniques and four different classification models were used to identify the best-performing embedding and classifier pair.

First, the TF-IDF technique was used for embedding and the performance of the Random Forest, XGBoost and LightGBM models was measured. Then, the Word2Vec embedding technique was applied and performance was evaluated using the same models. Table 4 presents the results of the models for the "Fit/Ölçü/Kalıp" label.

Table 4. Model Results for the 'Fit/Ölçü/Kalıp' Label

Embedding	Classifier	F1	Precision	Recall
TF-IDF	LightGBM	0.7581	0.7362	0.7814
TF-IDF	Random Forest	0.7951	0.6941	0.9304
TF-IDF	XGBoost	0.8224	0.7666	0.8871
Word2Vec	LightGBM	0.7381	0.6972	0.7841
Word2Vec	Random Forest	0.7696	0.6964	0.8600
Word2Vec	XGBoost	0.7749	0.7063	0.8582

Looking at Table 4, based on the F1 score of 0.8224 and the Precision score of 0.7666, the TF-IDF embedding method with the XGBoost classifier appears to be successful, while based on the Recall score of 0.9304, the TF-IDF embedding method with the Random Forest classifier is found to be effective. Table 5 presents the model results for the 'Ana Malzeme/Kumaş Kalitesi' label.

Table 5. Model Results for the 'Ana Malzeme/Kumaş Kalitesi' Label

Embedding	Classifier	F1	Precision	Recall
TF-IDF	LightGBM	0.5726	0.529	0.6239

TF-IDF	Rastgele Orman	0.6272	0.6092	0.6463
TF-IDF	XGBoost	0.6212	0.5799	0.6687
Word2Vec	LightGBM	0.5254	0.4842	0.5741
Word2Vec	Rastgele Orman	0.5381	0.4628	0.6426
Word2Vec	XGBoost	0.5456	0.4767	0.6376

Looking at Table 5, based on the F1 score of 0.6272 and the Precision score of 0.6092, the TF-IDF embedding method with the Random Forest classifier appears to be successful, while based on the Recall score of 0.6687, the TF-IDF embedding method with the XGBoost classifier is found to be effective. Table 6 presents the model results for the 'Genel Görüş (Belirsiz)' label.

Table 6. Model Results for the 'Genel Görüş (Belirsiz)' Label

Embedding	Classifier	F1	Precision	Recall
TF-IDF	LightGBM	0.458	0.481	0.4371
TF-IDF	Random Forest	0.6667	0.6138	0.7296
TF-IDF	XGBoost	0.6763	0.6895	0.6635
Word2Vec	LightGBM	0.3644	0.5114	0.283
Word2Vec	Random Forest	0.4465	0.5535	0.3742
Word2Vec	XGBoost	0.484	0.5574	0.4277

Looking at Table 6, based on the F1 score of 0.6763 and the Precision score of 0.6895, the TF-IDF embedding method with the XGBoost classifier appears to be successful, while based on the Recall score of 0.7296, the TF-IDF embedding method with the Random Forest classifier is found to be effective. Table 7 presents the model results for all labels.

Table 7. Model Results for All Labels

Embedding	Classifier	Macro F1	Weighted F1	Macro Precision	Weighted Precision	Macro Recall	Weighted Recall
TF-IDF	LightGBM	0.2124	0.5678	0.2445	0.57	0.2022	0.5793
TF-IDF	Random Forest	0.2591	0.6166	0.4053	0.6499	0.2327	0.6626
TF-IDF	XGBoost	0.3182	0.6598	0.3825	0.6597	0.2923	0.6831
Word2Vec	LightGBM	0.0825	0.4862	0.0945	0.4865	0.0823	0.5061
Word2Vec	Random Forest	0.1027	0.514	0.1254	0.5037	0.1067	0.5689
Word2Vec	XGBoost	0.1059	0.5235	0.133	0.5201	0.1083	0.5727

Looking at Table 7, the best-performing method was achieved by using the TF-IDF technique as the embedding method and the XGBoost model as the classifier, with a Weighted F1 Score of 0.6598, a Weighted Precision of 0.6597 and a Weighted Recall of 0.6831.

DISCUSSION AND CONCLUSION

In this study, 14,430 online customer reviews from the e-commerce platform of one of Turkey's leading apparel companies were analyzed. The classification results indicate that customer feedback predominantly focuses on the physical attributes of the products. In particular, the categories of 'Fit/Ölçü/Kalıp' and 'Ana Malzeme/Kumaş Kalitesi' accounted for 65.59% of the total comments, revealing that feedback is most concentrated in these areas. This finding suggests that customers tend to provide evaluations especially regarding fundamental product features such as sizing and fabric.

Among the algorithms used for the automatic classification of customer comments, XGBoost yielded higher accuracy compared to other methods. From a representation learning perspective, the TF-IDF method outperformed Word2Vec in terms of classification performance. Considering the effects of imbalanced class distributions on model performance, the use of weighted evaluation metrics made a significant contribution. Within this context, the developed model can automatically classify large-scale online customer reviews with high accuracy, thereby enabling real-time feedback analysis on e-commerce platforms.

It is anticipated that the model outputs can provide data-driven support for decision-making processes such as product development and marketing. Furthermore, the integration of sentiment analysis allows for the assessment of the emotional content of the texts, which may serve as a foundation for applications aimed at enhancing the customer experience. However, since the analysis was conducted solely on data from a single e-commerce platform, the generalizability of the findings may be limited. Additionally, the imbalanced distribution of labels may adversely affect classification performance for some classes, and the automatic labeling process may be prone to errors in the presence of subjective comments. To address these issues, future studies are recommended to employ methods such as data enrichment and synthetic data generation to balance label distributions.

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Investigation of the Structure and Function of Acid-Sensing Ion Channels (1017)

Ziya Cakır[1*]

[1]Tokat Gaziosmanpaşa University, Faculty of Health Services Vocational School, Department of Oral and Dental Health, 60000, Tokat, TÜRKİYE

*Corresponding author e-mail: ziya.cakir@gop.edu.tr

Abstract

Acid-sensing ion channels (ASICs) are members of the epithelial sodium channel/degenerin (ENaC/DEG) superfamily and are encoded by five distinct genes, giving rise to seven different subunits. These subunits predominantly assemble into trimeric ion channels that, upon activation by extracellular protons, generate a transient inward current, thereby enhancing cellular excitability. These ion channels, which are activated particularly by extracellular acidification (pH decrease), regulate intracellular ion balance and electrical activity. ASICs exhibit a broad range of tissue distributions and display diverse biophysical characteristics. Moreover, their capacity to form both homomeric and heteromeric trimers adds further complexity to their functional and pharmacological properties. Certain modulators have been identified that lower the proton concentration required for ASIC activation, thereby sensitizing these channels. The roles of ASICs in neurological diseases, pain mechanisms and psychiatric disorders are attracting increasing interest. Substantial evidence from transgenic mouse models and pharmacological investigations indicates that ASICs represent a promising target for therapeutic intervention in various pathological conditions. Further investigation of the molecular mechanisms of ASICs may enable the development of new therapeutic strategies in disease models. This review aims to summarize the current understanding of ASIC function, explore their physiological and pathological roles, discuss mechanisms of modulation, and identify critical gaps in knowledge that warrant further investigation.

Keywords: *Acid-sensitive ion channels, Acidosis, Structure and function.*

INTRODUCTION

Acid-Sensitive Ion Channels

Acid-sensitive ion channels were first discovered in rat sensory neurons in 1980 by Krishtal and Pidoplichko using the voltage clamp technique (Krishtal & Pidoplichko, 1980). Krishtal and Pidoplichko argued that these acid-evoked currents were mediated by a previously unidentified ion channel and thought that they might be acid-evoked Ca2+ channels. Lazdunski and colleagues later showed that these channels were sensitive to amiloride and permeable to Na+ ions, and described acid-sensitive ion channels for the first time (Waldmann, Champigny, Bassilana, Heurteaux, & Lazdunski, 1997). In 2007, the crystal structure of ASIC1a in chicken was described, and they showed that ASICs consist of trimers, i.e., three different subunits (Jasti, Furukawa, Gonzales, & Gouaux, 2007). Acid-sensitive ion channels, a subgroup of the voltage-insensitive proton-gated degenerin/epithelial sodium channel (DEG/ENaC) superfamily, were shown to be widespread in both the central and peripheral nervous systems (Grunder & Chen, 2010; Waldmann & Lazdunski, 1998). In subsequent studies, the basic properties of these channels were determined. These channels generally open with decreasing pH (Krishtal & Pidoplichko, 1980), are permeable to Na+ allow (Krishtal & Pidoplichko, 1981a), and are blocked by amiloride (Krishtal & Pidoplichko, 1981b). In normal cells, extracellular pH is between 7.3-7.4 (Calorini, Peppicelli, & Bianchini,

2012). ASICs are rapidly activated and desensitized when extracellular pH falls below the normal physiological value (approximately pH=7.4)) (Hesselager, Timmermann, & Ahring, 2004). ASIC subunits have different pH sensitivities, activation kinetics and desensitization rates (Korkushko & Kryshtal, 1984). ASIC members are named as "X-NaC". The abbreviation ''X" means that it is related to a basic property of the proteins (DEG, degenerin protein), while the abbreviation ''C" means 'channel'. For example; epithelial Na+ channel (ENaC), FMRF amide (Phe-Met-Arg-Phe-NH2) gated Na+ channel (FaNaC), Drosophila gonad Na+ channel (dGNaC), human intestinal Na+ channel (hINaC), and brain, liver, and intestine Na+ channel (BLINaC) (Grunder & Chen, 2010). At least 6 ASIC subunits encoded by 4 different genes have been identified in rodents. There are 6 known ASIC subgroups: ASIC1a (Waldmann, Champigny, et al., 1997), ASIC1b (Chen, England, Akopian, & Wood, 1998), ASIC2a (Price, Snyder, & Welsh, 1996), ASIC2b (Lingueglia et al., 1997), ASIC3 (Waldmann, Bassilana, et al., 1997) and ASIC4 (Akopian, Chen, Ding, Cesare, & Wood, 2000). ASIC subtypes can exist as homomers or heteromers. However, ASIC2b and ASIC4 subtypes are not functional as homomers; they function by forming heteromers with other ASIC subtypes. All ASIC subunits consist of two hydrophobic transmembrane domains (TM1 and TM2), a large extracellular loop and short intracellular N- and C-terminal regions. The typical structure of ASICs and their localization in the cell membrane are shown in Figure 1 (Osmakov, Andreev, & Kozlov, 2014).

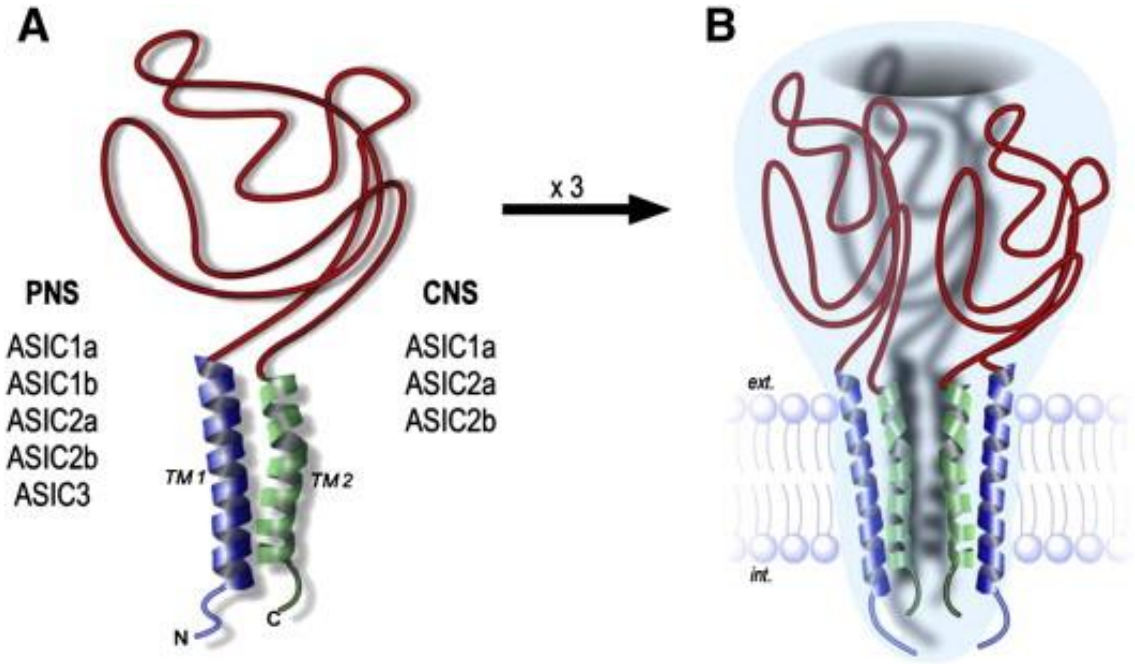

Figure 1. A. Schematic representation of ASIC subunits **B.** Trimeric structure of the ASIC channel (Osmakov, Andreev, & Kozlov, 2014).

ASICs have been shown to be present in most regions of the mammalian brain and in all sensory ganglia. However, it has been reported that ASIC channels are found in neurons but not in glial cells. Studies have shown that ASIC1a, ASIC2a, and ASIC2b genes are more expressed in the central nervous system, while ASIC1b and ASIC3 genes are more expressed in the peripheral nervous system (Chu et al., 2011). ASIC subtypes exhibit different electrophysiological and pharmacological properties (Hesselager, Timmermann, & Ahring, 2004). While all ASICs only conduct Na+ ions, ASIC1a conducts calcium ions along with sodium (Waldmann, Champigny, Bassilana, Heurteaux, & Lazdunski, 1997). Zinc (Zn2+) enhances homomeric and heteromeric ASIC2a currents, while attenuating other ASIC currents (110). Psalmotoxin1 is a specific ASIC1a inhibitor (Baron et al., 2001). Lead (Pb2+) inhibits ASIC1a currents but not other ASICs (Wang at al., 2006). Salicylic acid blocks only the ASIC3 subunit (Voileyet al., 2001). The kinetic

properties of the ion currents generated by ASIC subunits are different from each other (Figure 2) (Osmakov, Andreev, & Kozlov, 2014).

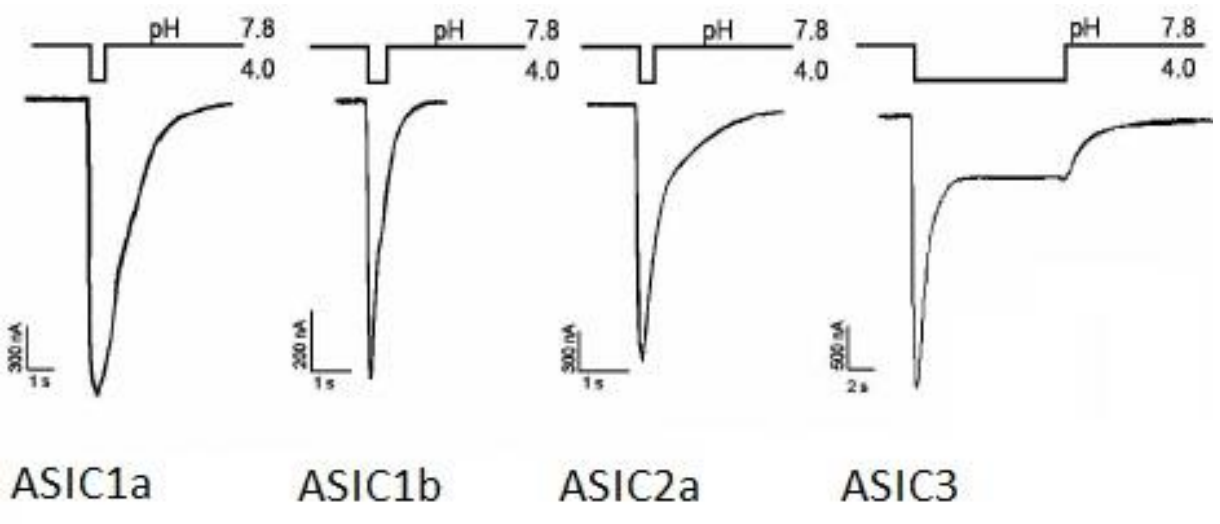

Figure 2. Currents generated by ASIC subunits expressed in frog (Xenopus laevis) oocyte cells (Osmakov, Andreev, & Kozlov, 2014).

Studies have shown that acid-sensitive ion channels play a role in many physiological events such as nociception (Chen et al., 2002), touch (Price et al., 2001), taste and smell transmission (Lin et al., 2002), long-term potentiation (LTP), synaptic transmission, memory/learning (Wemmie et al., 2002), sensory transmission and retinal integrity (Ettaiche et al., 2004) and in many pathological events such as pain (Duan et al., 2007), ischemia-related brain damage (Xiong et al., 2004), stroke and epilepsy (Biagini et al., 2001). In the auditory system, the presence of ASICs has been shown in hair cells in the cochlea (Ugawa et al. 2006), spiral ganglion neurons (Peng et al., 2004), vestibular organs (Mercado et al., 2006) and inferior colliculus neurons (Zhang et al., 2008). It has been determined that the ASIC2a subunit is related to noise sensitivity in mice (Peng et al., 2004), and that the absence of ASIC2 does not impair hearing (Roza et al., 2004). In a study conducted on mice that did not express the ASIC3 gene, hearing loss was found in mice lacking the ASIC3 subunit (h,ldebrand et al., 2004).

Acid-sensitive ion channels help in the transduction of stimuli in various physiological and pathophysiological conditions (Askwith et al., 2001). ASIC inhibitors are isolated from animal venoms, which are natural peptide toxins, as non-specific molecules (Diochot et al., 2007). Pepsin toxins, neuropeptides, organic compounds and some di/trivalent cations obtained from some animal venoms are involved in the modulation of ASICs.

MATERIAL AND METHODS

Material

Albino BALB/c mice aged 14-17 days were used in patch clamping studies. Cochlear nucleus brain sections were prepared to record patch clamping. Patch clamp recordings were taken using patch clamp micropipettes. These procedures were performed in constantly oxygenated normal CSF fluid.

Methods

Patch clamp technique

The patch clamp technique is a widely used electrophysiological technique to study ion currents of channels in the cell membrane. The patch clamp technique is based on measuring voltage changes in cells via the electrode in the pipette. This technique is important for determining the biophysical, physiological and

pharmacological properties of ion channels. When the patch clamp technique was first discovered, it was used to control the voltage of a small piece of cell membrane. Now, it is used for both voltage clamping and current clamping on the membrane with the help of a micropipette.

Statistical Analysis

Statistical evaluation in this study; It was done using the Statistical Package for Social Science (SPSS) Version 23.0 (SPSS inc, Chicago, USA) package program. In patch clamping studies, it was examined whether there was a difference between the recordings taken before the application of the acidic solution and the recordings taken during the application of the acidic solution. Student's t test was used to determine the difference during acidic solution. Descriptive statistics for numerical variables were expressed as group mean ± standard error (S.H.). In statistical evaluation, $p<0.05$ value was accepted as significant.

RESULTS

To investigate acid-induced currents in neurons of the cochlear nucleus, the responses of these neurons to a decrease in pH in the extracellular solution were recorded using the whole-cell patch-clamp technique. In the present experiments, the average resting membrane potential of stellate cells was found to be – 63.7 ± 0.71 mV (n = 42). Therefore, recordings were made by keeping the cells at – 62 mV holding potential. The acidic solution stimulated most of the neurons by generating inward currents. The amplitudes of acid-induced currents showed high sensitivity to pH (Figure 3). Acidic solutions with pH ranging from 7.4 to 4 were applied. The relationship between the peak values of the currents occurring in different acidic solutions and the pH values obtained was shown graphically.

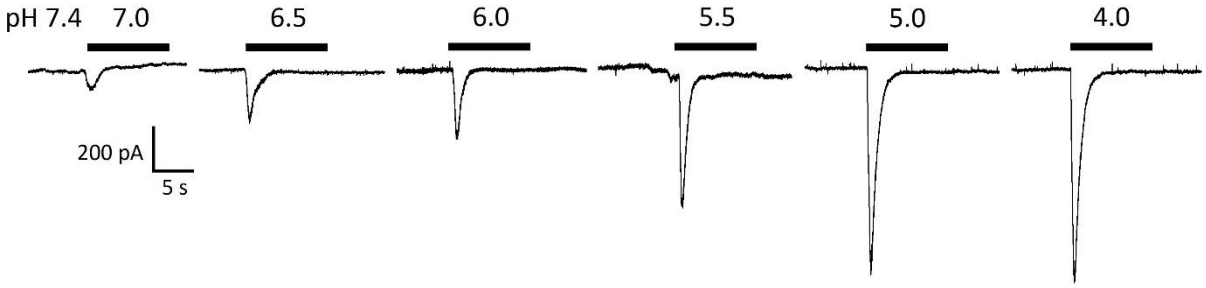

Figure 3. Typical traces showing the inward currents that are activated by extracellular solutions with different pH.

DISCUSSION AND CONCLUSION

Acid-sensitive ion channels (ASICs) are voltage-insensitive sodium channels that are activated by acidification of the extracellular environment. ASICs are widely expressed in the central and peripheral nervous systems. There are reports that ASICs activated by acid exposure play a role in various physiological mechanisms and pathophysiological events. Although local pH decreases in various cellular structures are important enough to activate ASICs, it is thought that pH in tissues is tightly regulated by homeostatic mechanisms (Chesler & Kaila, 1992). For example, synaptic vesicles are generally acidic and have a pH of 5.7 (Yuste et al., 2000). Studies in hippocampal neurons have shown that the extracellular pH in the synaptic cleft temporarily drops below 6 after vesicle release in synapses showing intense synaptic activity (Miesenbock et al., 1998). Again, studies in retinal cone receptors have shown that ASICs can affect synaptic transmission (DeVries SH., 2001). It is thought that many cellular stresses in the cochlea (e.g. ischemia and inflammation) can induce local acidosis. Although the cause is not well understood, it is known that inflammation or ischemia results in sudden hearing loss (Roza et al., 2004). In this context, it is possible that ASICs may underlie some hearing loss caused by non-mechanical causes. It is also thought that ASICs can act as a sensor against harmful stimuli and may have an important role in some pathological

cases. It has been shown that intercellular acidosis affects and activates ASICs, and ASICs trigger excessive excitatory activities associated with epileptic seizures and ischemia. Therefore, it is thought that ASICs play a role in the pathogenesis of these diseases (Varming T. 1999).

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Conflict of Interest

The authors have disclosed that they have no competing interests.

Author Contributions

ZC: Conceptualization, Project administration, Resources, Visualization, Data curation, Formal Analysis, Software, Resources, Writing – original draft, Writing.

Applying the γ-order Generalized Normal to: Cauchy distribution and the Hermite polynomials (1018)

Christos P. Kitsos[1*], Ioannis S. Stamatiou[2]

[1]University of West Attica, Department of Informatics,Greece
[2] University of West Attica, Deparment of Surveying and Geoinformatics Engineering, Greece

*Corresponding author e-mail: xkitsos@uniwa.gr

Abstract

Consider the γ-order Generalized Normal $N_\gamma(\mu, \sigma^2)$ and let $X \sim N_\gamma(0, 1)$, $Y \sim N_\gamma(0, 1)$. Then for their ratio $Cau_\gamma = X/Y$ it holds that $F_{x/Y}(c) = P(X \leq c|Y|)$. We prove that the ratio of two γ-order Generalized Normal distributions defines the γ-order Generalized Cauchy distribution and when γ=2, the classical Cauchy distribution is obtained.

The Hermite polynomials were defined by Laplace in 1811 and named on 1864 with a paper of Hermite referring to these polynomials. We consider the family of Hermite polynomials (HP) and we define the γ-order Generalized Hermite Polynomials $H_{r,\gamma}(w)$ and we prove that:

- *(i) The Differential of the γ-order Normal distribution is related to $H_{r,\gamma}(w)$*
- *(ii) The Generalized Heat equation, we have already defined, is an application of the $H_{r,\gamma}(w)$*
- *(iii) With γ=2 we are reduced to the classical case*

The applications of these results are immediate and can be easily obtained using R.

Keywords: *γ-order Generalized Normal Distribution, Cauchy Distribution, Heat Equation, Hermite Polynomial*

INTRODUCTION

The classical Chi-squared distribution, derived from the sum of squared standard Normal variables, plays a foundational role in statistical theory, particularly in inference, hypothesis testing, and variance analysis. However, in many practical applications - especially those involving heavy-tailed, or non-Gaussian data - the assumption of normality becomes restrictive (see Kitsos & Nyamsi, 2024). To address these limitations, Kitsos & Tavoularis (2009) have introduced generalizations of the Normal distribution via a shape parameter γ, leading to the family of γ-order generalized Normal distributions N_γ (μ,Σ).

Building upon this framework, Kitsos and Stamatiou (2024) introduce the γ-order generalized Chi-squared distribution, denoted _γ χ_n^2, as a extension of the classical χ_n^2 distribution. It is obtained by summing the squares of independent variables drawn from the γ-order generalized Normal distribution. The shape parameter γ controls the tail behavior and peakedness of the distribution, allowing a smooth transition between several known distributions. This flexibility enables modeling of non-standard behaviors that arise in real-world data.

This paper presents a simulation-based study of the γ-order generalized Chi-squared distribution (_γ χ_n^2). We employ an adjusted Ziggurat algorithm, adapted for the γ-order generalized Normal distribution, to generate random vectors and compute the resulting squared sums. Simulations are conducted for representative values around γ=2.

Our contribution is twofold. First, we numerically validate the theoretical properties of the _γ χ_n^2 distribution. Second, we explain with graphical illustrations how the extra shape parameter γ, behave on the shape of the distribution. These results demonstrate how this generalized Chi-squared family can be effectively used in applications requiring robust alternatives to classical Chi-squared models, particularly in the presence of heavy-tailed or non-Gaussian data.

THE γ-ORDER GENERALIZED CHI-SQUARE DISTRIBUTION

The γ-order generalized Normal distribution (γ-GN), denoted by $\mathcal{N}_\gamma(\mu, \sigma^2)$, was introduced to generalize the classical Normal distribution through a shape parameter $\gamma > 1$, controlling tail behavior and peakedness (Kitsos et al., 2014). For $\gamma = 2$, the γ-GN coincides with the standard Normal distribution. As $\gamma \to 1$, it converges to a uniform distribution; and as $\gamma \to \infty$, it approaches a Laplace distribution, Kitsos et al. (2011). The PDF of the univariate γ-GN is given by Kitsos & Stamatiou (2024):

$$\phi_\gamma(x; \mu, \sigma^2) = \frac{\lambda_\gamma}{\sqrt{\pi\sigma^2}} exp\left\{-\frac{\gamma-1}{\gamma}\left(\frac{|x-\mu|}{\sqrt{\sigma^2}}\right)^{\gamma/(\gamma-1)}\right\}, \qquad (1)$$

where the normalizing constant λ_γ is defined as:

$$\lambda_\gamma = \frac{\Gamma\left(\frac{1}{2}+1\right)}{\Gamma\left(\frac{\gamma-1}{\gamma}+1\right)}\left(\frac{\gamma-1}{\gamma}\right)^{\frac{\gamma-1}{\gamma}} \qquad (2)$$

Let $X \sim \mathcal{N}_\gamma(0, \sigma^2)$. We define the associated γ-order generalized chi-squared distribution ($_\gamma\chi_1^2$), as the distribution of the squared variable $Y = X^2$. This definition parallels the classical Chi-squared construction but retains the flexibility introduced by the γ parameter.
The probability density function of $Y_1 \sim {}_\gamma\chi_1^2$ is given by Kitsos & Stamatiou (2024):

$$f_{Y_1}(x) = \frac{\gamma_0^{\gamma_0}}{2\Gamma(\gamma_0+1)} \cdot \frac{1}{\sqrt{x}} \cdot exp\left\{-\gamma_0 \cdot \left(\sqrt{x}\right)^{\gamma_1}\right\}, \quad x > 0, \qquad (3)$$

where the shape-dependent parameters are:

$$\gamma_0 = \frac{\gamma-1}{\gamma}, \qquad (4)$$

$$\gamma_1 = \frac{1}{\gamma_0} = \frac{\gamma}{\gamma-1}. \qquad (5)$$

This density reduces to the classical chi-squared density when $\gamma = 2$ and generalizes it for other values of γ.
The m-th moment of Y_1 is:

$$\mathbb{E}(Y_1^m) = (\gamma_1)^{2m\gamma_0} \cdot \frac{\Gamma\left((2m+1)\gamma_0\right)}{\Gamma(\gamma_0)}. \qquad (6)$$

(Kitsos & Stamatiou, 2024)

The variance of Y_1 is given by:

$$Var(Y_1) = \frac{(\gamma_1)^{4\gamma_0}}{\Gamma(\gamma_0)}\left(\Gamma(5\gamma_0) - \frac{\gamma_0^2\Gamma^2(3\gamma_0)}{\Gamma(\gamma_0+1)}\right). \qquad (7)$$

(Kitsos & Stamatiou, 2024)

The application of equations (6) and (7) is illustrated in Table 1 below, where theoretical moments are validated against empirical estimates derived from simulation.

Let $X_1, X_2, \dots, X_n$ be i.i.d. random variables such that $X_i \sim \mathcal{N}_\gamma(0, \sigma^2)$ for all $i = 1, \dots, n$. We define the γ-order generalized Chi-squared distribution with n degrees of freedom as:

$$Y_n = \sum_{i=1}^{n} X_i^2 \sim {}_\gamma\chi_n^2. \qquad (8)$$

The probability density function of Y_n is given by (Kitsos & Stamatiou, 2024):

$$f_{Y_n}(z) = \frac{\gamma_0^{n\gamma_0}}{2\gamma_0\Gamma(n\gamma_0)} \cdot z^{\frac{n}{2}-1} \cdot exp\left\{-\gamma_0\left(\sqrt{z}\right)^{\gamma_1}\right\}, \quad z > 0. \qquad (9)$$

This generalization preserves the structure of the classical Chi-squared distribution while enabling much broader modeling flexibility via the shape parameter γ. The γ-order Chi-squared family provides a unified framework that includes the classical χ_n^2 distribution as a special case. For simulation purposes, the Ziggurat algorithm can be adapted to generate γ-GN samples, which are then squared and summed to produce ${}_\gamma\chi_n^2$ variates.

METHODOLOGY: THE ADJUSTED ZIGGURAT ALGORITHM

We introduce an adjusted version of the Ziggurat algorithm method (Marsaglia & Tsang, 2000) for efficiently sampling from the γ-GN, generalizing classical normal-distribution sampling to handle heavier or lighter tails through the shape parameter γ.

In the univariate case, we generate samples from the γ-generalized normal distribution by partitioning the target density $f_\gamma(x)$ into N horizontal rectangles of equal area A. The construction of the rectangles begins from a designated right-end point x_{tail}, which serves as the boundary of the last rectangle. This point is chosen such that the area under the density from x_{tail} to $+\infty$ equals the tail probability:

$$\int_{x_{tail}}^{\infty} f_\gamma(x)\, dx = A. \qquad (10)$$

Constructing the Ziggurat Layers

The PDF of the γ-GN is partitioned into N horizontal rectangles of equal area A, beginning from the tail cutoff point x_{tail}. This point is determined using a bounding strategy based on the CDF. We start with initial guesses for the lower and upper bounds, and expands them symmetrically until they bracket nearly all of the probability mass of the γ-GND distribution. For instance, $F_\gamma(x) = P(x < x_{\text{tail}}) = 0.9999$. Therefore, we compute

$$A = x_{tail} \cdot f_\gamma(x_{tail}) + \int_{x_{tail}}^{\infty} f_\gamma(x)\, dx \qquad (11)$$

A recursive scheme is then used to compute all remaining right-endpoints x_i and heights y_i, ensuring that each rectangle fits perfectly under the curve:

$$f_\gamma(x_i) = \frac{A}{x_{i+1}} + f_\gamma(x_{i+1}) \qquad \textit{and} \qquad y_i = f_\gamma(x_i) \qquad (12)$$

Selection of the Points

A rectangle is selected uniformly at random. A candidate point x is generated by drawing uniformly across the rectangle's width. This ensures that all regions under the density are sampled proportionally, maintaining the correct distributional shape.

Immediate Acceptance

If the point lies within the region strictly under the density curve (i.e., $x < x_{i+1}$), it is accepted immediately without evaluating the PDF. This improves efficiency by avoiding unnecessary function evaluations for a majority of samples (Marsaglia & Tsang, 2000).

Further Verification (Rejection Sampling)

If the sampled point lies in the upper portion of a selected rectangle—specifically, beyond the region guaranteed to be under the density curve—a second uniform random variable is used to compute a vertical coordinate. The candidate is accepted only if it lies below the value of the PDF at its horizontal coordinate. This rejection step ensures correctness in regions where the rectangle may extend above the target density.

Sampling from the Tail

For values beyond the rightmost rectangle ($x > x_{\text{tail}}$), the Ziggurat algorithm employs an exponential approximation of the tail behavior of the γ-generalized normal distribution. A transformation is applied to a uniform random variable using the inverse of the conditional tail CDF to generate a candidate sample. Specifically, the candidate x is computed as:

$$x = \left(x_{tail}^{\frac{\gamma}{\gamma-1}} - \frac{\gamma}{\gamma-1} ln(U_1) \right)^{\frac{\gamma-1}{\gamma}}, \qquad (13)$$

where $U_1 \sim \mathcal{U}(0,1)$. The sample is then subject to an acceptance-rejection test to ensure correctness:

$$U_2 < \frac{x_{tail}}{x}, \qquad (14)$$

where $U_2 \sim \mathcal{U}(0,1)$ is an independent uniform variable. This condition guarantees that the sampled value conforms to the true tail structure of the distribution.

Symmetry Considerations

Since the γ-GN is symmetric about its mean, the algorithm samples only on the positive half of the distribution. Once a value $x \geq \mu$ is accepted, a random sign is applied to ensure symmetry: $X = \mu \pm (x - \mu)$ with equal probability. This reduces computation while preserving distributional integrity.

Simulation Design

We generated n independent vectors $X_i \sim \mathcal{N}_\gamma(0,1)$, each of length $N = 1000$. Shape parameter values were set to $\gamma = 1.8$, 2 and 2.2. For each case, we computed:

$$Y_n = \sum_{j=1}^{n} X_j^2 \sim {}_{\gamma}\chi_n^2, \qquad (15)$$

a generalization of the classical Chi-squared distribution. Empirical histograms, theoretical PDF overlays, and summary statistics (mean, variance, skewness, kurtosis) were used to assess the generated data.

RESULTS

Table 1 illustrates how the cumulative distribution function (CDF) evolves for different values of the shape parameter γ and dimension n. For fixed n, we observe that increasing γ leads to a slower accumulation of probability mass as x increases. This behavior reflects the heavier tails of the distribution for lower values of γ, where more probability is allocated farther from the origin. For instance, at x=3 and n=2, the CDF values are 0.82, 0.78, and 0.75 for γ=1.8, 2.0, and 2.2, respectively. This consistent decrease highlights the tail-thinning effect of increasing γ.

Similarly, we observe that increasing the number of aggregated variables n leads to slower CDF growth at fixed x. For example, for γ=2.2 at x=3, the CDF decreases from 0.75 (when n=2) to 0.38 (when n=4). This behavior illustrates that as the number of variables increases, the distribution of their sum becomes more spread out, shifting probability mass toward larger values of x.

Table 1: CDF values for different values of γ, evaluated at several x points, for n = 2 and n = 4.

	$n=2$						$n=4$					
γ	$x=0$	$x=1$	$x=2$	$x=3$	$x=6$	$x=10$	$x=0$	$x=1$	$x=2$	$x=3$	$x=6$	$x=10$
1.8	0.00	0.42	0.67	0.82	0.97	1.00	0.00	0.11	0.32	0.52	0.88	0.99
2.0	0.00	0.39	0.63	0.78	0.95	0.99	0.00	0.09	0.26	0.44	0.80	0.96
2.2	0.00	0.38	0.60	0.75	0.93	0.99	0.00	0.08	0.23	0.38	0.73	0.92

Table 2 presents a comparison between the theoretical and empirical moments of the γ-generalized Chi-square distribution for selected values of γ and n = 1. The empirical values are computed from simulated samples, while the theoretical values are derived from analytical expressions. As shown, the empirical mean and variance closely approximate the theoretical counterparts across all values of γ, demonstrating the validity and accuracy of the proposed sampling algorithm. Notably, as γ increases, both the mean and variance increase, indicating that the distribution spreads more as its shape parameter changes. Minor discrepancies between empirical and theoretical values are expected due to finite sampling effects.

Table 2: Comparison of Theoretical and Empirical Moments for Different γ Values (n = 1)

γ	Theoretical Mean	Empirical Mean	Theoretical Variance	Empirical Variance
1.8	0.92	0.96	1.52	1.60
2.0	1.00	0.99	2.00	2.06
2.2	1.07	1.10	2.50	2.68

Figure 1 illustrates the performance of the adjusted Ziggurat algorithm in generating samples from the γ-generalized chi-squared distribution with γ = 2, which corresponds to the classical Chi-squared case. For n = 4, the distribution retains noticeable right skewness, as expected for lower degrees of freedom in the Chi-

squared family. Although minor deviations are visible in the tail—attributable to finite sample effects—the empirical PDF still follows the theoretical curve closely.

For n = 10, the distribution becomes significantly more symmetric and bell-shaped, closely resembling a normal distribution due to the central limit phenomenon. The empirical and theoretical densities align almost perfectly, indicating high sampling accuracy even in higher-dimensional settings.

These results confirm that when γ = 2, the generalized normal and associated chi-type distributions coincide with the classical normal and Chi-squared distributions. Importantly, this consistency demonstrates that the adjusted Ziggurat algorithm not only generalizes the classical case but also recovers it as a special instance when γ = 2.

We examine the effect of the shape parameter γ on the distribution of the generalized chi-square-type variable _γ χ_4^2, using fixed degrees of freedom n=4 and a sample size of N=1000. Figures 2 and 3 present the empirical and theoretical densities for γ=1.8 and γ=2.2, respectively. In Figure 2, the distribution for γ=1.8 appears more compact and sharply peaked, with thinner tails. The empirical histogram shows a rapid rise and fall around the mode, indicating less variability and stronger central concentration. This behavior is characteristic of lighter-tailed distributions. By contrast, Figure 3 shows the distribution for γ=2.2, which is more spread out and exhibits visibly longer tails. The density decays more slowly, allocating more mass in the tail regions.

Comparing the two, increasing γ from 1.8 to 2.2 leads to heavier tails and greater dispersion. This demonstrates how the parameter γ modulates the concentration and spread of the distribution, offering flexibility for modeling scenarios with different degrees of tail thickness.

These observations validate that the adjusted Ziggurat algorithm accurately captures the tail behavior and shape dynamics of the generalized Chi-squared distribution, adapting as γ varies.

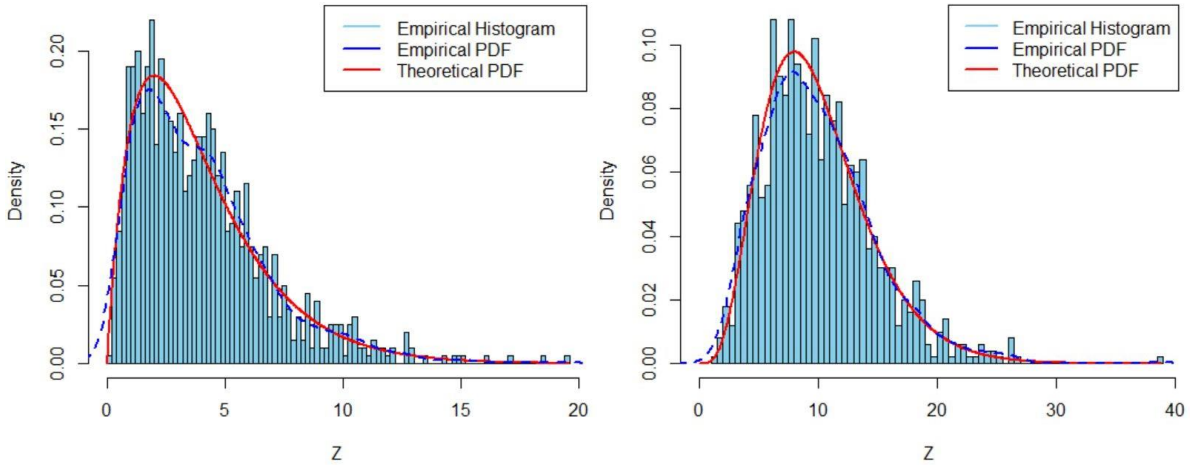

Figure 1: Distribution of generated samples from Chi-square-type gamma distributions with γ = 2, sample size N = 1000 and degree of freedom n.

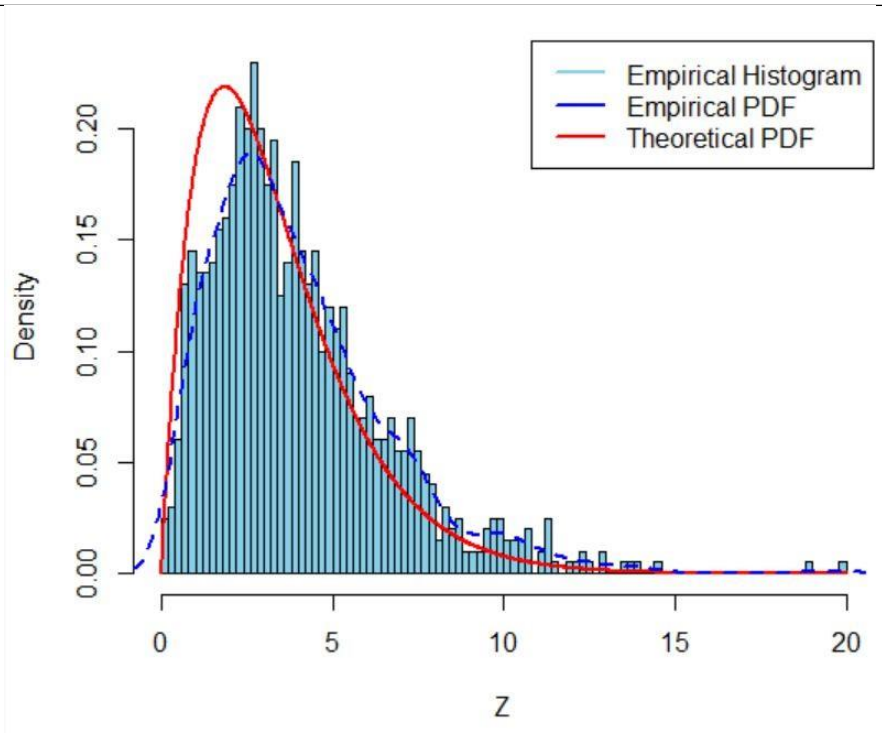

Figure 2: Distribution of generated samples from a Chi-square-type gamma distribution with $\gamma = 1.8$, sample size $N = 1000$, and degree of freedom $n = 4$.

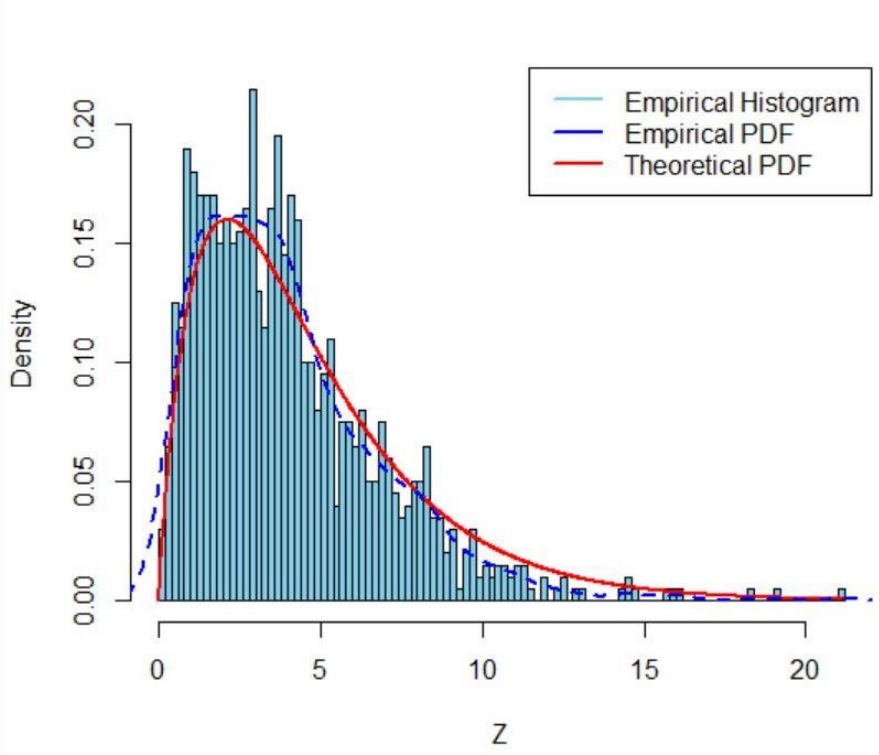

Figure 3: Distribution of generated samples from a chi-square-type gamma distribution with $\gamma = 2.2$, sample size $N = 1000$, and degree of freedom $n = 4$.

CONCLUSION

We chose the Ziggurat simulation algorithm due to its effectiveness in handling the tails of distributions. The γ-order Generalized Normal distribution exhibits fat tails for specific values of the shape parameter γ, making it well-suited for this simulation study. Accordingly, the algorithm was appropriately adjusted to generate simulation results for this particular family of distributions.

Regarding the γ-order Chi-square or γ distributions, the simulation results demonstrated satisfactory performance, particularly for parameter values around 2, where the underlying ”mother” distribution is Normal. The simulations provided evidence that for values of γ close to 1 and bigger than 1, (i.e., approaching the Uniform distribution), hardly we can obtain a Chi-square distribution.

This paper thus makes a meaningful contribution by providing a simulation algorithm tailored to the generalized forms of the γ-Chi-square family of distributions, with γ values around 2.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

The Need for Research on Scheduling Problems with Setup Times (1051)

Muberra Allahverdi[1], Ali Allahverdi[2*]

[1]Ankara Yıldırım Beyazıt University, Faculty of Engineering and Natural Sciences, Department of Mathematics, Turkiye
[2]Gazi University, Faculty of Engineering, Department of Industrial Engineering, Turkiye

*Corresponding author e-mail: aliallahverdi@gazi.edu.tr

Abstract

Setup times are defined to prepare resources for performing operations or tasks in a scheduling environment. The vast majority of research on scheduling problems ignore setup times or consider setup times as part of processing times. While this assumption may be valid for some scheduling environments, it is not valid for some other scheduling environments. Research on production scheduling, considering setup times as distinct from processing times, began as early as the mid-1960s. Since setup process is not a value-added factor, setup times need to be minimized in order to increase resource utilization, and to eliminate waste. The scheduling literature reports that in about 50 different manufacturing environments, there is a need for considering setup times as separate from processing times. Furthermore, it is reported in the scheduling literature that the majority of production schedulers have often faced with scheduling problems with setup times. However, the percentage of scheduling research papers treating setup times as separate from processing times to all the scheduling research papers is less than 1.5%. This clearly indicates that there is a great need for research on scheduling problems considering setup times as separate from processing times. Setup times can be treated as deterministic variables for some scheduling environments while for some other scheduling environments they are random variables. This paper addresses the need for research on setup times as well as the need to consider setup times as uncertain variables.

Keywords: *Scheduling, Setup Times, Uncertain*

INTRODUCTION

Allahverdi and Soroush (2008) state the significance of setup times in the scheduling environment. The recent literature review, Allahverdi et al. (1999, 2008) and Allahverdi (2015), on scheduling problems indicates that a very small percentage of the research consider setup time. However, it is reported that most of the schedulers in manufacturing environments face with setup times. Hence, ignoring setup times is not valid and and ignoring setup times increases the cost of manufacturing significantly. In this paper, we state the research in different manufacturing environments addressing setup times based on shop environemts, i.e., single machine, parallel machine, flowshop, job shop, and open shop. It should be noted that fort he last dacade, the percentage of research addressing scheduling problems with setup times is less than 1% of the whole scheduling problems with and without setup times. This shows the need to consider scheduling problems with setup times.

Let STsi and STsd denote sequence independent and sequence dependent setup times, respectively, and SCsd to denote sequence dependent setup cost. Moreover, let STsi,f and STsd,f denote sequence independent family setup time and sequence dependent family setup time, respectively.

SINGLE MACHINE

The percentage of research conducted on a single machine with respect to setup type (STsi, STsd, SCsd, STsi,f, STsd,f) is given in Figure 1. About 65% of the research addressed the STsd problem. This is good since the problem of STsi is a special case of the problem of STsd. In other words, the results for STsd are valid for STsi as well. It is also clear from the figure that the percentages of the conducted research are about 11 and 20 for the setup types STsi,f and STsd,f, respectively. It should be also noted that the problem STsi,f is a special case of the problem STsd,f. Hence, the results for STsd,f are valid for the problem STsi,f. The percentage of the research on setup cost (SCsd) is about 3%. When the main concern is related to the idle time of resource's, Allahverdi and Soroush (2008) stated that the setup time and the setup cost are proportional. Hence, it is sufficient to consider either the setup cost or the setup time. On the other hand, they reported when the changeover costs between certain jobs is high while the setup time is low, the type SCsd should be explicitly considered. In other words, considering STsd is not equivalent considering SCsd. There is a need to find out the percentage of the real-life applications where SCsd should be explicitly considered. Finally, there was only one research considering the problem of STsi, which was for the controllable setup time case.

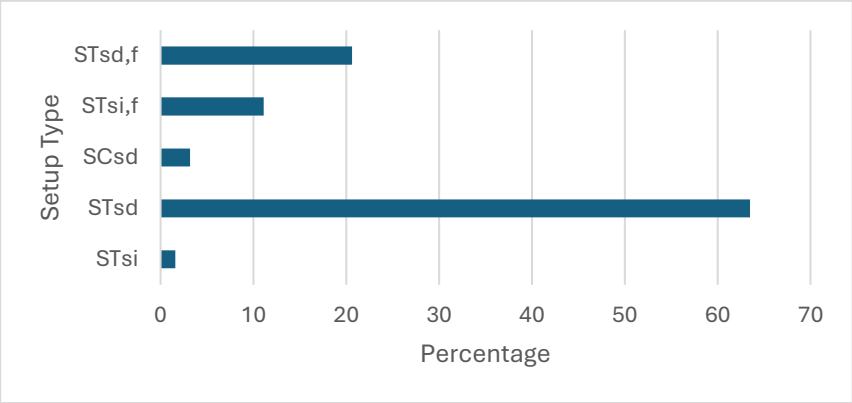

Figure 1: Single machine - % of research conducted based on setup type

PARALLEL MACHINES

Figure 2 shows the percentage of research carried out on parallel machines regarding setup type (STsi, STsd, SCsd, STsi,f, STsd,f). About 78% of the research studied the STsd problem while other type of problems were considered by less than 8%. Similar to the single machine case, the percentage of the research on the setup cost (SCsd) is about 3%. As stated earlier with the single machine case, there is a need to find out the percentage of real-life applications where SCsd should be explicitly considered.

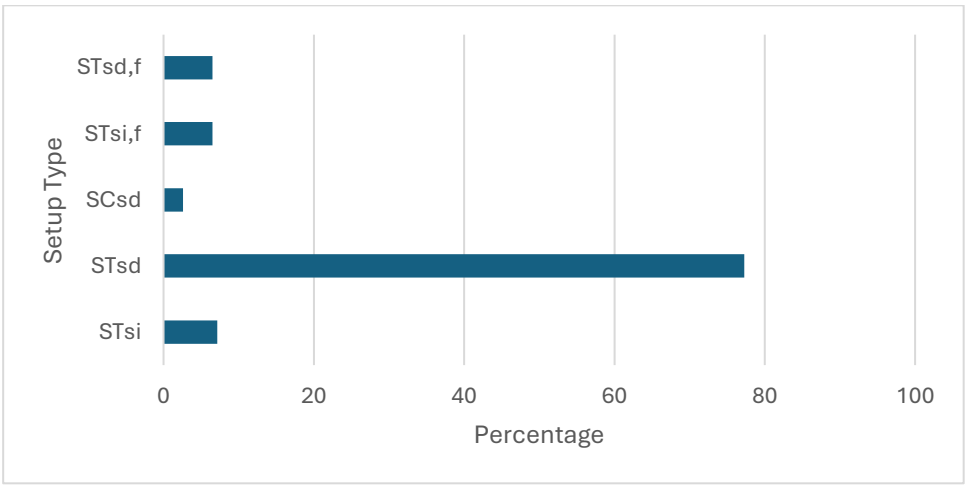

Figure 2: Parallel machine - % of research conducted based on setup type

Figure 3 indicates the percentage of research carried out on parallel machines regarding machine type (P, Q, R) where P denotes the identical machines, Q the uniform machines, and R the unrelated machines. About 54% of the research addressed the R problem, about 38% considered Q and 8% considered P. It should be noted that P is a special case of Q while Q is a special case of R. Therefore, the results obtained for R are valid for Q and P while the results obtained for Q are valid for the P.

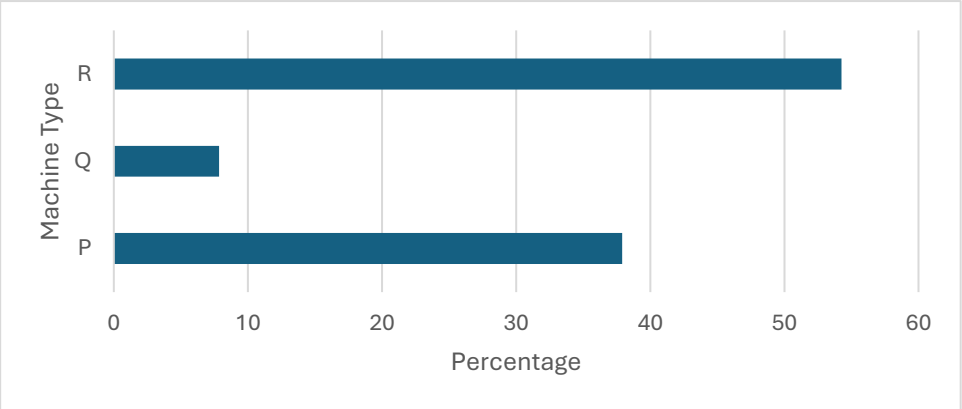

Figure 3: Parallel machine - % of research conducted based on machine type

FOLLOWSHOP

The percentage of research on flowshops which involve setup times are 71% for STsd, 22% for STsi, 6% for STsd,f, and finally 1% for STsd,f, as shown in Figure 4. That is the majority of the research considers STsd. Furthermore, the percentage of papers involving family setups is less than those of other shop environments.

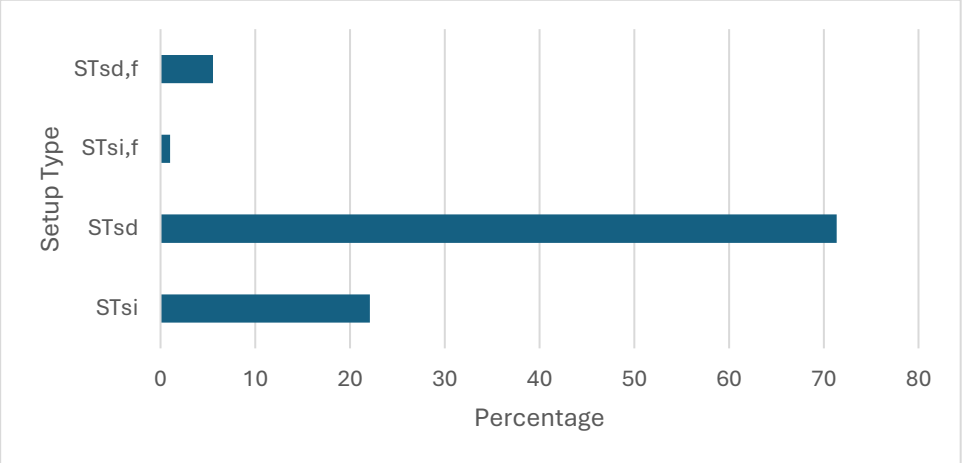

Figure 4: Flowhop - % of research conducted based on setup type

Figure 5 shows the the percentage of the research on flowshops based on the categories: (regular) flowshop, hybrid or flexible flowshop, and assembly flowshop. The percentage of papers considering flowshop, flexible flowshop, and assembly flowshop are 63, 28, and 9, respectively. The m-stage flowshop received more attention from researchers with about 53% of the papers. It has been observed that among the flowshop scheduling papers with setup times, about 16% considered no-wait flowshops while 24% addressed distributed flowshops.

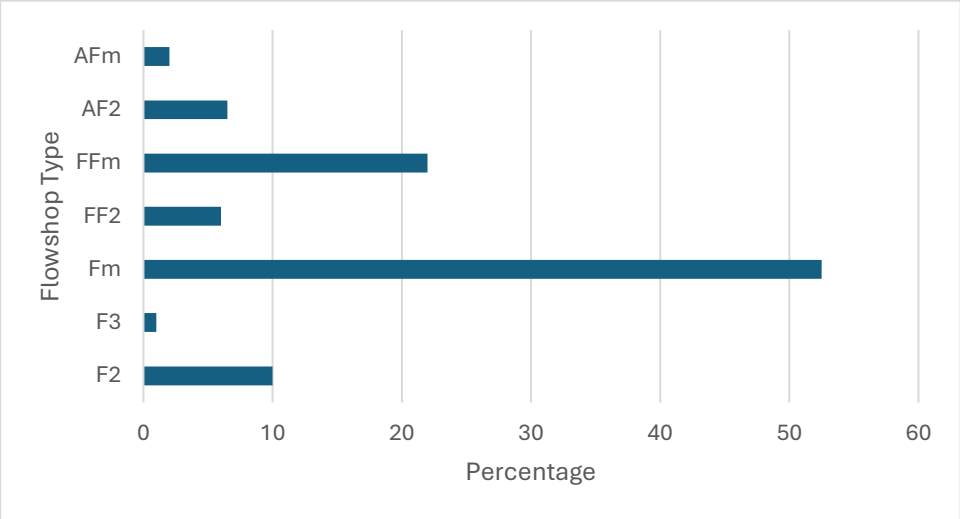

Figure 5: Flowshop - % of research conducted based on flowshop type

JOB SHOP AND OPEN SHOP

The percentages of papers on job shops and open shops involving setup types and the shop type are given in Figures 6 and 7, respectively. As seen in Figure 5, about 86% of the research is about STsd, which is very good as stated earlier, since STsi is a special case of STsd. The percentage of research on setup types are about 8, 3, 3 for STsi, SCsd, and STsd,f, respectively. The need for research involving SCsd was discussed earlier for other shop environments. Figure 7 shows that the percentage of research papers treating setup times separately, are 25, 64, and 11, for the shop types of J, FJ, and O, respectively. Again, needless to say, J is a special case of FJ.

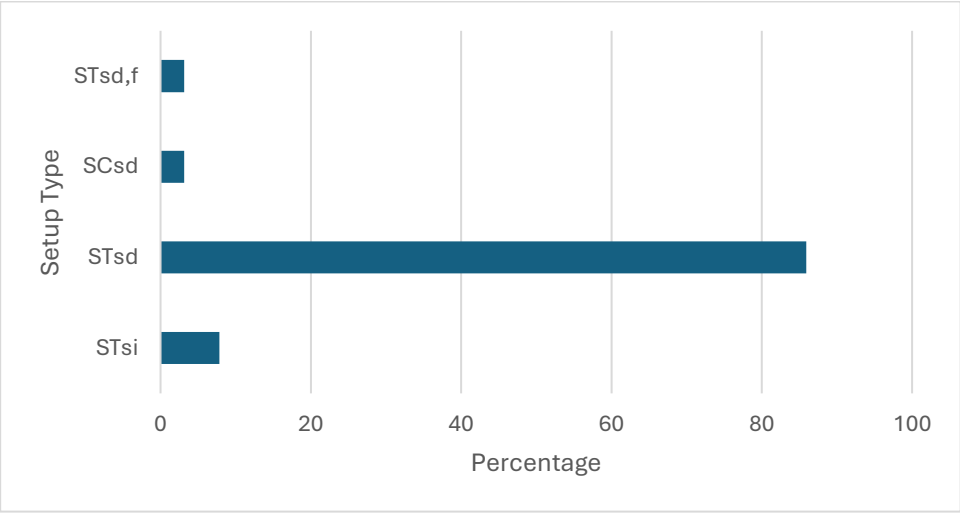

Figure 6: Job shop or open shop - % of research conducted based on setup type

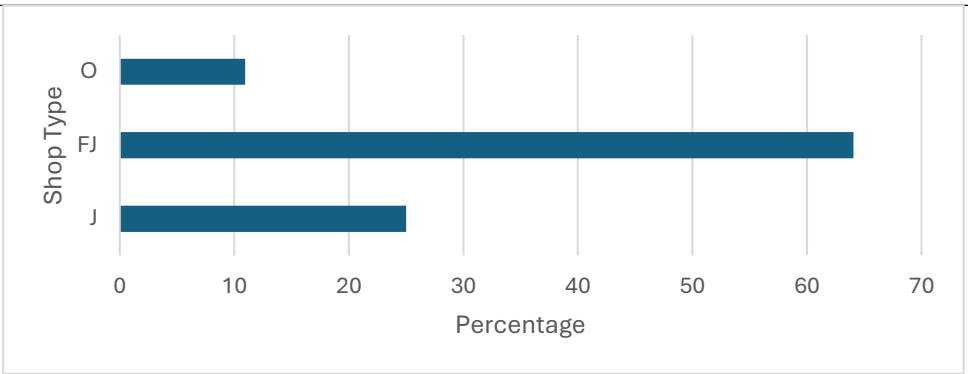

Figure 7: Job shop or open shop - % of research conducted based on shop type

DISCUSSION AND CONCLUSION

The vast majority of research on scheduling problems ignore setup times. While this assumption may be valid for some scheduling environments, it is not valid for some other scheduling environments. it is reported in the scheduling literature that the majority of production schedulers have often faced with scheduling problems with setup times. However, the percentage of scheduling research papers treating setup times as separate from processing times to all the scheduling research papers is less than 1% in the last dacade. This clearly indicates that there is a great need for research on scheduling problems considering setup times as separate from processing times. The classification of setup times based on setup type in different scheduling environments is given in this paper.

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A Hybrid KNN Model for Time Series Forecasting: An Experimental Study on Traffic Flow Prediction (1110)

İlker Arslan[1*], Ümit Işlak[2], Elif Yılmaz[3]

[1]MEF University, Dept. of Electrical and Electronics Engineering, Turkey
[2]Boğaziçi University, Dept. of Mathematics, Turkey
[3]University of Université de Neuchâtel, Institut d'informatique, Switzerland

*Corresponding author e-mail: arslanil@mef.edu.tr

Abstract

In this work, we introduce a hybrid KNN model and present an experimental study focusing on the traffic flow prediction problem. Leveraging the strengths of the KNN algorithm—which utilizes historical traffic patterns based on similarity—the hybrid model integrates individual KNN models with varying hyperparameters to enhance prediction accuracy. With the similarity measure held fixed, the hyperparameters used in the hybrid approach include the window size and the number of nearest neighbors. Experimental results indicate that this hybrid approach outperforms the standard KNN model, offering a promising alternative to traditional time series forecasting methods. An additional advantage of the proposed method is its interpretability in model design. In particular, the optimal models exhibit hyperparameters that align with intuitively expected traffic dynamics, and the overall method effectively captures both short-term and long-term similarities in traffic flow. Beyond standard accuracy metrics, statistical tests are also conducted to verify that the improvements are statistically significant. *Acknowledgement: This study has been supported by the Scientific and Technological Research Council of Turkey (TÜBİTAK) under the Grant Number 124F023.*

Keywords: *K-Nearest Neighbor, Hybrid Model, Time Series, Forecasting, Traffic Flow.*

INTRODUCTION

Predicting traffic flow accurately is essential to optimize transportation networks, improve navigation systems, and aid urban planners in designing more efficient road infrastructures. However, traffic dynamics is highly non-linear and is influenced by numerous external factors such as weather conditions, road incidents, and human driving behavior. Developing robust traffic flow prediction models requires methods capable of capturing these complex temporal and spatial patterns while remaining computationally feasible for real-time applications. See (Kashyap et al., 2022) and (Medina-Salgado et al., 2022) for general discussions on traffic flow prediction problems.

Over the years, various approaches for traffic prediction have been proposed, ranging from traditional statistical models to advanced machine learning techniques. Classical time-series models such as the AutoRegressive Integrated Moving Average (ARIMA) and Vector AutoRegression (VAR) have been widely used due to their interpretability and solid theoretical foundations (Chen et al., 2011; Zhang, 2020). However, these methods often struggle with non-stationary data and fail to capture long-range dependencies. More recently, deep learning models, including artificial neural networks, recurrent neural networks, and graph-based techniques, have demonstrated strong predictive capabilities. See (Kashyap et al., 2022) and (Sayed et al., 2023) for some exemplary work. Although effective, these models require extensive training data, significant computational resources, and often suffer from a lack of interpretability.

An alternative to these complex models is the K-nearest neighbors (KNN) algorithm, a nonparametric approach that leverages historical traffic patterns to make predictions based on similarity. The KNN model identifies past instances with similar traffic conditions and uses them to estimate future traffic states. The method has been extensively studied in traffic flow prediction literature. See, for example (Arslan et al., 2023; Habtemichael and Cetin, 2016; Smith et al., 2022). This method is useful in situations where time series exhibits recurring patterns, such as daily rush hours or seasonal fluctuations for the particular case of traffic flow. Additionally, KNN's simplicity and ease of implementation make it an attractive choice for real-time forecasting. Despite these advantages, standard KNN may struggle with large datasets and suboptimal neighbor selection, necessitating further optimization to enhance its performance.

In this paper, we propose a hybrid model that is within the KNN-based approach. The KNN model inherently has the hyperparameter K, the number of neighbors close to the window of interest, along with w, the size of the window considered. By varying (w,K), one obtains distinct KNN forecasts, and our goal here is to combine these for better accuracy. This, in turn, allows us to capture short-term dependencies with smaller values of w and long-term dependencies for larger values of it. As to be seen in the following, the resulting model is superior to the standard KNN, and competitive among other alternatives. Further, it gives more insight to the problem due to its interpretability.

The remainder of this manuscript is organized as follows. The next section provides a brief review of the literature. Section 3 is on the methodology, where the dataset is described and discussed. Also, in this section, we include background on KNN and introduce the suggested hybrid model. Section 4 is devoted to the experimental results, and lastly the paper is concluded in Section 5 with some concluding discussions.

LITERATURE REVIEW

Traffic flow prediction has been an active area of research for decades, with early approaches relying primarily on statistical time series models. Traditional models such as ARIMA and its extensions, including Seasonal ARIMA (SARIMA), have been widely applied due to their ability to capture temporal dependencies in traffic data (Chen et al., 2011; Zhang, 2020). Other statistical techniques, such as Vector AutoRegression (VAR) and Kalman filtering, have also been used for forecasting purposes (Lippi et al., 2013). These models provide interpretability and are computationally efficient, making them suitable for real-time applications. However, their main limitation is their reliance on stationarity assumptions and linear relationships, which often fail to capture the non-linear and chaotic nature of traffic flow. To address these shortcomings, hybrid models that combine ARIMA with machine learning techniques, such as ANN-ARIMA or SVR-ARIMA, have been explored, demonstrating improved accuracy by integrating statistical and nonlinear predictive capabilities. See, for example, (Chi and Shi, 2018) and (Yılmaz, 2022) for such combinations.

With the rise of machine learning, data-driven approaches have become increasingly popular for traffic forecasting. Traditional machine learning techniques such as K-nearest neighbors (KNN), decision trees, and support vector machines (SVM) have been employed to model complex traffic patterns. More recently, deep learning models, including recurrent neural networks (RNNs), long short-term memory (LSTM) networks, and gated recurrent units (GRUs), have demonstrated superior performance by effectively capturing temporal dependencies in traffic data. See (Kang et al., 2017), and (Kashyap et al., 2022) also for a general overview. Furthermore, convolutional neural networks (CNNs) have been utilized (Zhang et al., 2019) to extract spatial correlations in traffic networks when combined with graph-based models such as graph neural networks (Jiang and Luo, 2019). Despite their high predictive accuracy, deep learning methods require large-scale datasets, significant computational resources, and careful hyperparameter tuning, which can limit their practical deployment in real-time systems.

MATERIAL AND METHODS

The Dataset

The experiments in this study on traffic flow prediction are done with the Performance Measurement System (PEMS) dataset. This is a widely used traffic dataset in the literature collected by the California Department of Transportation. It consists of real-time traffic flow measurements obtained from loop detectors installed across the California freeway network. The dataset includes key traffic parameters such as vehicle counts, average speed, and occupancy, recorded in regular time intervals of 5 minutes. In our study these 5 minute lengths are combined, and the time flow data is treated in periods of 15 minutes. Due to its large scale and high granularity, it has been extensively used in machine learning and deep learning research to develop advanced traffic prediction models.

Our study below will focus on two specific stations that are labeled 716241 and 773535 in the PEMS database. During selecting these two, we pay particular attention to the occupancy of the corresponding road, as a very low occupancy one makes the problem less interesting. In both cases, the missing values were around 1% and these were filled by using the historical values. The following figure shows the behavior of the traffic flow time series corresponding to station 716241 for a one week period.

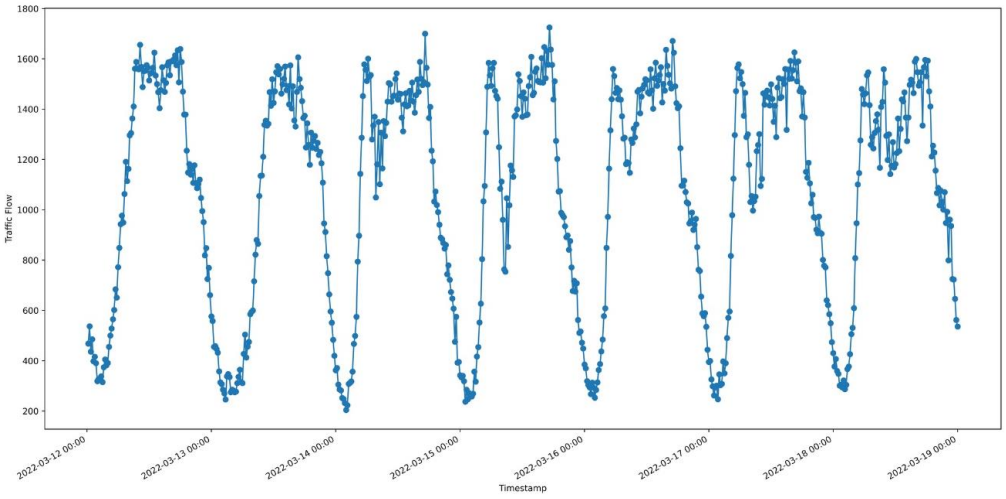

Figure 1. Weekly plot for Station 716241.

The dataset we use consists of 64 weeks. The first 40 weeks (Weeks 1-40) are used to get optimum values of w and K by experimenting with them over the next 12 weeks (Weeks 41-52). The remaining 12 weeks (Weeks 53-64) make up the test set, and the 40 weeks just before the 12 weeks of the test set are used for predictions for the test set.

KNN and a Hybrid Model

The hybrid model we use will be based on the K-nearest neighbor (KNN) model, which is also known as the method of similar trajectories in the literature. As noted earlier, this method has been extensively used in the literature in various forms. See (Arslan et al., 2023; Habtemichael and Cetin, 2016; Smith et al., 2002) for some exemplary papers. The KNN method in our case of time series forecasting can be described as follows. Consider a time series X_1,…,X_N of interest. Suppose that we are at time T and that we would like to provide a point forecast X^_(T+1) for time T+1. For this purpose, one chooses a window size w, a similarity measure d on R^w and also the number of nearest neighbors K to be used in the forecasting process.

In the setup just described, the KNN algorithm first forms the window (X_(T-w+1),…,X_T) of length w of the most recent observations. Afterwards a search of windows of length w of consecutive values in the past data gives the closest K windows with respect to the similarity measure d. Then, each of the values in the

time series right after the K windows is considered as a candidate for the forecast. The algorithm concludes by combining these candidates in a certain way to obtain a final forecast.

In the literature, the similarity measure d is often taken to be the weighted Euclidean distance as it outperforms the others (Arslan et al., 2023; Habtemichael and Cetin, 2016). The same selection is followed in the present work. Once the candidates via the closest windows are chosen, there is a numerous way of combining these; the simplest one being merely taking the average of the candidates, which we do in this study as well. In general, the hyperparameters w and K, together with the similarity measure d form a significant flexibility in the design of KNN-based algorithms. Our goal here is to focus on distinct KNN-models for varying w and K, and to combine the resulting models to obtain more accurate predictions without losing the advantage of high interpretability.

The overall method, demonstrated in Figure 2 below, can be summarized as follows. First, fix the number m of KNN models to be used in the hybrid model. Although, in principle m can any positive integer, we had our experiments with m∈{1,2,3,4} since the computational cost becomes high when m increases. After selecting m, choose the most optimal m tuples (w_i ,K_i), i∈{1,…,m}, in R^2 that best fit the data. These m KNN models then provide m candidates F_1,…,F_m as a forecast. The final forecast is then formed using these candidates. The model described will be denoted by KNNC.

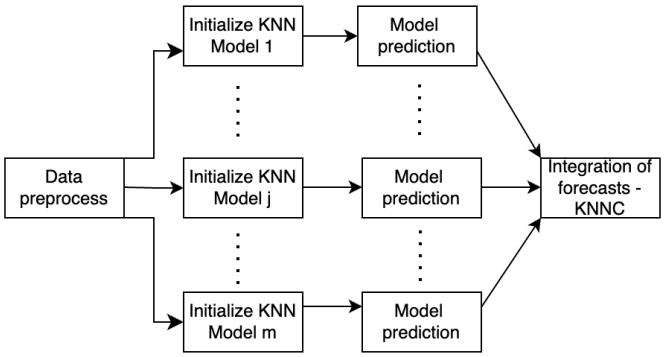

Figure 2. Flow Chart for the KNNC model

In general, KNNC((w_1 ,K_1), …, (w_m ,K_m)) is the combined KNN model where the jth individual model has a window size and the number of nearest neighbors w_j and K_j, respectively. In our case, once the candidates F_1,…,F_m from each of the KNN models are collected, the overall forecast is given by F ̂=1/m ∑_(j=1)^m F_j .

EXPERIMENTAL RESULTS

The models tested in this study will be compared by the corresponding mean absolute error (MAE) and mean absolute percentage error (MAPE). In addition to these, we also used the Diebold-Mariano test to see whether the differences in accuracies are statistically significant or not. The following table summarizes our results for the test set selected for station 716241.

Table 1. Prediction Results for Traffic Flow at Station 716241

Model	**MAE**	**MAPE**
Naive	70.29	7.22
ARIMA(2,0,5)	65.79	6.55
KNNC((22,30))	57.21	6.11
KNNC((7,25), (38,25))	54.7	5.90
KNNC((5,40), (16,10), (38,25))	54.75	5.87
KNNC((5,40), (16,10), (38,10), (38,25))	54.48	5.84

Here, besides the naive forecast, we also included the ARIMA result since it is another standard benchmark in traffic flow forecasting. The KNNC models we trained are all superior to both as expected. Also, in KNNC construction we see that the performance gets better as more KNN models are involved. In our case, the best results are obtained with KNNC((5,40), (16,10), (38,10), (38,25)). It is likely that increasing the number of KNN models used will improve the accuracy even further, but that comes with a corresponding computational cost. Next, we also present the following table, which summarizes the results for station 773535.

Table 2. Prediction Results for Traffic Flow at Station 773535

Model	**MAE**	**MAPE**
Naive	67.96	7.20
ARIMA(5,1,4)	63.40	6.60
KNNC((17,35))	55.34	5.80
KNNC((13,25), (35,40))	54.17	5.71
KNNC((7,10), (16,25), (35,25))	53.93	5.65
KNNC((7,10), (15,10), (16,25), (40,25))	53.91	5.65

Comments similar to Station 716241 also apply for Station 773535. After comparing the MAE and MAPE values of different models, we also implemented the Diebold-Mariano test (Diebold and Mariano, 2002) to see whether the improvements through the KNNC modes are statistically significant. The Diebold-Mariano (DM) test is a statistical test used to compare the predictive accuracy of two competing forecasting models. It evaluates whether the difference in forecast errors between two models is statistically significant, helping to determine if one model consistently outperforms the other. The test results revealed that all KNNC models were statistically superior to KNN((22,30)) and KNNC((17,35)) at a significance level of 0.001 for Station 716241 and Station 773535, respectively.

CONCLUSION

In this work, we introduced and studied a hybrid model in which the components are KNN with varying hyperparameters. The experimental results suggest that the hybrid model outperforms the standard KNN model. Although we did not include details for comparisons with other econometric or artificial intelligence models, the results we obtained are promising. Further, the hybrid model we analyze has the advantage of interpretability because of its construction. Indeed, the hyperparameters chosen agree with what one would expect intuitively. For example, focusing on the results for station 716241 with two forecasts, the ideal

model we had was KNNC((7,25), (38,25)). Recalling here that 7 and 38 are window sizes, we see that the introduced approach takes both short-term and long-term similarity when forecasting.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

İlker Arslan and Ümit Işlak formed the methodology of the algorithm. Elif Yılmaz took part in the implementations and statistical tests. The article was written by three authors together.

Accident Data of Istanbul Trolleybuses (1961-1976) (1034)

Murat Arısal[1*]

[1]Marmara Üniversitesi, İktisat Fakültesi, İktisat Bölümü, 34722, İstanbul

[*]Corresponding author e-mail: murat.arisal@marmara.edu.tr

Abstract

Following the 1950s, there was a rapid increase in the use of rubber-tired vehicles in urban traffic throughout Turkey. After the urban development initiatives implemented in Istanbul, tramways which had previously carried nearly half of the passengers transported by public transportation alone were phased out in favor of trolleybuses. Prior to Istanbul, trolleybus operations had already commenced in Ankara in 1947 and in İzmir in 1954. However, due to the widespread use of tramways in Istanbul's transportation system, the introduction of the first trolleybus service was postponed until May 27, 1961. Replacing the long-standing tramway system, which had constituted the backbone of urban transportation in Istanbul, trolleybuses assumed a critical role in the city's transit network. This significant role merits a thorough and detailed examination.

This study covers the period between 1961 and 1984, during which trolleybuses were actively operating in Istanbul's urban traffic. Accident data from the start of trolleybus operations in 1961 up to 1973 when the system transported the highest number of passengers have been extracted from archival documents and analyzed. Additionally, data from the years 1976 and 1977 were included, extending the statistical set to a fifteen-year period. Accidents have been categorized as involving injury, fatality, or property damage. Furthermore, the number of accidents per million kilometers traveled has been calculated.

With the future inclusion of accident statistics for buses and trams also operating in urban transportation, it will be possible to conduct comparative analyses among different modes of transit and to draw more meaningful conclusions. This study is considered a preliminary step toward that goal.

Keywords: *Istanbul, Transportation, Accident Data*

INTRODUCTION

Following 1950, there was a rapid increase in the use of rubber-tired vehicles in urban traffic in Turkey. Urban development initiatives carried out in Istanbul led to the abandonment of trams—which had transported half of all public transport passengers on their own—and paved the way for the adoption of trolleybuses. The operation of trolleybuses in urban transportation began in Ankara in 1947 and in İzmir in 1954, preceding Istanbul. The active presence of trams in Istanbul's transportation network delayed the launch of trolleybus services until May 27, 1961.

The trolleybus, which was introduced as a replacement for the tram—long the backbone of Istanbul’s public transportation—warrants a detailed analysis in terms of the role it assumed in urban transit. The trolleybus was launched with great expectations for resolving Istanbul’s traffic problems. The increase in rubber-tired vehicles, which could move more freely than trams in urban traffic, had notable effects on traffic flow and safety. Historical accident statistics regarding trolleybus operations are considered significant for evaluating the system’s performance and understanding the traffic regulations of the period.

MATERIAL AND METHODS

Within the scope of this study, accident data covering the 24-year operational period of trolleybuses were individually extracted through an archival document review and transferred into tables. A significant portion of the information regarding trolleybus operations was obtained from the IETT Budget Proposals and Balance Sheet & Operational Results books submitted to the Istanbul Municipal Council for the years 1961–1984. Missing data for certain operational years were supplemented using the final Istanbul Transportation Report, completed in August 1976 by the consultancy and engineering firm Sofretu, affiliated with the Paris transportation authority RATP (Régie Autonome des Transports Parisiens). The findings of the Istanbul Transportation Report also enabled cross-verification of results derived from archival documents.

The analysis of accident data provides insight into the development of policies concerning the flow and safety of urban transportation. This study examines the period between 1961 and 1984, during which trolleybuses were operational in Istanbul's urban traffic. Special focus is given to accident data from 1961, the year trolleybus operations began, to 1973, the year with the highest passenger numbers in Istanbul's public transit system. The study period was extended to fifteen years with the inclusion of data from 1976 and 1977. Accidents are classified into categories of injury, fatality, and material damage. Additionally, accident rates per 100,000 kilometers traveled by vehicles were calculated. Future studies may include the accident statistics of buses and trams operating in urban transport, making comparisons between different transportation systems possible and yielding more meaningful results. This study is considered a preliminary step in that direction.

TROLLEYBUS TRAFFIC

When the trolleybus network commenced operations on May 27, 1961, its fleet consisted of 100 vehicles (IETT Umum, 1964, p.6). The trolleybuses, each 10.5 meters in length and with a seating capacity of 80, were designed to carry approximately 150,000 passengers per day (Ağralı, 1959, pp.1, 7). The average commercial speed of trolleybuses operating on mixed-traffic roads was approximately 10–15 km/h (Jane's, 1991, p.141).

Although trolleybus operations began in 1961, the system did not reach full operational capacity until 1963 (IETT Umum, 1964, p.8). Upon the completion of its infrastructure, the trolleybus system was expected to significantly meet the city's transportation needs by providing service to various districts (IETT General Directorate, 1962, p.7).

Since 1961, IETT (Istanbul Electricity, Tramway and Tunnel General Directorate), the public authority overseeing transportation, primarily relied on buses and trolleybuses for urban transit, even though the use of shared taxis (dolmuş) and minibuses remained widespread (Istanbul Municipality IETT General Directorate, 1963, p.6). By the end of 1967, with the acquisition of new buses, the IETT fleet expanded to 489 buses and 100 trolleybuses, thereby enhancing the effectiveness of urban transportation (IETT General Directorate, 1968, p.42).

By the 1970s, reports from the IETT institution indicated a declining preference for trolleybuses compared to buses (Akıncı & Tuncer, 1976, p.3). The trolleybuses, which were significantly fewer in number than buses, mainly operated in central areas of the city (Kafalı et al., 1982, pp.190–192).

The number of trolleybus passengers rose from 4 million in its inaugural year, 1961, to 21.7 million in 1962, and reached 31.6 million in 1963, the year full capacity was achieved. In 1963, the average daily passenger count was 86,575. In 1967, when 88 out of the fleet's 100 vehicles were in operation, the annual number of passengers reached 40.4 million, equating to a daily average of 110,685 passengers. In 1968, with a service performance rate of 86%, 36.3 million passengers were transported. Up to 1980, the annual number of

trolleybus passengers remained around 30 million. Thus, the planned daily capacity of 150,000 passengers was never achieved on an annual average basis.

TROLLEYBUS ACCIDENT STATISTICS

It is understood that the majority of accidents involving trolleybuses during their operation in urban traffic consisted of non-injury property damage incidents. Accidents resulting in injury or fatality accounted for approximately 4% of all incidents on average. In certain years, such as 1969, this proportion decreased to around 1%, whereas in the early years of operation, it hovered around 6%.

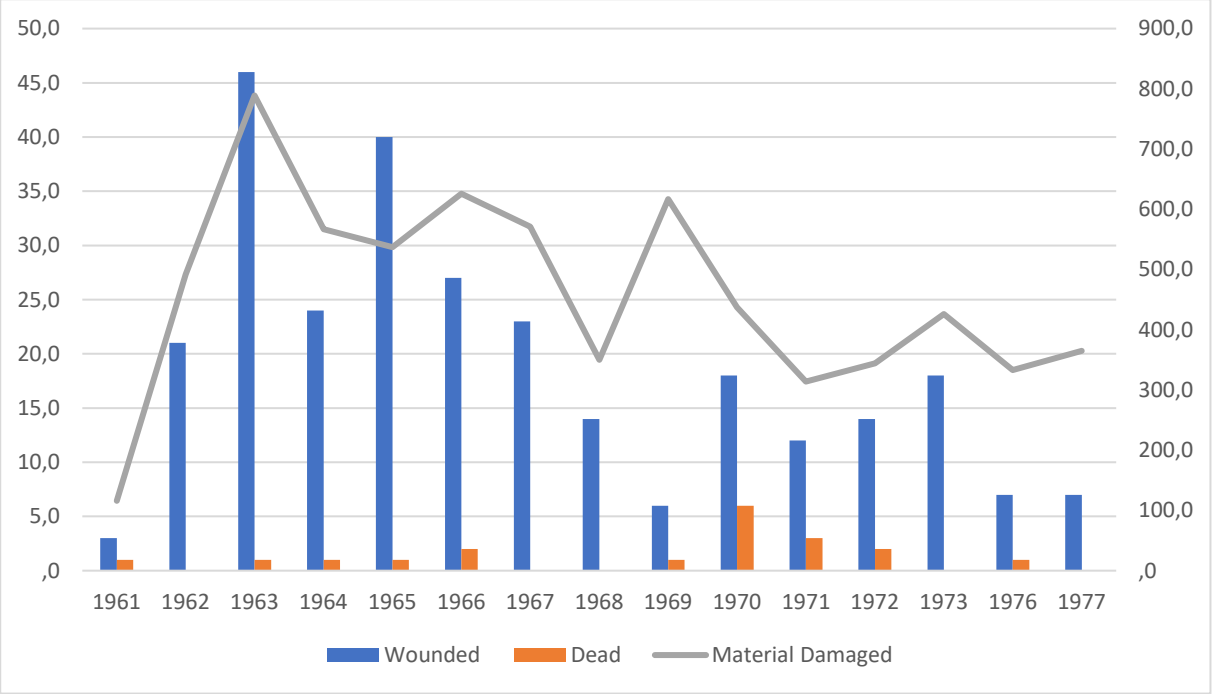

Figure 1. Types of Accidents Involving Trolleybuses (1961–1977)

It is clearly evident that the majority of trolleybus-related accidents were property damage incidents. The annual average number of injury-related accidents was calculated to be 19. The highest number of injury-related incidents occurred in 1963 with 46 accidents, followed by 40 accidents in 1965. In other years, the number of injury-related accidents remained around an average of 20. Fatal accidents were observed to average around one per year. While there were six fatal accidents in 1970, there were five years within the series in which no fatal accidents occurred.

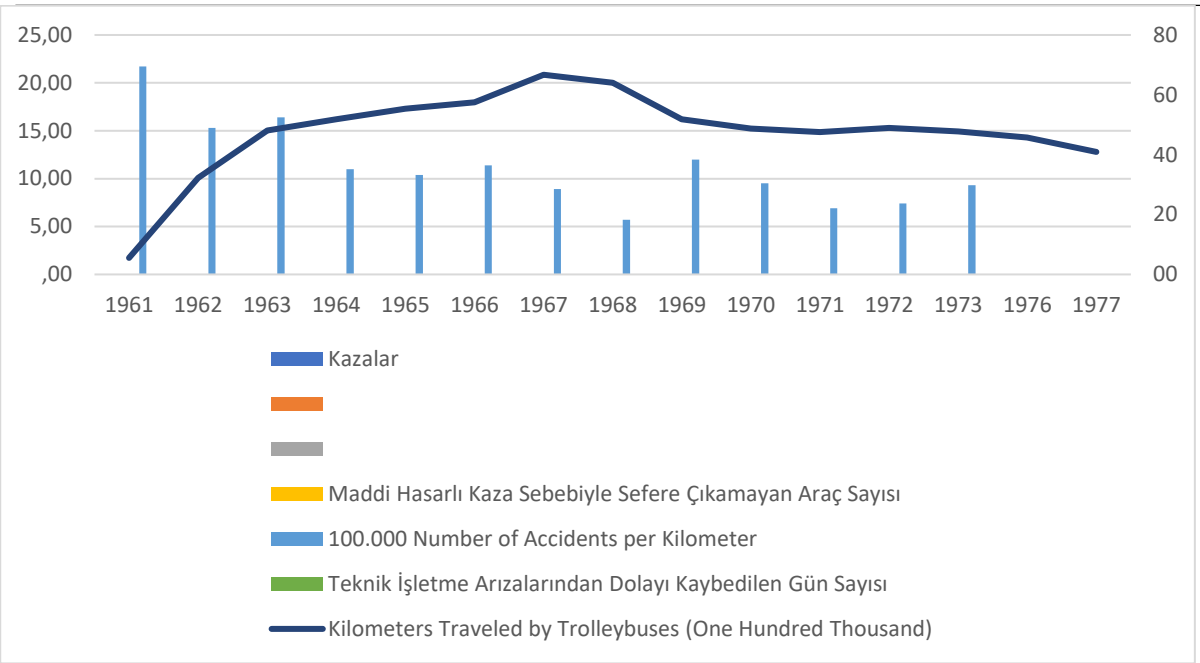

Figure 2. Number of Accidents per 100,000 Kilometers Involving Trolleybuses (1961–1977)

Trolleybuses in Istanbul traffic covered an annual average of 5.1 million kilometers. In 1967, this distance reached 6.7 million kilometers, and in 1968, 6.4 million kilometers were recorded. The annual average number of accidents per 100,000 kilometers traveled was calculated as 11.2. This figure remained relatively consistent around the average throughout the operational years. However, in the first year of service, 1961, the number reached 21.7. The fact that the number of accidents per 100,000 kilometers was more than twice the average in 1961 can be attributed to the public's unfamiliarity with the trolleybus system at the time. The corresponding figures were 15.3 in 1962 and 16.4 in 1963. From 1964 onwards, the number of accidents per 100,000 kilometers decreased to ten or fewer annually. In 1969, the reported number was 12.

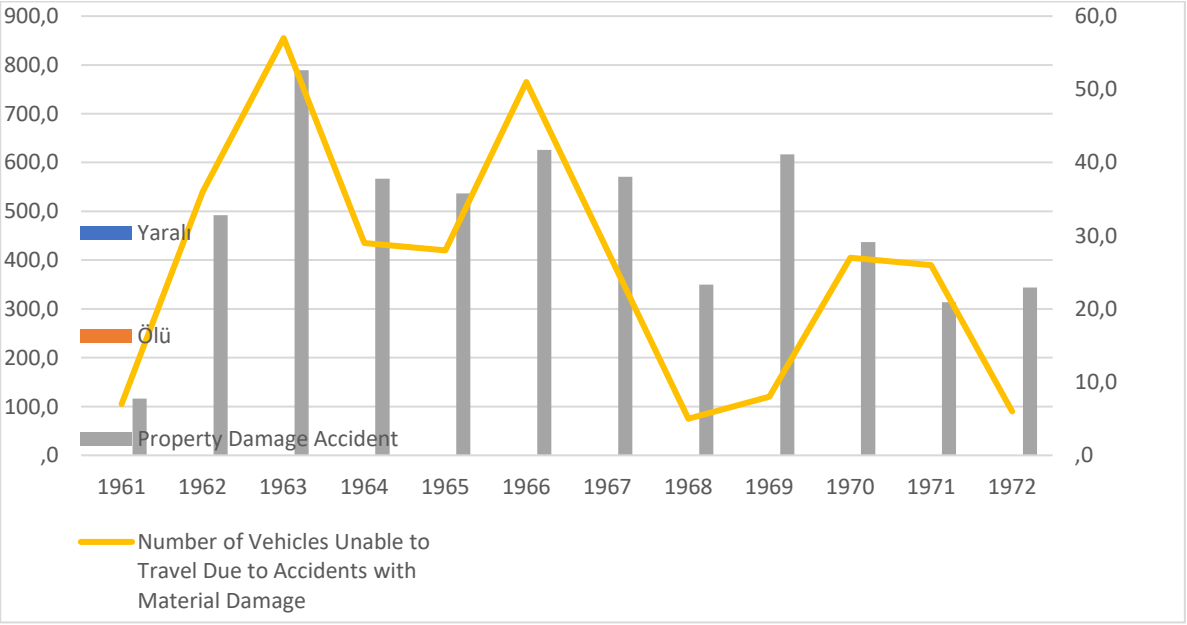

Figure 3: Number of Property Damage Accidents Involving Trolleybuses (1961-1972)

It is observed that the number of property damage accidents involving trolleybuses was relatively high in the early years of operation, but declined in the early 1970s. During the period under review, the annual average number of property damage accidents was 459. The average number of vehicles unable to operate due to damage resulting from accidents was approximately 26 per year. This figure, which is considerably high for a trolleybus fleet of 100 vehicles, indicates that the operable vehicle capacity was limited to approximately 74% on average. In 1968, only five vehicles were decommissioned due to property damage, enabling the highest operational performance in the history of trolleybus service, with a 86% service availability rate.

CONCLUSION

Of the accidents involving trolleybus vehicles operating in Istanbul traffic, 96% were classified as property damage incidents. The proportion of injury-related and fatal accidents, which affect passenger safety, was approximately 4% on average. Due to accidents, 26% of the fleet's vehicles were unable to operate. The fact that, on average, 26 out of 100 trolleybus vehicles were withdrawn from service annually due to property damage accidents not only indicates a significant operational loss, but also reflects a high accident rate per vehicle. For a transport mode like the trolleybus—characterized by limited maneuverability and operation along a fixed route—a 26% rate of accident involvement represents a notably high figure, offering further insight into the overall condition of Istanbul's traffic system.

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Attachments

EK 1. Trolleybus Loss Trip Statistics

Reason for Trolleybus Waste	**1976**		**1977**		**1982**		**1983**	
	Number of Trips	*Percantage*	*Number of Trips*	*Percantage*	*Number of Trips*	*Percantage*	*Number of Trips*	*Percantage*
Those who are late on the road	25.166	92,0	33.027	95,6	8.978	83,8	4.162	52,2
Due to Traffic	20	0,1	7	0,0	2	0,0	35	0,4
Due to Lack of Staff	559	2,0	266	0,8	121	1,1	891	11,2

Voyage Loss Due to Land	799	2,9	1.241	3,6	1.599	14,9	2.748	34,5
Don't Leave the Garage Late	12	0,0			20	0,2	86	1,1
Wire Breakage	88	0,3						
Power Outage	715	2,6					44	0,6
Total	27.359		34.541		10.720		7.966	

Ek 2: Trolleybus Operation Accident Statistics

Trolleybus Business																
		1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	1976	1977
Accidents	**Wounded**	3	21	46	24	40	27	23	14	6	18	12	14	18	7	7
	Dead	1	0	1	1	1	2	0	0	1	6	3	2	0	1	0
	Material Damaged	116	492	789	567	537	626	571	350	617	437	314	344	426	333	365
Number of Vehicles Unable to Travel Due to Materail Damage Accidenst	**Piece**	7	36	57	29	28	51	28	5	8	27	26	6			
Number of Accidents per 1.000.000 Km	**Piece**	21,7	15,3	16,4	11,0	10,4	11,4	8,9	5,7	12,0	9,5	6,9	7,4	9,3		
Number of Dyas Lost Due to Technical Business Supply	**Piece**	430	1.744	1.406	1.197	742	1.470	2.389	2.285	1.854	1.255	1.295	1.591	1.794	799	

Explainable Random Survival Forests Models for Predictive Maintenance of Aircraft Engines (1036)

Yigitcan Yardimci[*1], **Mustafa Cavus**[1]

[1]Eskisehir Technical University, Department of Statistics, 26555, Eskisehir, Turkiye

*Corresponding author e-mail: yigitcanyardimcii@gmail.com

Abstract

In recent years, predictive maintenance has become an essential component of operational reliability in aviation, where minimizing unplanned failures is critical. Estimating the remaining useful life of aircraft engines enables proactive scheduling of maintenance actions, thus improving efficiency and reducing costs. However, the presence of censored data in engine degradation datasets poses significant challenges for conventional machine learning models. Survival models are well-suited for handling censored data and generating risk predictions. Despite their statistical strength, state-of-the-art survival models are black boxes and often lack transparency, making it difficult for practitioners to interpret predictions and understand underlying decision mechanisms. This study proposes an explainable survival modeling framework for predictive maintenance. Specifically, the Commercial Modular Aero-Propulsion System Simulation dataset was used to train the Random Survival Forest model and the Cox Proportional Hazard model as a baseline to estimate the remaining useful life of aircraft engines. To explain the Random Survival Forest model, explainable artificial intelligence tools such as Time-Dependent Variable Importance and Partial Dependence Profile techniques were employed. These methods enable the uncovering of the reasoning behind model outputs and the identification of critical factors influencing engine degradation. The results demonstrate that sensor measurements related to engine temperature and pressure were the most influential variables in predicting failure times for the Random Survival Forest model, while they are less important for the Cox model. This indicates that the Random Survival Forest model can capture different relationships than the Cox model. Time-dependent partial dependence profiles indicate the fourth and fifteenth sensors showing stronger associations with increased failure risk. These findings highlight that integrating explanations of survival models improves the reliability of maintenance decisions.

Keywords: *Predictive maintenance, Survival model, Explainable artificial intelligence*

INTRODUCTION

Predictive maintenance systems are increasingly being adopted across various industries to enhance operational reliability while reducing maintenance costs. In aviation, predictive maintenance is crucial for cost reduction, safety, and system reliability (Hasib et al., 2023). However, censored data is one challenge in analyzing survival data (Yang et al., 2022).

Censored data refers to situations where it is not possible to determine with certainty whether a specific event, such as a failure, has occurred within a given timeframe. Such data introduces an additional uncertainty factor for standard models. The presence of censored data in survival analysis prevents the direct use of standard machine-learning models (Rahat et al., 2023). Survival models generate survival curves, also known as survival functions, and their primary purpose is to analyze censored data.

Remaining Useful Life (RUL) estimation plays a significant role in predictive maintenance applications (Pashami et al., 2023). It assists in scheduling maintenance operations at the optimal time, thereby preventing unnecessary downtime and system failures. In the field of machine prognostics, there is a limited number of studies utilizing survival analysis while considering censored data in the analysis of component failures (Yang et al., 2022). However, the presence of censored data in survival analysis makes it difficult

to apply commonly used machine learning models (Rahat et al., 2023). Furthermore, making predictions about how systems evolve is of great importance for maintenance planning and reliability analysis. In this context, survival modeling approaches used to estimate the lifetime distribution of systems have proven to be an effective method (Holmer et al., 2023).

The Cox Proportional Hazards (Cox) model (Cox, 1972) and Random Survival Forests (RSF) (Ishwaran et al., 2008) are two widely used survival analysis models in predictive maintenance applications. However, understanding and interpreting these models' prediction processes can often be challenging. In maintenance processes, evaluating predictions not only in terms of accuracy but also in terms of explainability contributes to more reliable and transparent decision-making.

Explainability is crucial for understanding the inner mechanism of the model and identifying the key factors that influence those predictions. In survival machine learning models, the ability to understand and interpret model outputs is a significant limitation. To address this issue, explainable artificial intelligence (XAI) algorithms are employed to enhance the transparency of complex prediction models. As a result, predictions derived from machine learning models can be evaluated more reliably in decision-making processes (Shi et al., 2024). However, explainability alone is not sufficient; the generated explanations must also be reliable and consistent to ensure that model outputs can be confidently used in real-world decision-making processes (Kundu & Hoque, 2023). In predictive maintenance applications, accuracy alone is not sufficient; it is also necessary to understand how models arrive at their predictions. Explainable models help build trust in maintenance processes and decision- making mechanisms, while also contributing to the detection of biases and errors within the model. However, the ambiguity in interpreting black-box models makes explainability critically important, especially in systems where reliability is essential. Indeed, Kundu and Hoque (2023), in their study using the C-MAPSS dataset, revealed that there are significant issues related to the explainability of the models used, and that these issues should not be overlooked. The main motivation of this study is not only to develop high-performing models, but also to make their decision-making processes understandable. For this reason, explainability has become the central aim of our work.

This study aims to train Cox and RSF models on the Commercial Modular Aero-Propulsion System Simulation (CMAPSS) dataset to analyze the key variables affecting survival predictions. Additionally, the survex library will be used to examine the impact of these models on survival time and to understand the reasoning behind their predictions, providing specialized explanations for survival model predictions despite their time-dependent changes (Spytek et al., 2023). Thus, we aim to enhance the explainability of survival models, providing decision-makers in predictive maintenance processes with a more transparent and interpretable framework.

MATERIAL AND METHODS

This study utilized survival analysis models to estimate the RUL of aircraft engines. Since conventional regression-based approaches cannot directly analyze censored data, the Cox and RSF models were chosen in this study, as they can produce more reliable estimates by incorporating censored data. These models can estimate survival times and evaluate censored observations, offering distinct analytical capabilities. However, it is more challenging to understand the prediction processes and decision mechanisms of survival models compared to other methodologies. Especially because the data are censored, it is necessary to use explainable artificial intelligence techniques specifically designed for survival analysis rather than general-purpose methods.

Methods

In this study, not only was the predictive performance of the models evaluated, but also the extent to which their decision-making processes could be understood was examined. In predictive maintenance applications, where failure predictions have a direct impact on operational decisions, model interpretability

is of great importance. Therefore, two primary explainability techniques were applied to the Cox and RSF models in this study: Variable Importance (VIMP) and Partial Dependence Plots (PDP).

To evaluate the contribution of variables to model performance, an out-of-bag permutation-based VIMP method was employed. VIMP is calculated by measuring the change in OOB prediction error when a variable's values are randomly permuted (Ishwaran, Lu & Kogalur, 2021). Furthermore, VIMP is defined as the difference between the prediction error of the original model and that of a new model in which the variable in question has been randomized (Ishwaran et al., 2008). Accordingly, VIMP analysis explains how much each sensor variable contributes to the model's overall prediction process. In this study, VIMP analyses were first used to identify which sensor measurements were considered more critical in the decision-making process of each model. Methods developed to explain the role of individual variables that significantly contribute to the model output have helped enhance the interpretability of VIMP analyses (Wies et al., 2023). These visualizations enabled a comparative analysis of the decision logic in both Cox and RSF models.

PDP plots, on the other hand, illustrate how changes in a given variable affect the model's predictions across various value ranges. Thus, the effects of different levels of selected sensor variables on the model predictions were analyzed through Partial Dependence Plots. These plots visualize how an increase or decrease in a variable influences the predicted RUL. PDP analysis made it possible to evaluate the effect of variables not only in terms of their importance, but also in terms of the direction and magnitude of their influence on the model outputs.

In conclusion, the combined use of VIMP and PDP methods provided a comprehensive and multifaceted understanding of the models' decision-making processes. This has made it possible to develop models that are not only highly accurate but also interpretable and reliable. Consequently, maintenance decisions can be made in a more informed and less risky manner.

Data

We used the CMAPSS dataset to analyze the operational performance of jet engines and, consequently, estimate the RUL. The dataset is designed to model engine performance under various operating conditions and failure modes. In this context, it consists of four subsets: FD001, FD002, FD003, and FD004, each representing a different operational scenario.

Each engine has been monitored over a certain number of operational cycles, and each data point has been collected through various sensors. The presence of censored data makes this dataset particularly suitable for survival analysis models. During the preprocessing phase, variables that remained constant throughout all cycles were removed to eliminate redundancy and enhance model performance.

The FD001 subset consists of 100 engines operating under a single, fixed environmental condition at sea level and includes only High-Pressure Compressor Degradation (HPCD) as the failure mode. The FD002 subset contains 260 engines operating under six different environmental conditions, all of which also exhibit only HPCD failure. FD003 includes 100 engines running under a fixed condition but with two types of failures: HPCD and Fan Degradation (FD). FD004, on the other hand, presents the most complex structure, with 248 engines operating under six varying environmental conditions and containing both HPCD and FD failure modes.

In this study, only the FD001 subset was considered. However, the proposed method can be extended to other subsets to broaden the scope of the analysis.

RESULTS

In this section, the findings of the study are presented under two main subheadings. The first subheading compares the performance of the Cox and RSF models. The second subheading focuses on the

explainability analysis of both models, where the effects of variables on survival are evaluated along with graphical representations.

Model Performance Evaluation

In this section, the performances of the Cox and RSF models were compared using only the FD001 dataset. The evaluation was carried out using two performance metrics: C-index and Brier Score.

The C-index is a metric ranging from 0 to 1 and reflects the model's ability to correctly rank survival times. As the value increases, the ranking accuracy of the model improves. On the other hand, the Brier Score represents prediction error, and therefore, a lower Brier Score indicates more accurate predictions.

The RSF model performed better than the Cox model on the FD001 dataset. The RSF model achieved a c-index of 0.730 and a Brier Score of 0.182, indicating superior results in terms of both ranking accuracy and prediction error. In contrast, the Cox model yielded a lower performance with a C-index of 0.600 and a Brier Score of 0.186. These results suggest that the RSF model was more effective on this dataset in terms of both ranking ability and predictive accuracy.

Explainability Analysis

In this section, we examined the decision-making processes of the Cox and RSF models in terms of XAI tools. The dataset was analyzed in detail to identify which variables held significance for the model and to assess how these variables influenced the prediction outcomes. We generated and interpreted VIMP and PDP.

VIMP was calculated to evaluate which sensor variables contributed the most to the model's predictive accuracy over time. In this analysis, each sensor variable was randomly permuted, and the resulting change in the Brier score was observed. The increase in the Brier score indicates the importance of that variable within the model. This method is particularly effective in survival models for examining how the influence of variables changes over time. In addition, only the most influential variables are presented in the plots. This allows for a clearer display and understanding of the variables that most affect predictive performance over time.

PDPs are created to visualize how different values of a sensor variable affect the model's survival prediction over time. In each plot, only the value of a single variable is changed while all other variables are kept constant, and the effect of that variable is observed across the time axis. The x-axis represents time, the y-axis indicates survival probability, and the colors show the values taken by the corresponding sensor. The variables shown in these plots were selected based on the results of the VIMP analysis, which identified the features that contributed most to the model's predictive performance. If the survival curve drops earlier and more sharply at higher sensor values, this indicates that the model associates these sensor values with a higher risk of failure. This feature of PDP plots is particularly valuable for improving the interpretability of survival models and for analyzing the effects of variables in greater detail.

As shown in Figure 1, Panel A presents the VIMP scores for the RSF model, while Panel B displays the same for the Cox model. In the RSF model, sensor_15 stands out as the most important variable throughout the entire period. In addition to this, sensor_3 and sensor_7 show notable effects on the Brier score, especially between cycles 175 and 220, with varying levels of influence over time. Other variables were influential only during limited time intervals.

In the Cox model, sensor_14 and sensor_9 appear as the most important variables. The permutation of these sensors led to noticeable increases in the Brier score. Moreover, some variables in the Cox model show a pattern of increasing or decreasing importance simultaneously over time. On the other hand, the RSF model exhibits a more fluctuating importance profile with a more dispersed pattern across time.

Figure 1 provides different perspectives on how variable effects behave over time in both models. While the RSF model generally shows greater fluctuation and higher variability, the Cox model tends to present more regular patterns. These differences emphasize that, in choosing between survival models, interpretability should be considered alongside predictive performance.

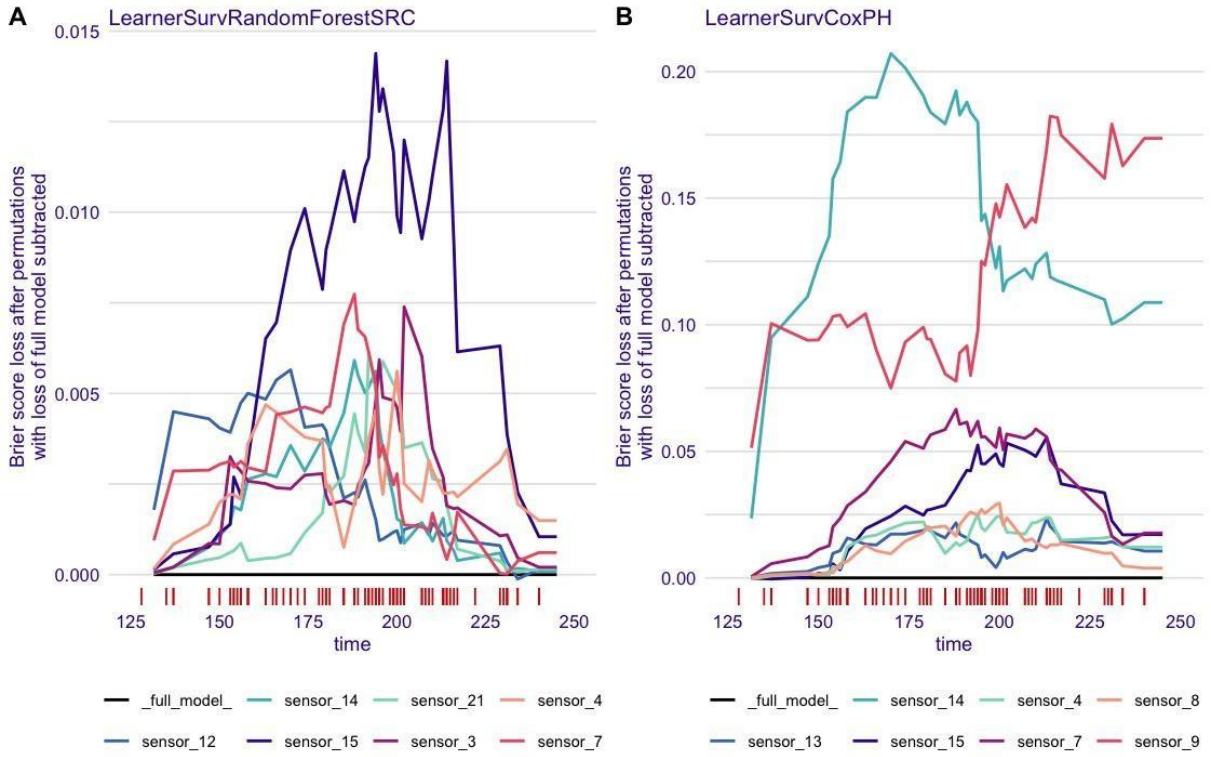

Figure 1. Time-dependent VIMP based on Brier score loss after permutation of the Cox and RSF models

Figure 2 shows the time-dependent PDPs produced using the Cox model with the FD001 dataset. These plots illustrate how the model's survival predictions vary over time across different value ranges of the selected sensor variables. In this way, it becomes possible to observe how the model interprets changes in sensor readings and how these changes affect the predicted survival probabilities.

In particular, for sensor_13 and sensor_9, a noticeable drop in survival probability is observed during the early periods when sensor values increase. This indicates that as the values of these two variables rise, the risk of engine failure also increases, and the model shows high sensitivity to changes in these features. Similarly, a decline is also observed for sensor_4. For the remaining sensors, the survival curves follow a nearly similar pattern, suggesting that the model is less sensitive to variations in those sensor values.

These plots demonstrate how the Cox model responds to different sensor values over time and offer meaningful insights for both model interpretability and technical analysis.

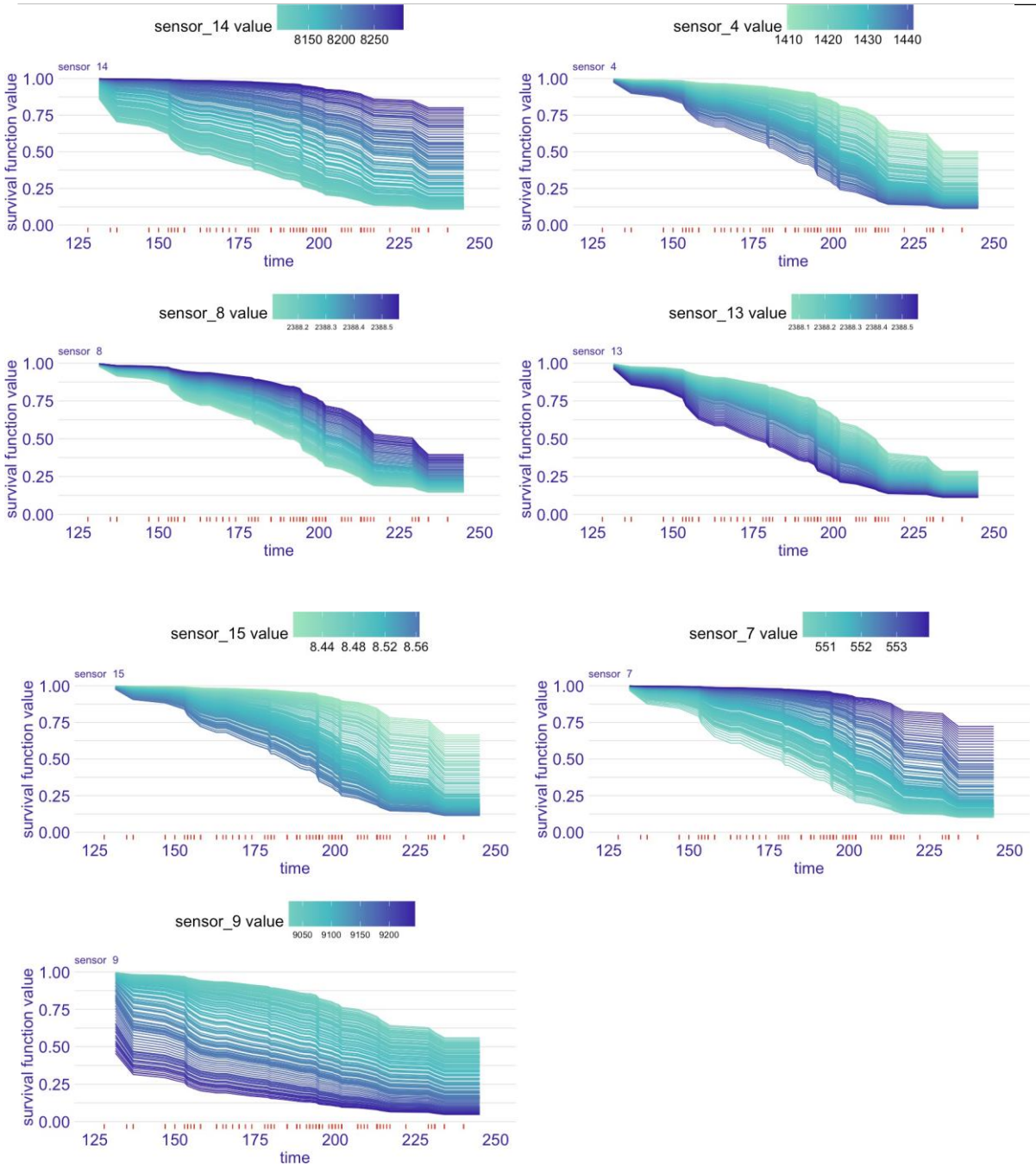

Figure 2. Time-dependent PDPs based on the survival probability of the Cox model

Figure 3 shows the time-dependent PDPs produced using the RSF model with the FD001 dataset. Upon examining the plots, it is understood that the model responds to varying sensor values in a balanced and consistent manner. As time progresses, the survival probability exhibits a steady decreasing trend across all sensor variables. This indicates that the RSF model reflects the engine degradation process with overall consistency.

However, in some sensors, particularly sensor_15 and sensor_4, the survival function shows a noticeable decline at higher sensor values. This suggests that the model associates elevated levels of these variables with a higher risk of failure. Nevertheless, instead of sharp or abrupt changes, these effects are reflected through smoother transitions, implying that the model accounts for interactions among variables and performs predictions in a balanced way.

In the plots of other sensors, the curves appear to follow each other closely. This suggests that the model responds with lower sensitivity to variations in these variables and that their influence in the decision-making process might be limited.

In conclusion, these plots show that the RSF model responds to time-dependent sensor data in a systematic and balanced manner, contributing to both the interpretability and predictive strength of the model.

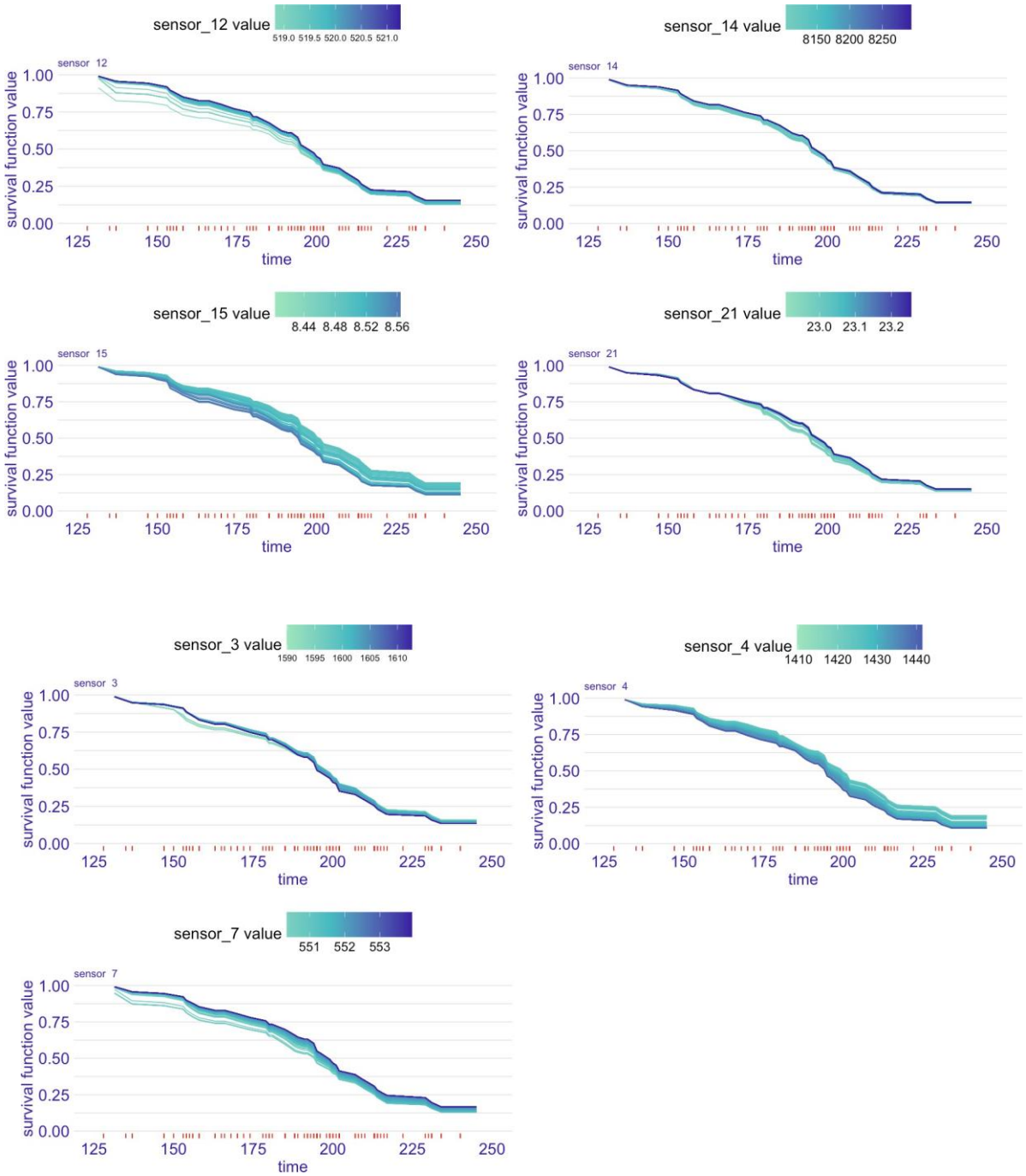

Figure 3. Time-dependent PDPs based on the survival probability of the RSF model

DISCUSSION AND CONCLUSION

In this study, survival analysis was conducted using the CMAPSS FD001 dataset with two models: the CoxPH model and the RSF model. The models were evaluated using standard performance metrics such as the C-index and Brier score; however, the study was not limited to these measures. The main objective was to understand the “black box” nature of the models — in other words, to uncover how each model generates

its predictions. Accordingly, interpretability analyses were carried out using VIMP and PDPs for both models.

The Cox model, due to its reliance on linear assumptions, reflects the relationships between covariates and the outcome through simpler and more interpretable patterns. While this allows the model to represent certain trends clearly and consistently, it can limit its ability to capture more complex or nonlinear relationships within the data. In contrast, the RSF model, with its tree-based structure, is capable of flexibly learning nonlinear relationships, time-dependent variable effects, and interactions between variables. This makes RSF more effective in uncovering hidden patterns in complex data structures that are often difficult to interpret.

Although both models achieve similar levels of predictive performance, the insights they provide differ. Their behavior can vary depending on the nature of the data, revealing fundamental differences in their internal mechanisms. Therefore, interpretability shifts the focus from what the model predicts to how the prediction is made.

In conclusion, especially in critical applications such as predictive maintenance where reliability is paramount, model selection should not be based solely on performance metrics. Understanding how a model operates, the transparency of its decision-making process, and its ability to model complex relationships are just as important as its predictive accuracy. This study, through an interpretability- focused comparison, highlights these aspects and offers valuable insights for more informed and reliable model selection.

Practical Implications

In this study, survival-based prediction models were not treated merely as black-box systems, but were also made interpretable. This outcome reveals which variables the models consider important, thereby providing a foundation for making more informed decisions in maintenance and monitoring processes. The proposed approach appears to be highly applicable in real-world contexts such as engine performance monitoring, fault detection, and various other domains. By prioritizing interpretability, this study offers a more reliable and transparent decision support process for both engineering teams and decision-makers.

Limitations and Future Work

In this study, only one subset of the CMAPSS dataset was used, and the analysis was limited to two survival models: Cox and RSF. On the other hand, there are many models available for survival analysis. This allows for future studies in the same research area to explore more diverse models and applications.

Furthermore, the models utilized in this study operated with default hyperparameter settings; no specific tuning of hyperparameters was undertaken. Nevertheless, adjustments in hyperparameters can impact both the model's performance and the results regarding interpretability. As a result, future studies might explore how varying hyperparameter settings affect both the accuracy and the explainability of the outcomes.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

All authors contributed to the study: Yigitcan Yardimci contributed to the implementation and results section and worked on the introduction and methods section, while Mustafa Cavus focused on the research question, design, and framework.

Simulink-Based Study of Pump-Driven Cooling: Understanding Water Circulation for Thermal Applications (1235)

Rahim Mammadzada[1*]

[1]Azerbaijan State Oil & Industry University, Faculty of Information Technologies and Control, Department of Instrumentation Engineering, AZ1010, Baku, Azerbaijan

*Corresponding author e-mail: rahim.mammadzada02@gmail.com

Abstract

Liquid cooling systems are widely used in thermal management across many industries, but the dynamic behavior of pumps is rarely modeled on its own. Most simulation models treat pumps as simple flow sources and do not capture how control inputs like voltage or speed affect flow rate and pressure over time. This creates a gap that makes it hard for engineers to design efficient and responsive cooling systems. While tools like Simscape can model pumps in detail, they often require complete technical data that is not always available or practical. In this paper, a theoretical pump model was made using Simulink, focusing on dynamic response based on the basics of liquid flow principles. The model shows how changes in input signals influence output flow and pressure, helping to predict performance more accurately under different conditions. By simulating the pump separately, this study offers a useful insight rather than a new tool, providing a practical way to build better control strategies and more realistic thermal models. The results highlight the benefit of using dynamic models instead of static ones and show how this approach can support future work in optimizing cooling systems, improving control design, and creating more flexible thermal management solutions.

Keywords: *Pump Dynamics, Water Cooling, Simulink Modeling, Input-Output Behavior, Thermal Management*

INTRODUCTION

Pumps play a vital role in various systems, including electronic cooling, HVAC, agricultural irrigation, and industrial processes. Although modeling tools such as Simulink and Simscape (MathWorks, 2025. Simulink Documentation; MathWorks, 2025. Simscape Documentation) offer built-in pump components, these blocks often require detailed internal specifications that are not made available by most manufacturers (EK Water Blocks, 2025). As a result, accurately simulating pump behavior becomes challenging, particularly for studies involving control, performance limits, or dynamic response.

In this paper, a simplified modeling approach is proposed to address this limitation. Using only manufacturer-provided data such as maximum head, rated flow, and speed, a basic linear model is formed based on fundamental fluid dynamics. Nonlinear effects are then introduced to better capture the pressure-flow relationship observed in real systems. This approach demonstrates that meaningful insight into pump behavior can be achieved, even with limited technical information.

MATERIAL AND METHODS

Material

In MATLAB, pumps are typically modeled using components from the Simscape Fluids library, such as the "Centrifugal Pump" or "Rotodynamic Pump" blocks (MathWorks, 2025. Simscape Documentation). These blocks require detailed input parameters like the pump head-flow curve, efficiency maps, and loss coefficients to accurately simulate performance across varying operating conditions. However, these inputs

assume access to comprehensive manufacturer data, which is rarely the case for small or commercially available pumps (Yüksel and Köseoğlu, 2019; Wang et al., 2021).

A commonly used liquid cooling pump is represented in Figure 1, the EK-D5 Vario (EK Water Blocks, 2025), which is widely applied in custom cooling and compact fluid systems. Its datasheet offers essential operating limits but lacks detailed hydraulic performance curves or internal loss coefficients. Like many commercially available pumps, it is designed for practical use rather than in-depth modeling, which limits its direct compatibility with detailed simulation blocks.

Figure 1. EK-D5 Vario Motor (EK Water Blocks, 2025)

Methods

The Collection of the Data

The simplified pump model uses basic specifications such as rated speed, maximum flow rate, and maximum head, all of which are taken from the manufacturer's datasheet (EK Water Blocks, 2025). These key values are summarized in Table 1 and serve as the basis for defining the mathematical relationships between flow, head, and rotational speed.

Table 1. Pump parameters provided by the manufacturer

EK Water Blocks EK-D5 Vario Motor Parameters	RPM		Flow Rate Q_{Max}	Head Pressure H_{Max}	Max Input Power	Adjustable Speed Steps
	Min	N_{Max}				
Values	1800	4800	1500 L/h	3,9 m	23W	5

Statistical Analysis

To model the relationship between pump rotational speed and flow rate, the classical affinity laws provide a starting point, stating that flow rate Q scales linearly with rotational speed N for geometrically similar pumps (Wikipedia, 2025; Intro to Pumps, 2025). Given manufacturer data specifying maximum flow $Q_{\max}$ at maximum speed $N_{\max}$, the linear approximation is expressed as:

$$Q = Q_{max} \cdot \frac{N}{N_{max}}$$

Although simple and intuitive, this model neglects the impact of system hydraulic resistance, which manifests as head loss increasing approximately with the square of flow. This loss is commonly modeled as:

$$H_{loss} = R \cdot Q^2$$

where R represents the system resistance coefficient, accounting for frictional losses in pipes, fittings, and valves. Physically, R encapsulates the hydraulic characteristics of the piping network and fluid properties, and thus must be evaluated or calibrated for each system.

The pump's characteristic head-flow relationship typically follows a parabolic curve, given by:

$$H = H_{max} \cdot \left(1 - \left(\frac{Q}{Q_{max}}\right)^2\right)$$

where H_{max} is the maximum pump head at zero flow. At steady state, the pump head balances the system head loss:

$$R \cdot Q^2 = H_{max} \cdot \left(1 - \left(\frac{Q}{Q_{max}}\right)^2\right)$$

To facilitate model implementation and parameter estimation, it is advantageous to define the pump curve constant:

$$K = \frac{H_{max}}{Q_{max}^2}$$

which quantifies the pump's characteristic head loss per unit squared flow. Using *K*, the balance equation simplifies to:

$$R \cdot Q^2 = H_{max} - K \cdot Q^2$$

or equivalently,

$$Q = \sqrt{\frac{H_{max}}{R + K}}$$

Moreover, considering the pump speed dependency, the affinity laws prescribe that the pump head scales with the square of the speed ratio Introducing a second constant:

$$H_{max,scaled} = H_{max} \cdot \left(\frac{N}{N_{max}}\right)^2$$

$$C = \frac{H_{max}}{N_{max}^2}$$

which represents the normalized head per squared speed, allows explicit incorporation of variable speed as follows:

$$Q = \sqrt{\frac{C \cdot N^2}{R + K}}$$

This formulation consolidates pump-specific parameters into the constants K and C, separating them from the system-dependent resistance R. Such separation enhances model clarity, eases parameter tuning, and simplifies implementation in simulation environments such as Simulink.

RESULTS

The pump model defines flow and head directly in terms of shaft speed using basic technical data. A quadratic term accounts for flow resistance, capturing typical pump behavior without added complexity. This approach supports fast and practical implementation in Simulink and connects easily to Simscape systems. The model allows users to estimate key performance characteristics using only RPM as the input. These results show that a reduced-variable formulation is effective for system-level studies, early design evaluations, and general-purpose use when full data is not available.

DISCUSSION AND CONCLUSION

This work proposes a basic yet effective pump model built on affinity laws, incorporating a quadratic system resistance relationship to reflect typical flow behavior. The formulation connects flow and head-to-pump speed, reducing the estimation problem to a single unknown variable. This makes the model highly suitable for direct use in Simulink and easy to integrate with Simscape networks. Its modular nature supports quick setup, parameter tuning, and design validation even when detailed manufacturer data is unavailable. While not intended to capture all physical complexities, the model is practical for steady-state evaluations, early-stage planning, and educational demonstrations. Future extensions could include performance adjustments based on test data.

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Acknowledgment

On March 18, 2025, MathWorks experienced a service disruption impacting several applications, including MATLAB and Simulink. As a result, simulations and plot generation for the pump model could not be completed during the preparation of this paper. Nevertheless, the developed model remains fully compatible with Simulink and can be validated once system access is resumed.

Conflict of Interest

The authors have declared that there is no conflict of interest.

Survival Analysis of Transition Time to Paid Version in Public Personnel Selection Examination (KPSS) Preparation Mobile App Users (1242)

Atilla Özdemir*[1,2] Mustafa Özaytaç[2], Damla Konya[2]

[1]Süleyman Demirel University, Faculty of Education, Department of Mathematics and Science Education, Isparta, Türkiye
[2]Hacettepe University, Department of Statistics, Ankara, Türkiye

*Corresponding author e-mail: atillaozdemir@sdu.edu.tr

Abstract

This study investigates the factors influencing the time it takes users of a KPSS preparation mobile application, HapNot, to upgrade to a premium membership. Employing survival analysis methods, particularly Kaplan-Meier estimates, Cox proportional hazards models, and parametric survival models, the study analyzes the behavioral patterns of 591 users, among whom only 7.3% upgraded to premium status. The analysis considered demographic and technological variables such as age, gender, education level, region, device brand, operating system, and acquisition channel. Findings indicate that premium users were slightly older on average and contributed more than twice the revenue of free users. Gender and education level significantly influenced upgrade likelihood, with female and university-educated users showing higher transition rates. Apple and iOS users also demonstrated greater initial risk of transitioning, although this effect diminished over time, suggesting violations of the proportional hazards assumption in some cases. Notably, most premium upgrades occurred within the first 30 days of use, particularly in the first 10–20 days. The effectiveness of Instagram advertising as a user acquisition channel was also highlighted, outperforming friend recommendations. Overall, the study provides evidence supporting the influence of user demographics, platform preferences, and acquisition channels on conversion timing, aligning with established technology adoption models such as UTAUT. The results offer practical insights for mobile learning app developers seeking to enhance monetization strategies through user segmentation and targeted engagement.

Keywords: *Survival Analysis, Public Personnel Selection Examination, Mobile Learning, User Behavior, Educational Technology*

INTRODUCTION

Mobile learning applications have become widely used among users with the rapid development of digital education technologies. A large portion of these applications adopt the "freemium" business model, where users can access basic educational content for free, but advanced content and features are offered for a fee. While the freemium model facilitates initial user acquisition, the conversion rates to paid subscriptions—which are critical for the sustainability of the application—tend to remain low, and these conversion decisions are often unpredictable.

Although the current literature includes various studies on revenue strategies and user behaviors in freemium models, the behavior of users transitioning to paid versions in mobile learning applications has been explored in a limited capacity. Particularly in mobile applications used for educational purposes, there is a need for comprehensive analyses to identify the demographic and technological factors influencing individuals' payment decisions.

The aim of this study is to examine the effects of users' demographic and technological characteristics on the time it takes them to switch to a paid subscription using time-based statistical methods, especially survival analysis. Within this scope, the factors influencing users to upgrade to a paid membership during

their free usage period were analyzed. The findings aim to contribute to the creation of more targeted user segmentations and the development of sustainable revenue models for mobile learning applications.

Survival analysis is a powerful method for modeling dynamic processes by considering not only whether an event occurs but also its timing (Box-Steffensmeier & Jones, 2004).

Objective: To analyze the time until an event occurs.

Duration (t): The time from the beginning to the occurrence of the event. Event: Death, illness, system dropout, etc.

Censoring: When the exact duration is unknown (e.g., participant leaves before study ends).

Survival Function (S(t)): The probability that the event occurs after a certain time.

Hazard Function (h(t)): The risk of the event occurring at a given time (instantaneous risk).

Although initially used in disciplines like medicine and engineering, it has also become widespread in fields such as marketing and behavioral economics. Topics like customer life cycle and churn risk can be analyzed more accurately using this time-based method (Leventhal, 2010).

Masarifoğlu and Büyüklü (2019) found that prepaid line users exited the system earlier and that customer loyalty increased with higher education levels. Within this framework, the behavior of transitioning from free to paid membership in mobile applications can also be meaningfully examined through survival analysis.

MATERIAL AND METHODS

Material

This study aims to analyze the time it takes for users of the Public Personnel Selection Examination (KPSS) mobile application named *HapNot* to purchase a premium membership and to identify the factors influencing this duration. The event is defined as the user purchasing a premium membership (event = 1). In this context, survival analysis methods will be used to evaluate the time until users switch to premium membership and the effects of demographic and technological variables influencing this process.

Methods

The Collection of the Data

The dataset used in the analyses includes the following variables related to individuals using the application:

- **Event Status (event):** Premium membership transition status (0: did not upgrade, 1: upgraded)
- **Time (time):** Number of days from the date the user downloaded the application to the date they purchased premium membership
- **Age:** User's age (18 years and older)
- **Gender:** Female / Male
- **Education Level:** High school graduate / University graduate
- **Region:** 12 regions according to NUTS1 classification (1–12)
- **Source of Awareness:** Instagram advertisement / Friend recommendation
- **Operating System:** iOS / Android
- **Device Brand:** Apple / Samsung / Xiaomi / OnePlus / Other

Statistical Analysis

In this study, the analysis began with descriptive statistics of the users, followed by an examination of the time to transition to premium membership using the Kaplan-Meier method. Kaplan-Meier survival curves were plotted for categorical variables to visually assess survival probabilities across groups, and the differences between these groups were tested using the log-rank test.

To evaluate the factors influencing the time it takes users to purchase a premium membership, the Cox proportional hazards model (Cox PH model) was applied. The proportional hazards assumption, which is fundamental to this model, was tested using Schoenfeld residuals and log-log survival plots. In cases where this assumption did not hold, an extended Cox model was utilized. If the hazard ratio of a variable was found to vary over time, an interaction term between the variable and time (e.g., variable × t) was added to the model to account for this change.

Additionally, parametric survival models based on Exponential, Weibull, and Log-Logistic distributions were constructed to compare model fit. The appropriateness of these models was assessed using the Akaike Information Criterion (AIC). Among these, the Weibull model was specifically examined for its ability to handle increasing or decreasing risk patterns over time. The data analysis was conducted using Stata and SPSS software. Model estimations and visualizations were generated through these programs.

RESULTS

Descriptive Statistics

The study sample comprises 591 users of the mobile application, of whom only 43 (7.3%) had upgraded to premium membership, while 548 users (92.7%) remained free users at the time of analysis. This high censoring rate strongly supports the use of survival analysis techniques, which are specifically designed to handle such cases where the event of interest (in this case, a premium upgrade) has not occurred for most individuals within the observation period.

In terms of demographic characteristics, the average age of users who upgraded to premium membership was slightly higher than that of those who did not (24.77 years vs. 24.03 years). Moreover, the average revenue generated per premium user ($6.11) was more than double the amount generated per free user ($2.44), suggesting a significant economic impact of conversion.

Gender distribution within the sample was relatively balanced, with female users making up 52.5% of the total. A similar balance was observed in device operating systems, although Apple users were slightly more dominant, constituting approximately 53% of the sample. When analyzed by device brand, Apple again stood out as the most commonly used brand, followed by Xiaomi, Samsung, OnePlus, and others.

In terms of education, university graduates (n = 366) made up a larger share of the user base than high school graduates (n = 225), which provides preliminary indications that education level may be linked to user behavior concerning premium adoption.

Regarding user acquisition channels, a significant majority (94%) reported discovering the app through Instagram advertisements, whereas only 5% had joined based on a friend’s recommendation. This highlights the strong effectiveness of paid digital marketing over organic or word-of-mouth growth in this context.

Finally, the geographic distribution of users, based on the NUTS1 classification system, revealed considerable imbalance. The largest number of users came from Region 12 (Southeastern Anatolia), with 79 users, while Region 1 (Istanbul) had only 6 users. This disparity should be taken into account when interpreting regional effects, as small group sizes may increase variance and reduce statistical power.

Overall, these descriptive statistics set the foundation for understanding the baseline user characteristics and point toward potential demographic and technological factors that may influence the likelihood and timing of transitioning to premium membership.

Kaplan-Meier Survival Analysis

Kaplan-Meier curves showed that most premium transitions occurred within the first 30 days, especially in the first 10–20 days. After day 100, transitions became sparse. The survival rate (i.e., staying as a free user) dropped to around 59% by day 300.

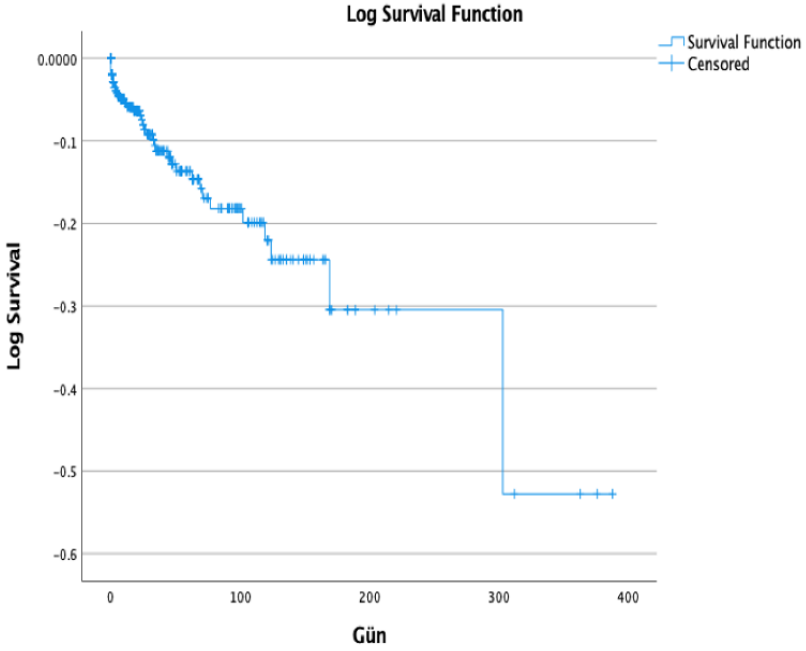

Figure 1. Kaplan-Meier survival curve

Gender Differences

Among male users, only 5% upgraded to premium, compared to 9.4% of female users. Log-rank, Breslow, and Tarone-Ware tests all indicated significant differences ($p < 0.05$). However, the proportional hazards assumption was violated ($p < 0.001$), and the effect of gender varied over time. Initially, women were 14 times more likely to upgrade (Exp(B) = 14.014).

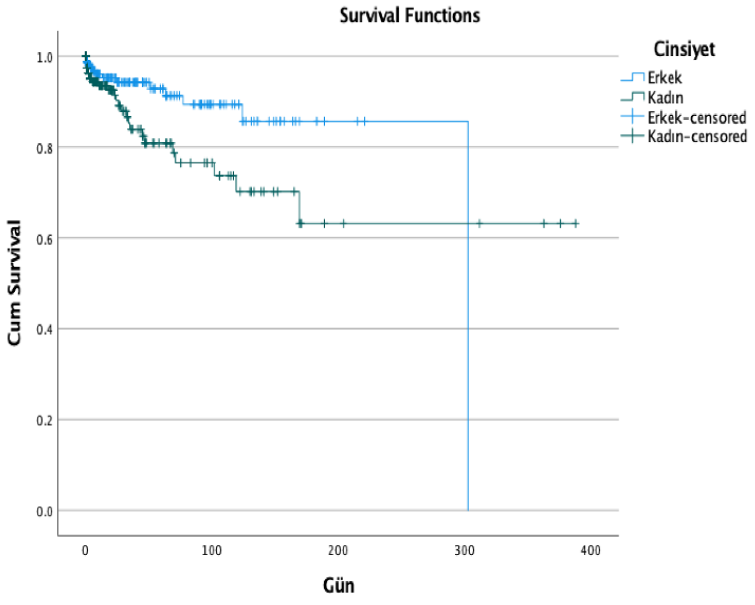

Figure 2. Kaplan-Meier survival curve by sex

Regional Differences

The highest upgrade rate was observed in the Northeastern Anatolia (TRA) region (18.7%), while some regions like Istanbul (TR1) had zero conversions. Significant differences were found between specific regions using log-rank and related tests (e.g., TR4–TRA: p = 0.017).

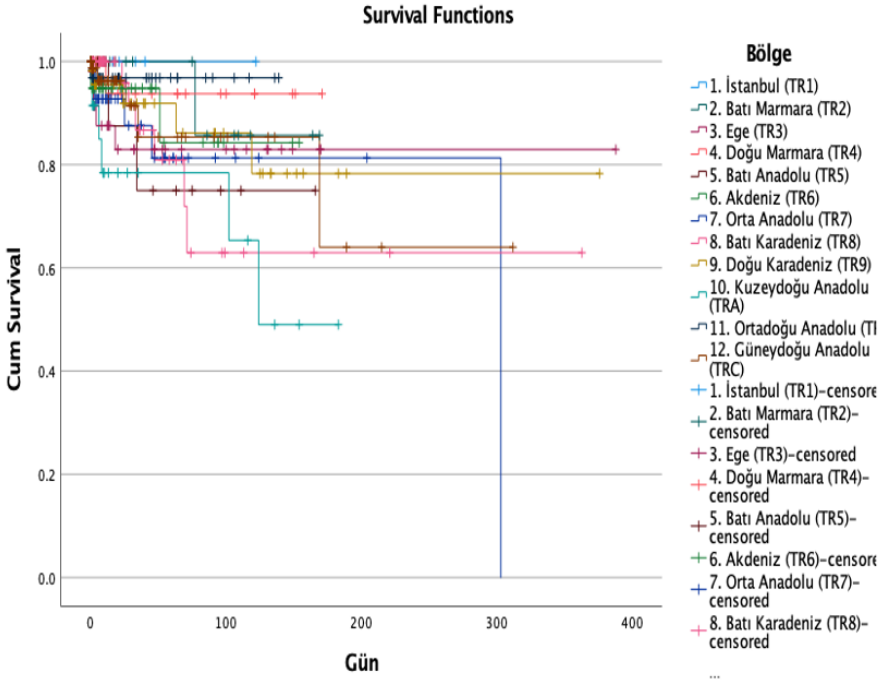

Figure 3. Kaplan-Meier survival curve by region

Operating System

iOS users exhibited a higher likelihood of upgrading than Android users (Exp(B) = 3.825, p = 0.001). However, this difference diminished over time, violating the proportional hazards assumption.

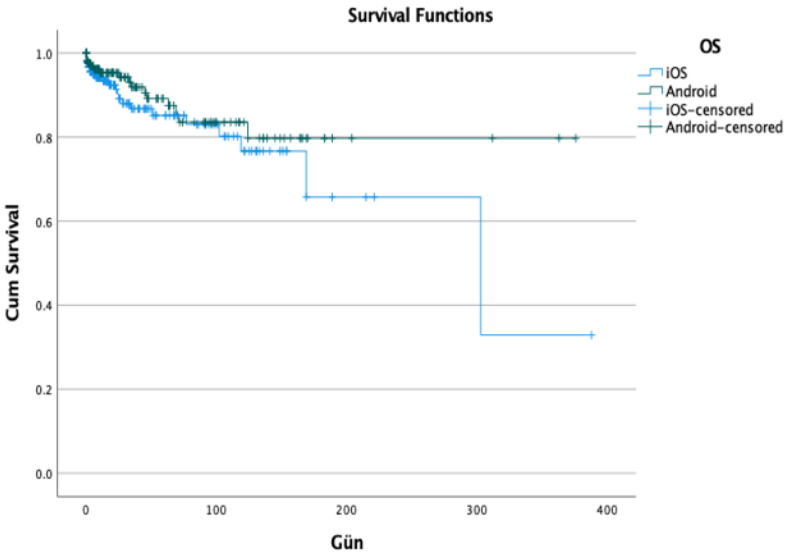

Figure 4. Kaplan-Meier survival curve by operating system

Device Brand

Apple users were the largest group but not the most likely to upgrade. Users of OnePlus, Xiaomi, and other brands showed significantly higher initial risks of transitioning to premium (Exp(B) values from 5.5 to 8.0). However, these differences narrowed as time progressed.

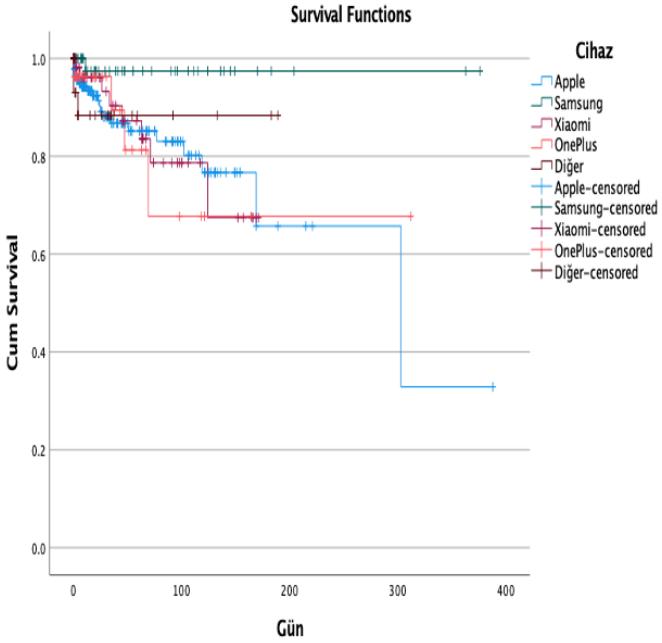

Figure 5. Kaplan-Meier survival curve by device brand

Acquisition Channel

Users who came via Instagram ads showed steadier upgrade behavior over time, while those who joined via friend recommendations transitioned earlier but with a sharply declining probability afterward. This variable also violated the proportional hazards assumption (Exp(B) = 0.058, $p < 0.001$).

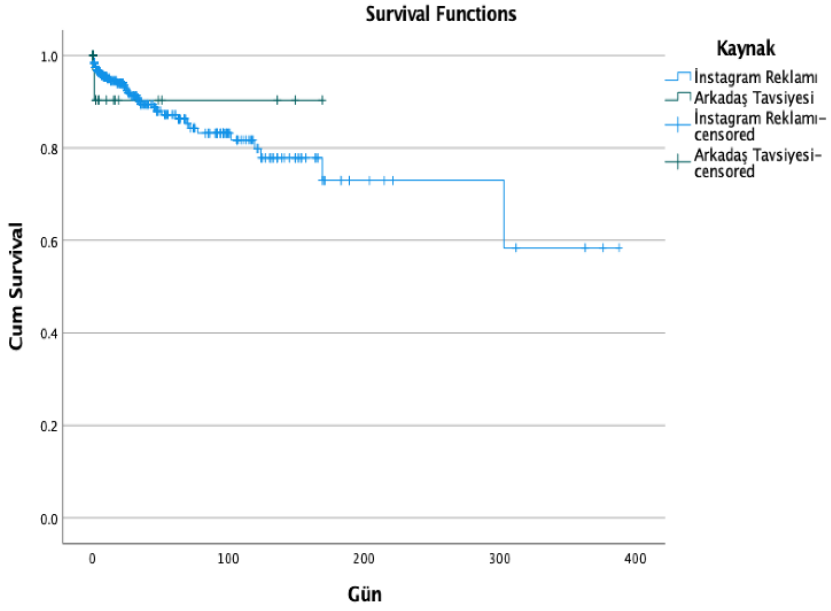

Figure 6. Kaplan-Meier survival curve by acquisition channel

Education Level

University graduates were significantly more likely to upgrade than high school graduates (Exp(B) = 13.36, $p < 0.001$), but this effect also declined over time (PH assumption violated). Around day 50, the risk equalized between the two groups.

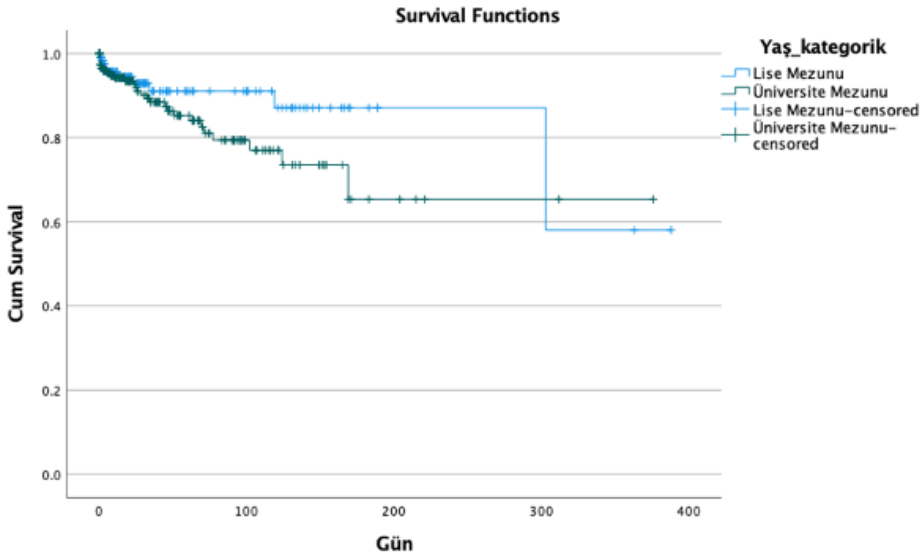

Figure 7. Kaplan-Meier survival curve by education level

DISCUSSION AND CONCLUSION

In this study, survival analysis methods were employed to examine the transition time to the premium version among users of *HapNot*, a mobile application designed for preparation for the Public Personnel Selection Examination (KPSS). The primary aim of the study was to evaluate the time-based factors influencing users' decisions to upgrade to premium membership. The findings broadly align with those reported in the existing literature across different contexts.

First, an analysis of users' age revealed that premium users had a significantly higher average age (24.77) compared to free users (24.03). This finding supports the conclusion of Hwang et al. (2024), who emphasized age as a demographic factor influencing technology adoption and payment intention. Similarly, the higher transition rate to premium among university graduates compared to high school graduates parallels the "performance expectancy" component of the Unified Theory of Acceptance and Use of Technology (UTAUT), indicating that education level plays a role in technology usage and financial decision-making.

Regarding the source of app discovery, the majority of users (n = 560) reported finding the app through Instagram advertisements. These users also showed higher rates of premium conversion compared to those who joined through friend recommendations. While this partially aligns with Hwang et al.'s (2024) findings on the role of social influence (r = .93), it also suggests that mass reach via social media platforms may be more effective than direct personal recommendations.

In terms of device type and operating system, Apple and iOS users exhibited a higher likelihood of transitioning to premium membership. This result aligns with Lin et al. (2020), who found that while Android devices might offer better retention rates, iOS users tend to be more engaged in payment behavior.

The impact of platform choice on monetization, as emphasized by Lee and Raghu (2014), is further supported by the findings of this study.

Although detailed data on initial usage time and user inactivity was not available, observations suggest that premium users tended to upgrade earlier and interacted with the app more frequently. This pattern is consistent with Hwang et al. (2024), who noted that recurring usage cycles and brief pauses followed by returns to the app significantly increase the likelihood of upgrading to a paid version.

Finally, in terms of average revenue per user, premium users contributed approximately 2.5 times more ($6.11) than free users ($2.44). This ratio resonates with Lee and Raghu's (2014) characterization of high-monetization user profiles. While the freemium model is effective in acquiring a broad user base, premium membership clearly offers a substantial advantage in terms of revenue generation.

In conclusion, this study supports the relevance of user demographics, technology acceptance, usage behavior, and platform preference—variables emphasized in the literature—as influential factors in premium conversion timing. Despite some contextual variations, the results largely validate theoretical frameworks such as UTAUT. The insights gained here are particularly valuable for developers of mobile learning applications seeking to better understand user behavior and optimize revenue strategies.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

All authors contributed equally to the conception, design, analysis, and writing of this study.

Visual and Descriptive Evaluation of Climate Studies Published Between 2015-2025 via Biblioshiny (1246)

Çiğdem Uçar[1*], Nur Kuban Torun[2]

[1]Bilecik Seyh Edebali University, Faculty of Economics and Administrative Sciences, Management Information Systems, Turkiye
[2]Bilecik Seyh Edebali University, Faculty of Economics and Administrative Sciences, Management Information Systems, Turkiye

*Corresponding author e-mail: cigdemucag@gmail.com

Abstract

Climate change constitutes a significant global challenge in modern society. Research and scholarly papers concerning this area are increasingly rising. This study aims to quantitatively analyze the scholarly literature on climate change. A search in the Web of Science database utilizing the keyword "climatechange" yielded 26,757 items published from 2015 to 2025, exclusively in article format, written in English, and indexed in SCI-E and SSCI. The data analysis was performed utilizing the Biblioshiny interface, which functions with the R programming language. The analysis encompasses the annual publication measure, the most referenced documents, the most cited sources, and author productivity within descriptive statistics. Visual analyses employed methods such as "TreeMap" and "Three-Field Plot" to examine the link between country, author, and word. The frequency analysis analyzed the most often utilized keywords, the fluctuation of terms over the years, and the most cited references and papers. The results were illustrated via graphs, a word cloud, and a global collaboration map. The findings indicate a substantial rise in publications about climate change, particularly during the past five years, with the majority of research originating from institutions in China, the United States, and Europe. This study enhances the comprehension of academic trends in the subject by examining the scientific reflections of climate change using statistical methods.

Keywords: *Climate Change, Descriptive Statistics, Frequency Analysis, Bibliometric Analysis*

INTRODUCTION

Climate change is an environmental concern stemming from the elevated carbon concentration in the atmosphere owing to anthropogenic greenhouse gas emissions, leading to enduring and worldwide consequences (IPCC, 2021). This phenomenon, characterized by elevated temperatures, altered precipitation patterns, glacial melt, and a heightened occurrence of extreme weather events, has recently emerged as a focal point in international policies and scientific inquiry, owing to its environmental and socioeconomic implications.

The rise in climate change publications necessitates comprehensive evaluations of the content, orientation, and distribution of academic works in this domain. Bibliometric analyses facilitate the systematic investigation of structural patterns, principal concepts, collaboration networks, and thematic trends in scientific literature via quantitative data, yielding objective insights into areas of focus, emerging topics, and the evolution of research. Consequently, bibliometric analysis approaches have emerged as a robust instrument commonly utilized in scientific research to elucidate the present status of the literature and prospective research trajectories (Donthu et al., 2021).

In the reviewed literature, in the study by Wang et al. (2014), provided a comprehensive overview of climate change vulnerability by bibliometric analysis, utilizing keywords such as "climatechange," "vulnerability," "bibliometric," and "backwardsearch" in the Web of Science (WoS) database. The authors analyzed the distribution of studies utilizing the required keywords over time, the participating nations, and the topical patterns in the research, thereby elucidating the evolution of the literature and the networks of international scientific collaboration. Fu and Waltman (2022) conducted an analysis of publications regarding global climate change from 2001 to 2018, investigating the extent of academic output, geographical emphases, and conceptual evolution on a worldwide scale. This study has elucidated the temporal evolution of climate change research focus points, the geographical distribution of research domains, and the research patterns among nations. The findings indicate a transition in research emphasis from physical sciences to climate technology and policies, reveal disparities in scientific output between developed and developing nations, and demonstrate that national agendas shape research directions. The research by Maharana and Pal (2023) analyzed literature on climate change and sustainable development approaches through bibliometric techniques. A total of 889 articles sourced from the Dimensions database were evaluated utilizing Biblioshiny and VOSviewer software in the study conducted. The investigations addressed bibliometric components like topic evolution, keyword co-occurrence maps, and the most prolific writers. The analysis indicates a consistent rise in publications within the pertinent subject, with an expectation of increased future research activity. The study by Marhana and Pal seeks to assist researchers in recognizing critical themes and research deficiencies in the pertinent subject. There has been a substantial rise in the quantity of scientific research undertaken in the domain of climate change in recent years. These articles mostly concentrate on particular subjects include greenhouse gas emissions, global warming, environmental sustainability, climate policies, and human health. Nevertheless, research on the interconnections across nations, authors, institutions, and concepts has been scarce. This condition reveals a substantial deficiency in both theoretical frameworks and knowledge generation within the discipline. In this setting, bibliometric analyses emerge as a crucial approach for assessing the overall structure of the literature and pinpointing deficiencies in the research domain. This study aims to analyze the organization and evolution of academic output on climate change through publications published from 2015 to 2025, employing bibliometric methodologies grounded in the Web of Science (WoS) database. This analysis encompasses the quantity of publications, prolific writers, keyword trends, collaborating nations, and cited papers. The results attempt to address the existing gap in the literature and to inform future research.

MATERIAL AND METHODS

Material

The research's material comprises academic publications regarding climate change. The primary data source for the study is the Web of Science (WoS) Core Collection database. Consequently, the search performed on WoS utilizing the keyword "climatechange" yielded publications in English, categorized as articles, published between 2015 and 2025, and indexed in SCI-E and SSCI. The acquired records were generated in BibTeX format and imported into the R application for analysis.

Methods

The study use bibliometric analysis as its methodology. Bibliometric analysis facilitates the quantitative assessment of the structural attributes, developmental trajectories, and collaborative dynamics of scholarly publications pertaining to a certain subject. This method facilitates the examination of factors including publication numbers, author productivity, highly cited publications, keyword density, and cooperation across countries and institutions.

The Collection of the Data

The dataset for the climate change literature study was generated by a search in the Web of Science (WoS) Core Collection database in May 2025. The Web of Science (WoS) is an interdisciplinary and international database for scientific literature that specifically includes articles from highly cited and respected journals. The study utilized exclusively the phrase "climatechange" for the search, thereby concentrating on material associated with climate change. Only publications categorized as "articles" from the years 2015 to 2025 were picked in the search results, with a preference for English as the language filter, and the Science Citation Index Expanded (SCI-E) and Social Sciences Citation Index (SSCI) were chosen as the indexes. The filtered dataset comprises bibliometric information (title, abstract, author names, author nations, journal of publication, publication year, citation counts, keywords, etc.) along with the fundamental text data of the articles. The bibliometric data has been exported in BibTeX format and is prepared for analysis.

Analysis of the Data

The R programming language and the Bibliometrix package (Aria and Cuccurullo, 2017) were employed for data analysis. R is an open-source statistical programming language preferred for data analysis, visualization, and modeling activities. Bibliometrix, a library for the R programming language, facilitates fundamental operations such the loading for bibliometric data, its summarization, the execution of network studies, theme analysis, and the production of graphical outputs. This study utilized Biblioshiny, the web-based and user-friendly interface of Bibliometrix, throughout the analysis procedure. Data in BibTeX format was uploaded to the system, thereafter analyzed through word clouds, annual publication distribution, identification of the most productive writers, cooperation visualizations, and keyword trends. The results are elaborated in the Results section, with supporting graphs and tables.

Statistical Analysis

The research on climate change literature provided descriptive statistics, including the annual publication count, the most prolific writers, and the most commonly utilized keywords. Furthermore, to visually substantiate the acquired data, word clouds, trend topic graphs, co-authorship visualizations, and thematic development maps have been created. The data acquired have served as a crucial reference for identifying the predominant study subjects throughout the literature, the evolving issues during time, and the most influential writers.

RESULTS

Number of Publications by Year (Annual Scientific Production)

The Annual Scientific Production graph illustrates the trends in the increase or decrease of academic publications on a certain topic over the years, so providing insights into the years with higher publication rates and the variability of scientific interest over time. Figure 1 illustrates a constant rising trend in the annual distribution of scholarly articles related to climate change from 2015 to 2022. The peak publication production was recorded in 2022. The primary reason for this increase is the expanded assessment of climate change, encompassing environmental, health, economic, and social implications in the post-COVID-19 age. Moreover, legal frameworks like the European Climate Law, enacted during the same period, provide other critical elements that have heightened scientific interest in this domain. A drop is noticed in the statistics for 2025. The reduction is attributed to the fact that the 2025 data encompasses only the first half of the year, as it remains incomplete.

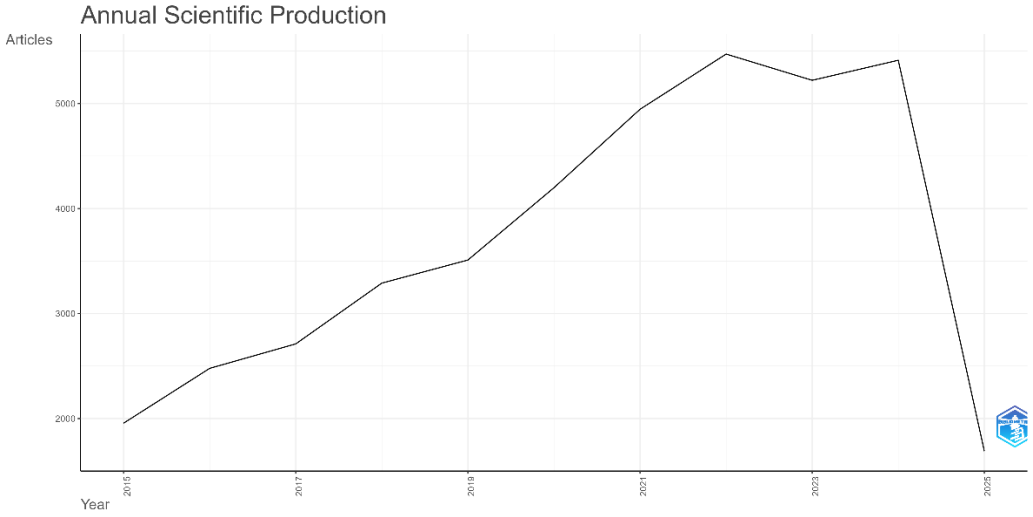


Figure 1. Annual distribution of climate change-themed publications from 2015 to 2025

Country – Author – Keyword Relationship (Three-FieldPlot)

The objective of the Three-Field Plot analysis in Figure 2 is to graphically illustrate the countries with active authors and the themes these authors are engaged in. Consequently, it facilitates the examination of research priorities, scholarly influence, and global trends concurrently. The three-field visualization study reveals notable correlations among the most prolific nations in climate change publications, the prominent writers from these nations, and the keywords they commonly utilize in their research. The statistics indicate that China and the United States are the foremost nations in publishing output. Prominent authors actively publishing in these nations include Zhang, Wang, and Li. Notable authors occupy a substantial position for both the volume of publications and their prominence in the literature. The studies predominantly feature the terms "climate change," "temperature," "precipitation," and "adaptation." These notions suggest that, alongside the environmental repercussions of climate change, adaptation strategies to the climate system are also significant in the literature.

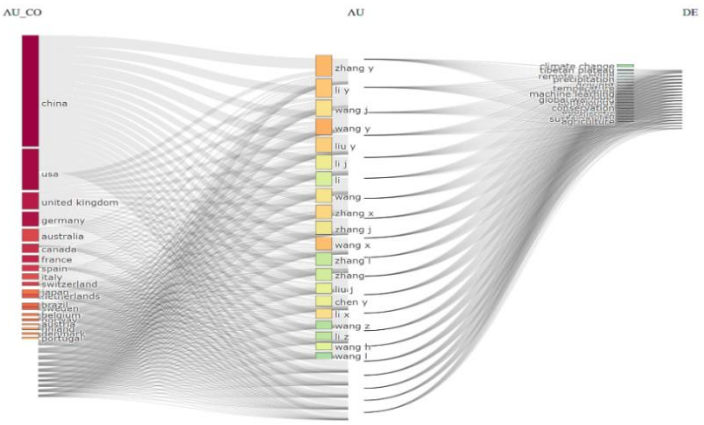


Figure 2. Country – Author – Keyword Triple Map

Effective Journals and Publication Performance

The graphs titled "Effective Journals and Their Annual Production Performance" in Figures 3 and 4 assess the most impactful publication sources based on their H-index[2] and productivity metrics in the climate change literature. Figure 3 illustrates the hierarchy of significant journals based on their H-indexes. This ranking identifies PNAS (Proceedings of the National Academy of Sciences of the United States of America), Nature Communications, Nature Climate Change, and Global Change Biology as the journals with the highest H-index. The elevated citation metrics and persistent publication frequency of these journals augment their influence in the literature.

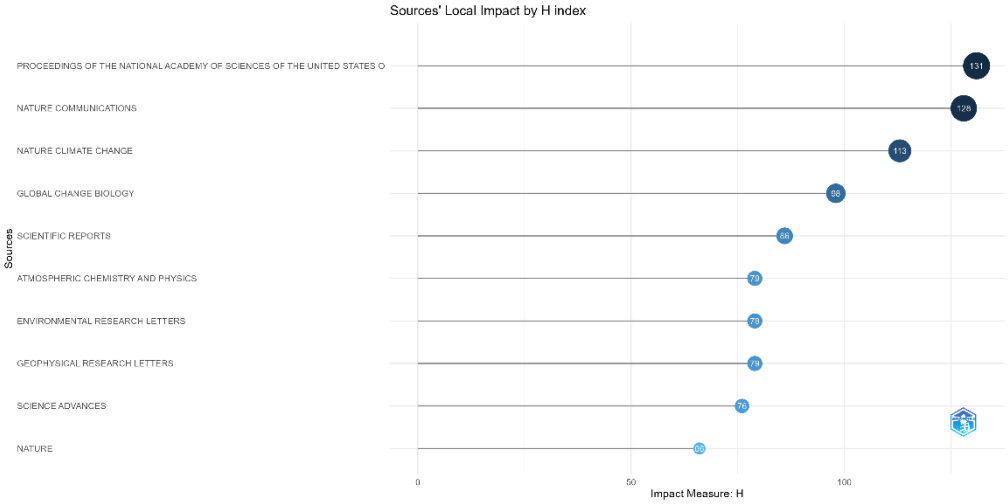
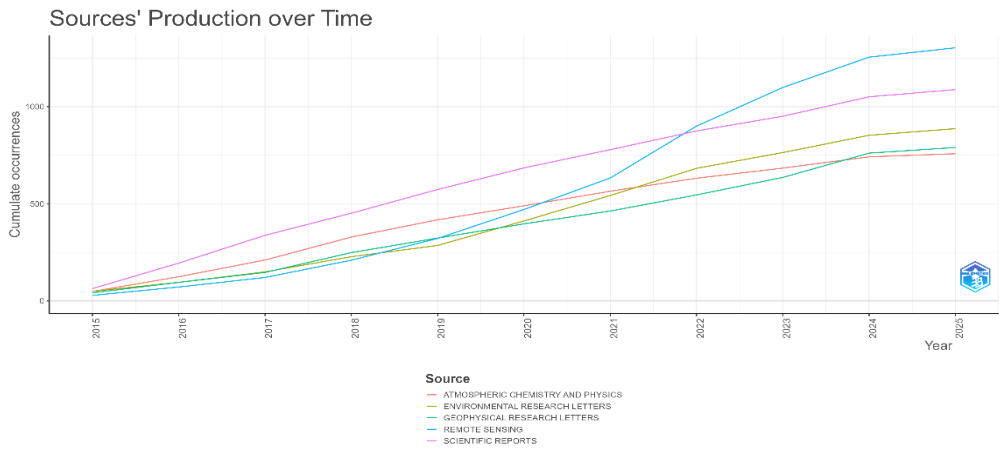


Figure 3. Influential Journals in Climate Change Literature by H-Index



Most Prolific Authors and Publishing Trends

The purpose of the Most Relevant Authors graph in Figure 5 is to analyze which authors have contributed the most to the field and the variability of their productivity over time. As a result of the bibliometric analysis, it has been determined that the majority of the most published authors in the field of climate change are of Chinese origin. Especially Wang Y. (822 publications) and Zhang Y. (813 publications) stand out as the most productive academics during the 2015–2025 period. These data reveal the strength of China's academic contributions to the climate change literature.

[2] H-index: It is an indicator that measures academic productivity and impact simultaneously. A high H-index means both a large number of publications and a high citation count.

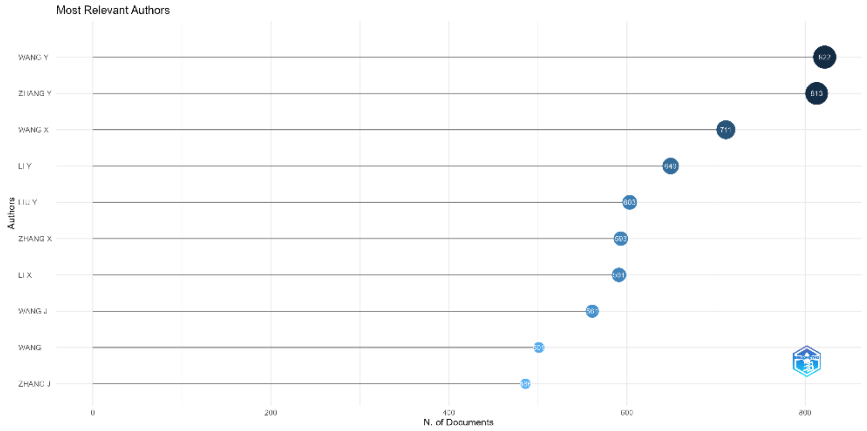


Figure 5. Authors with the Most Publications (2015–2025)

An analysis of the publishing distribution of the authors depicted in Figure 6 reveals a notable increase in the post-2019 timeframe. This escalation has been increasingly evident, particularly among authors including Wang Y., Zhang Y., Wang X., and Li Y. This circumstance signifies that scientific interest in climate change has intensified in recent years, as seen by increased productivity.

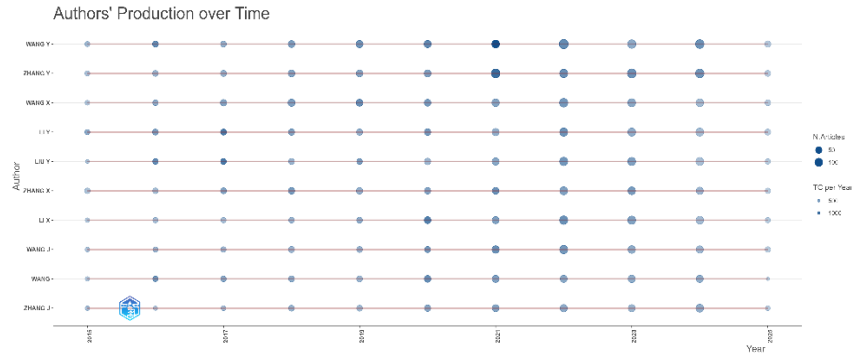


Figure 6. Temporal Distribution of Publications by the Most Prolific Authors

Scientific Cooperation Profile of Countries (Corresponding Author's Countries)

Figure 7 illustrates the distribution of countries' contributions to single-country publications (SCP) and multinational collaborations (MCP) in climate change research, indicating the national affiliations of corresponding authors (Baidya and Saha, 2024). Single-country contribution publications (SCP) denote works authored exclusively by scholars from a single nation, whereas multi-country contribution publications (MCP) signify works generated through collaboration among authors from many nations. The graph illustrates a significant disparity in collaboration between two leading nations, China and the USA:

China predominantly generates publications through its internal resources, signifying a robust capacity for academic production within the nation.

Conversely, the USA is distinguished by a higher prevalence of international joint publications, indicating a greater willingness to engage in scientific collaboration.

Furthermore, nations including France, Canada, Switzerland, and the United Kingdom participate in extensive global scientific collaboration. These countries are spearheading transdisciplinary and international initiatives to address global challenges. These findings demonstrate the extensive and critical nature of international research interactions regarding global challenges like climate change

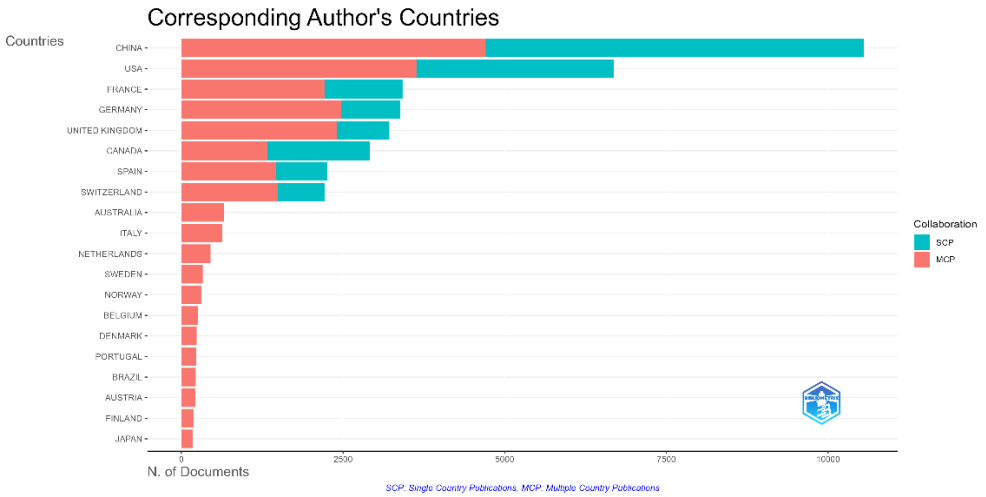


Figure 7. Distribution of Nations by SCP and MCP Publication Contributions

Global Publication Distribution in Climate Change Literature (Country Scientific Production)

The distribution of scholarly output about climate change by country indicates which nations have made more substantial academic contributions to the pertinent literature. Table 1 presents the "Country-Frequency Table of Global Publications in Climate Change Literature," while Figure 8 illustrates the "Publication Number Density" map, both ranking the top 10 countries by the volume of articles published on climate change from 2015 to 2025.

Table 1. Frequency Distribution of Global Publications in Climate Change Literature by Country

Country	Freq
CHINA	65876
USA	62772
GERMANY	26146
UK	25723
FRANCE	24245
CANADA	20609
SPAIN	15791
SWITZERLAND	15242
AUSTRALIA	9044
ITALY	6891

The analysis of academic papers on climate change by country from 2015 to 2025 reveals that scientific output is predominantly concentrated in nations with substantial research capabilities, notably China and the USA. China has the foremost position with 65,876 articles, however the United States holds the second position with 62,772 publications. These two countries represent a substantial share of total production and exert both a leadership role and a guiding effect on the climate change literature. Germany, the United Kingdom, France, Canada, and Switzerland are prominent representatives of Europe and North America in this domain. These countries consistently contribute to the global literature with a substantial volume of articles. Countries like Australia, Italy, and Spain, despite their restricted production, have been assessed to have a significant position at the regional level. Figure 8, entitled "Map of Publication Density by Country in Climate Change Literature at the Global Level," geographically illustrates this distribution, with publication density represented by dark blue hues. These results indicate that climate change is a prevalent scientific concern worldwide.

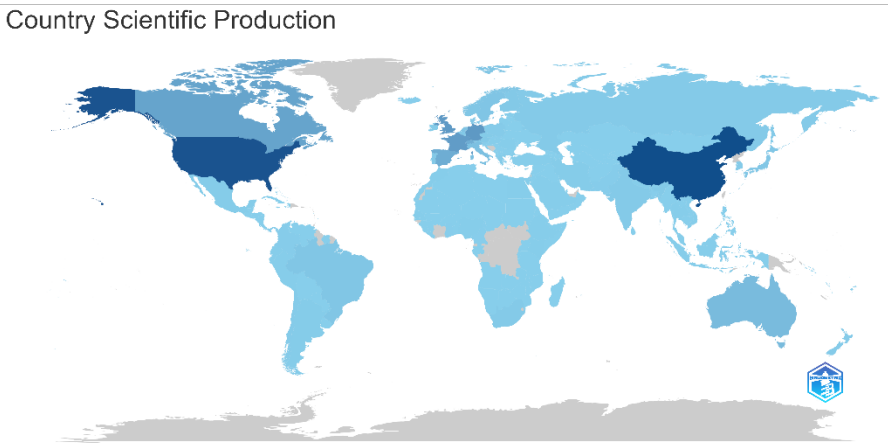


Figure 8. Worldwide distribution of publication density in climate change literature by nation (2015–2025)

Country Based Broadcasting Trend by Year (Country Production Over Time)

Figure 9 illustrates annual trends in climate change publication output for China, the USA, Germany, the United Kingdom, and France from 2015 to 2025. Upon analyzing the graph, it is obvious that China and the USA have markedly amplified their production momentum post-2020. The recent surge in publications from China has demonstrated the nation's academic rigor in the domain of climate change. Conversely, the production trajectories of nations such as Germany, the United Kingdom, and France exhibit a more equitable and sustained ascent. These countries have consistently contributed to the literature through a gradual increase in publications rather than abrupt surges. The graph indicates that scientific output concerning climate change has risen across all nations during the past decade, but at varying rates and via diverse methodologies.

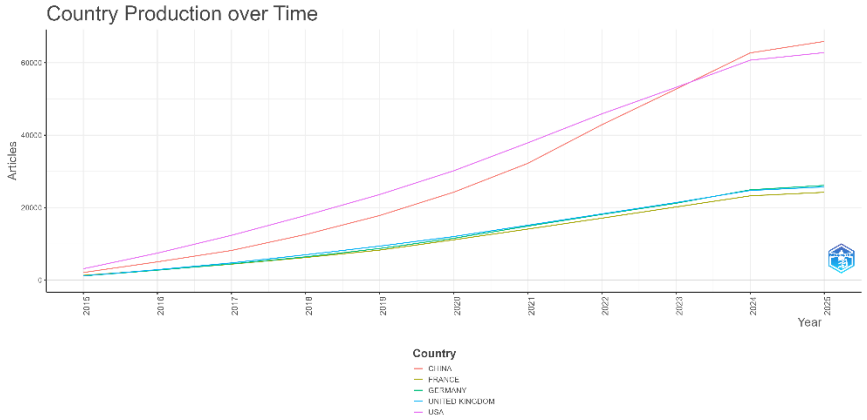


Figure 9. Annual trends in climate change publishing output in China, the USA, Germany, the United Kingdom, and France (2015–2025)

Most Cited Countries in Climate Change Literature

Figure 10 illustrates the examination of "Total citation counts for countries' academic publications focused on climate change." This examination demonstrates both the production volume and the quality of scientific impact. The graph indicates that the United States (US) possesses the highest impact level, with 295,465 citations. This scenario illustrates that the United States occupies a preeminent position not just in

production volume but also in scientific influence and citation frequency. China occupies the second position with more than 257,000 citations, signifying a substantial enhancement in its productivity and influence in recent years. The United Kingdom, Germany, and France are the most influential nations in Europe, garnering roughly 140,000 to 70,000 citations. Countries with lower yet significant citation numbers include Switzerland, Canada, Spain, Australia, and the Netherlands. These nations have assumed proactive positions in global academic engagement and have attained prominence in international literature through their contributions to climate change research. The statistics indicate that the correlation between publication quantity and scientific impact is not necessarily direct; high-quality, often referenced research is essential for scientific visibility.

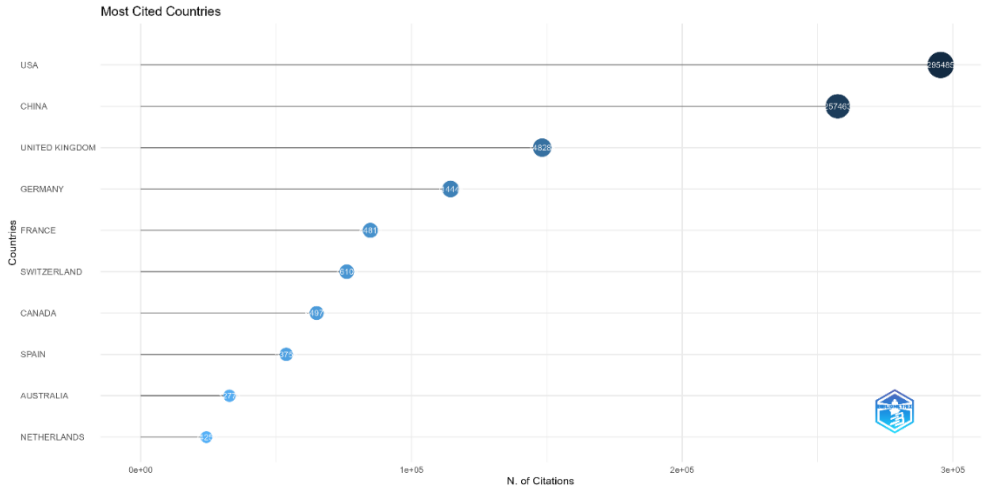


Figure 10. Aggregate Citations by Country (2015–2025)

Most Cited Articles

Figure 11 lists the documents that have garnered the highest number of citations worldwide in the domain of climate change. The enumerated documents not only denote the quantity of publications but also elucidate the extent of scientific influence. The most often referenced work is by Steffen W. (2015), published in the journal Science, with a total of 7,051 citations, much surpassing others. The subsequent works by authors Eyring V. (2016) and Reimer PJ. (2020) have garnered over 6000 citations. The results demonstrate that the pertinent articles are routinely utilized as reference materials in the literature and constitute essential foundations of the area.

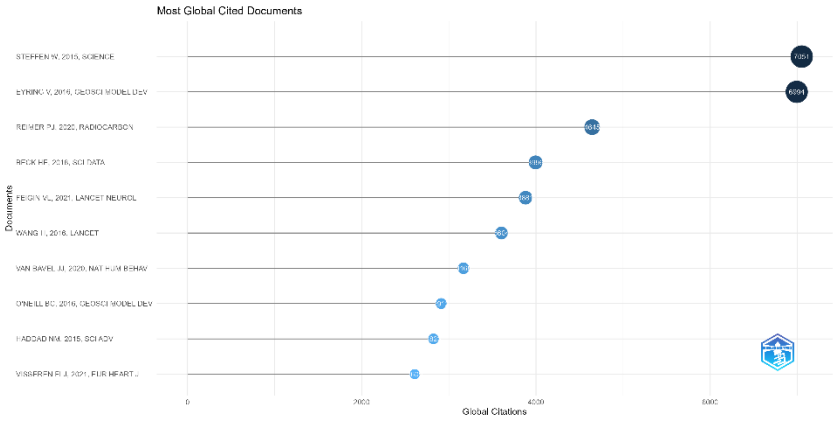


Figure 11. Preeminent Documents by Global Citation Frequency (2015–2025)

Most Frequently Used Concepts in Climate Change Literature

Figure 12 "TreeMap Visualization" and Figure 13 "Word Cloud of TreeMap Visualization," illustrate the most prevalent important topics in climate change literature from 2015 to 2025 in two distinct visual formats. In both graphics, the predominant term is "climatechange," succeeded by scientific and environmental themes including "temperature," "variability," "model," "impacts," and "precipitation." This scenario demonstrates that both climate effects and computer modeling occupy important positions in the literature. The recurrent utilization of terms such as "adaptation," "biodiversity," "emissions," and "management" underscores that the research concentrate not solely on the climate system but also on issues like adaptation, sustainability, and environmental management.

Figure 12. TreeMap Visualization According to Frequency of Keywords

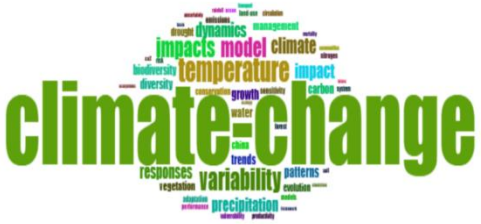


Figure 13. Word Cloud According to Keyword Density

Prominent Themes and Conceptual Density by Year

The Trend Topics graph illustrates the years in which significant topics in climate change literature from 2015 to 2025 were focused. The size of the circles indicates the quantity of publications in which the terms were utilized. Figure 14 "Trend Topics" diagram The annual concentration of major concepts that arose in the climate change literature from 2015 to 2025 is observable. During the years 2020–2021, it was noted that essential climate science terminology, including "climate change," "temperature," "model," "CO2," and "variability," was utilized with considerable regularity. The escalation of this intensity in response to global events, like the COVID-19 pandemic and the European Climate Law, is a significant aspect of the study. In recent years, notions such as "health," "stress," and "dataset," which emphasize public health,

psychosocial impacts, and data management, have gained prominence. This circumstance implies that climate change is being approached not alone as an environmental issue but also as an interdisciplinary challenge.

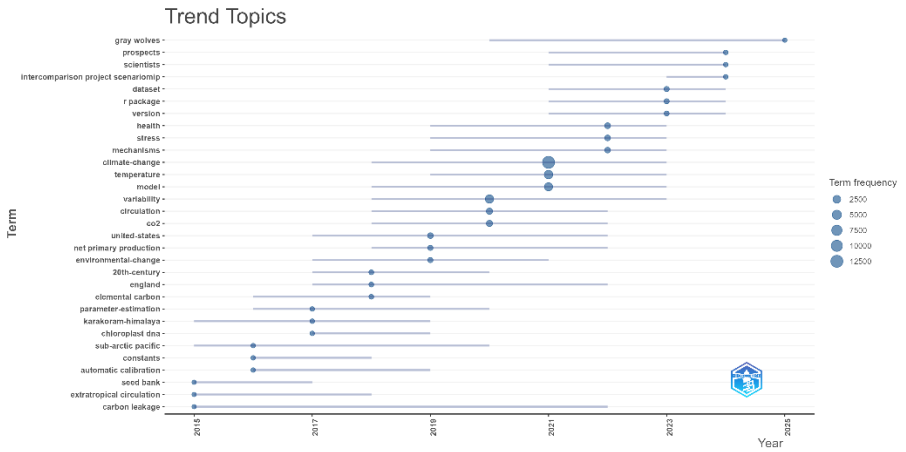


Figure 14. Annual Trends in Climate Change Literature

DISCUSSION AND CONCLUSION

The bibliometric study aimed to assess the structure, trends, and thematic development of academic literature on climate change from 2015 to 2025. The results indicate a notable rise in the volume of publications, particularly post-2020. This increase is directly associated with the heightened global environmental awareness during the COVID-19 pandemic and the implementation of legal frameworks such as the European Climate Law in 2021, which have intensified scholarly interest (Fu & Waltman, 2022). The prominence of China and the USA in publication output and citation metrics signifies that these nations occupy both a quantitative and a leadership role in climate change research. China's publications primarily adhere to the single-country contribution (SCC) model, whereas the United States is distinguished by its international collaborations (MCC), underscoring the disparities in scientific network structure (Glänzel, 2001). The results are congruent with the study by Wang et al. (2014), which underscores the importance of globalization in climate science. Anjum and Aziz (2025) similarly observe that nations such as the United States, the United Kingdom, Canada, and Australia possess a robust collaboration network in climate change literature, highlighting that the scientific contributions from these countries are distinguished by their quality and influence. The prevalence of the terms "climatechange," "temperature," and "model" in the conceptual analysis suggests that research predominantly centers on the physical and atmospheric dimensions of climate (Steffen et al., 2015). The growing prevalence of terms like "health," "stress," and "dataset" in recent years signifies the emergence of an interdisciplinary approach and the incorporation of social and technological influences in scholarly writings (Sweileh, 2020). The trend topic analysis reveals that concepts such as "climate change," "temperature," and "model" increased throughout the years 2020 to 2021. This era might be regarded as a phase in which global environmental crises and sustainable development policies directly mirrored scientific interest. Consequently, the literature on climate change has seen considerable growth in both subject diversity and the extent of international collaboration during the last decade. Future studies should prioritize adaption strategies, climate justice, local policy implementations, and interdisciplinary approaches to enhance contributions to the field.

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Conflict of Interest

The authors have declared that there is noconflict of interest.

Author Contributions

The authors have contributed equally.

An Efficiency Assessment of Technological Development Efficiency of NUTS-2 Regions Using Integrated Slack-Based Data Envelopment Analysis (1248)

Aydın Özdemir[1],

[1]Adiyaman University, Faculty of Economics and Administrative Sciences, Department of Business Administration, Türkiye

Corresponding author e-mail: aozdemir@adiyaman.edu.tr

Abstract

The purpose of this study is to assessment the technological development efficiency of 26 NUTS-2 regions in the Türkiye. For this purpose, an input-oriented Slack-Based Data Envelopment Analysis (DEA) model was applied using five input variables (Sectoral Structure, The Capacity of Research and Innovativeness, Digital Structure, Technology Outcomes and Life Quality and Labour Force Attractiveness) and one output variable (Ankara Chamber of Industry Technological Development Index of Provinces, in other words, ASO Iltek Score). The dataset belongs to 2024 or the nearest year, and it is gathered from TÜİK and ASO. The analysis was carried out using the deaR package in the R project. It was determined that 12 out of 26 NUTS-2 regions are efficient (TR10, TR21, TR22, TR31, TR32, TR51, TR62, TRA2, TRB2, TRC1, TRC2 and TRC3), while 14 out of 26 were found to be inefficient (TR33, TR41, TR42, TR52, TR61, TR63, TR71, TR72, TR81, TR82, TR83, TR90, TRA1 and TRB1). Besides, all of 12 efficient NUTS-2 regions ranking first, while TRA1 ranking last among the 26 NUTS-2 regions. As the technological development efficiency of 26 NUTS-2 regions remains largely unexplored in the existing literature, this study stands out by incorporating the Slack-Based Measure to fill this research gap.

Keywords: *Data Envelopment Analysis, Technological Development, NUTS-2 Regions.*

INTRODUCTION

Technology is recognized as the most important driving force for economic development and growth (Tunalı, 2024). Technlogy management can be defined as designing, managing and integrating an organization's technological fundemmentals to create a competitive advantage (Kalko et al., 2023). Since technology management is a hidden bridge between scieince and practice (Li-Hua & Khalil, 2007), although technology management has traditionally focused on product-oriented research and development (R&D), it is now increasingly recognized that resource perspective has vital importance for reaching comptetive advantage (Gregory, 1995). From resource perspective, technological capability is a set of resources that offers know-how to change existing products or create new product (Wu et al., 2019), therefore the accumulation of technological capability proceeds from simpler to more complex day by day (Liu et al., 2020). The increasing use of technology has resulted in a need for assessing the productivity impacts of technological capability (Chen et al., 2006).

In this context, the aim of the study is to assessment the efficiency of technological development of 26 NUTS-2 regions in the Türkiye. These regions, in other words the sample of this research, are TR10 (İstanbul), TR21 (Tekirdağ, Edirne, Kırklareli), TR22 (Balıkesir, Çanakkale), TR31 (İzmir), TR32 (Aydın, Denizli, Muğla), TR33 (Manisa, Afyonkarahisar, Kütahya, Uşak), TR41 (Bursa, Eskişehir, Bilecik), TR42 (Kocaeli, Sakarya, Düzce, Bolu, Yalova), TR51 (Ankara), TR52 (Konya, Karaman), TR61 (Antalya, Isparta, Burdur), TR62 (Adana, Mersin), TR63 (Hatay, Kahramanmaraş, Osmaniye), TR71 (Kırıkkale, Aksaray, Niğde, Nevşehir, Kırşehir), TR72 (Kayseri, Sivas, Yozgat), TR81 (Zonguldak, Karabük, Bartın),

TR82 (Kastamonu, Çankırı, Sinop), TR83 (Samsun, Tokat, Çorum, Amasya), TR90 (Trabzon, Ordu, Giresun, Rize, Artvin, Gümüşhane), TRA1 (Erzurum, Erzincan, Bayburt), TRA2 (Ağrı, Kars, Iğdır, Ardahan), TRB1 (Malatya, Elazığ, Bingöl, Tunceli), TRB2 (Van, Muş, Bitlis, Hakkari), TRC1 (Gaziantep, Adıyaman, Kilis), TRC2 (Şanlıurfa, Diyarbakır) and TRC3 (Mardin, Batman, Şırnak, Siirt).

Input Orientation Slack-Based Data Envelopment Analysis (DEA) model was employed using five input variables (Sectoral Structure, The Capacity of Research and Innovativeness, Digital Structure, Technology Outcomes, Life Quality and Labour Force Attractiveness) and one output variable (ASO Iltek Score) in this research.

Although there are numerous studies on the related topics, since the efficiency of technological development of 26 NUTS-2 regions is still a research gap, this research, using Slack-Based Measure as well, has a unique value.

The purpose of technology is to directly or indirectly improve productivity (T. R. Anderson & Hollingsworth, 1997) and Data Envelopment Analysis (DEA) can be regarded as a measuring tool of productivity in several research fields, including in economics, business, healthcare, public administration, etc. (Charles & Kumar, 2012; Emrouznejad & Cabanda, 2014; Gregoriou & Zhu, 2005). Some researches conducted using Data Envelopment Analysis in the technology management and related fields are presented in Table 1.

Table 1. Literature Summary

Title	Author(s)	Input(s)	Output(s)	DMUs	Analysis Procedure
Efficiency Analysis of Science and Technology Parks Using Data Envelopment Analysis: Evidence From Turkey	(Arslan & Belgin, 2021)	-Number of Personnel -Number of Firms -R&D Expenditure	-Total R&D Revenue -Number of Projects -Number of Patents	22 Science and Technology Parks in Türkiye	DEA and Cluster Analysis
Measuring R&D and Technopark Project Performance at University in Turkey Using Data Envelopment Analysis	(Özdemir & Selamzade, 2020)	-The Number of Professors -The Number of Associate Professors -The Number of Doctor Lecturers -The Number of Lecturers -The Number of research Assistants	-The Number of R&D Project - The Number of Publications - The Number of Publications per Academic Staff Published in Indexed Journals - The Annual Turnover Rate	45 Universities	CCR and CRS DEA

		-Total Students Studying -The Number of Students Participating in Technopolis Projects -The General Occupancy Rate in the Departments and Programs of the University -The Budget Spent on R&D	of the Technopark -International Symposiums, Congresses or Artistic Exhibitions and National Refereed Journals		
Evaluation of Management Efficiencies of Technology Development Zones Managing Company Using Data Envelopment Analysis	(Baykul et al., 2016)	-The Number of Capacity Devolopment Activity -The Number of Colloboration -The Number of Key Personnel	-The Number of Academic Spin-Off Firm -The Number of Foreign Firm -Total Employment Rate	39 Technology Development Zones Managing Companies	CCR DEA
Evaluation of Efficiency Measurement of Selected Technoparks with Data Envelopment Analysis (DEA)	(Demircioğlu & Özgüner, 2022)	-Infrastructure -Offered Advantages -Proximity to the University	-Colloboration -Patent Number	9 Technoparks	CCR DEA

In line with Table-1, the efficiency of 26 NUTS-2 regions is assessed using Slack-Based Data Envelopment Analysis. The five input variables and one output variable used during the analysis are explained in detail in the following sections.

MATERIAL AND METHODS

Data Envelopment Analysis (DEA)

Data Envelopment Analysis (DEA) is a nonparametric method of measuring and comparing the efficiency of decision-making units (DMUs), which have common input and output factors, such as firms, public agencies, and countries (Ray, 2004; Tone, 2017).

There are two types of model in data envelopment analysis (DEA): radial and nonradial (Tone, 2017).

The most basic Radial DEA model is the Constant Returns to Scale (CRS), also known as CCR (Charnes-Cooper-Rhodes) model, it was initially introduced by Charnes et al. in 1978 (Charnes et al., 1978). CCR (CRS) model yields an objective assessment of overall efficiency, identifies the sources, and estimates the number of inefficiencies in this way (Charnes et al., 1994).

As a representative extension of Radial DEA, Variable Returns to Scale (VRS), also known as BCC (Banker-Charnes-Cooper) model, was later developed by Banker et al. in 1984 (Banker et al., 1984). The BCC (VRS) model distinguishes between technical and scale inefficiencies by estimating pure technical efficiency and identifying possible increasing, decreasing, or constant returns to scale (Charnes et al., 1994). The BCC efficiency scores are considered pure technical efficiency, while CCR efficiency scores are considered technical efficiency (Ozcan, 2014).

While Input-Oriented DEA aims to produce the observed outputs with minimum inputs, Output-Oriented DEA aims to maximize output production, subject to the given resource level (Ramanathan, 2003).

Efficiency scores equal to 1.00 are defined as efficient in both models, between 0.00 and 1.00 are defined as inefficient in the input-oriented models, greater than 1 are defined as inefficient in the output-oriented models (Ozcan, 2014).

Proposed by (Tone, 2001) the Slack-Based Measure (SBM) DEA, as a Non-Radial DEA, is able to deal directly with the input excesses and the output shortfalls of the DMU under evaluation, since it satisfies the properties Units Invariant, Monotone, and Reference-Set Dependent which are not satisfied by CCR and BCC models.

The fractional programming formulas of CCR-Input Orientation (1), CCR-Output Orientation (2), BCC-Input Orientation (3), BCC- Output Orientation (4), SBM-CRS-Input Orientation (5), SBM-CRS-Output Orientation (6), SBM-VRS-Input Orientation (7) and SBM-VRS-Output Orientation (8) are as follows (Emrouznejad & Cabanda, 2014; Ozcan, 2009, 2014; Tone, 2001, 2017);

$$Eff = \min_{u_r v_i} \sum_{i} V_i \, X_{iJ_0}$$

s.t.

$$\sum_{r} u_r y_{rj} - \sum_{i} v_i x_{ij} \leq 0 \quad ; \forall j$$

$$\sum_{r} u_r y_{rj_0} = 1$$

$$u_r, v_i \geq 0 \quad ; \forall r, \forall i.$$

(1)

$$Eff = \max_{u_r v_i} \sum_{r} u_r y_{rj_0}$$

s.t.

$$\sum_{r} u_r y_{rj} - \sum_{i} v_i x_{ij} \leq 0 \quad ; \forall j$$

$$\sum_{i} v_i x_{ij_0} = 1$$

$u_r, v_i \geq 0 \quad ; \forall r, \forall i.$

(2)

$$\min_{\lambda, \emptyset, S_i^-, S_r^+} \emptyset$$

s.t.

$$\sum_j \lambda_j x_{ij} + S_i^- = \emptyset x_{ij_0} \quad \forall i$$

$$\sum_j \lambda_j y_{rj} - S_r^+ = y_{rj_0} \quad \forall r$$

$$\sum_j \lambda_j = 1$$

$$s_i^-, s_i^+ \geq 0 \quad \forall i, \forall r$$

$$\lambda_j \geq 0 \quad \forall_j.$$

(3)

$$\max_{\lambda, \emptyset, S_i^-, S_r^+} \theta$$

s.t.

$$\sum_j \lambda_j x_{ij} + S_i^- = \; x_{ij_0} \; \forall i$$

$$\sum_j \lambda_j y_{rj} - S_r^+ = \theta y_{rj_0} \quad \forall r$$

$$\sum_j \lambda_j = 1$$

$$s_i^-, s_i^+ \geq 0 \quad \forall i, \forall r$$

$$\lambda_j \geq 0 \quad \forall_j.$$

(4)

$$\rho_{\mathrm{I}}^* = \min_{\boldsymbol{\lambda}, \mathbf{s}^-, \mathbf{s}^+} 1 - \frac{1}{m} \sum\nolimits_{i=1}^{m} \frac{s_i^-}{x_{ih}}$$

$$\text{subject to}$$

$$x_{ih} = \sum\nolimits_{j=1}^{n} x_{ij} \lambda_j + s_i^- \quad (i = 1, \ldots, m)$$

$$y_{rh} = \sum\nolimits_{j=1}^{n} y_{rj} \lambda_j - s_r^+ \quad (r = 1, \ldots, s)$$

$$\lambda_j \geq 0 \;\; (\forall j), \;\; s_i^- \geq 0 (\forall i), \;\; s_r^+ \geq 0 (\forall r)$$

(5)

$$1/\rho_{\mathrm{O}}^{*} = \max_{\boldsymbol{\lambda},\mathbf{s}^{-},\mathbf{s}^{+}} 1 + \frac{1}{s}\sum_{r=1}^{s}\frac{s_r^{+}}{y_{rh}}$$
$$\text{subject to}$$
$$x_{ih} = \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^{-} \quad (i = 1,\ldots,m)$$
$$y_{rh} = \sum_{j=1}^{n} y_{rj}\lambda_j - s_r^{+} \quad (r = 1,\ldots,s)$$
$$\lambda_j \geq 0 \ (\forall j),\ s_i^{-} \geq 0 \ (\forall i),\ s_r^{+} \geq 0 \ (\forall r) \tag{6}$$

$$\rho_{\mathrm{I}}^{*} = \min_{\boldsymbol{\lambda},\mathbf{s}^{-},\mathbf{s}^{+}} 1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^{-}}{x_{ih}}$$
$$\text{subject to}$$
$$x_{ih} = \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^{-} \quad (i = 1,\ldots,m)$$
$$y_{rh} = \sum_{j=1}^{n} y_{rj}\lambda_j - s_r^{+} \quad (r = 1,\ldots,s)$$
$$\sum_{j=1}^{n}\lambda_j = 1,\ \lambda_j \geq 0 \ (\forall j),\ s_i^{-} \geq 0 \ (\forall i),\ s_r^{+} \geq 0 \ (\forall r) \tag{7}$$

$$\rho_{\mathrm{I}}^{*} = \min_{\boldsymbol{\lambda},\mathbf{s}^{-},\mathbf{s}^{+}} 1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^{-}}{x_{ih}}$$
$$\text{subject to}$$
$$x_{ih} = \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^{-} \quad (i = 1,\ldots,m)$$
$$y_{rh} = \sum_{j=1}^{n} y_{rj}\lambda_j - s_r^{+} \quad (r = 1,\ldots,s)$$
$$\sum_{j=1}^{n}\lambda_j = 1,\ \lambda_j \geq 0 \ (\forall j),\ s_i^{-} \geq 0 \ (\forall i),\ s_r^{+} \geq 0 \ (\forall r)$$

The Sample and Dataset

Decision Making Units (DMUs) are such units that utilize same inputs to produce same outputs (Cooper et al., 2006). The sample of the study consists of 26 NUTS-2 regions in the Türkiye and are shown in Table-1.

Since the sample of the study consists all 26 NUTS-2 regions in the Türkiye, the following three conditions related to the homogeneous group of DMUs are met (Golany & Roll, 1989);

- ✓ The DMUs under consideration perform the same tasks, with similar objectives,
- ✓ All DMUs perform under the same set of market conditions,
- ✓ The inputs and outputs of characterizing the performance of all DMUs in the group are identical.

Efficiency assessment depends on how the feasible set of input–output bundles is specified (Ray, 2004).

There are five input variables and one output variable adapted Ankara Chamber of Industry Technological Development Index of Provinces as the primary input/output framework. Inputs are i1: Sectoral Structure, i2: The Capacity of Research and Innovativesness, i3: Dijital Structure, i4: Technology Outcomes, i5: Life Quality and Labour Force Attractiveness. Output is ASO Iltek Score. The inputs and outpouts are in line with (Arslan & Belgin, 2021; Baykul et al., 2016; Demircioğlu & Özgüner, 2022; Özdemir & Selamzade, 2020), as depicted in Table 2.

The dataset, belongs to 2024 or the nearest year, is shown Table-1 and gathered from (Ankara Sanayi Odası, 2024; Bilgi Teknolojileri ve İletişim Kurumu, 2024; TÜİK, 2024a, 2024b, 2025; Türk Patent, 2025).

The sample size should be at least 2 or 3 times larger than the sum of the number of inputs and outputs (Ramanathan, 2003). This condition is met, since there are 6 total variables (5 input variables and 1 outputs variable) to measure the efficiency of 26 DMUs in this research.

Table 2. The Dataset

DMU Code	DMU Name	Inputs					Output
		i1	i2	i3	i4	i5	o1
TR10	İstanbul	8.018991	6,946.533308	1.372250	89.545003	52.800000	0.940000
TR21	Tekirdağ, Edirne, Kırklareli	3.895464	3,452.041598	1.022872	13.585316	54.600000	0.315294
TR22	Balıkesir, Çanakkale	2.655737	1,704.633265	1.087445	10.297757	50.700000	0.271552
TR31	İzmir	5.027550	3,757.145699	1.179914	37.834597	49.900000	0.530000
TR32	Aydın, Denizli, Muğla	2.225626	1,341.100035	1.084004	10.878825	51.100000	0.276370
TR33	Manisa, Afyonkarahisar, Kütahya, Uşak	2.846528	2,492.637262	0.991739	17.654844	51.800000	0.254413
TR41	Bursa, Eskişehir, Bilecik	4.833958	13,686.636821	1.110744	67.445280	50.500000	0.518579
TR42	Kocaeli, Sakarya, Düzce, Bolu, Yalova	7.509722	4,676.160456	1.113633	55.045324	53.500000	0.442878
TR51	Ankara	13.144501	19,133.659216	1.214834	107.434300	49.600000	1.000000
TR52	Konya, Karaman	3.414050	2,253.183748	1.037191	24.297916	48.800000	0.267054
TR61	Antalya, Isparta, Burdur	2.930607	1,946.635797	1.119499	13.355251	54.700000	0.301927
TR62	Adana, Mersin	1.770819	1,315.999515	1.047024	10.154051	47.200000	0.236879
TR63	Hatay, Kahramanmaraş, Osmaniye	1.286628	884.530002	0.984189	10.130932	44.200000	0.131576
TR71	Kırıkkale, Aksaray, Niğde, Nevşehir, Kırşehir	3.108462	1,637.316427	1.005215	10.858202	46.100000	0.187686
TR72	Kayseri, Sivas, Yozgat	3.476348	2,323.024983	1.004011	51.945437	46.400000	0.295259
TR81	Zonguldak, Karabük, Bartın	3.212659	1,805.997290	1.065419	14.367885	47.900000	0.282810
TR82	Kastamonu, Çankırı, Sinop	2.623222	1,258.995274	0.955190	4.944810	51.300000	0.196372
TR83	Samsun, Tokat, Çorum, Amasya	2.402787	1,349.538720	1.012052	11.193653	48.700000	0.205377
TR90	Trabzon, Ordu, Giresun, Rize, Artvin, Gümüşhane	2.843292	1,306.515096	1.003804	10.340586	51.200000	0.180903
TRA1	Erzurum, Erzincan, Bayburt	4.564827	3,070.002299	0.936254	19.627636	49.000000	0.166722
TRA2	Ağrı, Kars, Iğdır, Ardahan	1.796672	1,088.593600	0.891921	0.000000	49.300000	0.152083
TRB1	Malatya, Elazığ, Bingöl, Tunceli	3.285933	2,447.214379	0.960235	19.137979	45.700000	0.182564
TRB2	Van, Muş, Bitlis, Hakkari	1.308780	894.869688	0.780306	0.929201	39.500000	0.037629

TRC1	Gaziantep, Adıyaman, Kilis	1.423101	1,021.981342	1.044047	11.819776	45.000000	0.213886
TRC2	Şanlıurfa, Diyarbakır	0.792105	607.695424	0.759546	1.473684	40.900000	0.062949
TRC3	Mardin, Batman, Şırnak, Siirt	0.763310	425.554165	0.760745	2.034408	39.500000	0.043988
	Min	0.763310	425.554165	0.759546	0.000000	39.50	0.037629
	Max	13.144501	19,133.659216	1.37225	107.434300	54.70	1.000000
	Mean	3.506218	3,185.699823	1.020926	24.089717	48.46	0.295952
	Median	2.888568	1,755.315278	1.017462	12.587514	49.15	0.245646

According to Table 2;

i1: Sectoral Structure describes the technology intensity of production in the NUTS-2 regions (Ankara Sanayi Odası, 2024), it is represented by the *"R&D Human Resources (Per 1000 Capita)"* indicator in this article and the data is gathered from (TÜİK, 2024a).

i2: The Capacity of Research and Innovativesness describes the R&D intensity in the NUTS-2 regions (Ankara Sanayi Odası, 2024), it is represented by the *"R&D Expenditures (Per Capita)"* indicator in this article and the data is gathered from (TÜİK, 2024a).

i3: Dijital Structure describes the structure of internet and the usage of mobile device in the NUTS-2 regions (Ankara Sanayi Odası, 2024), it is represented by the *"Number of Broadband Subscriptions (Per Capita)"* indicator in this article and the data is gathered from (Bilgi Teknolojileri ve İletişim Kurumu, 2024).

i4: Technology Outcomes describes the patent intensity in the NUTS-2 regions (Ankara Sanayi Odası, 2024), it is represented by *"The Number of Patent Grant (Per 1 Mn Capita)"* indicator in this article and the data is gathered from (Türk Patent, 2025).

i5: Life Quality and Labour Force Attractiveness describes the availability of labour force market in the NUTS-2 regions (Ankara Sanayi Odası, 2024), it is represented by the *"Empoyment Rate"* indicator in this article and the data is gathered from (TÜİK, 2025).

o1: ASO Iltek Score describes a comprehensive and comparative analysis study that measures the level of technological development of provinces in the Türkiye (Ankara Sanayi Odası, 2024), it is represented by the *"Weighted ASO Iltek Score"* indicator in this article. It is calculated by weighted mean of ASO Iltek Score of each NUTS-2 region based on their GDP Per Capita. The data is gathered from (Ankara Sanayi Odası, 2024; TÜİK, 2024b).

Analysis Design

Since the managers of the NUTS-2 regions have more control over inputs than outputs (Ozcan, 2014), Slack-Based Measure deals directly with input excess and output shortfall (Tone, 2001), Input-Oriented Slack-Based Data Envelopment Analysis (DEA) model was employed in this research.

Reference sets were identified for each inefficient DMUs, so that they can become efficient and improvement options *(reducing of the inputs or increasing of the outputs)* for inefficient DMUs were calculated.

All analysis was performed using the deaR package in the R project (Coll-Serrano et al., 2018).

RESULTS

The efficiency scores, slacks (input excesses and the output shortfalls) and the ranking of the world's top investment migration destinations (DMUs) according to their efficiency scores are shown in Table-2.

Table 3. Efficiency Score, Ranking, and Slacks of DMUs

DMU	Efficiency Score	Efficiency Status	Rank	s_1^-	s_2^-	s_3^-	s_4^-	s_5^-	s_1^+
TR10	1.000000	**Efficient**	1	8.018991	6,946.533308	1.372250	89.545003	52.800000	0.940000
TR21	1.000000	**Efficient**	1	3.895464	3,452.041598	1.022872	13.585316	54.600000	0.315294
TR22	1.000000	**Efficient**	1	2.655737	1,704.633265	1.087445	10.297757	50.700000	0.271552
TR31	1.000000	**Efficient**	1	5.027550	3,757.145699	1.179914	37.834597	49.900000	0.530000
TR32	1.000000	**Efficient**	1	2.225626	1,341.100035	1.084004	10.878825	51.100000	0.276370
TR33	0.815130	Inefficient	18	2.426955	1,633.263169	0.991739	10.584877	50.156190	0.254413
TR41	0.764014	Inefficient	21	4.433916	3,776.427160	1.110744	46.343087	47.459082	0.518579
TR42	0.733883	Inefficient	24	3.758344	2,944.503281	1.113633	33.544322	49.748264	0.442878
TR51	1.000000	**Efficient**	1	13.144501	19,133.659216	1.214834	107.434300	49.600000	1.000000
TR52	0.769758	Inefficient	20	2.199139	1,495.840273	1.037191	13.139553	48.800000	0.267054
TR61	0.959867	Inefficient	15	2.715753	1,876.199036	1.080955	13.355251	51.596549	0.301927
TR62	1.000000	**Efficient**	1	1.770819	1,315.999515	1.047024	10.154051	47.200000	0.236879
TR63	0.843817	Inefficient	16	1.078999	796.057819	0.888899	6.177717	42.764137	0.131576
TR71	0.757747	Inefficient	18	1.560327	1,131.213868	0.963744	6.918116	46.100000	0.187686
TR72	0.790026	Inefficient	22	2.498184	1,937.296139	1.004011	20.651182	46.400000	0.295259
TR81	0.929210	Inefficient	14	2.303255	1,681.930440	1.065419	14.336498	47.900000	0.282810
TR82	0.898243	Inefficient	15	1.983229	1,211.822154	0.955190	3.959812	49.856178	0.196372
TR83	0.830534	Inefficient	17	1.846119	1,139.013228	0.975323	6.454667	48.700000	0.205377
TR90	0.738563	Inefficient	23	1.896141	1,147.146302	0.936463	2.522647	49.717395	0.180903
TRA1	0.565490	Inefficient	26	1.797001	1,096.741695	0.914933	1.949377	49.000000	0.166722
TRA2	1.000000	**Efficient**	1	1.796672	1,088.593600	0.891921	0.000000	49.300000	0.152083
TRB1	0.652303	Inefficient	25	1.482830	988.005709	0.957596	7.832654	45.700000	0.182564
TRB2	1.000000	**Efficient**	1	1.308780	894.869688	0.780306	0.929201	39.500000	0.037629
TRC1	1.000000	**Efficient**	1	1.423101	1,021.981342	1.044047	11.819776	45.000000	0.213886
TRC2	1.000000	**Efficient**	1	0.792105	607.695424	0.759546	1.473684	40.900000	0.062949
TRC3	1.000000	**Efficient**	1	0.763310	425.554165	0.760745	2.034408	39.500000	0.043988

According to Table 3, 12 NUTS-2 regions *(TR10, TR21, TR22, TR31, TR32, TR62, TRA2, TRB2, TRC1, TRC2 and TRC3)* have a score of 1 and they are relatively efficient by removing input excesses and augmenting the output shortfalls.

Besides, TR62 ranks first among the 12 efficient NUTS-2 regions while TRA1 ranks last among the 26 NUTS-2 regions.

Additionally, Table 3 provides the slacks of inputs and outputs so that each NUTS-2 regions can remove input excesses and augment the output shortfalls to achieve efficiency status.

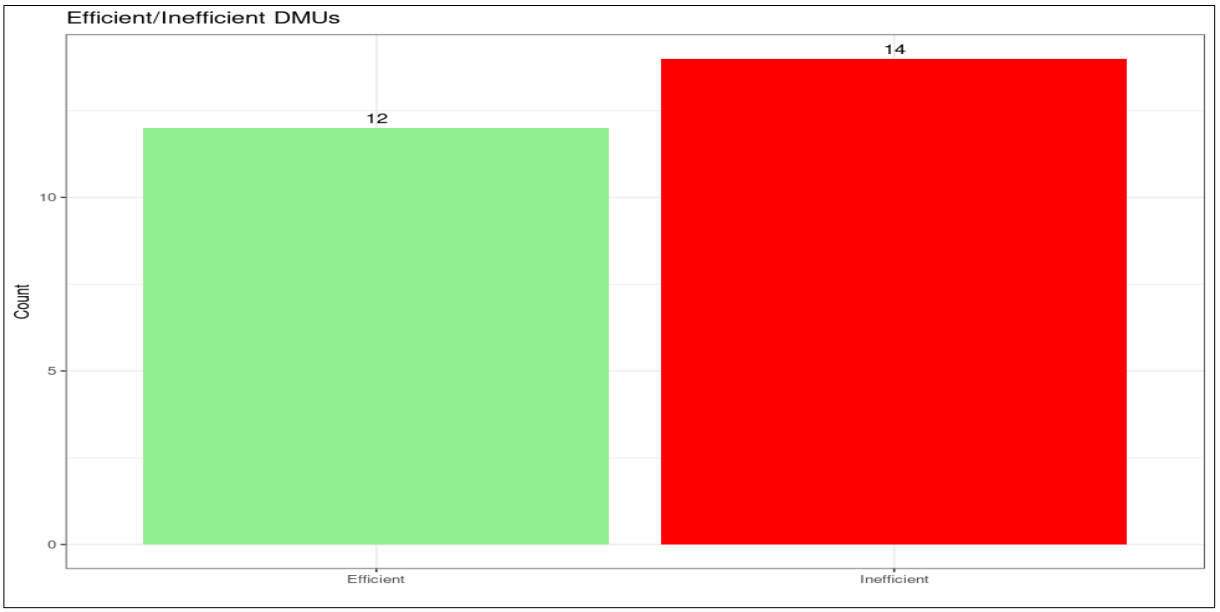

Figure 1. Efficiency Distribution

Figure 1 displays that 12 of the 26 NUTS-2 regions are efficient (yellow column) while 14 of these are inefficient (red column).

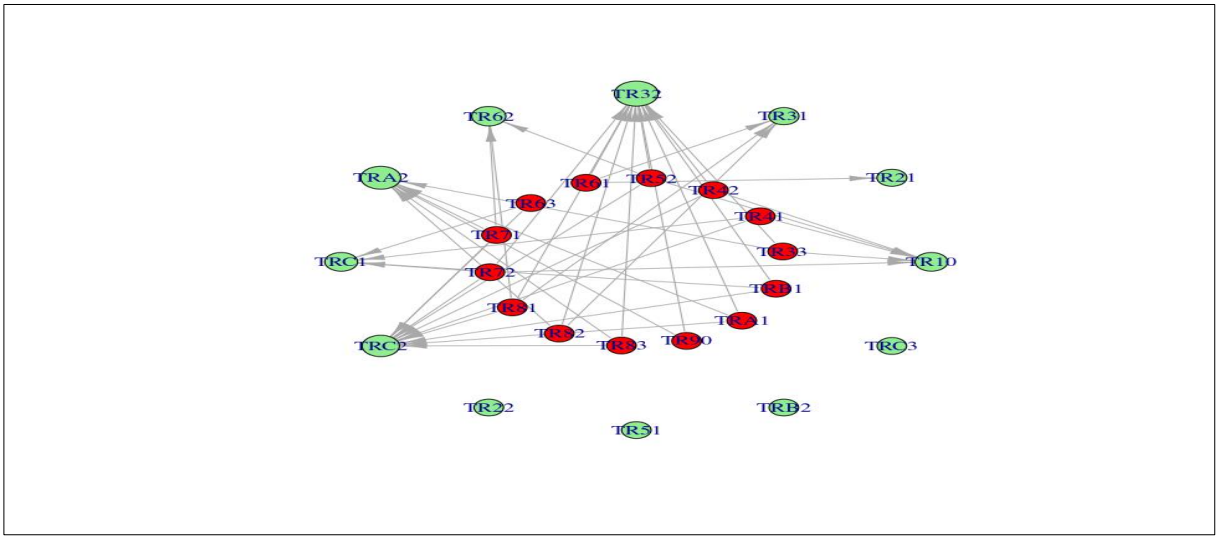

Figure 2. Network Graph

As seen in Figure 2 green circles represent the efficient NUTS-2 regions and the red circles represent the inefficient ones, and it displays Reference Sets defined so that the 14 inefficient NUTS-2 regions can become efficient.

Table 4 demonstrates the reference set weights, also known as Lambda -λ- values, which define the components of other producers used to construct the virtual producer (T. Anderson, 2003), of Reference Set so that the 3 inefficient NUTS-2 regions can become efficient.

Table 4. References Set of Inefficient States (λ)

DMU	TR10 (λ)	TR21 (λ)	TR31 (λ)	TR32 (λ)	TR62 (λ)	TRA2 (λ)	TRC1 (λ)	TRC2 (λ)
TR33	**0.0791**	0.0000	0.0000	**0.3218**	0.0000	**0.5990**	0.0000	0.0000
TR41	**0.4879**	0.0000	0.0000	0.0000	0.0000	0.0000	**0.1837**	**0.3284**
TR42	**0.3102**	0.0000	0.0000	**0.5056**	0.0000	0.0000	0.0000	**0.1842**
TR52	**0.0506**	0.0000	0.0000	**0.6126**	**0.1664**	0.0000	0.0000	**0.1703**
TR61	0.0000	**0.1676**	**0.0750**	**0.7574**	0.0000	0.0000	0.0000	0.0000
TR63	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0.4547**	**0.5453**
TR71	0.0000	0.0000	0.0000	0.0000	**0.6496**	**0.1318**	0.0000	**0.2186**
TR72	**0.1660**	0.0000	0.0000	**0.2656**	0.0000	0.0000	**0.1988**	**0.3696**
TR81	0.0000	0.0000	**0.1550**	**0.1067**	**0.7168**	0.0000	0.0000	**0.0214**
TR82	0.0000	0.0000	**0.0175**	**0.3032**	0.0000	**0.6794**	0.0000	0.0000
TR83	0.0000	0.0000	0.0000	**0.5672**	0.0000	**0.2398**	0.0000	0.1930
TR90	0.0000	0.0000	0.0000	**0.2319**	0.0000	**0.7681**	0.0000	0.0000
TRA1	0.0000	0.0000	0.0000	**0.1694**	0.0000	**0.7585**	0.0000	**0.0720**
TRB1	0.0000	0.0000	0.0000	**0.3522**	0.0000	0.0000	**0.2944**	**0.3533**

According to Table 4;

- ✓ The reference set of TR33 consists of TR10 (λ=0.0791), TR32 (λ=0.3218) and TRA2 (λ=0.5990).
- ✓ The reference set of TR41 consists of TR10 (λ=0.4879), TRC1 (λ=0.1837) and TRC2 (λ=0.3284).
- ✓ The reference set of TR42 consists of TR10 (λ=0.3102), TR32 (λ=0.5056) and TRC2 (λ=0.1842).
- ✓ The reference set of TR52 consists of TR10 (λ=0.0506), TR32 (λ=0.6126), TR62 (λ=0.1664) and TRC2 (λ=0.1703).
- ✓ The reference set of TR61 consists of TR21 (λ=0.1676), TR31 (λ=0.0750) and TR32 (λ=0.7574).
- ✓ The reference set of TR63 consists of TRC1 (λ=0.4547) and TRC2 (λ=0.5453).
- ✓ The reference set of TR71 consists of TR62 (λ=0.6496), TRA2 (λ=0.1318) and TRC2 (λ=0.2186).
- ✓ The reference set of TR81 consists of TR31 (λ=0.1550), TR32 (λ=0.1067), TR62 (λ=0.7168) and TRC2 (λ=0.0214).
- ✓ The reference set of TR82 consists of TR31 (λ=0.0175), TR32 (λ=0.3032) and TRA2 (λ=0.6794).
- ✓ The reference set of TR83 consists of TR32 (λ=0.5672) and TRA2 (λ=0.2398).
- ✓ The reference set of TR90 consists of TR32 (λ=0.2319) and TRA2 (λ=0.7681).
- ✓ The reference set of TRA1 consists of TR32 (λ=0.1694), TRA2 (λ=0.7585) and TRC2 (λ=0.0720).
- ✓ The reference set of TRB1 consists of TR32 (λ=0.3522), TRC1 (λ=0.2944) and TRC2 (λ=0.3533).

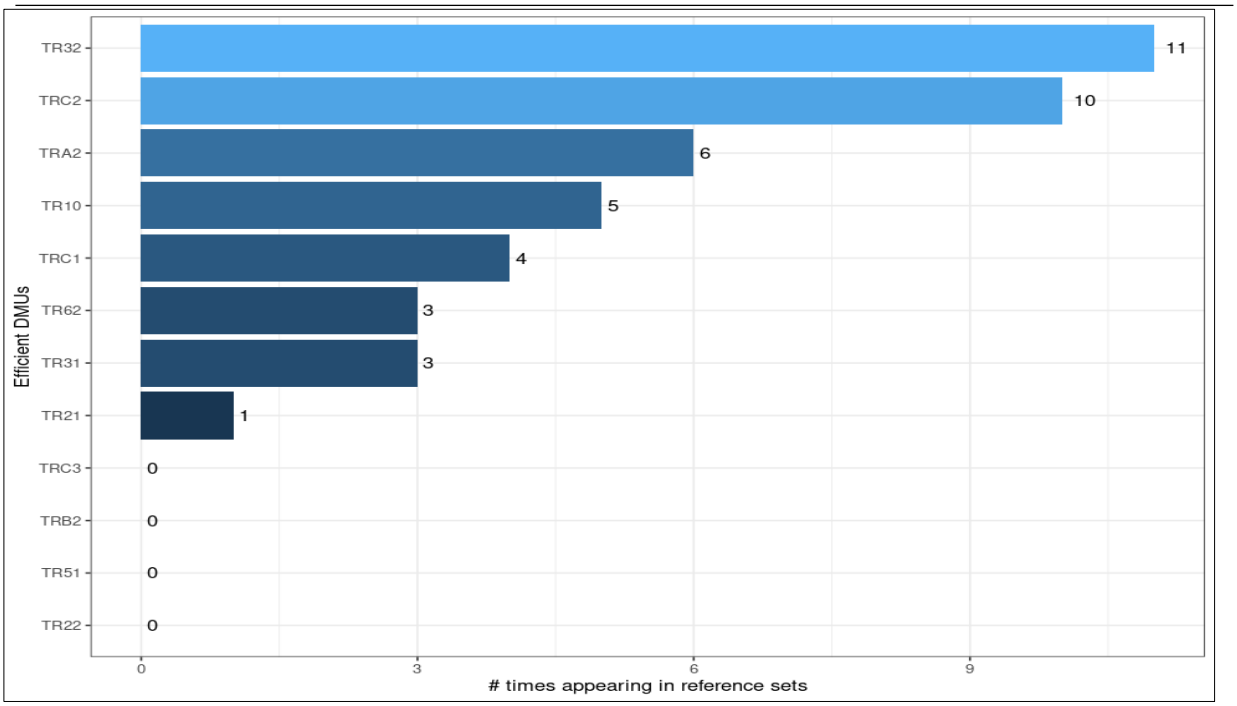

Figure 3. Reference (Peer) Counts

According to Figure-3; TR32 is the most appeared NUTS-2 region in the reference sets with 11 peer counts. It is followed by TRC2 with 10 peer counts. Besides, although TRC3, TRB2, TR51 and TR22 are efficient, they have not appeared in the reference sets.

Improvement options (reducing of the inputs or increasing of the outputs) for inefficient NUTS-2 region are shown in Table 5.

Table 5. Improvement Options for Inefficient Decision-Making Units (DMUs)

DMU	i*1	i*2	i*3	i*4	i*5	o*1
TR33	-0.419573	-859.374094	0.000000	-7.069967	-1.643810	0.000000
TR41	-0.400043	**-9,910.209661**	0.000000	-21.102193	-3.040918	0.000000
TR42	**-3.751377**	-1,731.657175	0.000000	-21.501002	**-3.751736**	0.000000
TR52	-1.214911	-757.343475	0.000000	-11.158362	0.000000	0.000000
TR61	-0.214853	-70.436761	-0.038544	0.000000	-3.103451	0.000000
TR63	-0.207630	-88.472183	**-0.095289**	-3.953214	-1.435863	0.000000
TR71	-1.548135	-506.102559	-0.041471	-3.940086	0.000000	0.000000
TR72	-0.978165	-385.728844	0.000000	**-31.294254**	0.000000	0.000000
TR81	-0.909404	-124.066850	0.000000	-0.031387	0.000000	0.000000
TR82	-0.639993	-47.173120	0.000000	-0.984998	-1.443822	0.000000
TR83	-0.556669	-210.525492	-0.036729	-4.738985	0.000000	0.000000
TR90	-0.947151	-159.368794	-0.067341	-7.817939	-1.482605	0.000000
TRA1	-2.767826	-1,973.260604	-0.021321	-17.678259	0.000000	0.000000
TRB1	-1.803103	-1,459.208670	-0.002639	-11.305325	0.000000	0.000000

According to Table 5;

- ✓ NUTS-2 region that needs the most improvement in terms of ***i1: Sectoral Structure*** is ***TR42*** with i*1 *(-3.751377)*.

- ✓ NUTS-2 region that needs the most improvement in terms of ***i2: The Capacity of Research and Innovativeness*** is ***TR41*** with i*2 *(-9,910.209661)*.
- ✓ NUTS-2 region that needs the most improvement in terms of ***i3: Dijital Structure*** is ***TR63*** with i*3 *(-0.095289)*.
- ✓ NUTS-2 region that needs the most improvement in terms of ***i4: Technology Outcomes*** is ***TR72*** with i*4 *(-31.294254)*.
- ✓ NUTS-2 region that needs the most improvement in terms of ***i5: Life Quality and Labour Force Attractiveness*** is ***TR42*** with i*5 *(-3.751736)*.
- ✓ In terms of **o1: ASO Iltek Score**, none of NUTS-2 region needs an improvement option.

DISCUSSION AND CONCLUSION

In this study, for the purpose of assessing the efficiency of technological development of 26 NUTS-2 regions in the Türkiye (TR10, TR21, TR22, TR31, TR32, TR33, TR41, TR42, TR51), TR52, TR61, TR62, TR63, TR71, TR72, TR81, TR82, TR83, TR90, TRA1, TRA2 , TRB1, TRB2, TRC1 and TRC3), Input-Oriented Slack-Based BCC-DEA model was employed using five input variables *(Sectoral Structure, The Capacity of Research and Innovativesness, Dijital Structure, Technology Outcomes, Life Quality and Labour Force Attractiveness)* and one output variable *(ASO Iltek Score).*

According to the analysis, 12 of the NUTS-2 regions *(TR10, TR21, TR22, TR31, TR32, TR62, TRA2, TRB2, TRC1, TRC2 and TRC3)* have a score of 1 and they are relatively efficient.

These findings are in line with (Özdemir & Selamzade, 2020) in terms of TR10, TR22, TR31, TR32, TR51, TR62 and TRC2; in line with (Demircioğlu & Özgüner, 2022) in terms of of TR10, TR31 and TR51; in line with (Baykul et al., 2016) in terms of TR42.

Besides, TR62 ranks first among the 12 efficient NUTS-2 regions while TRA1 ranks last among the 26 NUTS-2 regions. It is thought that this situation is resulting from the total population of these NUTS-2 regions. Also, the existence of the Mersin International Port in TR62 region and its multiplier effects may cause that interesting finding.

TR32 is the most appeared NUTS-2 region in the reference sets with 11 peer counts. It is followed by TRC2 with 10 peer counts. It is thought that this situation is resulting from TR32 (Aydın, Denizli and Muğla) and TRC2 (Şanlıurfa and Diyarbakır) are the attraction of center for both all of the Türkiye and their NUTS-1 (TR3-Agean and TRC-South Eastern Anatolian) region.

Although TRC3, TRB2, TR51 and TR22 are efficient, they have not appeared in the reference sets. TRC3 and TRB2 are being of border region may cause this situation. Also, TR51 known as Ankara region is a capital of Türkiye, therefore it has distintictive economic and technological environment. Besides TR22 is special region due to it is located in both Marmara and Aegean in Turkiye as well both Asia and Europe in the world.

When improvement options (reducing of the inputs or increasing of the outputs) for inefficient NUTS-2 region is examined;

NUTS-2 region that needs the most improvement in terms of i1: Sectoral Structure, is being represented by the *"R&D Human Resources (Per 1000 Capita)"* indicator in this article, is TR42. It is thought that TR42 region is being one of the most intenstive regions in terms of R&D activities (TÜİK, 2024a).

NUTS-2 region that needs the most improvement in terms of i2: The Capacity of Research and Innovativesness, is being represented by the *"R&D Expenditures (Per Capita)"* indicator in this article, is TR41. It is thought that TR41 region is being one of the most intenstive regions in terms of R&D activities (TÜİK, 2024a).

NUTS-2 region that needs the most improvement in terms of i3: Dijital Structure, is being represented by the *"Number of Broadband Subscriptions (Per Capita)"* indicator in this article, is TR63. It is thought that TR63 region is being one of the most intensive regions in terms of Syrian originated people.

NUTS-2 region that needs the most improvement in terms of i4: Technology Outcomes, is being represented by the *"The Number of Patent Grant (Per 1 Mn Capita)"* indicator in this article, is TR72. It is thought that TR72 region is being one of the most patent intensity regions (Türk Patent, 2025).

NUTS-2 region that needs the most improvement in terms of i5: Life Quality and Labour Force Attractiveness, is being represented by the *"Empoyment Rate"* indicator in this article, is TR42. It is thought that TR42 region is being one of the most intenstive regions in terms of industry and commerce (Kocaeli Valiliği, 2025).

None of NUTS-2 region needs an improvement option in terms of o1: ASO Iltek Score, is being represented by the *"Weighted ASO Iltek Score"* indicator in this article.

It is hoped that the results of this study will provide benefits, albeit relatively, to the institutions and organizations of NUTS-2 regions dealing with technology management in terms of carrying out more efficient technology polices. It is thought that the most important of these benefits is the opportunity for inefficient NUTS-2 regions to become efficient in terms of technology management, which includes financial and non-financial costs in terms of national accounts, by analysing in detail, comparing and simulating the practices of NUTS-2 regions in the reference sets defined for them.

As a result, it is recommended that researchers working in the field of technology management perform performance evaluation with Network Data Envelopment Analysis and Two-Stage Data Envelopment Analysis using different input/output bundles.

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Forecasting Turkey's Hazelnut, Pistachio, and Walnut Exports Using Time Series Analysis (2005–2030) (1255)

Ferda Nur Özdemir[1], Adem Aksoy[1*]

[1]Atatürk University, Faculty of Agriculture, Department of Agricultural Economics, Türkiye

*Corresponding author e-mail: ferdanur.ozdemir@atauni.edu.tr

Abstract

This study aims to forecast the export volumes of Turkey's key nut products—hazelnuts, pistachios, and walnuts—using time series data from 2005 to 2024. The Autoregressive Integrated Moving Average (ARIMA) model was employed to analyze past trends and provide projections for the 2025–2030 period. Monthly export data were obtained from national and international statistical databases, seasonally adjusted, and tested for stationarity using the Augmented Dickey-Fuller (ADF) test. The optimal ARIMA models were selected based on the Akaike Information Criterion (AIC), and model adequacy was confirmed through residual diagnostics. Forecast results indicate that hazelnut exports are expected to exhibit moderate growth, while pistachio exports may fluctuate due to yield variability. In contrast, walnut exports are projected to rise steadily, reflecting increasing global demand. These findings provide valuable insights for policymakers, exporters, and stakeholders in the Turkish nut industry and highlight the utility of ARIMA models in agricultural export forecasting.

Keywords: *Time Series Analysis, Agricultural Trade, Arima Model, Nut Exports, Forecasting*

INTRODUCTION

Thanks to its agro-ecological diversity and favorable climatic conditions, Turkey is a major global actor in the production and export of tree nuts, particularly hazelnuts, pistachios, and walnuts. The country dominates the global hazelnut market, accounting for an average of 64% of world production and 67% of exports during the 1961–2018 period (Uzundumlu et al., 2022). Hazelnut cultivation is concentrated in the Black Sea region, yet it spans 37 provinces nationwide and represents a cornerstone of Turkey's agricultural economy in terms of both production volume and export income (Uzundumlu et al., 2022). Nevertheless, structural issues such as aging orchards, yield instability, and climate-related fluctuations pose ongoing challenges for sustainable hazelnut production (Bak, 2021).

Pistachio production has also experienced a consistent upward trajectory. Turkey ranked second globally in pistachio production in 2020, with a 26.33% share of the market, following the United States (Uzundumlu et al., 2024). According to the Turkish Statistical Institute, 119,000 tons of pistachios were produced in 2021, with the number of bearing trees reaching 55.5 million (Kirca & Bak, 2022). Forecasts based on ARIMA models predict that this upward trend will continue through 2025 (Uzundumlu et al., 2024). Similarly, Turkey produced 325,000 tons of walnuts in recent years, placing it among the top five walnut producers globally (Kirca & Bak, 2022).

The export performance of these high-value commodities is strongly influenced by factors such as climate variability, changes in production efficiency, global market dynamics, and international pricing trends (Çelik et al., 2017; Uzundumlu et al., 2022). Therefore, generating accurate and data-driven export forecasts is essential for effective agricultural policy-making, production planning, and global competitiveness. Among the various forecasting techniques, the Autoregressive Integrated Moving Average (ARIMA) model, developed by Box and Jenkins, stands out as a statistically robust and widely accepted method for modeling time-dependent data and predicting future trends (Box et al., 2016; Alabdulrazzaq et al., 2021).

In this study, Turkey's monthly export data for hazelnuts, pistachios, and walnuts from 2005 to 2024 are analyzed using ARIMA models. Based on these models, forecasts are generated for the 2025–2030 period. The study aims to offer actionable insights for policymakers, agricultural strategists, and industry stakeholders to support evidence-based export planning and sustainable trade development.

What distinguishes this study from prior research is its integrated and comparative application of ARIMA models to three of Turkey's most strategically important nut commodities—hazelnuts, pistachios, and walnuts—simultaneously. While earlier studies have primarily focused on single-crop analyses or national-level production trends, this study uniquely combines long-term export data with tailored time series models for each crop. The forecasts provided herein extend beyond generalized projections and offer product-specific, year-by-year insights for the 2025–2030 horizon. By aligning empirical modeling with sectoral realities, this research fills a gap in the literature and contributes to both academic understanding and policy formulation in agricultural export forecasting.

MATERIAL AND METHODS

Material

This study utilized annual export quantity data (in kilograms) for hazelnuts, pistachios, and walnuts from Turkey. The dataset covers the period from 2005 to 2024 and represents official national statistics on tree nut exports. These time series served as the primary input for forecasting analysis and were organized and cleaned prior to model implementation.

Methods

The Collection of the Data

The annual export data used in this study were obtained from the Turkish Statistical Institute (TURKSTAT). Covering the years 2005 to 2024, the dataset includes reported export volumes for hazelnuts, pistachios, and walnuts. The data were reviewed for completeness and consistency before being transferred into EViews 12 for time series modeling and ARIMA-based forecasting.

Statistical Analysis

In the time series, the first step for selecting the most suitable model involves checking whether the series is stationary, and various unit root tests are applied to non-stationary series. The Box-Jenkins model assumes that time series are stationary. If a series is stationary, it has a constant mean, constant variance, and constant autocorrelation. Once stationary is achieved, the next step is to determine the values of p and q in the ARIMA (p, I, q) model, where I indicate how many differences need to be taken to obtain a stationary series (Anonim, 2022). The ARIMA (p, d, q) Box-Jenkins Model, proposed by Box and Jenkins, is a widely used method for constructing a univariate time series forecasting model (Mensah 2015).

The Box-Jenkins ARIMA model approach was defined in a book published by statisticians George Box and Gwilym Jenkins in the 1970s. An ARIMA process is a mathematical model used for prediction. The Box-Jenkins modeling involves identifying an appropriate ARIMA process, fitting it to the data, and then using the suitable model for forecasting. One attractive feature of the Box-Jenkins approach for forecasting is that ARIMA processes encompass a rich class of models and often provide adequate explanations for the data (Hyndman 2001). According to Box and Jenkins (1976), a non-seasonal ARIMA model is represented by ARIMA (p, d, q), where d represents the difference, p represents autoregressive coefficients, and q represents moving average coefficients (Dasyam et al 2015).

These values are determined using autocorrelation functions (ACF) and partial autocorrelation functions (PACF) and the Dickey Fuller (ADF) test (Awal and Sıddıque, 2011). ACF and PACF shapes are used as tools to determine the stationarity of variables and the lag length of the ARIMA model when predicting ARIMA models. The Dickey Fuller (ADF) test is applied to non-stationarity during stationarity testing.

PACF or partial correlogram is used to determine the appropriate number of lags for the AR model. The number of non-zero relationships in PACF determines where AR lags should be included. The ACF correlogram is used to determine the number of lags for the MA model, where again, non-zero relationships indicate where lags should be included.

An AR model function is as follows:

$Yt = c + \phi\ 1\ Yt\text{-}1 + \phi\ 2\ Yt\text{-}2 + \ldots + \phi\ p\ Yt\text{-}p + \varepsilon_t$

Here, *Yt* is the value of the time series at time *t*, *c* is a constant, $\phi 1, \phi 2,\ldots,\phi p$ are autoregressive parameters, and εt is the white noise error term.

Integrated (I) Component: The integrated component involves differencing the time series to achieve stationarity. The order of differencing is denoted by *d*, and the integrated component is represented by:

$Yt' = Yt - Yt-1$

The MA model function is: The moving average component involves modeling the relationship between the current value of the time series and past forecast errors. The MA(q) component is represented by:

$Yt = \mu + \theta 1 \varepsilon t-1 + \theta 2 \varepsilon t-2 + \ldots + \theta q \varepsilon t-q + \varepsilon t$

Here, μ is the mean of the time series, $\theta 1, \theta 2,\ldots,\theta q$ are moving average parameters, and εt is the current error term.

Generally, the ARIMA model is as follows:

$\Delta d\ Zt = c + (Ø1\ \Delta d\ Zt\text{-}1 + \ldots + Øp\ \Delta d\ Zt\text{-}p) - (\theta 1\ \varepsilon_t\text{-}1 + \ldots + \theta q\ \varepsilon_t\text{-}q) + \varepsilon_t$

Here, Δ denotes the difference as shown below: Zt-1, Zt-p, are the values of the past series with delays of 1, p, respectively.

$\Delta\ Zt = Zt - Zt\text{-}1$

$\Delta 2\ Zt\text{-}1 = \Delta Zt - \Delta Zt\text{-}1$

RESULTS

This section presents the results of the ARIMA models constructed using annual export data for hazelnuts, pistachios, and walnuts in Turkey for the period 2005–2024. For each commodity, the most appropriate model was selected based on statistical evaluation criteria such as the Akaike Information Criterion (AIC), coefficient of determination (R^2), and residual diagnostics. Following model validation, annual forecasts were generated for the period 2025–2030 using both static and dynamic forecasting methods. The forecast results presented below reveal the expected trends and potential fluctuations in the export volumes of each product.

Table 1. Augmented Dickey-Fuller (ADF) Unit Root Test Results for Hazelnut, Pistachio, and Walnut Export Series

UNIT ROOT TEST RESULTS TABLE (ADF)
Null Hypothesis: the variable has a unit root

	At Level			
		HAZELNUT_EXPKG	PISTACHIO_EXPKG	WALNUT_EXPKG
With Constant	t-Statistic	-2.7750	3.3455	-0.1951
	Prob.	***0.0806***	***1.0000***	***0.9238***
		*	n0	n0
With Constant & Trend	t-Statistic	-4.2485	1.0437	-2.2024
	Prob.	***0.0172***	***0.9997***	***0.4549***

		**	n0	n0
Without Constant & Trend	t-Statistic	0.3128	4.6653	0.6323
	Prob.	***0.7650***	***1.0000***	***0.8440***
		n0	n0	n0
	At First Difference			
		d(HAZELNUT_EXPKG)	d(PISTACHIO_EXPKG)	d(WALNUT_EXPKG)
With Constant	t-Statistic	-7.4123	-2.0508	-4.2189
	Prob.	***0.0000***	***0.2647***	***0.0048***
		***	n0	***
With Constant & Trend	t-Statistic	-7.1144	-3.4652	-4.3907
	Prob.	***0.0001***	***0.0741***	***0.0140***
		***	*	**
Without Constant & Trend	t-Statistic	-7.5781	-1.4683	-3.8926
	Prob.	***0.0000***	***0.1286***	***0.0006***
		***	n0	***

According to (Table 1.) the results of the Dickey-Fuller unit root test, the export series for hazelnuts (HAZELNUT_EXPKG), pistachios (PISTACHIO_EXPKG), and walnuts (WALNUT_EXPKG) are not stationary at level. However, when first differences are taken, the hazelnut and walnut series become stationary at the 1% significance level, while the pistachio series becomes stationary at the 10% level. In this context, since stationarity could not be achieved at level for any of the series, taking the first differences of the variables was deemed appropriate before proceeding with the ARIMA modeling.

Table 2. Arıma Models Result for Hazelnut, Pistachio, Walnut

Model	**AR(1) (p-value)**	**MA(1) (p-value)**	**R^2**	**Adj. R^2**	**AIC**	**D-W Stat**
WALNUT_EXPKG	0.902 (0.001)	0.081 (0.899)	0.796	0.758	34.118	1.869
HAZELNUT_EXPKG (1st Diff)	–0.567 (0.039)	–	0.304	0.218	36.606	2.089
PISTACHIO_EXPKG	0.951 (0.000)	0.710 (0.017)	0.912	0.896	33.564	1.816

The estimation results indicate that each export series exhibits distinct structural characteristics over time, which are captured to varying extents by the applied ARIMA models. For the walnut and pistachio series, the models produced statistically significant parameters and acceptable levels of model fit, suggesting relatively stable autoregressive and/or moving average patterns in these data. Although the hazelnut export series, modeled in first difference, demonstrated a comparatively lower explanatory power, the significance of the autoregressive component indicates that relevant time-dependent dynamics were still captured. Overall, the models—validated through residual diagnostics—can be considered suitable tools for generating short-term export forecasts.

Figure 1 illustrates the actual, static, and dynamic forecast values for Turkey’s hazelnut exports between 2005 and 2030. The actual series exhibits a highly volatile structure with no clear long-term trend, particularly between 2010 and 2017, where several notable decreases are observed. A moderate upward trend appears after 2018, but fluctuations remain evident. The static forecast model, which bases each year's estimate on the previous actual value, produces a stable and conservative projection for the 2025–2030 period. In contrast, the dynamic forecast model, which builds each new prediction on the preceding forecast,

shows a slightly increasing but more variable trajectory. This reflects the potential for cumulative forecasting errors but also better captures medium-term uncertainty. Overall, both models align reasonably well with historical data up to 2024. Given the erratic nature of hazelnut exports, which are often influenced by climate variability, yield instability, and international market conditions, the static model offers a more risk-averse outlook, while the dynamic model presents a broader scenario range. These results suggest that incorporating additional contextual variables into the forecasting framework may further enhance predictive accuracy for commodities with such irregular export behavior.

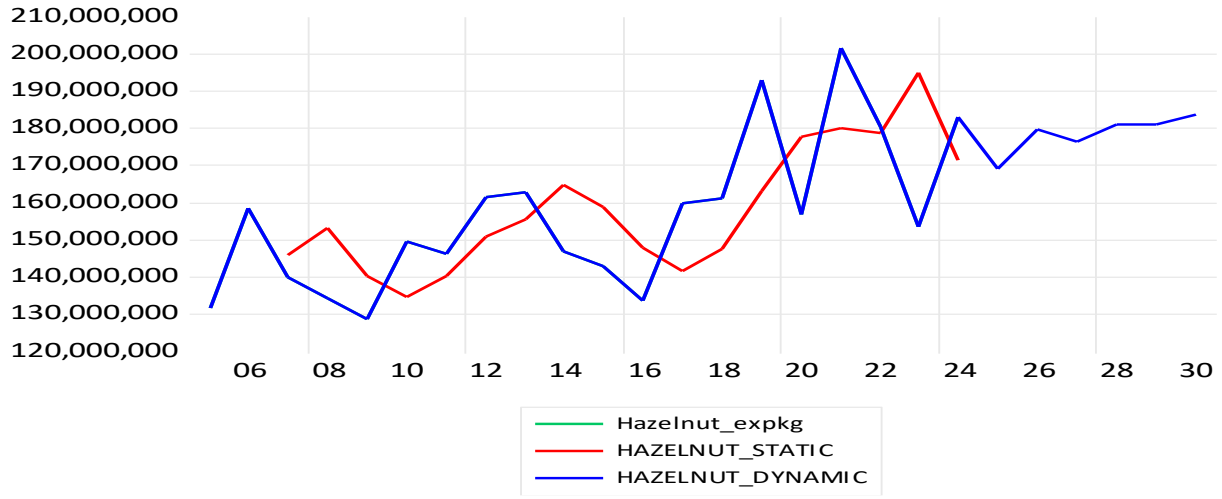

Figure 1. Actual and Forecasted Values of Turkey's Hazelnut Exports (2005–2030)

Figure 2 displays the actual, static, and dynamic ARIMA forecasts for Turkey's pistachio exports from 2005 to 2030. The actual series remained relatively flat and volatile until 2020, after which a significant upward trend emerged. The static forecast model reflects this pattern and projects a more cautious but steady growth trajectory for the post-2025 period. In contrast, the dynamic forecast model captures the upward trend more aggressively, generating higher estimated values through 2030. Both models closely align with the actual data through 2024, successfully reflecting the sharp increase in exports after 2020. This indicates that the ARIMA model performs well in capturing short-term dynamics in pistachio exports and provides a reliable basis for policy and planning decisions. However, projections beyond 2025 should be interpreted with caution, as external factors such as climate conditions, supply-demand shifts, and global pricing may significantly influence future export patterns.

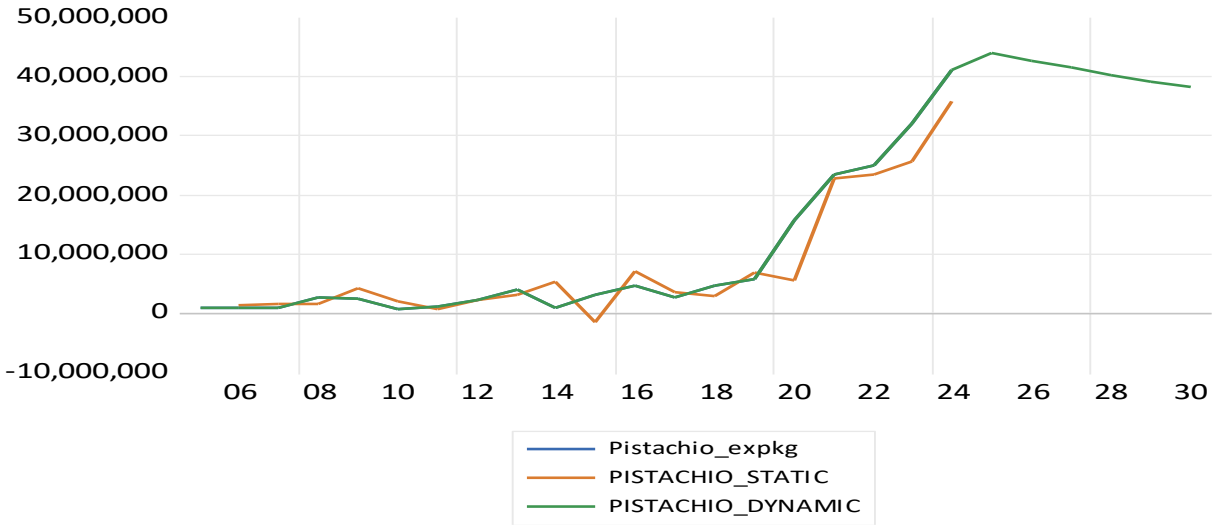

Figure 2. Actual and Forecasted Values of Turkey's Pistachio Exports (2005–2030)

Figure 3 presents Turkey's walnut export trends from 2005 to 2030, including actual export data and forecasts generated using ARIMA models. Until 2020, the export volume follows a steady upward trend, followed by a sharp and notable increase between 2021 and 2023. This spike may be attributed to external factors such as policy shifts, exchange rate fluctuations, or temporary market dynamics. The static forecast model incorporates this surge and projects a plateau at elevated levels for the 2025–2030 period. In contrast, the dynamic forecast model reflects a downward trend, possibly due to cumulative forecast adjustments stemming from a high baseline. While dynamic forecasts are more prone to compounding error, they may better capture potential market corrections. Both models track the actual data closely up to 2024 but diverge afterward, offering alternative future scenarios. These results emphasize the importance of interpreting model-based forecasts cautiously and supplementing them with context-specific external variables when projecting walnut export performance.

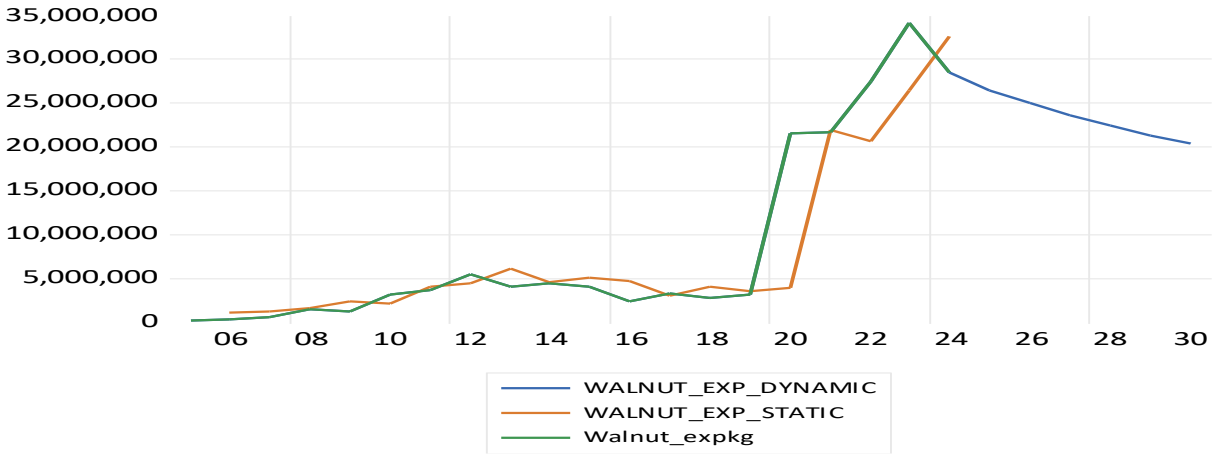

Figure 3. Actual and Forecasted Values of Turkey's Walnut Exports (2005–2030)

Figure 4 presents the inverse root plots of the characteristic polynomials for the ARIMA models estimated for hazelnut, pistachio, and walnut export series. In all three graphs, the inverse roots of both autoregressive (AR) and moving average (MA) components lie strictly within the unit circle, indicating that the estimated models satisfy the stationarity and invertibility conditions. For the pistachio and walnut series (shown in the first and third plots), both AR and MA roots are well-behaved and positioned comfortably inside the circle, suggesting a stable time series structure with reliable forecasting performance. In the hazelnut model (second plot), only an AR component is present, and its inverse root is similarly located within the circle, confirming the stationarity of the differenced series. These root plots support the statistical validity of the selected ARIMA models and demonstrate that the forecasts derived from them are based on stable underlying processes.

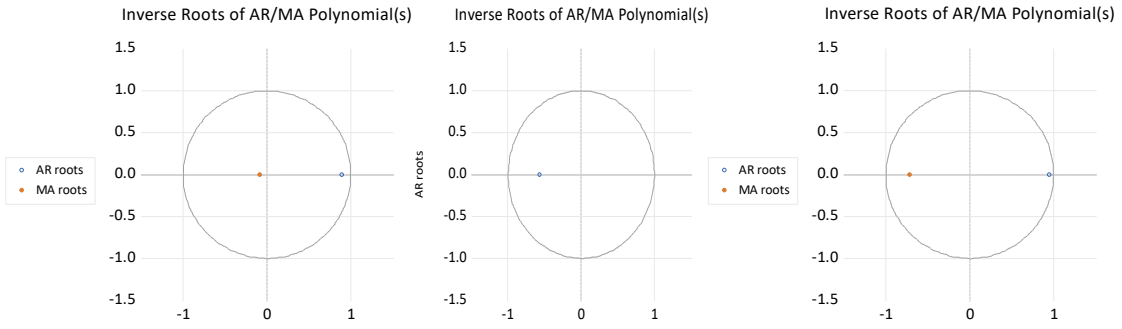

Figure 4. Inverse Root Graphs of Characteristic Polynomials for Hazelnut, Pistachio, and Walnut ARIMA Models

DISCUSSION AND CONCLUSION

This study aimed to forecast the export trends of hazelnuts, pistachios, and walnuts in Turkey using ARIMA models based on historical export data from 2005 to 2024. The stationarity of the series was confirmed through unit root tests, and appropriate ARIMA configurations were selected for each commodity. The model outputs demonstrate differing dynamic structures and volatility levels for each product, reflecting their unique production and trade characteristics.

For hazelnuts, the ARIMA model was effective in capturing the irregular yet gradually increasing trend observed in recent years. This aligns with earlier findings by Uzundumlu et al. (2022), who emphasized that Turkey's dominance in hazelnut exports is both structural and market-driven, accounting for over 64% of global production and 67% of exports in the 2011–2018 period. Forecasting models such as ARIMA have also been successfully applied to hazelnut production and export data by Kilic and Turhan (2020), who confirmed the model's robustness in medium-term predictions.

In contrast, pistachio exports demonstrated a more volatile historical pattern. The static and dynamic ARIMA forecasts diverged after 2025, reflecting uncertainty driven by market expansion and yield variability. While Turkey ranks among the top global producers, its export share remains limited compared to the USA and Iran (FAO, 2022). Nonetheless, ARIMA-based modeling has been successfully applied in earlier studies such as those by Karacan and Ceylan (2017), who examined price dynamics, and by Uzundumlu et al. (2024), who modeled global production and export trends for pistachio between 2020 and 2025. The forecasts in this study complement those findings, indicating a potential for moderate but steady export growth under stable economic and climatic conditions.

As for walnuts, the time series displayed a steady upward trend with a sudden rise after 2021, likely reflecting market shocks or policy changes. The divergence between static and dynamic models post-2025 suggests potential volatility. Previous modeling efforts by Sacti and Kilci (2016) support the use of ARIMA for walnut production forecasting in Turkey, with results showing high predictive accuracy in short-term projections.

The results of this study are consistent with global research advocating the application of ARIMA models in agricultural forecasting. Celik (2013) and Amin et al. (2014) demonstrated the model's adaptability across various agricultural commodities, emphasizing its strength in capturing non-linear trends in production and export data. Moreover, Kaynar and Tastan (2009) reported that while machine learning models may offer marginally higher precision, ARIMA remains preferable for its interpretability and ease of use in policy contexts.

Overall, the projections obtained in this study provide valuable insights for policymakers and sectoral planners. They suggest that while Turkey maintains structural advantages in hazelnut exports, pistachio and walnut exports could further benefit from targeted investments, promotional strategies, and climate-resilient agricultural practices. Nevertheless, future forecasting efforts would benefit from integrating exogenous variables such as climate indicators, exchange rates, and global demand indices to enhance the models' explanatory power and applicability.

This study employed ARIMA models to forecast Turkey's hazelnut, pistachio, and walnut export volumes using annual data from 2005 to 2024. The findings revealed that each nut product exhibits distinct export behaviors and time-dependent structures. Hazelnut exports followed a relatively fluctuating pattern with mild upward movement, pistachio exports reflected more volatile dynamics, and walnut exports showed a steady upward trend, especially after 2021.

The selected ARIMA models were found to be statistically adequate and structurally stable, with root diagnostics confirming model validity. The export forecasts for the 2025–2030 period suggest continued

moderate growth for all three commodities, though with varying degrees of confidence and expected volatility across products.

Overall, the results indicate that ARIMA modeling provides a useful framework for short- and medium-term export forecasting in agricultural markets. These projections can serve as a decision-support tool for policy-makers, exporters, and agricultural planners. To enhance export potential, it is important to focus on strategies such as improving supply chain efficiency, ensuring production sustainability, and adapting to global market conditions through proactive planning.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

Both authors contributed equally to this work (50% – 50%).

Examining the Relationship Between Hazelnut Prices and Exchange Rates in Türkiye: A Granger Causality Test (2016M01–2025M03) (1256)

Vahit Cem Tüzemen[1], Ferda Nur Özdemir[1*]

[1]Atatürk University, Faculty of Agriculture, Department of Agricultural Economics, Türkiye

*Corresponding author e-mail: ferdanur.ozdemir@atauni.edu.tr

Abstract

This study aims to examine the causality relationship between hazelnut prices and the USD/TRY exchange rate in Türkiye using the Granger causality test. Monthly data covering the period from January 2016 to March 2025 were obtained from national statistical databases and include average hazelnut prices in Turkish Lira and nominal USD/TRY exchange rates. Prior to analysis, the stationarity levels of the variables were determined using the Augmented Dickey-Fuller (ADF) test. Appropriate lag lengths were selected based on the Akaike Information Criterion (AIC), and the Granger causality test was applied to the stationary series. The results indicate a unidirectional causality relationship between the two variables. This finding suggests that changes in exchange rates may influence hazelnut prices and highlights the significance of macroeconomic indicators in agricultural markets.

Keywords: *Granger Causality, Hazelnut Prices, Exchange Rate, Time Series, Agricultural Economics*

INTRODUCTION

The formation of agricultural product prices is influenced not only by internal supply-demand dynamics but also by external macroeconomic variables such as exchange rates, energy prices, and global market conditions. Accordingly, the idea that agricultural commodity prices are shaped by both the level and volatility of energy markets and exchange rate movements has been explored in the literature since the 1980s. For example, Hamilton (1983) and Pindyck & Rotemberg (1990) were among the first to demonstrate that oil price volatility plays a significant role in macroeconomic fluctuations.

One of the most comprehensive early analyses of the relationship between oil and agricultural prices was conducted by Baffes (2007), who emphasized that this linkage is particularly evident in developing economies. Similarly, Dehn (2000) showed that oil price shocks generate strong but lagged effects on commodity markets. More recent studies have highlighted not only the price-level effects but also the volatility transmission between energy and food markets (Esmaeili & Shokoohi, 2011; Chen et al., 2007), confirming a co-movement between oil and agricultural prices (Abbott et al., 2008).

Exchange rates also represent a critical macroeconomic determinant, especially in the context of agricultural exports. Mundlak & Larson (1992) provided structural evidence of exchange rate pass-through into domestic agricultural prices.

In the case of Türkiye, where hazelnuts are both domestically consumed and heavily exported, the interaction between energy prices, exchange rates, and agricultural pricing becomes more complex. Türkiye accounts for approximately 70% of global hazelnut production and relies heavily on this product for foreign exchange income. However, hazelnut prices are influenced not only by domestic supply-demand dynamics but also by external pressures such as energy costs and currency volatility.

In this context, Erdal and Uzunöz (2008) analyzed the relationship between Turkish and European hazelnut prices and the exchange rate, revealing a one-way causality from the exchange rate to hazelnut prices. In a more recent study, Aşkan, Gök, and Sevinç (2022) employed a VECM-BEKK MGARCH approach and demonstrated that volatility in fuel prices and exchange rates has both direct and indirect (conditional variance) effects on hazelnut prices. The theoretical foundation of these models is based on the ARCH/GARCH framework developed by Engle (1982) and Bollerslev (1986).

In this light, the present study aims to empirically examine the causal relationship between real Brent oil prices, the exchange rate, and real hazelnut prices in Türkiye using econometric methods. By investigating this specific triangle of agriculture–energy–exchange rate interactions in the context of the hazelnut market, the study seeks to contribute to both the academic literature and the decision-making processes of policymakers and stakeholders.

MATERIAL AND METHODS

Material

The material used in the scientific study should be given clearly depending on its sources. In addition, the statistical models used in the study should be expressed clearly and precisely. The first paragraph after the title should begin under the title and not be indented. The text of the proceedings should be written in Times New Roman, 11 font size, 1.15 line spacing and justified to the column. There should be 6 nk spaces between the paragraphs.

For the second and subsequent paragraphs, the beginning of the paragraph should be indented 0.5 cm. Body of text Body of text Body of text Body of text Body of text Body of text body of text (Author's Surname, 2021; Author's Surname and Author's Surname, 2021).

Methods

The Collection of the Data

Dataset

This study uses a monthly time series dataset covering the period from 2016M01 to 2025M03. The variables included in the analysis are:

Hazelnut_Real: Real hazelnut prices (in TL/kg), deflated by the Consumer Price Index (CPI)

Brent crude oil: Real exchange rate (USD/TRY), also deflated by CPI

Statistical Analysis

The real series were calculated using the following formulas:

$_t = (HAZELNUT\_NOM_t / CPI_t) \times 100$

Brent crude oil $_t$ = (Brent crude oil $\_NOM_t / CPI_t) \times 100$ Here, CPI represents the Turkish Consumer Price Index.

Stationarity Test (ADF Test)

The Augmented Dickey-Fuller (ADF) test is applied to determine whether each series is stationary. The general regression equation for the ADF test is as follows:

$\Delta Y_t = \alpha + \beta t + \gamma Y_{t-1} + \Sigma(\delta_i \Delta Y_{t-i}) + \varepsilon_t$

Δ: first difference operator

α: constant term

βt: deterministic trend

γ: lagged level term

ε_t: white noise error term

Hypotheses:

H_0: $\gamma = 0 \rightarrow$ the series has a unit root (non-stationary)

H_1: $\gamma < 0 \rightarrow$ the series is stationary

Granger Causality Test

Granger causality is used to investigate whether one time series can predict another. The following Vector Autoregressive (VAR) model is estimated:

$$Y_t = \alpha_0 + \Sigma(\alpha_i Y_{t-i}) + \Sigma(\beta_j X_{t-j}) + \varepsilon_{1t}$$

$$X_t = \delta_0 + \Sigma(\delta_i X_{t-i}) + \Sigma(\theta_j Y_{t-j}) + \varepsilon_{2t}$$

Null Hypothesis:

H_0: $\beta_j = 0$ (X does not Granger cause Y)

H_1: $\exists\ \beta_j \neq 0$ (X Granger causes Y)

The test is based on the F-statistic for joint significance.

Impulse Response Function (IRF)

The Impulse Response Function traces the effect of a one standard deviation shock in BRENT_REAL on HAZELNUT_REAL over time. The mathematical expression is:

$IRF_h = \partial Y_{t+h} / \partial \varepsilon_{xt}$, for h = 0, 1, ..., H

h is the forecast horizon (in months)

ε_{xt} represents the structural shock in the exchange rate series

Forecast Error Variance Decomposition (FEVD)

Variance decomposition quantifies how much of the forecast error variance of a variable (e.g., HAZELNUT_REAL) is explained by shocks to another variable (e.g., EXCHANGE_REAL):

$FEVD_h(Y)$ = [Variance explained by X / Total forecast error variance of Y] × 100

This decomposition is reported across a forecast horizon of h = 1 to 10 months.

RESULTS

In this study, the causal relationship between the real exchange rate and real hazelnut prices in Türkiye was examined using monthly data covering the period from 2016M01 to 2025M03. The stationarity of the series was tested using the Augmented Dickey-Fuller (ADF) method, followed by the application of the Granger

causality test. To further investigate the direction and persistence of the relationship, impulse response functions and forecast error variance decomposition analyses were conducted. The empirical findings indicate that fluctuations in the exchange rate have a statistically and economically significant effect on hazelnut prices.

The results of the Augmented Dickey-Fuller (ADF) unit root test indicate that both BRENT_REAL and HAZELNUT_REAL series are non-stationary at their level forms but become statistically significant and stationary after first differencing. In the level form, the ADF test statistics for both series are above the critical values, and the corresponding p-values do not allow for the rejection of the null hypothesis of a unit root at the 5% significance level. However, after first differencing, the p-values are found to be 0.0000, and the ADF statistics are well below all critical thresholds. These results confirm that both variables are integrated of order one (I(1)) and are therefore suitable for use in VAR and Granger causality analyses. Consequently, the stationarity tests conducted prior to the empirical modeling support the methodological validity of the analysis.

Table 1. Panel Unit Root Test Results for BRENT_REAL and HAZELNUT_REAL

UNIT ROOT TEST RESULTS TABLE (ADF)
Null Hypothesis: the variable has a unit root

		At Level	
		BRENT_REAL	HAZELNUT_REAL
With Constant	t-Statistic	-0.7553	-2.4621
	Prob.	***0.8272***	***0.1276***
		n0	n0
With Constant & Trend	t-Statistic	-3.5211	-3.3931
	Prob.	***0.0421***	***0.0576***
		**	*
Without Constant & Trend	t-Statistic	-0.9086	-0.1078
	Prob.	***0.3206***	***0.6443***
		n0	n0
		At First Difference	
		d(BRENT_REAL)	d(HAZELNUT_REAL)
With Constant	t-Statistic	-8.8136	-7.9785
	Prob.	***0.0000***	***0.0000***
		***	***
With Constant & Trend	t-Statistic	-8.8349	-7.9349
	Prob.	***0.0000***	***0.0000***
		***	***
Without Constant & Trend	t-Statistic	-8.8174	-8.0127
	Prob.	***0.0000***	***0.0000***
		***	***

As shown in Table 2, in determining the optimal lag length for the VAR model, the Akaike Information Criterion (AIC) and Final Prediction Error (FPE) indicate the lowest values at lag 3. In contrast, the Schwarz Criterion (SC) and Hannan-Quinn (HQ) suggest lag 2 as optimal. Since AIC and FPE prioritize forecasting performance, lag 3 was selected for the subsequent analysis.

Table 2. VAR Lag Length Selection Criteria for BRENT_REAL and HAZELNUT_REAL

VAR Lag Order Selection Criteria
Endogenous variables: BRENT_REAL HAZELNUT_REAL
Exogenous variables: C

Sample: 2016M01 2025M03
Included observations: 105

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-390.5026	NA	6.052593	7.476240	7.526791	7.496724
1	-175.8702	417.0001	0.109533	3.464194	3.615848	3.525647
2	-165.2609	20.20804*	0.096587	3.338304	3.591062*	3.440726*
3	-161.0080	7.938820	0.096147*	3.333486*	3.687347	3.476877
4	-160.0700	1.715240	0.101967	3.391809	3.846774	3.576170
5	-159.5714	0.892753	0.109076	3.458502	4.014570	3.683832
6	-156.3329	5.675006	0.110782	3.473008	4.130179	3.739307

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

The Granger causality test results presented in Table 3 reveal a unidirectional causality running from BRENT_REAL to HAZELNUT_REAL at the 1% significance level (F = 4.078, p = 0.0089). This indicates that past values of the real Brent oil price significantly help explain variations in real hazelnut prices, with an effect observed over a three-period lag. In contrast, the null hypothesis that HAZELNUT_REAL does not Granger cause BRENT_REAL cannot be rejected at the 5% level (F = 2.256, p = 0.0865), implying no statistically significant reverse causality. These findings support the view that fluctuations in global energy prices play a significant role in shaping agricultural product prices in Türkiye.

Table 3. Granger Causality Test Results between BRENT_REAL and HAZELNUT_REAL

Sample: 2016M01 2025M03
Lags: 3

Null Hypothesis:	Obs	F-Statistic	Prob.
HAZELNUT_REEL does not Granger Cause BRENT_REAL	108	2.25569	0.0865
BRENT_REEL does not Granger Cause HAZELNUT_REAL		4.07843	0.0089

Null Hypothesis	**F-Stat.**	**p-value (Prob.)**	**Comment**
HAZELNUT_REAL does not Granger Cause BRENT_REAL	2.25569	0.0865	Cannot be rejected (marginal)
BRENT_REAL does not Granger Cause HAZELNUT_REAL	4.07843	**0.0089**	Rejected (significant causality exists)

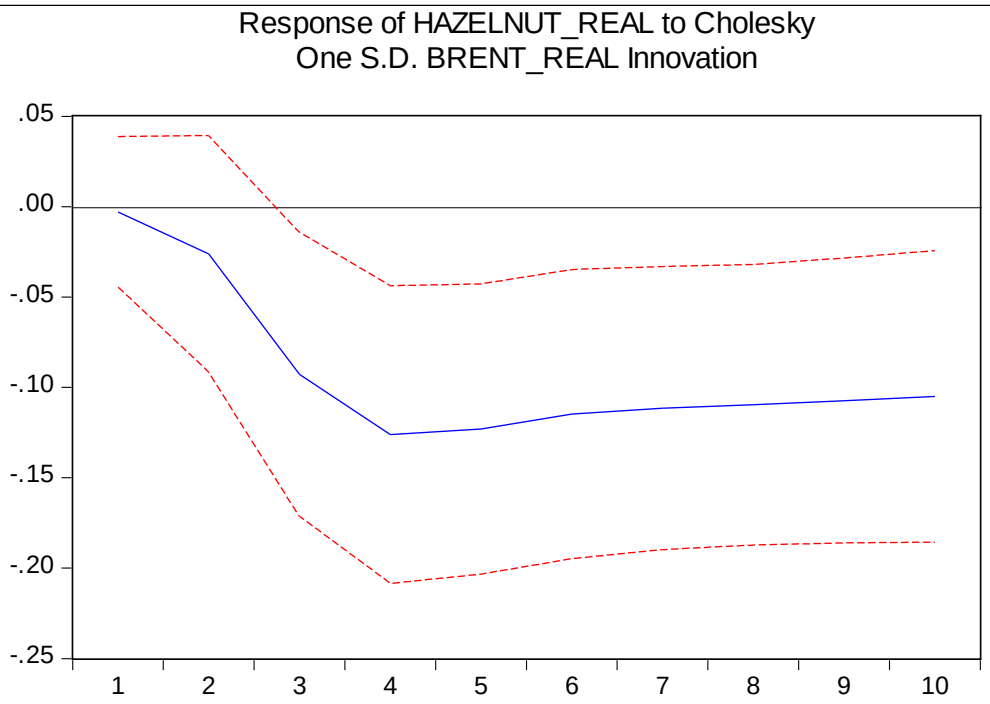

Figure 1. Interpretation of the Impulse Response Analysis

Figure 1. illustrates the impulse response of real hazelnut prices (HAZELNUT_REAL) to a one standard deviation positive shock in Brent oil prices (BRENT_REAL) over a 10-month forecast horizon. The results reveal a negative response: following the shock, hazelnut prices begin to decline from the first month, reaching their lowest point (approximately -12 units) in the third month. This effect slows during the fourth and fifth months but stabilizes around -15 units from the sixth month onward, persisting through the end of the forecast period. These dynamics suggest that the shock induces a medium-term, persistent negative effect on hazelnut prices.

The red dashed lines in the graph represent the 95% confidence bands. While the blue impulse response line does not cross the lower bound of these bands, it approaches the threshold of statistical significance particularly between the second and seventh months. Therefore, the effect can be described as economically meaningful and statistically marginally significant.

These findings indicate that increases in Brent oil prices can influence agricultural markets in Türkiye through both direct and indirect channels. Rising energy, fuel, and logistics costs can raise production expenses, which may reduce producers' profit margins and exert downward pressure on supply-side prices. Alternatively, inflationary pressures and a contraction in consumer demand triggered by oil price increases may contribute to weakening prices on the demand side.

Supported by the Granger causality test, the impulse response analysis confirms that fluctuations in global oil markets exert lagged, persistent, and negative effects on real hazelnut prices. This implies that the transmission of energy price shocks to agricultural commodity prices is not only immediate but also evident in the medium term.

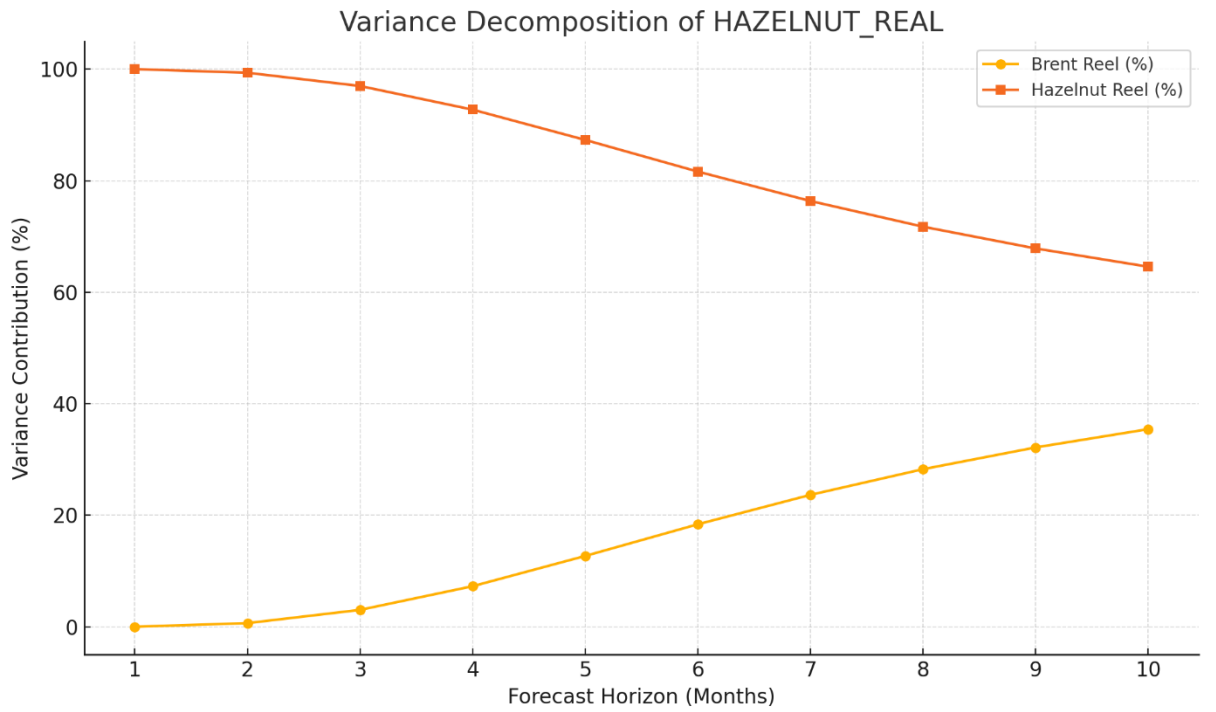

Figure 2. Forecast Error Variance Decomposition

The variance decomposition graph above demonstrates how the influence of real Brent oil price shocks on real hazelnut prices evolves over a 10-month forecast horizon. The Y-axis shows the percentage of the forecast error variance in HAZELNUT_REAL explained by BRENT_REAL and its own lags, while the X-axis represents the time horizon in months.

In the first period, Brent oil prices explain an almost negligible portion of the variance in hazelnut prices. However, this contribution increases significantly over time, reaching 12.7% by the 5th month and 35.4% by the 10th month. Meanwhile, the share of the variance explained by HAZELNUT_REAL's own past values gradually declines, although it remains dominant overall.

This finding indicates that fluctuations in global energy markets gradually exert a stronger explanatory power over agricultural commodity prices. By the 10th month, over one-third of the uncertainty in hazelnut prices can be attributed to shocks in Brent oil prices. These results underscore the growing external influence of energy markets on agricultural pricing dynamics, particularly in the medium term.

DISCUSSION AND CONCLUSION

The analyses conducted in this study reveal that macroeconomic indicators such as energy prices and exchange rates have significant effects on hazelnut prices in Türkiye. The Granger causality test confirms that changes in Brent oil prices significantly influence hazelnut prices. Additionally, impulse response analysis shows that this impact is negative and persistent in the medium term. Variance decomposition results indicate that Brent oil prices account for more than 35% of the forecast error variance in hazelnut prices by the tenth month.

These findings are consistent with the results of Nazlioglu et al. (2013), who reported volatility spillovers from energy prices to agricultural commodity markets. Similarly, Baffes (2007) emphasized that oil price shocks transmit to agricultural product prices with a lag, which can be attributed to the energy-dependent structure of agricultural production systems.

Serra et al. (2011) also found that energy and biofuel markets play a significant role in shaping food price dynamics. The negative response observed in this study may reflect a supply-side pressure, where increased production costs resulting from energy shocks lead to downward pressure on agricultural prices.

The results are also in line with the findings of Erdal and Uzunöz (2008), who analyzed the relationship between exchange rates and hazelnut prices in both Türkiye and Europe. Their study revealed a unidirectional Granger causality from exchange rates to hazelnut prices, highlighting the role of exchange rate fluctuations in determining export prices.

Furthermore, the study by Aşkan et al. (2022) using a VECM-BEKK MGARCH framework demonstrated that volatility in fuel prices and exchange rates exerts both direct and spillover effects on hazelnut prices in Türkiye. They emphasized that negative shocks in these macroeconomic indicators generate persistent uncertainty and long-term effects in the hazelnut market.

Taken together, these results suggest that fluctuations in energy and exchange rate markets act as significant external and often lagged determinants of agricultural price dynamics in emerging economies such as Türkiye. For high-export commodities like hazelnuts, such externalities should be carefully considered by policymakers when designing strategies to stabilize agricultural markets.

This study examined the causal relationship between real Brent oil prices, exchange rates, and real hazelnut prices in Türkiye using monthly data from 2016M01 to 2025M03. The results of the Granger causality test show that real Brent oil prices have a statistically significant effect on hazelnut prices, while no significant reverse causality was found. This indicates that energy markets act as a unidirectional and external determinant of agricultural commodity prices.

The impulse response analysis reveals that shocks in Brent oil prices create a negative and persistent impact on hazelnut prices, while the variance decomposition results show that by the 10th month, Brent oil prices account for over 35% of the forecast error variance in hazelnut prices. These findings suggest that energy cost shocks affect not only agricultural production inputs but also market pricing with a noticeable lag. For export-dependent products such as hazelnuts, the relationship between global energy volatility and price instability becomes even more pronounced.

From an economic standpoint, rising energy prices increase agricultural production and logistics costs, exerting upward pressure on the cost side while potentially leading to downward price rigidity due to weakened producer margins. Exchange rate volatility simultaneously influences export revenues and pricing behaviors, threatening overall price stability. This dual vulnerability of agricultural markets to external shocks is especially evident in sectors integrated into global trade dynamics.

At the policy level, these findings highlight the structural sensitivity of agriculture to macroeconomic fluctuations, particularly in energy and exchange rate markets. Therefore, agricultural policy must extend beyond conventional production supports to include institutional and financial mechanisms that enhance price stability. These may include:

- Energy input price stabilization and targeted subsidies,
- Exchange rate protection instruments for exporters,
- Expansion of agricultural insurance systems to cover exchange rate and fuel price shocks.

In conclusion, this study presents a micro-level example of the agriculture–energy–economy nexus, emphasizing that close monitoring of energy and currency markets is essential for food security, income stability, and agricultural sustainability. The evidence underscores the importance of integrating macroeconomic risk management tools into agricultural policy frameworks.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

Both authors contributed equally to this work (50% – 50%).

Residential Rental Market in Albania: A Case Study of Tirana (1273)

Ilirjana Kraja[1]*, Doriana Matraku[2], Abdulmenaf Sejdini[3]

[1] University of Tirana, Faculty of Economics, Department of Economics, Albania
[2] University of Tirana, Faculty of Economics, Department of Economics, Albania
[3] Higher Colleges of Technology, UAE,

*Corresponding author e-mail: ikraja@instat.gov.al

Abstract

The residential rental market in Albania, particularly within urban areas such as Tirana, has experienced substantial transformation in recent years. This shift can be attributed to various demographic and economic factors, including a notable influx of students, professionals, workers, etc. seeking housing. The growing preference for urban living is driven by the desire for proximity to educational institutions and employment opportunities. Consequently, there has been a marked increase in demand for housing rentals, which encompasses a range of options from different type of apartments and houses.

This research examines the key determinants influencing the residential rental market in Albania, employing a mixed-methods approach that integrates quantitative data analysis with qualitative insights. The paper aims to identify the factors influencing the house rental market in Albania, with a specific focus on the city of Tirana, where the majority of rental transactions occur. Utilizing data collected from a survey conducted within local real estate agencies, the study analyzes key determinants of rental prices.

Keywords: *Real estate, Residential rental market, Rental prices, Demographic trends.*

INTRODUCTION

The rental housing market in Albania has undergone significant and complex changes over the past few decades, influenced by a range of economic, social, and demographic shifts. These transformations are rooted in the country's transition from a centrally planned economy to a market-oriented system, a process that began in the early 1990s. This transition marked a pivotal moment in Albania's economic development, triggering structural shifts not only in the overall economy, but also in key sectors such as housing. The rapid urbanization that accompanied these changes has had a profound impact on the demand for rental properties, particularly in major urban centres like Tirana, Durrës, and Vlorë, where population growth has been most pronounced. As cities grow, demand for housing increases, which puts upward pressure on rents. Glaeser et al. (2005) and Saiz (2010) argue that population growth in metropolitan areas leads to higher rental prices due to the scarcity of land and housing supply constraints, especially in cities with high levels of in-migration. Changes in demography, such as changes in household formation, significantly impact rental prices. A study by Green, Malpezzi, and Mayo (2005) found that increases in household formation can drive up demand for rental units.

As more people migrate from rural areas to cities in search of better economic opportunities and education, the demand for housing in urban areas has risen significantly. At the same time, Albania has seen increased foreign investment, particularly in real estate and tourism, which has further strained the housing market. Cities like Tirana, the capital, have become key destinations for both local and international investment, driving up rental prices as demand outpaces supply. Additionally, the growth of the tourism sector has led to a rise in short-term rentals, which has further inflated prices and made long-term housing less affordable

for local residents. Research by Zervas, Proserpio, and Byers (2017) demonstrated that the growth of Airbnb listings in major tourist cities such as New York, Paris, and London led to significant increases in nightly rates for short-term rentals. As more homeowners converted traditional rental units into short-term vacation rentals, prices were driven up by the heightened demand during peak tourism periods.

The combination of these factors has led to rising rental prices, particularly in the most desirable urban areas. As a result, renting a home in Albania has become more expensive, especially for lower-income residents, while property owners and investors are benefiting. This growing pressure on both housing supply and affordability is putting strain on the housing market, creating challenges for policymakers, urban planners, and residents. Understanding what is driving these price increases is essential for creating solutions that will lead to a more balanced and sustainable housing market in the future. Rental prices tend to rise as household incomes increase, particularly in growing urban centers where economic opportunities attract people. However, rising incomes do not always lead to an equal increase in rent affordability, especially in markets where demand outpaces supply (Green, Malpezzi, & Mayo, 2005)

The main objective of this paper is to identify and analyze the key factors that have influenced the demand for and the price of rental housing in Albania. This analysis will focus on both domestic and external factors that have contributed to changes in the rental market, particularly in urban areas like Tirana.

LITERATURE REVIEW

Rental prices for houses are influenced by a complex interplay of factors spanning economic, social, and geographical dimensions. These factors can be broadly categorized into housing market dynamics, macroeconomic conditions, and specific property characteristics. Understanding these influences is crucial for both tenants and landlords navigating the housing market.

The fundamental economic principle of supply and demand significantly impacts rental prices. An increase in the rental housing supply can exert downward pressure on house prices (Li et al., 2022), while a decrease can lead to increased rental rates. Conversely, higher demand for rental properties, driven by factors like population growth or increased urbanization, can drive prices up (Mao, 2024)

The size and development of the rental market itself play a crucial role. A developed rental market can mitigate fluctuations in the housing sector (Czerniak & Rubaszek, 2018), suggesting that a larger, more diverse rental market can offer stability and potentially more competitive pricing. In China, the influence of the housing rental market on economic operations is a significant concern, and the rental market has been undergoing a transition from being disregarded to being valued (Kong & Dong, 2024)

The presence of short-term rental platforms like Airbnb can impact housing prices and liquidity (Alvarez & Pennington-Cross, 2022). For instance, one study found that a doubling of Airbnb properties in a neighborhood can increase housing prices by approximately 11% (Alvarez & Pennington-Cross, 2022). This effect is heterogeneous, being more pronounced in less dense areas (Alvarez & Pennington-Cross, 2022).

Rent prices are strongly linked to economic factors and the structural and environmental characteristics of housing stock (Subaşı & Baycan, 2022). Unexpected events, such as wars and epidemics like COVID-19, can significantly impact urban housing markets by affecting the economy (Subaşı & Baycan, 2022).

The characteristics of a rental property, such as its size, number of rooms, and overall condition, significantly affect rental prices (Ezebilo & Ezebilo, 2017). Studies using multi-linear regression have analyzed these factors based on large datasets of rental listings (Mao, 2024)

Location is a critical determinant of rental prices, with properties in more desirable areas commanding higher rents (Mao, 2024). Factors such as proximity to employment centers, schools, amenities, and

transportation hubs can all influence rental rates. The distance from the city center also plays a crucial role, with closer proximity often correlating with higher rental prices (Pormon et al., 2023).

RENTAL HOUSING MARKET IN ALBANIA

The Albanian rental housing market, with a particular emphasis on the capital city Tirana, has experienced considerable fluctuations in recent years. This evolution is driven by multiple demand-side and supply-side factors, notably the influence of short-term rental platforms, foreign professionals, internal migration patterns, etc.

Based on the last Census of 2023 population of Albania is 2.402.113 inhabitants. Approximately, 25 % of the population lives in Tirana and 79 % of population owns their dwellings. The population of Tirana in the 2001 Census was around 11%, and in the 2011 Census it reached approximately 15%, indicating a migratory movement of the population toward Tirana. The structure of Albanian families has undergone notable changes in recent decades, leading to an increased demand for housing. According to the results of Census 2023, the average household size has decreased to 3.2 persons. This is a continuation of a long-term trend, as the average was 3.9 persons per household in Census 2011 and 4.2 in Census 2001. The move toward smaller household units driven by social, economic, and demographic shifts means that more housing units are now required to accommodate the population. As a result, this transformation has contributed significantly to the rising demand for residential properties, particularly in urban areas.

According to Consumer Price Index (CPI) data from the Albanian Institute of Statistics (INSTAT), the division 04 "Housing, water, electricity, gas and other fuels" have the weight around 20% in the total basket of this index. This large weight obviously demonstrates how much of the household's budget spent ward on housing services is. As such, it has an impact on the overall inflation rate, which highlights the significant share of housing and utilities in the cost of living for Albanian families.

The figure 1 shows the Rent Price Index for whole Albania with a significant upward trend in the last periods. The continued rise is evidence of the increasing burden housing costs, and especially rental costs, put on household wallets. The increasing share of the household budget occupied by rent long-stood as an indicator of the household budget constraint and its relationship to the overall price of living or cost of living, particularly in urban areas where housing is in great demand and the supply remains constrained.

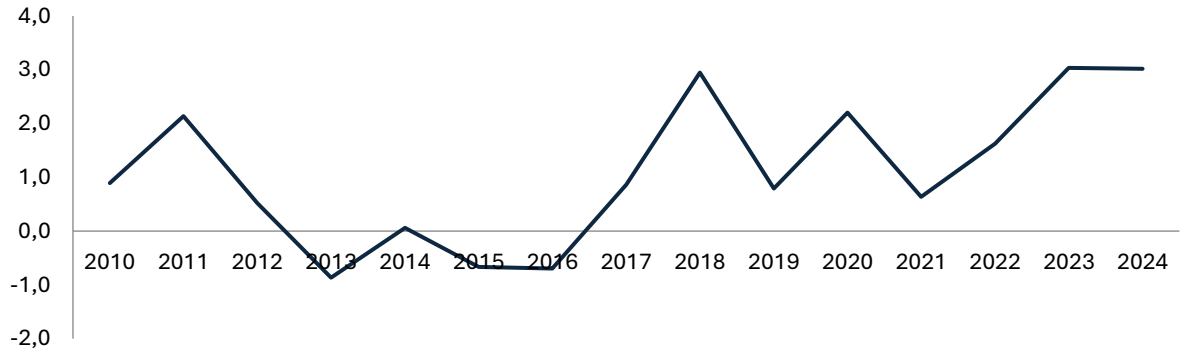

Figure 1. Rent Price Index

The significant rise in tourism in Albania over recent years has drawn increasing attention from foreign visitors. Many of them, impressed by the natural beauty, low cost of living, and welcoming culture, have shown interest in renting or purchasing property, especially in coastal and touristic areas such as Saranda, Vlora, and Tirana. This growing foreign interest has directly influenced the local housing market, leading to a noticeable increase in the demand for rental homes. Both short-term rentals for holiday stays and long-term rentals for those considering relocation have become more competitive. As a result, rental prices have

risen in many areas, especially during the high tourist season, creating new opportunities for property owners and investors. The expansion of short-term rental platforms such as Airbnb has led many property owners to shift their houses from long-term rentals to short-term leases, especially in touristic zones. This shift has reduced the availability of rental housing for residents, further driving up long-term rental prices.

Tirana is the city with the highest number of public and private universities in the country, as well as vocational secondary schools, attracting a large student population each academic year. The influx of students creates seasonal pressure on the rental market, particularly for affordable, shared housing.

Overall economic conditions in Albania, including GDP growth, wage levels and employment opportunities, have a significant impact on the rental housing market. Tirana, as the country's economic engine, continues to attract individuals from all over Albania seeking better job prospects and living standards. This migration from rural to urban intensifies housing pressure in the capital, where supply often lags behind demand. In addition, sectors such as services, administration, IT and construction largely concentrated in Tirana play a key role in driving internal migration and reshaping the dynamics of urban rental housing. As more individuals and families move to the city for work, the demand for affordable and accessible rental housing increases, contributing to rising prices and decreasing availability.

METHODOLOGY AND RESULTS

This section provides an evidence-based overview of recent market developments, based on field survey data and complemented by qualitative insights from local estate agents. The methodology used to analyze the factors influencing residential rental prices in Albania involved collecting data from real estate agencies. These agencies were asked to report rental prices for various types of residential properties, including apartments, houses, and other dwelling types. Rental prices were further categorized according to key property characteristics such as location (zone)[3], size, property type, furnished or unfurnished status, year of construction, floor, number of rooms, and number of bathrooms. Data collection occurred over a seven-year period, from 2017 to 2023, capturing key fluctuations in rental prices and demand patterns. The data are collected only for city of Tirana, where the majority of rental transactions occur. The numbers of observations are 750. Also, estate agencies have shared their perspectives on the future of the rental market, offering insights into potential trends and challenges that could shape the industry in the coming months and years. According to their reports, several key factors are expected to influence rental demand, pricing, and availability.

According to agents, some owners prefer to wait for the right tenant instead of lowering the asking price; they trust that demand will grow in the coming weeks and that they will be able to make a deal at "their" price. Survey findings reveal a marked upward trend in rents for all types of apartments between 2024 and 2025, while rents for detached houses remained relatively stable, registering a slight decrease of -1%. Two-bedroom apartments demonstrated the highest rental growth (+21%), followed by three-bedroom (+16%) and one-bedroom (+10%) units. This sharp rise in apartment rents is attributed to a triad of factors:

- High general demand, partly fueled by the influx of professionals in the IT and medical sectors.
- A significant contraction in long-term rental supply, as a growing number of properties are diverted to the short-term rental market (e.g., Airbnb).
- An increase in the share of newly constructed, higher-quality dwellings, often developed in collaboration with international architectural firms.
- İncrease of the tourism has become a key driver of housing demand in Albania, reshaping the real

[3] Zone 2/1 (Rruga e Elbasanit), Zone 2/2, Zone 5/1 (Blloku), Zone 5/2, Zone 7/1, Zone 7/2, Zone 10/3, Zone Selitë, Zone Kodra e Diellit, Zone Sauk, and Zone Komuna e Parisit

estate market and contributing to the country's economic growth.

These dynamics indicate a structural shift in the rental housing stock, where newly built properties dominate, often with elevated standards and pricing.

For analysing of the data the O'Sullivan (2006) model is applied. The O'Sullivan model applies a log-linear approach to analyze spatial variation in housing prices in Ireland.

Hypothesis for this analyse are:

- H_0 (null): The characteristics do not have effect in rent prices
- H_a (alternative): The characteristics (some or all) have effect in rent prices

OLS regression model has this form (Log-linear model):

$$\ln Price_{it}=\beta_0+\beta_i zone_t+ \beta_i size_t + \beta_i Prop.type_t + \beta_i No.Room_t + \beta_i NoBath_t + \beta_i Furnished_t + \beta_i yearC_t + \beta_i floor_t +u_i$$

where: lnPrice-rent price of the dwellings, zone-(are included 11 zones of Tirana considers as main areas from real estate's agents), No. Room-number of rooms, NoBath-numer of toilets, Furnished-if dwelling is furnished or not, yearC-year of construction and floor.

Table 1. Model summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	0.794	0.63	0.625	0.423

As shown in table 1 the model is explained by 63 %. So, 63% of the variance in the log of the rent price is explained by the variables included in the model.

Table 2. ANOVA table

Model	Sum of Squares	df	Mean Square	F	Sig.
Regression	125.782	8	15.723	87.593	0.0001
Residual	73.601	741	0.099		
Total	199.383	749			

The ANOVA table indicates the statistical significance of the regression model under study. In this case, $p < 0.0001$, which is less than 0.01, indicating that the regression model as a whole statistically significantly predicts the dependent variable (rent price).

Table 3. Ordinary least squares (OLS) results for factors influencing house rent price.

Variables	B	t	Sig.
Constant	2.112	10.891	0.000
Size (m²)	0.01	5.793	0.000
PT1	0.092	2.706	0.007
PT2	-0.134	-4.061	0.000
No. of Rooms	0.068	4.250	0.000
No. of Bathrooms	0.091	4.550	0.000

Furnished (1=Yes)	0.178	6.846	0.001
Year of Construction	0.004	3.008	0.001
Floor	0.017	4.250	0.000
Z2/1	-0.243	-6.231	0.000
Z2/1	-0.187	-4.921	0.000
Z2/2	-0.021	-0.583	0.560
Z5/1	0.066	1.783	0.075
Z5/2	-0.165	-4.125	0.000
Z7/1	-0.098	-2.579	0.010
Z7/2	0.142	3.463	0.001
Z10/3	-0.201	-5.289	0.002
Zone Kodra e Diellit	-0.231	-6.243	0.000
Zone Sauk	-0.189	-4.846	0.001

Size (m²): The coefficient of 0.01 ($p < 0.001$) suggests that a 1 m² increase in apartment size is associated with approximately a 1% increase in rental price, holding other variables constant. This aligns with theoretical expectations, confirming size as a primary price determinant.

Property Type[4] (PT1 and PT2): PT1 has a positive and significant effect ($B = 0.092$, $p = 0.007$), while PT2 is negatively associated with rental price ($B = -0.134$, $p < 0.001$). These results imply heterogeneity in rental value depending on property classification, with PT1 (e.g., premium types) commanding higher rents than the base category, and PT2 (e.g., less desirable units) attracting discounts.

Number of Rooms and Bathrooms: Both variables have positive and statistically significant effects. Each additional room increases the rent by 6.8%, and each additional bathroom increases it by 9.1%, respectively. This highlights that tenants value both spaciousness and utility features when making rental decisions.

Furnished (1 = Yes): A furnished apartment is associated with 17.8% higher rent compared to an unfurnished one ($p = 0.001$), reflecting tenant preferences for move-in-ready units.

Year of Construction: The coefficient ($B = 0.004$, $p = 0.001$) indicates that newer apartments tend to have slightly higher rents, consistent with the desirability of modern construction and amenities.

Floor Level: The positive coefficient ($B = 0.017$, $p < 0.001$) suggests that higher floors yield higher rents, potentially due to better views, noise insulation, or perceived status.

The model includes several zone-based dummy variables to capture spatial heterogeneity in rent values. These coefficients are interpreted relative to a base (reference) zone:

Zones with Significantly Lower Rent: Z2/1 ($B = -0.243$ and -0.187): Appears duplicated; both coefficients show significant rent reductions.

Z5/2, Z7/1, Z10/3, Kodra e Diellit, and Sauk all have negative and significant coefficients, indicating they are less preferred locations compared to the base.

For example, Kodra e Diellit shows a reduction of about 23.1% in rent relative to the reference zone.

Zones with Higher Rent: Z7/2 has a positive and significant coefficient ($B = 0.142$, $p = 0.001$), indicating it is more desirable.

Insignificant Zones: Z2/2 and Z5/1 are not statistically significant, suggesting their rent levels do not differ meaningfully from the base category.

[4] PT1-apartments and PT2-houses

CONCLUSIONS

Rental prices and property values are tightly interlinked: as property values rise, more households are pushed into renting, further boosting rental demand. The rental price is influenced by most of the characteristics of the dwelling.

This study provides an in-depth analysis of the residential rental market in Tirana, Albania, highlighting the main factors driving rental price fluctuations. The findings confirm that rental prices are significantly influenced by property characteristics such as size, number of rooms and bathrooms, furnishing status, year of construction, floor level, and location. The regression model explains a substantial portion (63%) of the variation in rental prices, demonstrating strong predictive power.

The rapid urbanization, demographic shifts, and economic migration toward Tirana have intensified housing demand, leading to a sharp increase in rents especially for apartments. Short-term rental platforms and increased foreign interest in real estate have further constrained the availability of long-term rentals, pushing prices even higher.

Results show a structural transformation in the rental market, with a growing preference for modern, well-equipped dwellings. While landlords and investors benefit from these trends, the rising rental prices pose challenges to affordability, particularly for lower-income residents.

In summary, the study underscores the importance of comprehensive housing policies and urban planning strategies to balance growth, investment, and affordability in Albania's rental housing market.

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Conflict of Interest

If there is no conflict of interest of the authors, it should be written as "The authors have declared that there is no conflict of interest".

Author Contributions

All authors worked jointly on all stages of this study.

Performance Review of OECD Countries' Merchant Fleets (1282)

Maruf Gögebakan [1*], Tarık Erdoğmuş[1]

[1]Bandırma Onyedi Eylül University, Maritime Faculty, Department of Maritime Business Administration, Turkiye

*Corresponding author e-mail: mgogebakan@bandirma.edu.tr

Abstract

Countries' ship fleets play a crucial role in foreign trade via maritime routes. Countries with larger and newer fleets capture a greater share of maritime trade, contributing to their economies. A country's fleet directly affects both international trade and logistics efficiency. This study evaluates the trade fleet performance of OECD countries between 2014-2024 using multi-criteria decision-making (MCDM) methods, considering three variables: the number of ships, total carrying capacity (DWT), and the average age of ships. The number of ships represents the fleet size, while carrying capacity reflects the fleet's cargo transport potential, and the average age indicates the fleet's modernity. The importance weights of these variables were determined using the Entropy method, and fleet performances were ranked using the TOPSIS and VIKOR methods. These rankings were combined with the Borda method to produce the final performance ranking. Greece ranked highest due to its large fleet and high carrying capacity. Japan and Germany followed with large fleets and modern ships. South Korea, the United States, and the United Kingdom also showed strong performance, with high carrying capacity and low average ship age. Denmark, Turkey, and Switzerland were in the middle ranks, while the Netherlands, Belgium, and Italy ranked lower. Countries with lower performance tended to have smaller, older, and lower-capacity fleets, which negatively impacted trade and logistics efficiency. In conclusion, fleet performance differences are primarily influenced by carrying capacity, fleet size, and the average age of ships. Fleets with higher capacity, newer ships, and larger sizes demonstrated better performance.

Keywords: *Marıtıme Fleet Performance, Mcdm, Entropy, Topsıs, Vıkor*

INTRODUCTION

Over 80% of global trade is carried out through maritime transportation; this mode of transport stands out due to advantages such as economies of scale, low cost, reliability, and continuity (UNCTAD, 2024; Balık, Aksay, & Şenbursa, 2015). Countries seek to play an active role in the modern maritime market through their owned ships and maritime transport enterprises, which in turn intensifies competition among nations (Öçal, 2023). In this context, one of the most important factors determining a country’s effectiveness in global maritime trade is its merchant fleet. Studies evaluating fleet performance generally focus on multiple variables such as fleet size, age structure, tonnage, ship types, technological level, and maritime policies.

Akgül and colleagues examined the age, tonnage, technological level, and financial constraints of the Turkish fleet, highlighting the aging fleet structure and technological deficiencies (Akgul, Akdamar, & Gögebakan, 2024). Bayraktar evaluated the types, numbers, tonnage values, and aging of the Turkish fleet, emphasizing the importance of fleet renewal strategies (Bayraktar, 2022). Lee and Kim analyzed the performance of ship management companies in South Korea and linked it to the national fleet size, noting the positive impact of fleet size on growth (Lee & Kim, 2024). Balık and colleagues analyzed the current status and potential of the maritime transport sector in Turkey, stressing the importance of fleet renewal and modernization (Balık, Aksay, & Şenbursa, 2015). Park et al. investigated the economic, social, and sectoral impacts of South Korea’s Jeju ship registration system, demonstrating its contributions to fleet tonnage and employment (Park, Kim, Ryu, Kim, & Kwon, 2022). Öçal compared Turkey’s maritime trade

status with that of the G8 countries, evaluating Turkey's number of ships and port performance (Öçal, 2023). Nguyen analyzed country-based factors affecting the size of national maritime fleets, including costs, shipbuilding, maritime history, and financial systems. Murat Çiftçi examined the importance of Turkey's merchant fleet as a national asset and its international position according to income levels (Nguyen, 2011). Yang studied the shrinkage of the Taiwanese fleet and its effects, providing policy recommendations (Yang, 2014). Zhang and Drumm analyzed Germany's maritime sector and fleet structure (Zhang & Drumm, 2020). Puscaciu and colleagues investigated the structural characteristics and dynamic trends of the world merchant fleet (Puscaciu, Viorica, & Puscaciu, 2015). Naletina and Perkov examined the maritime transport sector in Croatia and compared it with the global situation (Naletina & Perkov, 2017).

This study aims to analyze the performance of the merchant fleets of OECD countries and provide insights into their maritime performance. Using the criteria of the number of ships, carrying capacity (DWT), and average ship age, the performance of the fleets was evaluated through multi-criteria decision-making methods. In this context, the objective was to reveal the performance differences among countries based on the obtained rankings.

MATERIALS and METHODS

Materials

In this study, the performance of the merchant fleets of OECD member countries was evaluated. The analysis used data for the period 2014–2024 obtained from the UNCTADStat (2024) database based on beneficial ownership. However, due to missing data for some countries, only 35 countries with complete data were included in the analysis. The Czech Republic, Hungary, and Slovakia were excluded from the evaluation due to insufficient data.

In the performance evaluation, each country was considered as an alternative, and the factors determining performance were treated as variables. The merchant fleets of the countries were represented by the following three variables:

- Number of Ships (X_1): Refers to the total number of commercial vessels owned by the country.
- Average Ship Age (X_2): An indicator reflecting the dynamism and modernity level of the fleet.
- Total DWT (X_3): Represents the country's total cargo-carrying capacity.

The decision matrix constructed using these variables served as the primary data set for the performance analysis of the countries' merchant fleets. For each variable, the annual values for the period 2014–2024 were averaged, and the analyses were conducted based on these 11-year average values.

Entropy method, one of the multi-criteria decision-making (MCDM) techniques, was used to objectively determine the relative importance weights of the variables. Subsequently, the performance rankings of the countries were conducted using the TOPSIS and VIKOR methods. The ranking results from both methods were then combined using the BORDA count method to obtain the final ranking.

Methods

ENTROPY Method

ENTROPY method, an analytical weighting technique, was used to determine the changes in the number of ships, ship age, and DWT variables of countries over the years. It is a method developed by Shannon to analytically measure the disorder within a system (Shannon, 1948). The analytical weighting steps of the Entropy method are presented below in Algorithm 1.

Algorithm 1. Entropy analitical weighted method	
1.Step: Decision Matrix,	$A = [a_{ij}]_{mxn}$, where a_{ij}correspond to each element of criteria.
2.Step: Normalized Matrix	$r_{ij} = a_{ij} / \sqrt{\sum_{i=1}^{m} a_{ij}^2}$, where r_{ij} denotes the elements of the normalized matrix $N = [r_{ij}]_{mxn}$.
3.Step: Entropy values	$e_j = -h \sum_{i=1}^{m} a_{ij}\, ln(a_{ij})$, It is calculated as $h = \frac{1}{ln(m)}$, where m is the number of criteria and h is the entropy constant.
4.Step: Entropy weight vector	$w_j = \frac{1-e_j}{\sum_{j=1}^{n}(1-e_{ij})}$, where d_j is called the entropy information difference and is obtained by $d_j = 1 - e_j$.

Multi-Criteria Decision Making Methods

Multi-Criteria Decision-Making (MCDM) methods allow for the comparison, evaluation, and prioritization of alternatives in situations involving multiple criteria and options (Bayram & Eren, 2023). In this study, countries are considered as the alternatives, while DWT, the number of ships, and the average ship age are treated as the criteria.

The decision matrix consists of the average values of each performance criterion for the countries. In the decision matrix, the rows represent the alternatives (classification methods), while the columns represent the criteria (performance measures).

$$D = \begin{bmatrix} d_{11} & \cdots & d_{1m} \\ \vdots & \ddots & \vdots \\ d_{n1} & \cdots & d_{nm} \end{bmatrix}$$

TOPSIS Method

The TOPSIS method is an approach used to select the most suitable alternative by identifying the one that is closest to the ideal solution and farthest from the negative ideal solution (Tzeng & Huang, 2011).

TOPSIS Application Steps

Algorithm 2. TOPSIS sorting method	
1. Step: Decision Matrix	$A = [a_{ij}]_{mxn}$, where a_{ij}correspond to each element of criteria
2. Step: Normalized Matrix	$r_{ij} = a_{ij} / \sqrt{\sum_{i=1}^{m} a_{ij}^2}$, bura r_{ij} denotes the elements of the normalized matrix $N = [r_{ij}]_{mxn}$.
3.Step: Weighted Normalize Matrix	It consists of the elements $v_{ij} = w_j r_{ij}$ and is denoted as $V = [v_{ij}]_{mxn}$, where $\sum_{j=1}^{n} w_j = 1$ and $r_{ij} \in N$.
4. Step: Ideal and Non-Ideal Solutions	Ideal and negative ideal separation measures are calculated with weighted matrix elements as $S_i^+ = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^+)^2}$ and $S_i^- = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^-)^2}$, respectively.
5. Step: Similarity to Ideal Solution measure	The optimum solution is determined by the $C_i^+ = S_i^- / (S_i^- + S_i^+)$ ratio, where $0 \le C_i^+ \le 1$.

VIKOR Method

VIKOR method is a technique developed by Opricovic and Tzeng for solving multi-criteria decision-making problems in complex systems involving conflicting criteria (Opricovic & Tzeng, 2007).

VIKOR Application Steps

Algorithm 3. VIKOR Sorting Method

1.Step: Decision Matrix	$A = [a_{ij}]_{mxn}$, where a_{ij}, corresponds to each element of the criteria.
2.Step: Determination of the Best f_{i^*} and Worst f_j^- Values	The best f_{i^*} and worst f_i–values for all criterion functions must be determined. For benefit criteria: $f_j^* = \max_i X_{ij}$ ve $f_j^- = \min_i X_{ij}$ For cost criteria: $f_j^* = \min_i X_{ij}$, $where$ $j = 1,2 \ldots n$ indicating that the maximum and minimum values of the criteria are selected accordingly.
3.Step: Calculation of Normalized Values	$r_{ij} = \frac{f_j^* - X_{ij}}{f_j^* - f_j^-}$ (benefit criteria) $r_{ij} = \frac{X_{ij} - f_j^*}{f_j^- - f_j^*}$ (cost criteria) Here r_{ij}, is the normalized distance value.
4.Step: Calculation of Total and Maximum Distances	$S_i = \sqrt{\sum_{j=1}^{m} w_j \,.\, r_{ij}}$ $R_i = max_j(\, w_j \,.\, r_{ij})$ where w_j $is\ the\ weight\ of\ the\ jth$ criterion.
5.Step: Calculation of the VIKOR Compromise Measure	$Q_{i= v \,.\, \frac{S_i - S^*}{S^- - S^*} + (1-v) \,.\, \frac{R_i - R^*}{R^- - R^*}}$ $S^* = min_i\, S_i, S^- = mak_i\, S_i$ $R^* = min_i\, R_i, R^- = mak_i\, R_i$ Typically, $v = 0.5$ is used.
6.Step: Ranking of the Alternatives	The alternatives are ranked in ascending order based on their Q_i values. The smallest Q_i indicates the best alternative.
7.Step: Checking the Acceptability Conditions	In order for the best alternative to be selected as the sole solution, the following two conditions must be satisfied: Condition 1: $Q(A_2) - Q(A_1) \geq \frac{1}{m-1}$ Condition 2: A_1 must be ranked first in both the S and R rankings.

Borda Count Method

Borda Count method is a data aggregation technique that allows the integration of multiple ranking lists into a single, more reliable ranking. During the application of this method, each alternative is scored based

on preferences. The least preferred alternative is assigned zero (0) points, while the other alternatives receive increasing points according to their preference order, with the most preferred alternative receiving the maximum score of $n-1$. Here n represents the total number of alternatives. The scores received by each alternative are summed to calculate the Borda score for that alternative. Based on these scores, an overall ranking is established, and the alternative with the highest Borda score is considered the most advantageous or the best option (Meşe & Özdemir, 2022).

FINDINGS

ENTROPY Analysis of Decision Variables

Table 1. Variable Weights Using the ENTROPY Method

Variables	wj
DWT (X1)	0,578572843
Number of Ships (X2)	0,402804521
Average Ship Age (X3)	0,018622636

When examining the weights obtained using the ENTROPY method, the DWT (X1) variable has the highest weight of 0.5785, making it the most decisive factor in the decision-making process due to having the greatest information variation among the measured alternatives. The Ship Number (X2) variable ranks second with a weight of 0.4028, providing a significant contribution but playing a less influential role compared to DWT. On the other hand, the Average Ship Age (X3) variable has a very low weight of only 0.0186, indicating that it does not exhibit meaningful variation among the alternatives and therefore contributes very little to the decision-making process.

TOPSIS Sorting

Table 2. TOPSIS Sorting

Rank	Country	C_i^+
1	Greece	0,99999
2	Japan	0,86279
3	Germany	0,25674
4	Republic of Korea	0,12572
5	United States	0,09062
6	Norway	0,08909
7	United Kingdom	0,04993
8	Turkiye	0,04411
9	Denmark	0,02517
10	Netherlands (Kingdom of the)	0,02145
11	Italy	0,00866
12	Switzerland	0,00837
13	Belgium	0,00605
14	France	0,00359
15	Canada	0,00222
…	…	…
34	Iceland	0,00001
35	Slovenia	0,00000

According to the performance scores obtained using the TOPSIS method, Greece stands out as the country with the highest performance by a wide margin, achieving an almost perfect score of 0.99999. This indicates

that Greece holds a very strong position compared to other countries in terms of the size and efficiency of its maritime trade fleet. Japan ranks second with a score of 0.86279, while Germany is third with a score of 0.25674. These countries are notable for their high carrying capacity and advanced maritime infrastructure. Countries positioned in the middle and lower parts of the ranking exhibit relatively lower performance. Countries at the bottom of the list, such as Iceland (0.00001) and Slovenia (0.00000), have scores that reveal their maritime trade fleets are quite limited and insufficient according to the evaluation criteria. Overall, the results demonstrate significant differences in maritime performance among countries, showing that only a few possess fleet structures that are competitive on a global scale.

3. VIKOR Sorting

Table 3. VIKOR Sorting

Rank	Country	Q_i
1	Greece	0,00000
2	Japan	0,15695
3	Germany	0,38815
4	Republic of Korea	0,47719
5	United States	0,64117
6	Norway	0,65196
7	United Kingdom	0,67672
8	Denmark	0,74762
9	Turkiye	0,80326
10	Switzerland	0,82247
11	Belgium	0,83460
12	Netherlands (Kingdom of the)	0,87877
13	Italy	0,88097
14	France	0,91007
15	Canada	0,94075
	…	…
34	Iceland	0,99742
35	Slovenia	0,99872

As a result of the evaluation conducted using the VIKOR method, Greece ranks first as the country with the best performance, standing out with its balanced structure and proximity to the ideal solution. Japan and Germany also rank among the top-performing countries, holding strong positions in the maritime sector. Countries such as South Korea, the United States, and Norway are positioned in the middle ranks, indicating that while they have relatively strong maritime structures, they remain somewhat distant from the ideal solution. At the lower end of the list, Iceland and Slovenia draw attention with their low and unbalanced performance, reflecting a significantly weaker position in terms of merchant fleet compared to other countries. These results reveal substantial structural and performance differences between countries in the maritime domain.

Final Ranking Based on the Borda Count Method

Table 4. Final Sorting Obtained by the Borda Count Method

Country	**TOPSIS Rank**	**Borda Score (T)**	**VIKOR Rank**	**Borda Score (V)**	**Total Score**	**Final Rank**
Greece	1	34	1	34	68	1
Japan	2	33	2	33	66	2
Germany	3	32	3	32	64	3

South Korea	4	31	4	31	62	4
Unidet States	5	30	5	30	60	5
Norway	6	29	6	29	58	6
Unidet Kingdom	7	28	7	28	56	7
Turkiye	8	27	9	26	53	8
Denmark	9	26	8	27	53	8
Netherlands	10	25	12	23	48	10
Switzerland	12	23	10	25	48	10
Italy	11	24	13	22	46	12
Belgium	13	22	11	24	46	12
France	14	21	14	21	42	14
Canada	15	20	15	20	40	15
Sweden	16	19	16	19	38	16
Australia	18	17	17	18	35	17
Spain	17	18	19	16	34	18
Luksembourg	23	12	18	17	29	19
Mexico	19	16	23	12	28	20
Poland	21	14	22	13	27	21
Ireland	22	13	21	14	27	21
Israel	24	11	20	15	26	23
Finland	20	15	26	9	24	24
Chile	25	10	24	11	21	25
Portugal	27	8	25	10	18	26
Estonia	26	9	29	6	15	27
Latvia	28	7	27	8	15	27
Austria	30	5	28	7	12	29
Lithuania	29	6	32	3	9	30
Colombia	31	4	31	4	8	31
New Zealand	32	3	30	5	8	31
Costa Rica	33	2	33	2	4	33
Iceland	34	1	34	1	2	34
Slovenia	35	0	35	0	0	35

The final ranking obtained through the Borda Count method was created by combining the rankings derived from the TOPSIS and VIKOR methods. Greece secured the highest total score by ranking first in both methods, making it the clear leader. Japan and Germany followed in second and third place, respectively, due to their consistently strong performance. These top-ranking countries stand out for possessing robust maritime trade fleets. Countries in the middle ranks displayed relatively balanced performance, whereas those at the bottom of the list—such as Slovenia, Iceland, Costa Rica, and Colombia—received low scores in both methods, resulting in the lowest total scores. This indicates that these countries have very limited maritime capacities and weak overall performance. The Borda method provided a more balanced and comparative result regarding the overall maritime performance of countries by integrating multiple rankings.

RESULTS

In this study, the trade fleet performance of OECD countries between 2014 and 2024 was evaluated using the TOPSIS and VIKOR methods, based on variable weights determined by the Entropy weighting method. The rankings obtained from these methods were then combined using the Borda Count method to produce

a final performance ranking. The analysis revealed that the number of ships, total carrying capacity (DWT), and average ship age are key variables that play a decisive role in determining the fleet performance of countries.

According to the ranking, Greece demonstrated the highest performance thanks to its large carrying capacity and number of ships. Japan and Germany also ranked at the top due to their extensive fleets and modern ship structures. South Korea, the United States, and the United Kingdom showed strong performance as well, with high carrying capacities and low average ship ages. Denmark, Türkiye, and Switzerland were placed in the middle ranks, while the Netherlands, Belgium, and Italy were positioned slightly lower.

Countries with lower performance scores exhibited weaker results in terms of fleet carrying capacity, number of ships, and average ship age. These countries generally possess smaller, older, and lower-capacity fleets, which negatively impacts trade and logistical efficiency. In conclusion, performance differences are primarily based on three key factors: carrying capacity, number of ships, and fleet age. Fleets that are large, modern, and have high carrying capacities tend to perform better, whereas fleets composed of fewer, older, and lower-capacity vessels show lower performance.

DISCUSSION AND CONCLUSION

An aging fleet structure increases insurance and maintenance costs, and the difficulty older ships face in complying with international standards weakens the competitive power of countries (Akgül et al.). Therefore, replacing old ships with new ones, supporting domestic shipyards, reducing costs, and investing in modern technologies are of critical importance (Natelina & Perkov, 2017). Countries with strong shipbuilding infrastructure are better able to modernize their fleets using local resources and gain an advantage in global competition. Indeed, China, Japan, and South Korea continue to lead the sector by accounting for 85% of global shipbuilding activities (Ahmed, 2025). Shipbuilding activities play a significant role in Japan and South Korea's high rankings in the performance evaluation.

In conclusion, young, large, and high-capacity fleets demonstrate higher performance in maritime trade, whereas older, smaller, and lower-capacity fleets reduce trade efficiency. In this context, it is of great importance for countries to focus on fleet renewal strategies, promote domestic production, and strengthen their technological infrastructure in order to gain a competitive advantage in the maritime sector.

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Conflict of Interest

"The authors have declared that there is no conflict of interest"

Author Contributions

The authors contributed equally to the study.

Overview of New Generation Employment Models and Developments in Their Measurement (1292)

Güzin Erdoğan[1*]

[1]Turkish Statistical Institute, Expert, Türkiye

Corresponding Author e-mail: guzin.erdogan@tuik.gov.tr

Abstract

The classical employment model of working full-time in the place where the job belongs to the employer and under his/her supervision has now begun to give way to new generation employment models. This article covers the historical development of the point reached with the industrial revolutions on the way to new generation working models, where work relations, working hours, workplaces or countries of work have changed, work is carried out electronically such as on digital platforms, and payment and compensation methods have changed. In addition, information technologies, digitalization, globalization, demographic and sociological reasons, the phenomenon of migration, sustainability and environment, and the factors affecting employment models in working life with the global pandemic such as Covid-19 have been tried to be summarized. This article addresses new generation employment models and the professions of the future, and reveals what kind of developments should be made within the scope of household labour force survey for measurement.

Keywords: *Employment models, Remote work, New generation, Business world*

INTRODUCTION

Transformative global dynamics—particularly those accelerated by the Covid-19 pandemic—have fundamentally reshaped the world of work and paved the way for new employment models. Systematic literature reviews that examine what these models entail, how they operate, and the outcomes they generate are vital for forecasting the future of labour markets and informing policy-making processes. Moreover, the intergenerational differences that influence these shifts underscore the need for a rigorous analysis of emerging employment patterns. Accordingly, this study seeks to develop concrete recommendations on data-collection procedures, classification approaches, and methodological frameworks for measuring new generation employment models.

The Concept of Employment

Due to its broad scope, the concept of employment is addressed differently across various disciplines and is interpreted from multiple perspectives. In contemporary usage, employment can be defined as the participation of a country's available labour force in productive activities. Individuals who contribute to the production process through their labour in various forms are considered to be employed.

According to the definition provided by the International Labour Organization (ILO), individuals of working age who engage in the production of goods or services with the aim of earning income (profit or wage) during a specified reference period are counted as employed. Under this definition, individuals who have worked at least one hour during the reference week in any economic activity—or who, despite being temporarily absent from work, retain their employment status—are considered employed. This includes wage earners, salaried employees, casual workers, self-employed individuals, employers, and unpaid family workers (Kandemir, 2020).

Similarly, according to the Turkish Statistical Institute (TurkStat) Household Labour Force Survey, employment encompasses individuals who, during the reference period, engage in any productive or

service-related activity with the aim of earning profit or wages. These individuals are classified into two main groups:

- **Those at work**: Individuals who worked at least one hour during the reference period,
- **Those temporarily absent from work**: Individuals who were away from their job during the reference period due to temporary reasons but still maintained a formal employment relationship (TurkStat, 2021).

Historical Development of Employment Models

The concept of work can be examined in three major historical phases: the pre-Industrial era, the Industrial Revolution, and the post-Industrial period. Each of these stages brought about profound changes and transformations in employment structures. This evolution has progressed within a framework of continuous historical development and transformation, a trajectory that continues to shape the modern world (Ören & Yüksel, 2012).

In the pre-Industrial era, labour was primarily regarded as an activity undertaken to fulfil basic and immediate needs. During this period, the central concern of societies was day-to-day survival, and economic activities were limited to sustaining life. There was no distinct separation between domestic and professional life, and production was typically confined to meeting individual or household needs (Ören & Yüksel, 2012).

The Industrial Revolution, symbolized by James Watt's invention of the steam engine in 1768, marked the transition from labour-intensive to capital-intensive production. This transformation facilitated the integration of technology into manufacturing processes, positioning technological advancement as a core determinant of national economic competitiveness. In a narrow sense, industrialization refers to the increasing mechanization of production and the rising share of industry in national income; more broadly, it encompasses enhancements in production quality, the adoption of innovative techniques, and structural transformations in social and working life (Ören & Yüksel, 2012).

Following the Industrial Revolution, the era of information technologies emerged. In this phase, developments in fields such as computing, telecommunications, microprocessors, and genetic engineering gained prominence. A shift from physical labour to intellectual labour occurred, highlighting the importance of knowledge-based work. Known as the "Information Age," this era introduced a fourth stage to the traditional "Three-Sector Model" of economic development, outlining a transition from agriculture to industry, from industry to services, and ultimately from services to the knowledge sector (Ören & Yüksel, 2012).

Factors Influencing Employment Models,

The transformations brought about by the Industrial Revolutions offer a valuable framework for understanding how and why employment models have evolved over time. Identifying the key drivers of these changes is essential for analysing the conditions under which new forms of employment have emerged.

Technological advancements, the integration of robotics and artificial intelligence into work processes, the accelerated digitalization initiated by the Fourth Industrial Revolution, the facilitative aspects of globalization, the implications of an aging population, the changing expectations of new generations

regarding work, the impact of migration on labour markets, environmental concerns, and sustainability demands—as well as the profound disruptions caused by the Covid-19 pandemic—have all contributed to a shift away from traditional modes of employment toward more diverse and flexible working models.

This section provides a detailed examination of these factors, offering insights into how each has influenced the evolution of contemporary employment structures.

Information Technologies

Technological advancements—particularly in computer and communication technologies—have played a crucial role in the transformation of labour and the changing skill composition of the workforce. These developments have significantly impacted societal structures and laid the foundation for the emergence of the information age. Unlike in the agricultural or industrial periods, knowledge has become the central element of contemporary society. In this era, where knowledge generates economic value, businesses that are capable of producing, utilizing, and transferring knowledge have gained a significant competitive edge (Yüksel, 2014).

The changing nature of work, fuelled by robotics and artificial intelligence, has led to increased expectations of productivity and efficiency through automation in various professions. However, these changes have also given rise to concerns and uncertainties within the labour force. Technology has not only altered how work is performed but has also transformed job-matching processes, employer expectations, and the skill sets demanded from workers. In this context, platforms such as LinkedIn have digitized recruitment and matching processes, while services like Uber, Trendyol, and Amazon have brought many service-based activities into the digital realm (Kandemir, 2020).

Digitalization

Digitalization, in its broadest sense, refers to the transformation of information from analogue to digital formats. This transformation entails the increased use of databases and the storage, processing, and transmission of information through digital tools. At its core, digitalization involves the representation of objects, images, sounds, or documents in a digital environment through numerical encoding. The internet, in this context, provides the global infrastructure that enables access to and dissemination of digital data. In the context of working life, digitalization is defined as the process of utilizing digital technologies and information to transform business operations. As digital tools increasingly penetrate production processes, significant changes are expected in both the structure of production and occupational classifications. With the advancement of automation and robotization, it is anticipated that certain occupations will become obsolete, while entirely new professions will emerge as a direct outcome of digital transformation (Kandemir, 2020).

Globalization

According to the Turkish Language Association, globalization is defined as the process of becoming global, synonymous with the term “globalization” itself (TDK, 2024). In its modern context, globalization emerged notably after World War II and gained momentum in the 1980s with the revival of neoliberal policies and significant advancements in communication technologies. This process has led to widespread effects in economic, political, cultural, and ecological spheres. While globalization can be examined from various perspectives, it is generally understood as the transformation of activities such as travel, communication, finance, trade, sports, and professional practices from nationally confined operations to ones that inherently require international interaction. In other words, globalization refers to the cross-border integration process through which individuals from different regions engage in the exchange of goods, ideas, and services, thereby contributing to the formation of a new global order (Kandemir, 2020).

Demographic and Sociological Factors (Changing Generations, Aging Population)

Among the dynamics of transformation that impact the social and economic structure are changes in time, space, opportunities, and general patterns of thought. These shifts have led to the emergence of generations with significantly different characteristics. When examining individuals who have joined the workforce particularly since the early 1900s, six main generational cohorts stand out: the Traditionalist Generation, the Silent Generation (1930–1944), the Baby Boomer Generation (1945– 1964), Generation X (1965–1979), Generation Y (1980–1999), and Generation Z (2000–present). Intergenerational differences in values, attitudes, and behaviours have also shaped divergent perspectives on working life.

Generations prior to Generation Y are generally characterized by a strong sense of loyalty, compliance with rules, respect for authority, and a strong work ethic. In contrast, Generation Y values flexible work arrangements, autonomy, and living a meaningful life. They tend to have lower organizational commitment than Generation X, switch jobs more frequently, and are often driven by aspirations to become managers or start their own businesses. Generation Z, on the other hand, has grown up immersed in digital technologies, adept at using mobile devices and the internet, and maintains constant global connectivity. They are also prone to rapid consumption habits (Yılmaz & Aktaş, 2018).

While research indicates that older workers may face greater difficulties adapting to flexible work models and automation—resulting in reduced productivity and increased costs—their experience remains a valuable asset. Developing strategies to integrate their expertise into the new world of work can enhance societal welfare and facilitate intergenerational knowledge transfer. In this context, analysing educational needs and equipping older workers with new skills through lifelong learning stands out as a critical approach (Kandemir, 2020).

Migration

Migration is defined as the movement of individuals or communities from one country to another, or within the borders of the same country, due to economic, social, or political reasons. Regardless of the causes of migration, migrants play a transformative role in the labour markets of host countries; they influence and are influenced by the economic and social systems of those countries. The presence of migrant labour directly impacts the local workforce, particularly by displacing low-skilled local workers from certain job sectors in a permanent manner. Consequently, this dynamic necessitates that the unskilled native labour force acquires specific competencies and develop new skills to remain employable (Kandemir, 2020).

Sustainability and Environment

The fight against changing environmental conditions is closely linked to the increase in employment generated in this field. In particular, in developed countries that have intensified their environmental investments, climate change mitigation efforts have had a positive effect on the creation of new jobs. These environmental developments have paved the way for the emergence of the "green economy" in labour markets, along with a rise in "green jobs" within this framework.

It is widely recognized that the green economy will lead to the creation of many new professions while rendering some traditional occupations obsolete. In this context, according to projections made by the International Labour Organization (ILO) based on the year 2030, approximately 25 million new jobs are expected to be created in sectors such as construction, manufacturing, mining, and electronics. Conversely, nearly 6 million jobs are anticipated to disappear in sectors like oil extraction and refining, coal mining, and coal-based electricity production (Kandemir, 2020).

Covid-19

Following the classification of Covid-19 as a pandemic and the recognition of its highly contagious and severe impact, numerous measures were implemented in working life by both governments and businesses

on their own initiatives. Among these, remote working emerged as the most prominent and rapidly adopted practice. Additionally, many wholesale and retail companies increasingly turned to online sales and payment channels—tools that were already in use before the pandemic—leading to their widespread adoption during the crisis (Kandemir, 2020).

New Generation Employment Models and Future Professions

Unlike traditional employment forms such as remote work, flexible work, and working from home, new-generation employment models have brought about significant transformations in labour markets. This section provides a detailed analysis of these emerging models, while also exploring the professions expected to come to the forefront in the future in light of factors such as digitalization, automation, and sustainability.

New Generation Employment Models

In the evolving global order shaped by the widespread adoption of digital tools, the advantages of remote work have become increasingly evident. Remote work refers to a work arrangement in which employees perform their duties outside of a traditional office setting—typically from home or any other geographic location. Enabled by internet connectivity and digital communication technologies, this model offers employees the opportunity to achieve a better balance between work and personal life. Among the key advantages of remote work are flexible working hours, the elimination of commuting time, the ability to personalize the workspace, and the minimization of time lost in traffic. Furthermore, remote work removes geographic barriers, enabling job seekers to access positions across various regions and offering broader employment opportunities (Çetinoğlu, 2024).

Behbahani (2023) highlights that researchers have identified eight new work models, each representing various degrees of current and future employment structures. These models not only reflect the integration of advanced technologies but also introduce new perspectives on how and where work is conducted.

Table 1. New Working Models (Behbahani, 2023)

New Work Model	Description
Flexible Work Schedules	Employees can arrange their work hours based on preferences, fulfilling a set number of weekly hours (e.g., 40), with flexible start/end times or compressed weeks.
Location Independence	Employees can choose where to work—office, home, or co-working spaces. This includes remote and hybrid working options.
Task Autonomy	Employees have greater control over the tasks and projects they work on, aligning them with their skills and interests, and are empowered in decision-making.
Results-Oriented Work	Focus is placed on outputs rather than fixed schedules. Employees manage their time and workflows to meet defined goals.
Gig and Freelance Work	Individuals engage in project-based or short-term contracts instead of full-time employment, often working for multiple employers simultaneously.
AI/Automation Integration	Employees work alongside AI systems and robots to increase efficiency. These technologies serve to assist rather than replace human labour.
Virtual Reality Collaboration Spaces	Teams collaborate in digital environments that simulate physical proximity, enabling effective interaction regardless of geographical separation.
Augmented Reality Work Environments	Using AR devices like smart glasses, employees access real-time data and interactive elements within their field of vision while performing tasks.

According to the report titled “Next Generation Working Models” published by the Turkish Confederation

of Employer Associations (TİSK), the onset of the Covid-19 pandemic has triggered a rapid wave of digital transformation in the world of work. This shift has led to the widespread adoption of remote/home-based working across various sectors. However, next-generation working models are not limited to remote or home-based work alone. The report emphasizes that the transformation in employment structures is multifaceted and encompasses a range of innovative work arrangements.

Table 2 below outlines the new working models identified by TİSK (2022).

Table 2. New Generation Working Models (TİSK, 2022)

New Generation Working Models	Description
Remote/Home-Based Work	Performing all or part of the job outside of the conventional workplace, enabled by digital tools. This model promotes flexibility and eliminates geographic constraints.
Part-Time work	Employment under working hours shorter than the standard full-time arrangement. It is particularly appealing for those seeking work-life balance.
Marginal Part-Time Work	A variant of part-time work that includes significantly fewer working hours, typically adopted for supplementary income or temporary engagements.
Mini Jobs/Micro Jobs	As defined in Germany, jobs that earn less than €450 monthly and last no more than 3 months or 70 working days per year. These jobs are not subject to social security contributions.
On-Call Work	A non-regular model where the employee is called to work only when needed, based on a pre-agreed contract. It allows high employer flexibility.
Voucher-Based Work	A system where employers pay independent workers through intermediaries, often with state oversight, for specific tasks or services.
Freelance Work	Individuals work independently without forming a company, invoicing the client for services rendered. It offers autonomy and project-based flexibility.
Online Platform Work	Jobs performed via digital platforms like Uber or food delivery apps, often requiring low skill and attracting underemployed or unemployed individuals.
Rotational Work	A flexible model where multiple employers share a workforce, assigning tasks on a rotational and demand-driven basis.
Job Sharing	A full-time position is divided among two or more individuals, each sharing duties and benefits, improving flexibility and productivity.

Future Professions and Disappearing Occupations

With the rapid advancement of technologies such as digitalization, artificial intelligence, automation, and machine learning, it is predicted that many of the professions currently employing millions of people will undergo transformation or disappear entirely. In this transformation process, some occupations are expected to evolve and continue to exist in new forms, while others will be replaced by new job categories that are more technology-driven and based on digital skills.

Professional fields such as medicine, law, and engineering-due to their structure requiring long-term education and high levels of expertise-are anticipated to be directly affected by this change. Many tasks in these fields are expected to be performed more efficiently and accurately through AI-assisted systems. This situation necessitates the restructuring of the educational processes, qualification criteria, and professional standards associated with these occupations.

Moreover, raising public awareness of these emerging job profiles, developing strategies to help the workforce adapt to this transformation, and institutionalizing the concept of lifelong learning are of great importance. In this context, it is becoming increasingly critical for both individuals and organizations to continuously update their digital competencies in order to remain competitive in the labour market.

Table 3. Occupations Expected to Gain Importance and Experience Increased Employment Rates (Kandemir, 2020)

Category	Occupations
Occupations Expected to Gain Importance	Nutrition and dietetics specialist, aesthetician, food engineer, organic farming specialist, sports, yoga and meditation instructor, art and craft educators (e.g. painting, ceramics, tailoring)
Occupations Expected to Have a High Number of Employees in the Future	Climate engineer, robotic veterinarian, dream facilitator, 3D production engineer, artificial intelligence trainer, drone operator, waste engineer, personalized body part fabricator
Occupations Anticipated to be in High Demand	Smart home design manager, carbon engineer, digital tailor, waste data engineer, artificial intelligence-based healthcare technician
Occupations with the Highest Expected Increase in Employment Rates	Solar energy and wind energy technicians
Occupations Likely to Retain Their Relevance	Specialized nursing, home healthcare services, elderly care services, customer relations specialist
Occupations with Very Low Risk of Disappearance in the Next 20 Years	Dentistry, clergy, psychotherapist, fitness instructor, all creative professions
Emerging Occupations	3D production engineer, robot technician, artificial intelligence trainer, bio-technologist (DNA programmer), climate engineer, data analyst, industrial design engineer, digital rehabilitation consultant, drone pilot, personal privacy consultant, nanotechnology engineer, technical ethics specialist, nano-medical expert, vertical farmer

In addition to the increasing demand for certain professions in the future, occupations that are most at risk due to automation include low-skill jobs such as cashiering, waiting tables, and taxi driving. Furthermore, mid-skill jobs such as data entry clerks, accounting personnel, and secretarial positions are also predicted to disappear over time (Kandemir, 2020).

During this transformation process, individuals possessing high levels of perception and manipulation capabilities, creative intelligence, and strong social skills are expected to retain their employment. Conversely, those lacking these competencies are likely to experience job loss over time (See Table 4).

Table 4. Occupations at High Risk of Disappearence (Yılmaz and Aktaş, 2018)

Category	Occupations
Occupations with high risk of disappearance	Telemarketer, accountant, taxi driver, bartender, housekeeper, computer programmer
Occupations projected to disappear within 20 years	Laboratory technician, accounting, taxi and bus drivers, telemarketing, sales clerk, technical journalism, real estate agents, sales personnel, hospital workers (excluding doctors)

Developments and Evaluations Regarding the Measurement of Employment Forms

In order to ensure the comparability of international labour statistics, the use of a standardized classification system is essential. In this context, the International Classification of Status in Employment (ICSE), initially introduced in 1993, was revised and updated in 2018 to reflect the changing nature of employment, particularly with the emergence of new forms of work influenced by digitalization and flexible working arrangements.

The subsections below provide a comparative overview of ICSE-1993 and ICSE-2018, highlighting structural differences, expanded scope, and implications for data comparability. Furthermore, the main topics and conclusions of the International Conference on the Measurement of New Forms of Employment, held in July 2024, are discussed in the relevant sections.

International Classification of Status in Employment (ICSE-1993 and ICSE-18)

The Household Labour Force Survey constitutes the primary source of data regarding the supply side of a country's labour market. This survey is designed to collect information on various aspects such as the economic activity, occupation (or job performed), employment status, and working hours of the employed, as well as the duration of job search and type of occupation sought by the unemployed. Within this framework, both individuals currently employed and those who are not employed during the reference period but have worked previously are classified according to economic activity, occupation, employment status, and educational attainment. The classification system traditionally used is the International Classification of Status in Employment (ICSE-1993).

ICSE-2018 includes two distinct frameworks:

- ICSE-18-A: Classification based on the type of authority
- ICSE-18-R: Classification based on the type of economic risk

Table 5. Comparison of ICSE-93 and ICSE-18

ICSE-93: Employment status	ICSE-18-A: Employment status by type of authority	ICSE-18-R: Employment status by type of economic risk
Paid employment 1. Employees ***Self-employment*** 2. Employers 3. Own-account workers 4. Members of producer's cooperatives 5. Contributing family workers	***Self-employed*** **A. Employers** 11- Employers in companies 12- Employers in household market enterprises **B. Self-employed workers without employees** 21- Owner-operators of companies without employees 22- Own-account workers in household market enterprises without employees ***Dependent workers*** **C. Dependent contractors** 30-Dependent contractors **D. Employees** 41-Permanent employees 42-Fixed-term employees 43-Short-term and temporary employees 44-Paid apprentices, interns and interns **E. Contributing family workers** 51-Contributing family workers	***For-profit workers*** **F. Independent workers in household market businesses** 12-Employers in household market businesses 22-Own-account workers in household market businesses without employees **C. Dependent contractors** 30-Dependent contractors **E. Contributing family workers** 51-Contributing family workers ***Paid workers*** **G. Owner-operators of companies** 11-Employers in companies 21-Owners of companies without employees **D. Employees** 41-Permanent employees 42-Fixed-term employees 43-Short-term and temporary employees 44-Paid apprentices, interns and interns

According to the International Labour Organization (ILO), "statistics on employment relationships provide essential information on the nature of economic risk and authority experienced by individuals in their work, as well as on the strength and nature of their attachment to the economic unit in which they work." Due to the significant transformation in employment forms worldwide, ICSE-1993 was revised and expanded, resulting in the more inclusive International Classification of Status in Employment – ICSE-2018.

The structural differences between the ICSE-1993 and ICSE-2018 systems-particularly with regard to their ability to capture new forms of employment emerging in the digital age-are outlined in Table 5 (ILO, 2023).

The Conceptual Framework of Employment Forms aims to classify jobs using a two-dimensional approach that distinguishes between employment relationships and work modalities.

Employment relationships refer to the structural link between workers and the economic units for which they work. In contrast, work modalities describe how work is organized, performed, and fulfilled in terms

of time and space. There is currently no universal statistical standard governing this conceptual distinction. However, the framework draws upon the "working patterns" approach developed by Eurofound and the "employment performance forms" concept introduced by the French National Statistical Information Council (UNECE, 2022).

In addition to the basic evaluation of employment relationships and work modalities, the framework includes four complementary dimensions to better assess and measure employment forms and their impacts on individual well-being:

1. The stability and permanence of the employment relationship;
2. The social protection system under which the employment form is regulated;
3. The socioeconomic status of individuals engaged in such forms of employment;
4. The impact of employment forms on individual well-being, as captured by the broader concept of job quality.

The International Classification of Status in Employment (ICSE-18) provides a foundational structure for measuring and interpreting employment forms by classifying jobs based on the nature of the employment relationship. Its main categories include employees, independent workers, and dependent contractors. Employment relationships are further classified according to their continuity and stability, with ICSE-18 identifying four key types of employees: permanent, fixed-term, short-term, and temporary workers.

The relationship between employment relationships and work modalities is conceptually illustrated in Table 6, which includes examples such as working hours, primary place of work, and use of digital platforms.

Table 6. Relationship Between Employment Relationships and Work Modalities at Job Level

Employment Relationships (ICSE-18)	Work modalities: example					
	Working-time		Working location		Digital platforms	
Main ICSE-18 categories	Length of work hours		Main work location		Work through a digital platform	
	Part-time	Full-time	At home	Outside the home	Yes	No
Employees						
Dependent contractors						
Employers in corporations						
Employers in household market enterprises						
Owner-operators of corporations without employees						
Own-account workers in household market enterprises without employees						
Contributing family workers						

Measuring the degree of permanence or stability experienced by workers in their jobs is of interest to both national statistical offices and researchers. While employment permanence refers to the presence of open-ended and continuous job security, stability concerns the duration of employment (tenure) and the regularity with which workers remain employed or engaged in income-generating activities. Indicators of

permanence and stability can be integrated with the classification of employment relationships and work modalities that underpin the measurement of employment forms, as illustrated in Table 7, covering all the main dimensions of interest.

Table 7. Employment Relationship, Work Modalities and Job Permanence and Stability

Employment relationships (ICSE-18)		**Work modalities: example**					
		Working-time		**Working location**		**Digital platforms**	
Main ICSE-18 categories	Permanence and stability	Length of work hours		Main work location		Work through a digital platform	
		Part-time	Full-time	At home	Outside the home	Yes	No
Employees	Permanent						
	Fixed-term						
	Short-term and casual						
	Paid apprentices, trainees and interns						
Dependent contractors	More permanent or stable						
	Less permanent or stable						
Employers in corporations	More permanent or stable						
	Less permanent or stable						
Employers in household market enterprises	More permanent or stable						
	Less permanent or stable						
Owner-operators of corporations without employees	More permanent or stable						
	Less permanent or stable						
Own-account workers in household market enterprises without employees	More permanent or stable						
	Less permanent or stable						
Contributing family workers	More permanent or stable						
	Less permanent or stable						

All these indicators will enable us to obtain comprehensive and summarized information that reveals the current status of measuring and identifying new generation employment forms.

Global Conference on Measuring New Forms of Employment

The ongoing and rapid transformation of the world of work lies at the core of many issues concerning policymakers. These include the impact of artificial intelligence, the increasing digitalization of jobs, platform-based employment, and evolving work relationships. It is evident that understanding these transformations requires reliable data. Accordingly, most countries and international organizations are seeking to enhance the quality and comparability of information on changing work activities, regulations, and conditions.

In this context, Eurostat and the International Labour Organization (ILO) jointly organized a global conference in Brussels on 4–5 July 2024. The event brought together policymakers and statistical experts

from around the world to explore current challenges and prospects in measuring new forms of employment.

The conference addressed the policy demand for reliable statistical information concerning the ongoing transformation of labour markets at national and international levels. It also discussed potential statistical responses, experiences, data sources, and measurement issues. The opening and closing sessions helped frame measurement efforts within policy needs and supported the development of a forward-looking roadmap.

Furthermore, the 21st International Conference of Labour Statisticians (ICLS) mandated the ILO to develop statistical standards for digital platform work, to be presented at the 22nd ICLS in 2028. This topic will also be included in the agenda of the 113th International Labour Conference in 2025. These global deliberations are critical for defining accurate statistical concepts for diverse and emerging forms of work. The European Commission has likewise prioritized the working conditions of digital platform workers as part of its broader vision for a fair and human-centred labour market. High-quality statistical data will be essential for designing such policies.

The emergence and expansion of new forms of employment have been supported by the publication of the Handbook on Forms of Employment in 2022, led by UNECE. This document has facilitated the implementation of a coherent statistical framework. Specifically, the International Classification of Status in Employment (ICSE-18) introduces two dimensions—type of authority and economic risk— for classifying employment statuses. Technological progress and digitalization, in turn, require new data collection methods and the development of novel measurement topics. Following a 2022 pilot study, the EU Labour Force Survey is set to begin collecting data on platform work in 2026. A handbook developed for this purpose was made available to users in 2023 (OECD, ILO, Eurostat, 2023).

In parallel, the U.S. National Statistics Committee and the Bureau of Labour Statistics have conducted significant work on contingent labour, including studies published in 2023. A quality framework for assessing the reliability and comparability of data on digital platform work is under development and will be discussed with international experts. Additionally, the use of algorithmic management and AI- based tools in workplaces and their implications will form a critical area of future statistical research.

The 21st ICLS has also initiated the development of standards for measuring informal employment and the informal economy. Given that informality increases workers' vulnerability and reduces job quality, Eurostat has begun work on new standards to support the formulation of policies aimed at increasing the formality of the European labour market.

Lastly, other major transformations—such as the decarbonisation of the economy, digitalization (including telework), and the development of the social economy—are also reshaping labour markets and lifelong learning needs, requiring new efforts in measurement.

This global conference provided a valuable platform to review the implementation of recently developed statistical tools and to reflect on outstanding and emerging statistical needs in the field (Eurostat, ILO, 2024).

CONCLUSION

This study has examined the definition and historical development of employment, while introducing contemporary new-generation employment models. The transition from traditional employment types to more flexible, location-independent, and technology-driven models has been analysed in the context of increasing use of information technologies, digitalization, globalization, demographic shifts, migration, sustainability concerns, environmental challenges, and global crises such as the COVID-19 pandemic.

Prominent new employment forms include remote work, flexible scheduling, task autonomy, outcome-

oriented performance, digital platform labour, and the integration of artificial intelligence applications.

This transformation signals the gradual decline of certain traditional occupations due to automation and digitalization, while emphasizing the emergence of roles such as robotics technician, AI trainer, climate engineer, industrial design engineer, and nanotechnology engineer. These developments necessitate the reskilling of the workforce and may require revisions to legal frameworks and definitions to ensure decent work standards in emerging professions.

For countries like Türkiye, which possess a young and dynamic population structure, the implementation of new employment models requires the development of appropriate legal regulations and incentive mechanisms. As physically intensive labour is increasingly replaced by roles demanding skills and competencies, preparing the workforce for this transition and maximizing societal benefits becomes a critical issue.

Numerous international initiatives are underway to define and statistically measure new employment forms more accurately. The global conference organized by Eurostat and the ILO in July 2024 has laid the groundwork for ongoing dialogue, and further discussions are expected during the 2025 International Labour Conference and the 2028 ICLS session. These will focus on integrating new concepts and definitions into international statistical standards. In this regard, the Handbook on Forms of Employment, published in 2022 under the leadership of UNECE, marked a significant step forward, with the ICSE- 18 classification system forming its basis. Additionally, Eurostat plans to collect data on digital platform work through the EU Labour Force Survey starting in 2026.

This paper can be considered a preliminary effort to explore what constitutes new-generation employment models, and what should be taken into account in the process of their adoption and statistical measurement. By shedding light on the historical transition from traditional to modern employment forms, and identifying key drivers of this shift, the study contributes to a deeper understanding of ongoing labour market transformations. Furthermore, by highlighting conceptual and methodological needs in the area of statistical measurement, it may support the formulation of future roadmaps at national and international levels.

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The Relationship Between Consumer Confidence Index and Selected Economic Indicators in Türkiye: 2005–2024 (1144)

Helga Şimşek*

*Corresponding author e-mail: helga.tural@tuik.gov.tr

Abstract

The consumer confidence index (CCI), which reflects consumers' assessments, expectations, and tendencies regarding economic conditions, is considered one of the leading indicators used to predict the economic outlook. It plays a critical role in forecasting the economic cycle and is frequently utilized in macroeconomic analyses and decision-making processes. As it represents consumer sentiment, which directly affects consumption behavior, it is closely monitored by analysts, policymakers, and the general public. This study aims to examine the relationship between the consumer confidence index, exchange rate, stock market index (BIST-100), and consumer price index (CPI) in Türkiye, using data from the period 2005–2024. According to the results of the ARDL analysis, a long-term cointegration relationship was found among the variables. In the long run, an increase in the exchange rate negatively affects consumer confidence, while the BIST-100 index has a positive effect. Although the effect of CPI is negative, it is not statistically significant. Diagnostic tests on the model's residuals indicated that the assumptions of no autocorrelation, homoskedasticity, and normality were satisfied.

Keywords: *Consumer Confidence, ARDLl, Exchange Rate, BIST-100, Inflatıon*

INTRODUCTION

Many studies suggest that consumer behavior has significant effects on the economic conjunture. Consumers' feelings and thoughts are affected by many macroeconomic factors as well as psychological, social and political factors (Topuz, 2014). The general economic situation in the country is also a factor that can affect consumer behavior.

The issue of consumer confidence is also addressed within the discipline of behavioral economics. In this context, consumer confidence can be influenced by psychological prejudices and emotional states in decision-making processes rather than rational expectations. The behavioral economics discipline argues that household behavior depends both on purchasing power and the willingness to purchase (Katona, 1968). While purchasing power is dependent on objective factors, willingness to purchase is shaped by subjective factors (Roos, 2008). The income level of the consumer is not sufficient to explain the changes in consumer behavior. . The consumer confidence index is of great importance in measuring both purchasing power and willingness to purchase, as it reflects information about consumer attitudes (Tunalı and Özkan, 2016). Optimism in consumer confidence can lead to an increased willingness to spend and tendency to get into debt, while pessimism can cause consumers to reduce their spending and reassess their financial situation (Arısoy and Aytun, 2014). In this regard consumer confidence can be affected by developments in financial markets as well as macroeconomic developments.

The consumer confidence index (CCI), which summarizes consumer assessments, expectations and tendencies, is one of the indicators designed to measure changes in the economy and is widely used in macroeconomic evaluations and forecasts. Therefore, the index is closely followed by analysts, policymakers and the public. Analysts tend to associate changes in consumer confidence with economic indicators such as private consumption demand, economic growth or inflation. In this context, the index

is considered a leading indicator that measures consumer attitudes.

The aim of this study is to analyze the relationships between the consumer confidence index and the exchange rate, stock market index (BIST-100) and consumer price index (CPI) in Türkiye within the framework of both long-term cointegration and short-term dynamics. The second section of the study introduces the data set used and provides theoretical information regarding the methodology and model selection. It also includes descriptive statistics on the variables and a discussion of the statistical methods employed. The third section presents the findings obtained through the analyses. The final section offers a general evaluation and policy recommendations for decision-makers.

MATERIAL AND METHODS

Material

This study utilizes data covering the period 2005–2024 for Türkiye. The primary variables include the consumer confidence index (CCI), the monthly change rate of the consumer price index (CPI), the exchange rate and the stock market index (BIST-100). To smooth fluctuations in the variables, reduce temporary effects and obtain a more stable trend, the series were adjusted using the three-month moving average method. Additionally, to reduce data variability and obtain more robust interpretations in the analysis, the natural logarithms of the variables were used. In this way, linear relationships between the variables were strengthened.

Methods

The Collection of the Data

The study is based on data from the 2005–2024 period. Two of the variables—the consumer confidence index and the monthly change rate of the consumer price index (CPI)—are derived from the consumer tendency survey conducted in cooperation with Turkish Statistical Institute (TurkStat) and the Central Bank of the Republic of Türkiye (CBRT). The other two variables are the stock market index (BIST- 100) and the exchange rate. For the exchange rate, the average quarterly value of the U.S. dollar selling rate was used. Data for both of these variables were obtained from the CBRT's electronic data distribution system.

Statistical Analysis

The study initially examines the descriptive statistics of the variables. In the second stage, relationships between the variables are analyzed using correlation coefficients. One of the fundamental assumptions in time series analyses is the stationarity of the series. Non-stationary series can lead to misleading regression results due to changes in statistical properties such as mean and variance over time. Therefore, it is critical to determine the stationarity of the series before proceeding with the modeling process. In this context, the stationarity levels of the series were examined using both the Augmented Dickey-Fuller (ADF) (Dickey and Fuller, 1979) and the Phillips-Perron (PP) tests (Phillips and Perron, 1988).

The results obtained revealed that some of the series are stationary at level I(0), while others are stationary at the first difference I(1). Since the variables have different degrees of integration, the ARDL (Autoregressive Distributed Lag) model was preferred, as it allows for the joint analysis of both short- and long-term relationships. This model offers flexibility in working with variables that have different levels of integration. Furthermore, short-term causality relationships were also analyzed within the framework of the error correction model (ECM), an extension of the ARDL approach. The statistical significance of the error correction term indicates that deviations from the long-term equilibrium in the model are corrected at a certain rate in each period and the system tends to return to equilibrium. This supports the existence of a long-term relationship between variables, even in the presence of short-term fluctuations. All analyses were carried out using the EViews package program.

RESULTS

The basic descriptive statistics related to the data used in the study were examined and the findings obtained are given in Table 1.

Table 1. Basic descriptive statistics

	LOG_CCI	LOG_RT	LOG_CPI	LOG_BIST_100
Mean	4.464182	1.238375	-0.119976	6.836661
Maximum	4.594330	3.540427	2.051128	9.273119
Minimum	4.190664	0.169867	-2.238047	5.531530
Standard Deviation	0.092261	0.994612	0.902966	0.906637
Number of Obver.	80	80	80	80
	LOG_CCI	LOG_RT	LOG_CPI	LOG_BIST_100
LOG_CCI	1	-0.72	-0.59	-0.57
LOG_RT	-0.72	1	0.71	0.95
LOG_CPI	-0.59	0.71	1	0.67
LOG_BIST_100	-0.57	0.95	0.67	1

The correlation coefficients between the variables used in the study indicate the presence of significant linear relationships. The consumer confidence index (LOG_CCI) is negatively correlated with all other variables. In particular, it has a strong negative correlation with the exchange rate (LOG_RT), with a coefficient of -0.72. This suggests that increases in the exchange rate are associated with a decrease in consumer confidence. Similarly, a negative correlation of -0.59 is observed with the consumer price index (LOG_CPI) and -0.57 with the BIST-100 index (LOG_BIST_100).

Among the independent variables, strong positive correlations are observed. In particular, the exchange rate and the BIST-100 index show a high correlation of 0.95, indicating a parallel movement between these two financial indicators. The exchange rate also has a positive correlation of 0.71 with the CPI rate. These findings suggest that the variables may be affected by the same economic shocks and could have a structural relationship among them. However, by definition, correlation analysis does not reveal any causal relationships between variables. Therefore, the existence of a positive or negative relationship between variables does not imply a causal connection. In economic analyses, it is crucial to determine whether a change in one variable affects another and, if so, the direction and magnitude of that effect. In this context, the study investigates the causality relationships among the variables.

In time series analysis, it is essential to determine the stationarity levels of the variables to obtain meaningful and reliable results. Therefore, the stationarity of all variables in the study, whether at levels or in differences, was tested using both the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests. Table 2 presents the unit root test results for the variables under consideration.

Table 2. Unit root test results

Models	**ADF**		**PP**	
Variables	**Constant**	**Constant-Trend**	**Constant**	**Constant-Trend**
LOG_CCI	0.1366	0.1047	0.1366	0.0920
ΔLOG_CCI	0.0000*	0.0000*	0.0000*	0.0000*
LOG_RT	1.0000	0.9824	1.0000	0.9900
ΔLOG_RT	0.0000*	0.0000*	0.0000*	0.0000*
LOG_CPI	0.6721	0.0001*	0.0001*	0.0000*
ΔLOG_CPI	0.0000*	0.0000*	0.0000*	0.0000*
LOG_BIST_100	0.9953	0.9754	0.9945	0.9648
ΔLOG_BIST_100	0.0000*	0.0000*	0.0000*	0.0000*

(*) indicates statistical significance at the 1% level.(Δ) symbol denotes the first difference of the variable.

Both tests were evaluated with models that include a constant and both a constant and trend. The results indicate that the LOG_CPI variable is stationary at level I(0), while all other variables become stationary after first differencing, indicating integration at order I(1). Due to the differing degrees of stationarity among the variables, the ARDL bounds testing approach was preferred for analyzing long- term relationships. The ARDL model is advantageous in terms of being flexible to dependent and independent variables having different integration levels (I(0) and I(1)), providing reliable estimates in small samples, and being able to examine both short-term and long-term relationships within the same model. Furthermore, unlike classical cointegration tests, the ARDL approach does not require all variables to be integrated of the same order (Pesaran and Shin, 1997).

Using the ARDL model, the study examined the short- and potential long-term relationships between the consumer confidence index (LOG_CCI) and macroeconomic indicators such as the exchange rate, BIST-100 index, and CPI. The lag lengths of the model were determined through automatic selection using the Akaike Information Criterion (AIC) in the EViews program to enhance model accuracy and fit. According to the AIC, the optimal model structure was selected as ARDL(1, 1, 0, 0). The model estimation results are presented in Table 3 and the model statistics are given in Table 4.

Table 3. ARDL (1,1,0,0) model estimation results (dependent variable: LOG_CCI)

Variable	Coefficient	Std. Error	t- statistic	Prob.
LOG_CPI(-1)	0.7003	0.0719	9.733	0.0000*
LOG_RT	-0.3015	0.0705	-4.276	0.0001*
LOG_RT(-1)	0.2624	0.0753	3.484	0.0008*
LOG_BIST_100	0.0382	0.0170	2.246	0.0277*
LOG_CPI	-0.0072	0.0075	-0.958	0.3412
Constant (C)	11.332	0.3005	3.772	0.0003

(*) indicates statistical significance at the 1% level.

Table 4. ARDL model statistics

R^2	0.8496
Adjusted R^2	0.8393
Akaike Information Criterion	-3.6919
Standart Eror	0.0368
F-statistic	82.4486
p (F-statistic)	0.0000
Durbin-Watson İstatistiği	1.9727

According to the model results, the lagged dependent variable LOG_CCI(-1) is statistically significant at the 1% level ($p = 0.0000$), with a coefficient of approximately 0.70. This suggests that consumer confidence is largely influenced by its own past values, indicating a persistent pattern over time.

Among the independent variables, the exchange rate (LOG_RT) is statistically significant in both its current ($p = 0.0001$) and one-term lagged values ($p = 0.0008$). While the current exchange rate has a negative effect on consumer confidence, its lagged value has a positive effect. This suggests that increases in the exchange rate in the short term reduce consumer confidence but the effects can be partially compensated by market adjustment after a while.

The BIST-100 index (LOG_BIST_100) has a positive and statistically significant coefficient ($p = 0.0277$). This result can be interpreted as positive developments in financial markets increasing consumer confidence.

The consumer price index (LOG_CPI) has a negative sign but is not statistically significant (p = 0.3412), suggesting that changes in price levels do not significantly affect consumer confidence in the short term.

The overall model demonstrates high explanatory power (R^2 = 0.849), and the Durbin-Watson statistic (1.97) indicates that there is no autocorrelation problem.

Following the ARDL model estimation, a bounds test was applied to determine whether a long-term cointegration relationship between variables. The results of the ARDL bounds test are shown in Table

5.The F-statistic value of 5.304459 exceeds the critical values at the 5% (4.35) and 10% (3.77) significance levels. Therefore, the null hypothesis of no long-term relationship is rejected. It is concluded that there is a long-term cointegration relationship among the variables. However, while the ARDL bounds test confirms a long-term equilibrium, it does not imply that each explanatory variable has a statistically significant effect on consumer confidence in the long term.

Table 5. ARDL bounds test results

	Model	F-statistic
	ARDL(1, 1, 0, 0)	**5,304459**
Critical Value Bounds	**I(0)**	**I(1)**
5%	3,23	4,35
10%	2,72	3,77

After confirming the existence of the cointegration relationship, the direction of this relationship and the long-term effects of the variables on consumer confidence were evaluated based on the long-term coefficient estimates of the ARDL model. According to the long-term coefficient estimates given in Table 6, it is seen that the exchange rate, BIST-100 index and CPI variables do not have statistically significant effects on consumer confidence. This situation reveals that the relevant macroeconomic indicators have limited and statistically unreliable effects on consumer confidence rather than being determinative in the long term. The fact that only the constant term is found to be statistically significant, indicating that consumer confidence tends to fluctuate around a certain long-term average level. It is estimated that a 1% increase in the exchange rate increases consumer confidence by approximately 0.13%, while a 1% increase in the BIST-100 index decrease consumer confidence by about 0.79% in the long term. Although the effect of inflation (CPI) is negative, it is not statistically significant. However, all these effects do not carry statistical reliability due to high p-values.

Table 6: Long-term coefficients estimates

Variable	Coefficient	Std. Error	t- statistic	Prob.
LOG_RT	0.001320	0.009410	0.140277	0.8888
LOG_BIST_100	-0.007924	0.008752	-0.905433	0.3682
LOG_CPI	-0.003634	0.003793	-0.958274	0.3411
C(Constant)	0.575124	0.132629	4.336328	0.0000*

(*) indicates statistical significance at the 1% level.

In light of the findings on the long-term balance relationship, the error correction model (ECM) was used to reveal the short-term deviation process of the system and how the balance was restored. Through the ECM, the sustainability of the relationship between the variables and the speed of adjustment back to equilibrium were examined. The results are presented in Table 7.

Table 7: Error correction term and short-term coefficient estimates

Variable	Coefficient	Std. Error	t- statistic	Prob.
LOG_RT	0.001538	0.070518	0.021817	0.9827
LOG_BIST_100	-0.015615	0.017002	-0.918426	0.3614
LOG_CPI	-0.007162	0.007475	-0.958052	0.3412
ECM (t-1)	-1.970437	0.071947	-27.387.525	0.0000*

(*) indicates statistical significance at the 1% level.

The error correction term ECM(t–1) is negative with a value of –1.97 and statistically significant at the 1% level. This finding indicates that the system returned to balance in the long term and short-term deviations were quickly corrected. The coefficient being between –1 and –2 indicates that the system exhibits an over-adjustment process, potentially leading to a volatile recovery in the short term. Although the exchange rate has a positive coefficient, its short-term effect is not statistically significant. Similarly, the BIST-100 index and the CPI rate also do not have a statistically significant effects in the short term.

DISCUSSION AND CONCLUSION

Consumer confidence is an importent economic indicator that reflects individuals' assessments of current economic conditions and their expectations for the future. This indicator directly influences household tendencies regarding consumption, saving and borrowing, thereby playing a critical role in determining aggregate demand and overall economic growth. The level of confidence consumers have in the economy serves as a powerful signal that can shape the direction of economic activity in both the short and long term. For this reason, consumer confidence is closely monitored by policymakers and investors.

The aim of this study is to analyze the relationship between the consumer confidence index in Türkiye and main macroeconomic indicators—namely the exchange rate, the stock market index (BIST-100), and the consumer price index (CPI)—within the framework of both long-term cointegration and short- term dynamics. Data covering the period 2005–2024 were used in the study.

Prior to the analysis, the stationarity of the series was examined using the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests. The results indicated that the variables are stationary at different levels, either I(0) or I(1). Based on these findings, the ARDL (Autoregressive Distributed Lag) model was selected, as it allows for estimation with variables integrated at different levels, making it well-suited for this type of econometric analysis.

In this study, the ARDL (Autoregressive Distributed Lag) model was applied to analyze the short- and long-term relationships between the consumer confidence index and main macroeconomic indicators: the exchange rate, BIST-100 index and consumer price index (CPI). Optimal lag lengths were determined based on the Akaike Information Criterion (AIC) and the model was structered as ARDL(1, 1, 0, 0). The ARDL bounds test applied revealed the existence of a long-term cointegration relationship between the dependent variable (consumer confidence index) and the explanatory variables. The F-statistic exceeded the critical threshold at the 1% significance level, statistically validating the presence of a long-term equilibrium relationship among the variables. This finding indicates that consumer confidence tends to move in tandem with the selected macroeconomic indicators over time.

However, long-term coefficient estimates revealed that none of the explanatory variables had a statistically significant effect on consumer confidence. This suggests that, over the long term, the macroeconomic indicators examined in the study exert limited and statistically unreliable impacts on consumer confidence. The significance of only the constant term implies that the consumer confidence index stabilizes around a certain average level in the long term. When the coefficient values are examined, while the exchange rate

increases consumer confidence very limitedly, the effect of BIST- 100 index and inflation (CPI) is negative. However, all these effects do not carry reliability due to high p-values. Accordingly, no strong or causal long-term relationship was found between consumer confidence and the selected macroeconomic variables during the study period.

The short-term and error correction model (ECM) results are largely consistent with the long-term findings. Short-term coefficient estimates indicated that the explanatory variables did not have a statistically significant impact on consumer confidence in the short term. Nevertheless, the error correction term (ECM) was found to be statistically significant at the 1% level with a coefficient of –

1.97. This implies that the model possesses a strong adjustment mechanism, correcting deviations from the long-term equilibrium in each period. The fact that the absolute value of the coefficient exceeds 1 suggests an aggressive adjustment process, meaning the system tends to return rapidly to equilibrium, albeit with potential short-term fluctuations.

Based on the findings, it is concluded that financial market stability may have a short-term impact on consumer confidence. Therefore, investor and consumer sentiment should be taken into account by policymakers. In particular, the adverse short-term effects of exchange rate volatility on consumer confidence highlight the need for policies aimed at reducing currency fluctuations and stabilizing economic expectations. The lack of a statistically significant effect of inflation on consumer confidence underscores the importance of communication strategies that enhance public perception regarding price stability. Future studies are recommended to expand the model by incorporating additional variables such as interest rates, household income levels, and unemployment to achieve a more comprehensive analysis of the determinants of consumer confidence.

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The Impact of Selected Macroeconomic Indicators on Economic Growth: The Case of Türkiye (1295)

Mükerrem Can[1*], Özge Kuran[2]

[1] Dicle University, Graduate School of Natural and Applied Science, Department of Statistics, 21280, Diyarbakır, Türkiye
[2] Dicle University, Faculty of Science, Department of Statistics, 21280, Diyarbakır, Türkiye

*Corresponding author e-mail: canmukerrem@gmail.com

Abstract

Economic growth, one of the most important macroeconomic indicators of countries, has been the subject of numerous studies in the literature, with many researchers investigating the factors that influence it. Similarly, the aim of this study is to examine the effects of inflation, employment, tourism revenues, exports, and imports on economic growth in Türkiye. In this study, economic growth is represented by the chained volume index of gross domestic product (GDP); the export variable is measured by the export unit value index, and the import variable by the import unit value index. Other variables used in the study include the inflation rate, employment rate, and tourism revenues, all of which were obtained from the official website of the Türkiye Statistical Institute (TurkStat). To obtain more robust results, the natural logarithms of the series were taken, and their stationarity was examined using graphical methods and unit root tests. It was determined that the series were not stationary at their level values but became stationary after first differencing. In order to apply multiple linear regression analysis with the stationary series, the study examined whether there was a multicollinearity problem among the explanatory variables, whether the error terms were normally distributed, and whether there were issues of autocorrelation and heteroskedasticity. These assumptions were tested using appropriate statistical methods, and all variables were found to meet the required conditions. After confirming the model assumptions, a multiple linear regression analysis was conducted to examine the relationship between economic growth and the explanatory variables. The estimated model coefficients were found to be statistically significant. It was observed that, during the analyzed period, inflation and exports had a negative effect on economic growth, whereas tourism revenues, imports, and employment had a positive impact.

Keywords: *Economic Growth, Stationarity, Regression Analysis, Unit Root*

INTRODUCTION

Economic growth is defined as the increase in a country's real Gross Domestic Product (GDP) over a specified period and is conceptually regarded as the expansion of production capacity, reflected through the rising volume of goods and services generated within the economy (Düzgün, 2015).

Economic growth is commonly defined as the increase in real GDP compared to the previous year. GDP represents the monetary value of all final goods and services produced within a country's borders over a specific period. It serves as a measure of the value of the flow of goods and services in an economy. Economic growth can be categorized as real or nominal. When GDP is calculated at current prices, it reflects nominal growth; when calculated at constant prices, it reflects real growth (Eğilmez and Kumcu, 2011).

Real GDP measures the changes in the quantity of goods and services produced over time by valuing all goods produced in different periods at constant prices. In contrast, nominal GDP measures the value of final goods and services produced within a specific period using current prices of that period. However, due to price fluctuations, nominal GDP does not accurately reflect changes in the actual quantity of goods and services produced. Therefore, when comparing the volume of final goods and services produced across different years, real GDP is used as the primary indicator (Dornbusch et al., 2007).

In order to identify the real changes in the quantity of goods and services produced, it is necessary to adjust their prices for inflation. This is achieved by multiplying the quantities of goods and services produced in different years by the prices of a fixed base year, allowing for the calculation of the actual change in production volume. In Türkiye, GDP is periodically calculated by the Türkiye Statistical Institute (TurkStat) in both nominal and real terms. For the calculation of real GDP, a relatively recent method known as the chained volume index is employed (Deniz and Koç, 2019).

Economic growth is regarded as one of the most important economic indicators of social welfare and development. It is among the primary macroeconomic objectives pursued by both developed and developing countries. In recent years, interest in economic growth has increasingly focused on policies aimed at achieving long-term growth and ensuring its sustainability over time (Berber, 2017).

Differences in the level of development and the diversity of resources among countries lead to variations in their economic growth rates. Numerous macroeconomic indicators contribute to a country’s long-term economic growth, either individually or through mutual interaction, supporting both the realization and the sustainability of economic growth (Gallup et al., 1999).

Regarding some of the macroeconomic indicators included in this study that are believed to be related to economic growth;

Inflation is a macroeconomic indicator that leads to a decrease in the real value of money due to a continuous increase in the general price level of consumer goods and services over time. Typically expressed as a rate, inflation is tracked through various indices reflecting different dimensions. Price changes encountered by consumers are measured through the Consumer Price Index (CPI), while the costs faced by producers are measured using the Producer Price Index (PPI). The overall price level in the economy is monitored via the deflator (Düzgün, 2015). An increase in the inflation rate leads to a reduction in individuals' purchasing power due to the rise in the general price level. On the other hand, a decrease in inflation is associated with a slowdown in the rate of price increases, an increase in real income, and consequently, an improvement in both purchasing power and employment levels (Eygü, 2018).

The relationship between economic growth and inflation remains a subject of ongoing debate within the economics literature, with no definitive consensus having been reached. While it was widely accepted until the 1980s that inflation could act as a supportive factor for economic growth, more recent empirical studies have increasingly highlighted the adverse effects of high inflation on growth performance. This shift in perspective reflects a growing recognition of the potential distortions inflation can introduce into resource allocation, investment decisions, and overall macroeconomic stability (Karaca, 2003). The depreciation of the national currency leads to a decline in the value of certain goods, adversely affecting not only producers and sellers but society as a whole. This process underscores the emergence of inflation as a significant socioeconomic issue, with wide-ranging implications for income distribution, consumption patterns, and overall economic welfare (Orhan, 1995). The formation of negative expectations regarding future inflation constitutes one of the key factors adversely affecting economic growth. Such expectations can lead to inefficient allocation of

economic resources, contribute to the depreciation of the national currency, and result in a decline in net exports. Collectively, these dynamics undermine macroeconomic stability and hinder sustainable economic performance (Berber and Artan, 2004).

Export, by expanding the volume of production through the foreign trade multiplier, is regarded as one of the fundamental drivers of economic growth. Provided that domestic demand remains constant, an increase in exports leads to higher levels of production and employment (Ağayev, 2011). Foreign exchange revenues generated through exports contribute to meeting a country's foreign currency needs by facilitating the import of intermediate goods and raw materials that are essential for production but cannot be sourced domestically. In addition, export-oriented production processes promote an expansion in economic output by enhancing the productivity of production factors through the adoption of new knowledge and technologies (Yapraklı, 2007).

Tourism, as an invisible export item, contributes to foreign exchange earnings under the category of service exports, thereby raising national income. The relationship between tourism and economic growth is largely based on the literature examining the impact of foreign trade on growth. In the balance of payments, tourism activities, which fall under the services section of the current account, have an export-like effect due to the foreign exchange revenues they generate. An increase in tourism revenues boosts foreign currency inflows into the sector, which, in turn, enhances competitiveness and productivity. This leads to the realization of economies of scale, resulting in increased employment, production, and ultimately, economic growth (Özcan, 2015).

For tourism to contribute to a country's long-term economic growth, it is essential that foreign exchange earnings exceed foreign exchange expenditures. Developments in the tourism sector promote competition both internationally and domestically, enabling more efficient allocation of economic resources. Moreover, investments in infrastructure within tourism regions (such as airports, water, sewage, communication, and energy systems) not only support tourism activities but also provide socio-economic benefits by improving the quality of life for local populations (Kara et al., 2012).

Employment refers to the total number of individuals who are currently employed as well as those actively seeking work and willing to work, while the labor force encompasses both employed and unemployed individuals. Full employment is a situation in which all individuals seeking employment at the prevailing wage rate can find work. However, the inability of all individuals who desire to work at the prevailing wage to find employment represents a significant issue faced by the economy. Full employment also includes frictional unemployment (Parasız, 2010).

There is a direct relationship between employment and GDP. An increase in a country's real GDP leads to a rise in the level of production, which in turn increases the demand for production factors, particularly labor. The employment rate is calculated by dividing the number of individuals employed by the working-age population (ages 15-64). Changes in employment levels are important indicators for assessing developments in national income (Eğilmez and Kumcu, 2011).

The relationship between imports and economic growth has been extensively explored in the economic literature and explained through various theories. This relationship can generate both positive and negative effects, with the magnitude and direction of these effects varying depending on the economic conditions. Technological products and machinery imported from developed countries may provide domestic producers with access to new technologies, thereby enhancing productivity and contributing to long-term economic growth. Similarly, imports of raw materials and intermediate goods, which are necessary for domestic production, can stimulate the expansion of domestic production capacity, thereby supporting economic growth. Furthermore, imports intensify competition in the domestic market, compelling local firms to improve their productivity, which can lead to lower production costs

and the production of higher-quality goods. As a result, economic growth may be facilitated.

An increase in imports, in the absence of sufficient exports, can lead to a widening trade deficit. This situation may raise the country's external debt, potentially exerting a negative impact on long-term economic growth. Excessive imports can weaken the competitiveness of domestic industries, particularly as low-cost imports may cause local producers to lose market share. Consequently, this can result in a decline in production levels and a reduction in employment. Additionally, a rise in imports increases the demand for foreign exchange, which may lead to depreciation of the local currency. This, in turn, can complicate the repayment of external debt and place further pressure on economic growth.

For developing countries, particularly economies like Türkiye, imports can often serve as a catalyst for economic growth. These countries, due to their reliance on technology and capital goods, may find that such imports enable the modernization of production processes and enhance productivity. However, the potential negative impacts of excessive imports on domestic production must also be considered.

In conclusion, the relationship between imports and economic growth is complex and multifaceted. Imports can foster technological progress, the expansion of production capacity, increased productivity, and improved living standards. However, excessive imports may negatively impact the growth process by leading to a decline in domestic production, an expansion of the trade deficit, and the onset of currency crises. The net effect of imports on economic growth varies depending on the country’s economic structure, the nature of the imports, and its foreign trade policies.

In this study, the effects of inflation, exports, imports, tourism revenues, and employment on economic growth in the Turkish economy, covering the period from 2013-Q1 to 2024-Q4, are examined using multiple linear regression analysis in time series.

Literature Review

In their influential study, Khan and Senhadji (2001) investigated the relationship between inflation and economic growth across 140 developed and developing countries over the period 1960–1998. Utilizing the Ordinary Least Squares (OLS) estimation technique, the authors identified a statistically significant negative correlation between inflation and growth. Their analysis also revealed the presence of threshold effects: inflation rates above approximately 11–12% for developed economies and 1–3% for developing economies were associated with a notable decline in economic growth.

Karaca (2003), examined the relationship between economic growth and inflation in Türkiye, analyzing data from the period 1987-2002 using time series analysis. The study, which applied regression and Granger causality analysis, found a negative relationship between growth and inflation, asserting that this relationship runs from inflation to growth.

Berber and Seyfettin (2004), examined the relationship between inflation and economic growth in Türkiye for the period 1987:1-2003:2 using quarterly data on CPI, WPI, and GDP, employing the ECM method and Granger causality test. The analysis revealed a negative relationship between economic growth and inflation, indicating that a 10% increase in inflation led to a 1.9% decrease in economic growth, and confirmed the existence of a negative relationship from inflation to economic growth.

In a study by Demirhan (2005), the relationship between economic growth and exports in Türkiye was analyzed using quarterly data on exports and economic growth from 1990Q1 to 2004Q1. The study employed cointegration and the vector error correction model (VECM) as the analytical methods. The results revealed a unidirectional causality running from exports to economic growth, and according to

the cointegration equation, exports were found to have a positive impact on economic growth in the long run.

In a study by Aslan (2008), the relationship between economic growth and tourism revenues in Türkiye was examined using data from 1992Q1 to 2007Q2, employing the Johansen cointegration test and the Granger causality test as the analytical methods. The analysis revealed that tourism revenues supported economic growth during the analyzed period, and a positive relationship was found between the two variables.

Saraç (2009), using quarterly data of CPI, PPI, and GDP for Türkiye between 1988 and 2007, examined the relationship between economic growth and inflation employing the ARDL model cointegration approach (Bounds Testing). The study identified a negative relationship between economic growth and CPI both in the short and long term, while a negative relationship was found between economic growth and PPI in the short term.

In a study by Muratoğlu (2011), the relationship between employment and economic growth in Türkiye was examined for the period from 2000Q1 to 2011Q4 using GDP and employment data. The study employed the Engle-Granger cointegration method and Granger causality tests for the analysis. The results indicated that there was no significant relationship between economic growth and employment during the studied period.

Altuntepe and Güner (2013), examined the relationship between employment and economic growth in Türkiye using data from 1988 to 2011, employing the Engle-Granger cointegration method. The study constructed and analyzed two separate models: one for the relationship between employment and economic growth, and the other for economic growth and employment. The results indicated that while employment has an impact on economic growth, only the service sector, on a sectoral basis, was found to significantly influence economic growth.

Another study that examines the impact of exports and imports on economic growth in Türkiye is that of Saraç (2013), who analyzed quarterly data from 1989Q2 to 2011Q4 using a nonlinear econometric approach. The findings indicate that both exports and imports have a positive effect on economic growth during both expansionary and contractionary periods.

Bozkurt and Topçuoğlu (2013), investigated the relationship between economic growth and tourism revenues in Türkiye using data from 1970 to 2011, applying the Engle-Granger cointegration test and the error correction model (ECM) as their analytical methods. The study found a bidirectional relationship between tourism revenues and economic growth in both the short and long run.

Another study examining the relationship between economic growth and inflation in Türkiye is the work of Süleymanov and Nadirov (2014), which employed the Granger causality test. The study utilized quarterly data on the CPI and GDP for the period 2003-2013, and the analysis revealed a unidirectional relationship from economic growth to inflation.

Özcan (2015), analyzed the relationship between tourism revenues and economic growth in Türkiye using data from 1963 to 2010, applying both asymmetric and symmetric causality methods. The study found a causal relationship between tourism revenues and economic growth, with causality running from tourism revenues to economic growth.

Korkmaz and Aydın (2015), investigated the relationship between foreign trade and economic growth in Türkiye using quarterly data on GDP, exports, and imports for the period 2002Q1–2014Q2. Employing the Granger causality test as the methodological approach, the study revealed a bidirectional causality between imports and economic growth, while no causal relationship was found between economic growth and exports.

In the study conducted by Elhdiy et al. (2015) on Egypt, which examined the relationship between economic growth and employment, data from the period of 2006Q1-2013Q2 was used. The analysis employed the Dickey-Fuller (DF) unit root test and the standard Granger causality test. The findings revealed that there was no short-term relationship between economic growth and employment, but a long-term relationship was observed.

Arvas and Torusdağ (2016), analyzed the relationship between imports, exports, and economic growth in Türkiye using data from 1987 to 2015 and employing the OLS analysis method. The study identified a positive relationship between imports and economic growth, while no significant relationship was found between exports and economic growth.

Özpençe (2016), using data from the period between 2003:1 and 2015:4, analyzed the relationship between economic growth and inflation through the Vector Autoregressive Model (VAR) and the Granger causality test, finding a unidirectional causality from economic growth to inflation.

MATERIAL AND METHODS

In this study, the seasonally and calendar-adjusted GDP chained volume value (in thousands of Türkiye lira, with 2009 as the base year) is used to represent economic growth. Inflation is measured as the percentage change compared to the same quarter of the previous year, using the 2003 base year. Tourism revenues are represented in thousands of US dollars, exports are measured using the export unit value index (with 2015 as the base year), imports are represented using the import unit value index (with 2015 as the base year), and employment is represented by employment rates.

The data for GDP, employment rates, and tourism revenues were sourced quarterly from the official website of the TurkStat. The inflation rate, export, and import unit value index data were also obtained monthly from TurkStat's official website and then converted into quarterly data using the geometric mean transformation. Furthermore, to improve the accuracy of the results, the natural logarithms of all variables used in the study were taken for further analysis.

Table 1: Description of Variables

Variable	Definition
gsyh	Chained Volume Measure of Gross Domestic Product (base year 2009)
ihBDE	Export Unit Value Index (base year 2015)
ist	Employment Rate
itBDE	Import Unit Value Index (base year 2015)
TGeliri	Tourism Revenues
enf	Inflation Rate (base year 2003)

The expressions and definitions in the table are at the level of variables, with the notation 'L' used for series whose natural logarithms have been taken, and 'Δ' used for series from which the difference has been taken

Multiple Linear Regression Analysis

In econometric analyses, the accurate collection of data related to dependent and independent variables from reliable sources, along with the appropriate organization of the data in line with the model specifications, is a critical factor that directly influences the consistency and reliability of the predictions made (Gujarati, 2003).

The time series refers to a set of observations of a variables values at different points in time (Akın, 2009). In this study, time series data is utilized. The analysis of the data in this study is conducted using the multiple linear regression analysis method.

Regression analysis is used to examine the relationship between two or more variables. While the relationships between two variables are investigated using simple regression analysis, relationships involving multiple variables are addressed using multiple regression analysis. In such analyses, the mathematical form of the relationship between variables is sought. The relationship can be either linear or take a non-linear form (Çakıcı et al., 2020).

In regression analysis, the variables need to be defined as dependent and independent variables. The dependent variable is the one that is explained by the independent variables. In this study, the independent variables are the inflation rate, employment rate, export unit value index, import unit value index, and tourism revenues, while the dependent variable is the GDP variable

The multiple linear regression model that will be included and discussed in the study;

$$gsyh_t = \beta_0 + \beta_1 enf + \beta_2 ist + \beta_3 ihBDE + \beta_4 itBDE + \beta_5 TGeliri + \varepsilon_t$$

will be as follows.

The definitions of the variables presented here are provided in Table 1, where β_0, β_1, β_2, β_3, β_4 and β_5 represent the unknown parameters of the model to be estimated, and ε_t is the error term, which is independently and normally distributed with zero mean and constant variance (Akın, 2009). In this study, the estimation of the parameters is carried out using the Eviews econometrics software.

In addition to these assumptions regarding the error term, it is also essential that there is no multicollinearity problem in multiple linear regression analysis. The absence of multicollinearity is characterized by correlation coefficients between the independent variables being either zero or very close to zero (Arı and Önder, 2013).

The Concept of Stationarity and Stationarity Analysis

Time series data generally do not exhibit the necessary characteristics for stationarity. If a series contains factors such as an increasing or decreasing trend, cyclical and seasonal fluctuations, these elements cause the mean of the series to change. In such cases, the series loses its stationarity property, and autocorrelations may deviate significantly from zero, resulting in notable differences (Sevüktekin and Nargeleçekenler, 2007).

In analyses performed with non-stationary time series, a number of issues can arise. One of the primary problems is that the t-test values, which indicate the significance level of the model parameters, and the R^2 value, which reflects the explanatory power of the independent variable on the dependent variable, may yield erroneous results. This situation is referred to as spurious regression. Therefore, future forecasts based on non-stationary data are likely to be unreliable (Sevüktekin and Nargeleçekenler, 2007).

For time series to be effectively modeled, the series must first be rendered stationary (Ekmekçi, 2016). Furthermore, since a large portion of probability theory for time series is concerned with stationary series, non-stationary series must be transformed into stationary ones for these theories to be applicable in time series analysis. In the literature, graphical analysis, correlogram tests, and unit root tests are used to investigate stationarity. In practice, the unit root test is the most commonly applied method.

Correlogram Test

Through correlograms, sample autocorrelations, partial correlations, and Q statistics can be calculated together. This process can be particularly used to determine whether the series generates a statistically significant coefficient for a given number of lags (Sevüktekin and Nargeleçekenler, 2007).

A correlogram presents the predicted or calculated values of the autocorrelation function for a specific lag in the column labeled AC. If a series demonstrates a continuous upward or downward trend, rather than fluctuating around a specific mean, and the correlogram of the autocorrelation function initially shows a high value that decreases over time, this indicates the series is non-stationary. The closer the AC column approaches zero, the more the stationary nature of the series becomes evident. The presence of statistically significant autocorrelations indicates that the series is non-stationary (Sevüktekin and Nargeleçekenler, 2007).

Stationarity Analysis: Unit Root Test

Unit root tests are among the fundamental methods used to determine whether a time series is stationary or to identify its degree of stationarity (Gujarati, 2003). Time series generally have a unit root. An important criterion in determining the stationarity properties of time series is the number of unit roots present in the series. The number of unit roots in a non-stationary time series is equal to the number of differences required for the series to achieve stationarity. In this context, if a time series becomes stationary after being differenced once, it is defined as being integrated of order one, or I(1). More generally, if d differences are needed for the series to become stationary, the series is said to be integrated of order d and is represented as I(d). On the other hand, series that do not require differencing and are stationary by definition are classified as I(0). This classification serves as a fundamental reference for analyzing time series using appropriate modeling methods (Torun, 2015).

Dickey-Fuller (DF) and Augmented Dickey-Fuller (ADF) Test

The first empirical tests for determining stationarity in time series were developed by Fuller (1976) and Dickey and Fuller (1979). These tests were specifically designed to test whether the series contains a unit root. In the DF test, the null hypothesis posits that the series contains a unit root, meaning it is non-stationary; in contrast, the alternative hypothesis represents the assumption that the series is stationary. This test has been widely accepted as a fundamental method for statistically determining the presence of randomness or trends in time series data (Harris, 1995). The analysis of the long-term characteristics of time series can be conducted by examining the effect of the series' previous period values on the current period. In this context, presenting the relationship between the current value of the series and its past values through regression analysis is a fundamental step in understanding the structural dynamics of the series. Unit root tests, developed within this framework, are frequently used to determine the stationarity level of the series and to lay the groundwork for the appropriate modeling process (Torun, 2015).

The equation showing the relationship between the variable Y_t and its previous period Y_{t-1} is as follows;

$$Y_t = \phi_1 Y_{t-1} + u_t$$

is expressed by the equation. Here "u_t", represents the residual term, and the model expresses a first-order autoregressive process, AR(1). If the coefficient ϕ_1 found to be equal to one, the unit root problem, meaning non-stationarity, arises. In this case, the model;

$$Y_t = Y_{t-1} + u_t$$

takes the form and means that the economic values and characteristics of the previous period of the series remain in the system as they are, influencing the long-term behavior of the series. If the coefficient ϕ_1 is found to be less than one, it indicates that shocks occurring in the past periods of the series will continue to have an effect for a certain period, but their impact will gradually decrease and eventually dissipate after a short time (Tanrı, 2005).

The equation $Y_t = Y_{t-1} + u_t$ can also be written in another form as follows:

$$\Delta Y_t = (1 - \phi_1) Y_{t-1} + u_t$$

$$\Delta Y_t = \delta Y_{t-1} + u_t$$

Here;

$$\Delta Y_t = Y_t - Y_{t-1} \text{ ve } \delta = 1 - \phi_1$$

In this case, the null hypothesis is formulated as H_0: $\delta = 0$. Because $\phi_1 = 1$ when $\delta = 0$ and thus;

$$\Delta Y_t = Y_t - Y_{t-1} = u_t$$

the first difference of the Y_t series will be stationary.

The hypotheses to be set are based on the model as follows H_0: $\phi_1 = 1$ or H_0: $\delta = 0$ and the respective hypotheses indicate the situation of non-stationarity. For this purpose, the test applied is the DF test. In this test, the well-known t-statistic is referred to as the "τ (tau)" statistic (DF-test statistic), and due to the fact that the calculated t-value does not follow the t-distribution even in large samples, the t-test cannot be applied in the evaluation of τ Statistics (Enders, 2004). Therefore, the τ statistic is compared with the critical values of MacKinnon. The critical values of τ (tau) statistics are tabulated by Dickey and Fuller through Monte Carlo simulations (Dickey and Fuller, 1979). If the absolute value of the τ statistic (|τ|) is smaller than the absolute value of the MacKinnon critical value, the null hypothesis H_0 cannot be rejected, and it is concluded that the series is non-stationary, meaning it contains a unit root (Ertek, 1996). Additionally, by examining the corresponding probability (p-value) of the obtained test statistic, a decision can be made whether to reject or accept the hypothesis.

The main regression models used in the DF test are as follows:

- Model without a constant term;

$$\Delta Y_t = \delta Y_{t-1} + u_t$$

- Model with a constant term;

$$\Delta Y_t = \beta_0 + \delta Y_{t-1} + u_t$$

- Model with a constant term and trend;

$$\Delta Y_t = \beta_0 + \beta_1 t + \delta Y_{t-1} + u_t$$

Here, "*t*" is the time variable. If the error term u_t is serially dependent, the regression model to be used is;

$$\Delta Y_t = \beta_0 + \beta_1 t + \delta Y_{t-1} + \alpha_i \sum_{i=1}^{m} \Delta Y_{t-i} + u_t$$

takes the form, and if the DF test is applied to this model, it is called the Augmented Dickey-Fuller (ADF) test (Gujarati, 2003).

Differencing Process

In time series analysis, predicting future values of a random variable based on past observations is a forecasting process that leverages the autocorrelation structure of the series (Sevüktekin and Nargeleçekenler, 2007). The simplest form of difference equations in time series modeling is the random walk process, where each value is obtained by adding a random shock to the previous value (Kutlar, 2005). For example, in this study, if the value of GDP in any given month is assumed to be zero, the model representing the GDP is based on the foundation of a stochastic difference equation:

$$Y_t = Y_{t-1} + e_t$$

$$Y_t - Y_{t-1} = e_t$$

$$\Delta Y_t = e_t$$

Here "Y_t", represents the GDP value in month t, and, "e_t" is the random error with a mean of zero.

The equations used in time series analysis are based on the mathematics of difference equations. The first difference of a series is expressed in the form of;

$$\Delta Y_t = Y_t - Y_{t-1}$$

The second difference of the series refers to taking the difference of the differenced series once again.

Key Assumptions and Desired Properties in Regression Analysis

In order for regression analysis to produce reliable and valid results, the error terms in the model must satisfy certain assumptions. In this context, it is commonly assumed that the error terms have a mean of zero, exhibit constant variance (homoskedasticity), and follow a normal distribution. Additionally, the independence of the error terms is of critical importance. This is particularly relevant in models based on time series data, where the issue of autocorrelation should not be overlooked. The presence of a sequential relationship among error terms can negatively impact the model's predictive power, as well as lead to incorrect estimation of standard errors for the coefficients. Consequently, this may result in invalid statistical inferences, such as t-tests and confidence intervals. For these reasons, it is essential to test whether the aforementioned assumptions are met after the regression model has been constructed (Kadılar and Çekim, 2024).

Normality Assumption and Testing of Model Error Terms

In time series analysis, the normality test is applied to the error series and is based on the skewness and kurtosis coefficients. In this study, the Jarque-Bera test is employed to assess the normality of the model's error terms. The test is calculated as follows:

$$JB = \frac{T-p}{6}(S^2 + \frac{K^2}{4})$$

This test is named after the econometricians A.K. Bera and C.M. Jarque, who first introduced it (Erdoğan, 2016). In this formulation, "*T*" denotes the number of observations, while "*p*" indicates the number of parameters—two in the case of a normal distribution, namely the mean and the variance. "*S*" represents the skewness coefficient, and "*K*" denotes the kurtosis coefficient. It should be noted that in the calculation of kurtosis, the value 3 is subtracted to obtain excess kurtosis.

Hypotheses to be tested;

H_0: There is no difference between the distribution of the series and a normal distribution (The series follows a normal distribution).

H_1: The distribution of the series differs from the normal distribution (The series does not follow a normal distribution).

The calculated JB statistic is compared to the χ^2 value with p degrees of freedom. If the calculated test statistic is smaller than the critical value from the table, the null hypothesis is accepted, and it can be concluded that the series follows a normal distribution at the specified significance level (Kadılar and Çekim, 2024).

Additionally, a decision regarding the hypothesis can also be made by examining the probability values corresponding to the obtained JB test statistic.

Autocorrelation Assumption and Testing

One of the key assumptions for ensuring the validity of regression analysis is the absence of autocorrelation among the error terms in the model. For this assumption to hold, it is expected that the predicted error terms be independent and sequentially unrelated. In other words, each error term should represent a random deviation specific to that period. However, when the error terms are statistically related to the error values from previous periods, an autocorrelation problem arises. This situation not only violates the model's assumption of independence but also leads to incorrect estimation of the standard errors of the regression coefficients, thus compromising the validity of statistical tests (Kadılar and Çekim, 2024).

Various factors can contribute to the emergence of autocorrelation. The most prominent of these is the selection of an inappropriate functional form for the model, such as incorrectly modeling nonlinear relationships between variables as linear. Additionally, the omission of significant independent variables that effectively explain the variance of the dependent variable (i.e., model misspecification) and the presence of high multicollinearity among the independent variables can trigger the autocorrelation problem. The presence of autocorrelation in the model violates the assumed independence of the error terms. This results in the standard errors of the regression coefficients being overestimated, leading to lower absolute values for t-statistics. Consequently, the rejection of null hypotheses becomes more difficult, and the statistical significance of the model can be questioned (Ünver et al., 2011).

In this study, the presence of autocorrelation among the error terms will be examined using the Breusch-Godfrey LM test.

Hypotheses;

H_0: There is no autocorrelation among the model’s error terms.

H_1: There is autocorrelation among the model’s error terms.

he hypothesis will be tested, and the decision to accept or reject the null hypothesis will be based on the computed p-value from the test.

Heteroscedasticity (Changing Variance) Assumption and Testing

The problem of non-stationarity in a time series is generally addressed through differencing. However, in some cases, this method may not be sufficient. Specifically, some time series may be stationary in terms of their mean, yet their variances may exhibit variability over time. In other words, while the mean of the series remains constant, its variability (variance) may increase or decrease over time. In such cases, differencing may not be adequate to achieve stationarity. For such series, particularly those that are 'heteroscedastic'—i.e., with changing variances—more advanced transformation techniques and tests may be necessary for proper modeling (Kadılar and Çekim, 2024).

In this study, the Harvey heteroscedasticity test will be used to check whether the assumption of heteroscedasticity is met. The theoretical steps for applying the Harvey test are as follows;

- Creation of the initial model and application of regression.
- Obtaining the residuals from this regression and calculating the squared residuals
- Using a new regression model where the squared residuals are the dependent variable, and the other independent variables are included in the regression
- Finding and comparing the log-likelihood values of the first and second models, performing a likelihood ratio test, and calculating the corresponding p-value.

- A decision regarding the presence of changing variance is made based on the computed p-value.

he hypotheses to be tested for detecting the issue of changing variance are as follows;

H_0: There is no changing variance among the error terms.

H_1: There is changing variance among the error terms.

Assumption of Multicollinearity

The presence of a linear or near-linear relationship among the explanatory variables in a model is referred to as multicollinearity. This issue is particularly common in time series data due to the influence of trends. In such cases, the problem of multicollinearity can often be addressed by revising the model and removing theoretically insignificant variables, thereby initiating a new modeling process (Gujarati and Porter, 2009).

In this study, the presence of multicollinearity will be examined by analyzing the Variance Inflation Factor (VIF) values. VIF is a quantitative measure commonly used to detect multicollinearity problems. In a regression model, high correlation among independent variables leads to inflated standard errors of the regression coefficients, which in turn reduces the reliability of the parameter estimates (Wooldridge, 2016).

The VIF is calculated as;

$$VIF_j = \frac{1}{1 - R_j^2}$$

Where "VIF_j", is the variance inflation factor for the j-th independent variable and "R_j^2", is the coefficient of multiple determination obtained from regressing that variable on all other independent variables.

The Significance Test of the Regression Coefficients

One of the crucial steps in constructing a model that appropriately fits the series is ensuring that the model coefficients are statistically significant.

The hypotheses to test the significance of the regression coefficients are as follows:

$$H_0: \beta = 0$$

$$H_1: \beta \neq 0$$

The test statistic to be used is the t-test, with the formula:

$$t = \frac{b - \beta}{Se(\beta)}$$

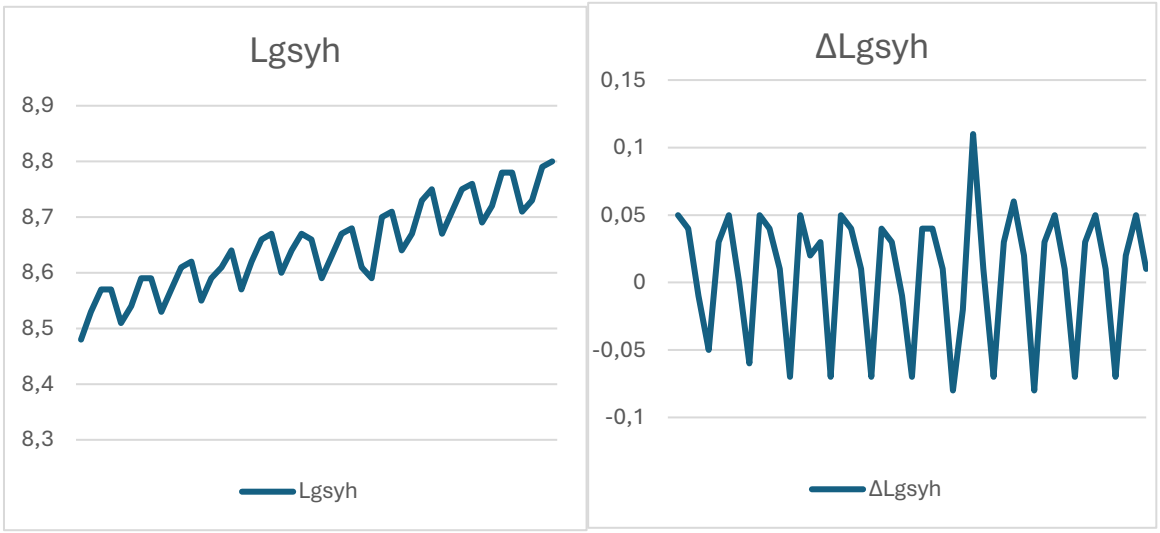
The computed value is then compared with the critical value from the table, and the hypotheses are interpreted accordingly (Kadılar and Çekim, 2024). Additionally, in statistical software packages, the decision is made by comparing the calculated p-value for the coefficient with the significance level, and the hypothesis is accepted or rejected accordingly.

RESULTS

Table 2: Descriptive statistics for the variables

Definition	**Variables**					
	Lgsyh	**Lenf**	**List**	**LTGeliri**	**LihBDE**	**LitBDE**
Number of Observation	48	48	48	48	48	48

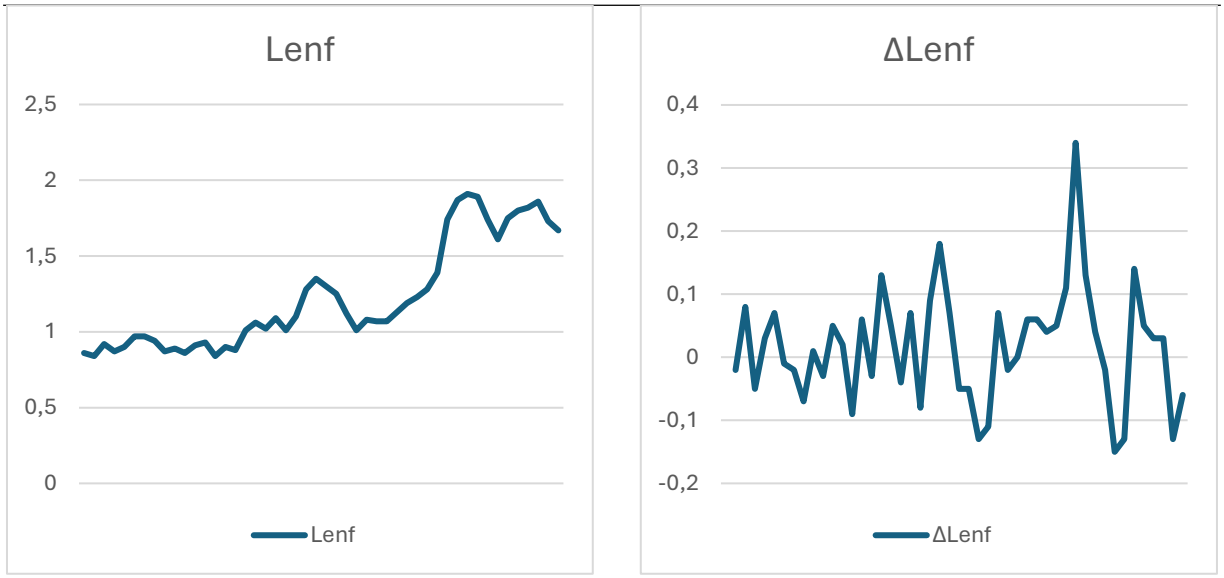
Mean	8,6470	1,2249	1,6638	6,9112	2,0170	2,0496
Std. Error of the Mean	,01158	,05129	,00281	,04162	,00509	,00957
Median	8,6399	1,0860	1,6663	6,9298	2,0109	2,0241
Mode	8,48[a]	,84[a]	1,62[a]	5,81[a]	1,97	1,95[a]
Std. Deviation	,08023	,35537	,01948	,28833	,03529	,06629
Variance	,006	,126	,000	,083	,001	,004
Skewness	,064	,783	-,321	-1,044	,068	,298
Std. Error of the Skewness	,343	,343	,343	,343	,343	,343
Kurtosis	-,728	-,878	-,453	3,022	-1,693	-1,064
Std. Error of the Kurtosis	,674	,674	,674	,674	,674	,674
Range	,32	1,07	,08	1,55	,10	,24
Minimum	8,48	,84	1,62	5,81	1,96	1,95
Maximum	8,80	1,91	1,70	7,37	2,07	2,19
Toplam	415,06	58,79	79,86	331,74	96,82	98,38



(a) **(b)**

Figure 1: Time series plots of the Lgsyh series level value and its first differenced series.

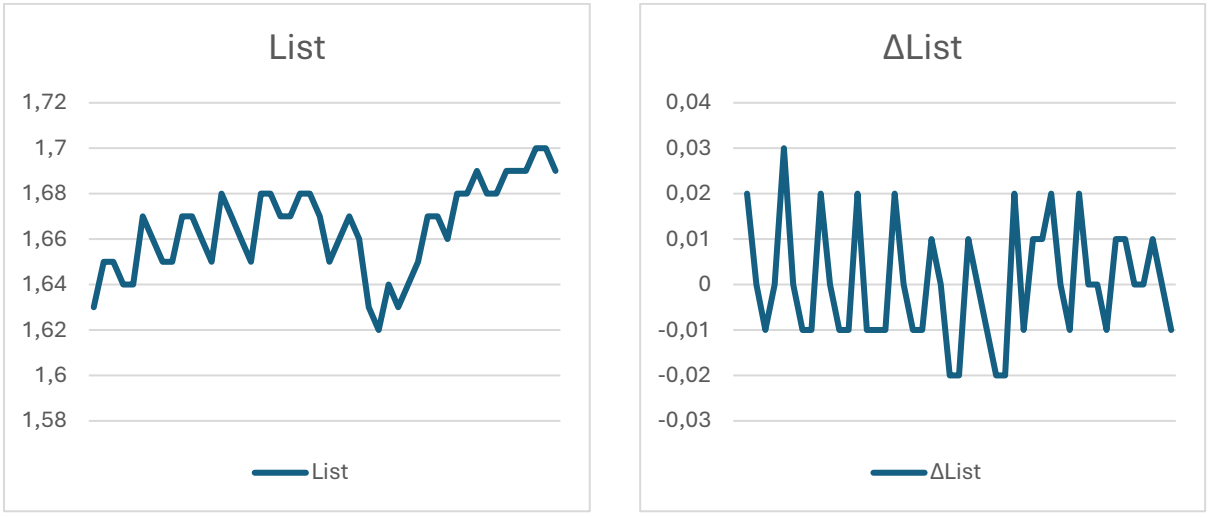
The time series plots of the GDP series for the period between 2013Q1 and 2024Q4, showing both the level value and the first differenced series, are presented in Figures 1a and 1b. Upon examining the plots, it can be observed that the series at the level value exhibits a trend, whereas the differenced series appears to move around a mean.



(a) **(b)**

Figure 2: Time series plots of the Lenf series level value and its first differenced series.

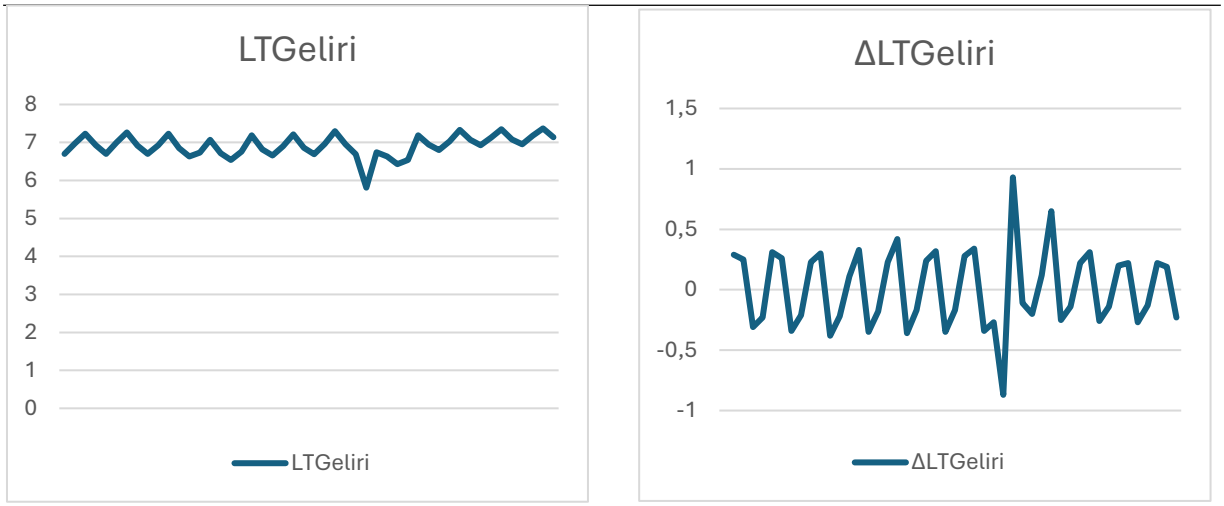
The time series plots of the inflation series for the period between 2013Q1 and 2024Q4, showing both the level value and the first differenced series, are presented in Figures 2a and 2b. Upon examining the plots, it can be observed that the series at the level value exhibits a trend, whereas the differenced series appears to move around a mean.



(a) **(b)**

Figure 3: Time series plots of the List series level value and its first differenced series.

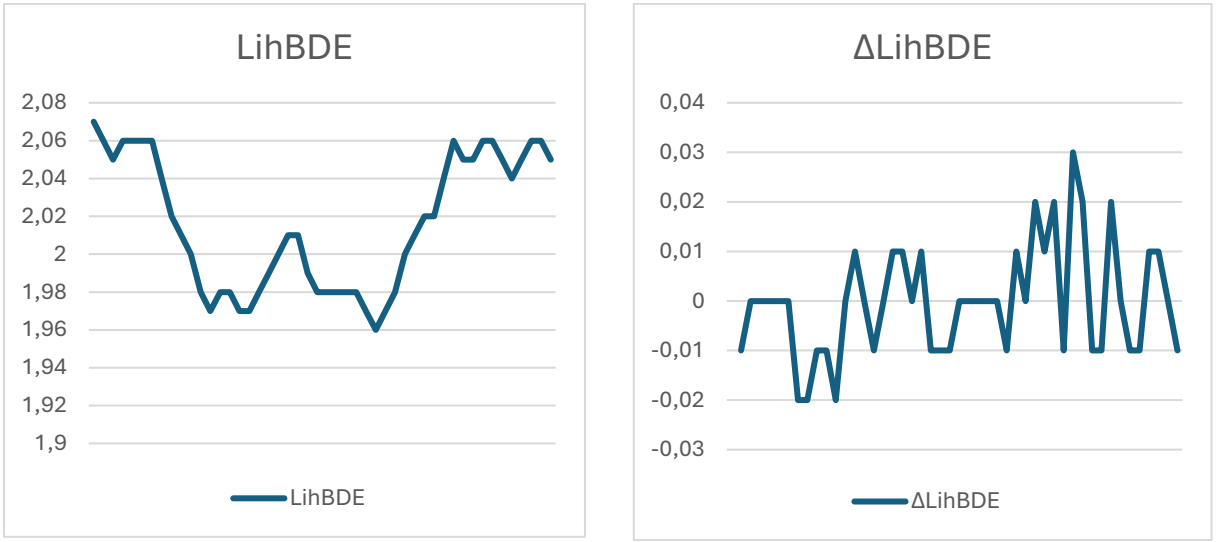
The time series plots of the employment series for the period between 2013Q1 and 2024Q4, showing both the level value and the first differenced series, are presented in Figures 3a and 3b. Upon examining the plots, it can be observed that the series at the level value exhibits a trend, whereas the differenced series appears to move around a mean.



(a) **(b)**

Figure 4: Time series plots of the LTGeliri series level value and its first differenced series.

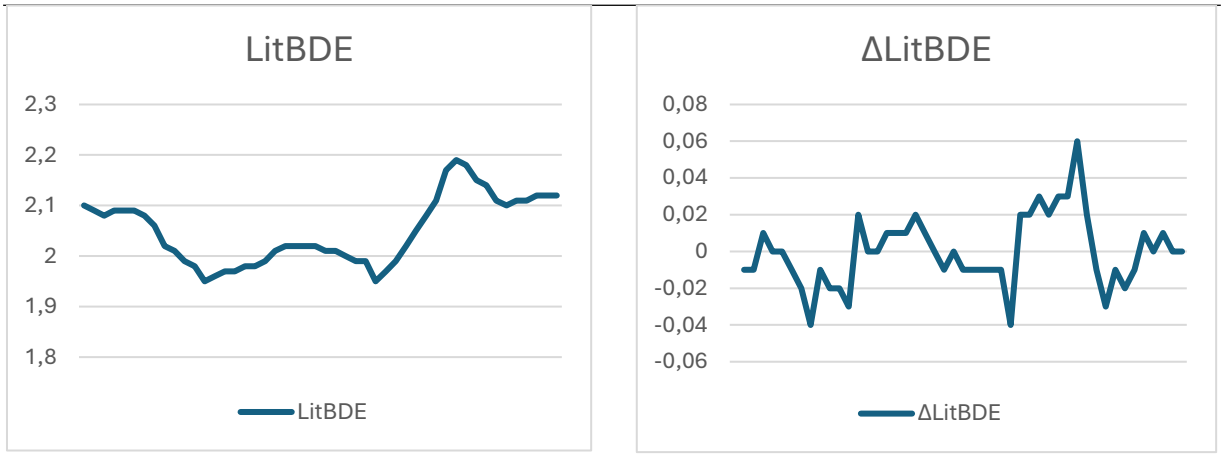
The time series plots of the tourism income series for the period between 2013Q1 and 2024Q4, showing both the level value and the first differenced series, are presented in Figures 4a and 4b. Upon examining the plots, it can be observed that the series at the level value exhibits a trend, whereas the differenced series appears to move around a mean.



(a) **(b)**

Figure 5: Time series plots of the LihBDE series level value and its first differenced series.

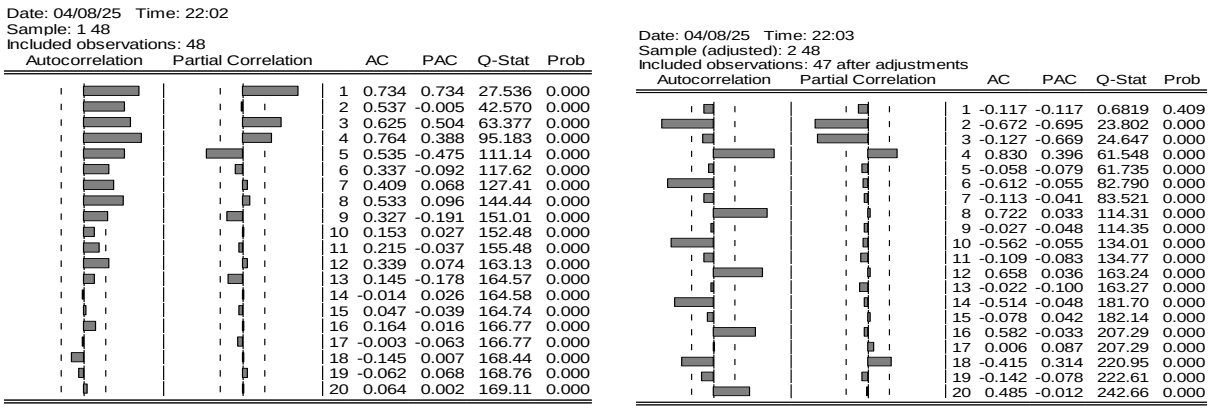
The time series plots of the export unit value index series for the period between 2013Q1 and 2024Q4, showing both the level value and the first differenced series, are presented in Figures 5a and 5b. Upon examining the plots, it can be observed that the series at the level value exhibits a trend that first decreases and then increases, whereas the differenced series appears to move around a mean.



(a) **(b)**

Figure 6: Time series plots of the LitBDE series level value and its first differenced series.

The time series plots of the import unit value index series for the period between 2013Q1 and 2024Q4, showing both the level value and the first differenced series, are presented in Figures 6a and 6b. Upon examining the plots, it can be observed that the series at the level value exhibits a trend that first decreases and then increases, whereas the differenced series appears to move around a mean.



Date: 04/08/25 Time: 22:02
Sample: 1 48
Included observations: 48

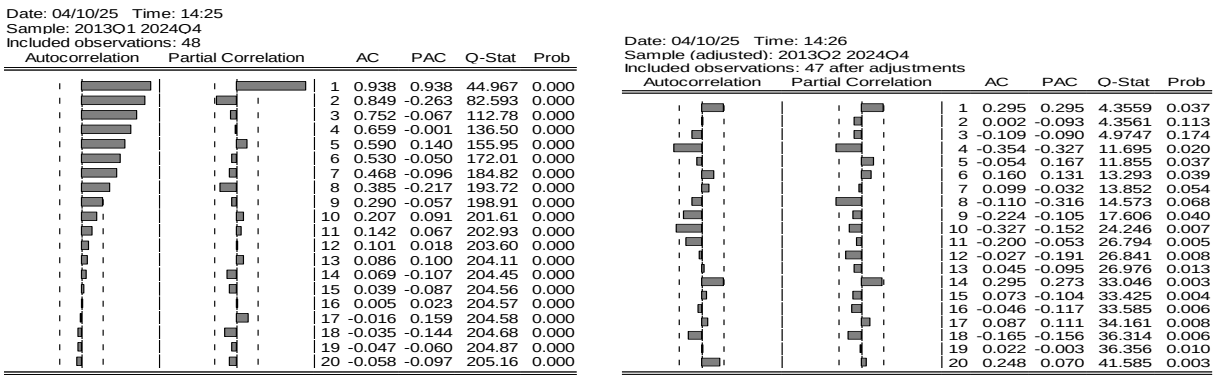
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.734	0.734	27.536	0.000
		2	0.537	-0.005	42.570	0.000
		3	0.625	0.504	63.377	0.000
		4	0.764	0.388	95.183	0.000
		5	0.535	-0.475	111.14	0.000
		6	0.337	-0.092	117.62	0.000
		7	0.409	0.068	127.41	0.000
		8	0.533	0.096	144.44	0.000
		9	0.327	-0.191	151.01	0.000
		10	0.153	0.027	152.48	0.000
		11	0.215	-0.037	155.48	0.000
		12	0.339	0.074	163.13	0.000
		13	0.145	-0.178	164.57	0.000
		14	-0.014	0.026	164.58	0.000
		15	0.047	-0.039	164.74	0.000
		16	0.164	0.016	166.77	0.000
		17	-0.003	-0.063	166.77	0.000
		18	-0.145	0.007	168.44	0.000
		19	-0.062	0.068	168.76	0.000
		20	0.064	0.002	169.11	0.000

Date: 04/08/25 Time: 22:03
Sample (adjusted): 2 48
Included observations: 47 after adjustments

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.117	-0.117	0.6819	0.409
		2	-0.672	-0.695	23.802	0.000
		3	-0.127	-0.669	24.647	0.000
		4	0.830	0.396	61.548	0.000
		5	-0.058	-0.079	61.735	0.000
		6	-0.612	-0.055	82.790	0.000
		7	-0.113	-0.041	83.521	0.000
		8	0.722	0.033	114.31	0.000
		9	-0.027	-0.048	114.35	0.000
		10	-0.562	-0.055	134.01	0.000
		11	-0.109	-0.083	134.77	0.000
		12	0.658	0.036	163.24	0.000
		13	-0.022	-0.100	163.27	0.000
		14	-0.514	-0.048	181.70	0.000
		15	-0.078	0.042	182.14	0.000
		16	0.582	-0.033	207.29	0.000
		17	0.006	0.087	207.29	0.000
		18	-0.415	0.314	220.95	0.000
		19	-0.142	-0.078	222.61	0.000
		20	0.485	-0.012	242.66	0.000

(a) **(b)**

Figure 7: ACF and PACF plots of the Lgsyh series at the level value and its first differenced series.



Date: 04/10/25 Time: 14:25
Sample: 2013Q1 2024Q4
Included observations: 48

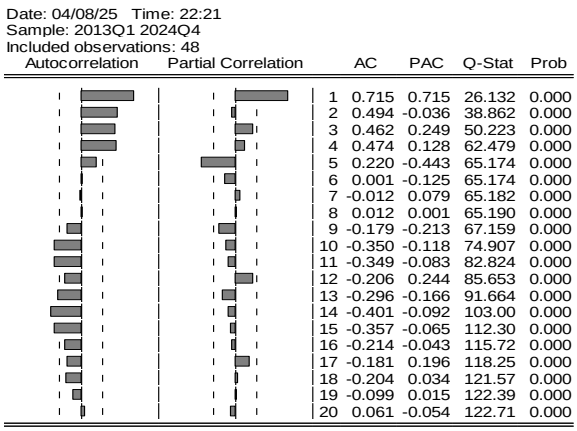
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.938	0.938	44.967	0.000
		2	0.849	-0.263	82.593	0.000
		3	0.752	-0.067	112.78	0.000
		4	0.659	-0.001	136.50	0.000
		5	0.590	0.140	155.95	0.000
		6	0.530	-0.050	172.01	0.000
		7	0.468	-0.096	184.82	0.000
		8	0.385	-0.217	193.72	0.000
		9	0.290	-0.057	198.91	0.000
		10	0.207	0.091	201.61	0.000
		11	0.142	0.067	202.93	0.000
		12	0.101	0.018	203.60	0.000
		13	0.086	0.100	204.11	0.000
		14	0.069	-0.107	204.45	0.000
		15	0.039	-0.087	204.56	0.000
		16	0.005	0.023	204.57	0.000
		17	-0.016	0.159	204.58	0.000
		18	-0.035	-0.144	204.68	0.000
		19	-0.047	-0.060	204.87	0.000
		20	-0.058	-0.097	205.16	0.000

Date: 04/10/25 Time: 14:26
Sample (adjusted): 2013Q2 2024Q4
Included observations: 47 after adjustments

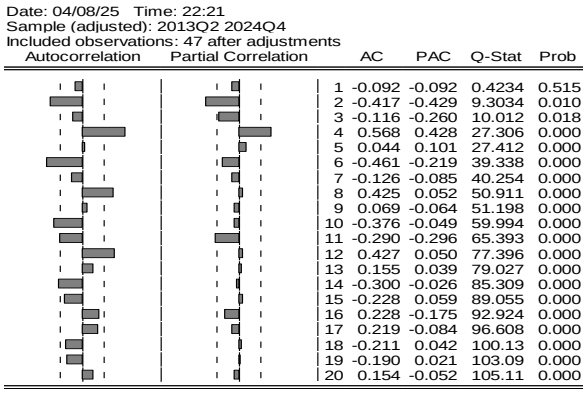
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.295	0.295	4.3559	0.037
		2	0.002	-0.093	4.3561	0.113
		3	-0.109	-0.090	4.9747	0.174
		4	-0.354	-0.327	11.695	0.020
		5	-0.054	0.167	11.855	0.037
		6	0.160	0.131	13.293	0.039
		7	0.099	-0.032	13.852	0.054
		8	-0.110	-0.316	14.573	0.068
		9	-0.224	-0.105	17.606	0.040
		10	-0.327	-0.152	24.246	0.007
		11	-0.200	-0.053	26.794	0.005
		12	-0.027	-0.191	26.841	0.008
		13	0.045	-0.095	26.976	0.013
		14	0.295	0.273	33.046	0.003
		15	0.073	-0.104	33.425	0.004
		16	-0.046	-0.117	33.585	0.006
		17	0.087	0.111	34.161	0.008
		18	-0.165	-0.156	36.314	0.006
		19	0.022	-0.003	36.356	0.010
		20	0.248	0.070	41.585	0.003

(a) **(b)**

Figure 8: ACF and PACF plots of the Lenf series at the level value and its first differenced series.


Date: 04/08/25 Time: 22:21
Sample: 2013Q1 2024Q4
Included observations: 48

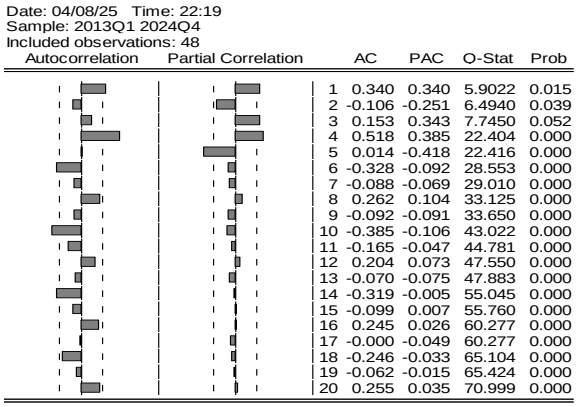
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.715	0.715	26.132	0.000
		2	0.494	-0.036	38.862	0.000
		3	0.462	0.249	50.223	0.000
		4	0.474	0.128	62.479	0.000
		5	0.220	-0.443	65.174	0.000
		6	0.001	-0.125	65.174	0.000
		7	-0.012	0.079	65.182	0.000
		8	0.012	0.001	65.190	0.000
		9	-0.179	-0.213	67.159	0.000
		10	-0.350	-0.118	74.907	0.000
		11	-0.349	-0.083	82.824	0.000
		12	-0.206	0.244	85.653	0.000
		13	-0.296	-0.166	91.664	0.000
		14	-0.401	-0.092	103.00	0.000
		15	-0.357	-0.065	112.30	0.000
		16	-0.214	-0.043	115.72	0.000
		17	-0.181	0.196	118.25	0.000
		18	-0.204	0.034	121.57	0.000
		19	-0.099	0.015	122.39	0.000
		20	0.061	-0.054	122.71	0.000


Date: 04/08/25 Time: 22:21
Sample (adjusted): 2013Q2 2024Q4
Included observations: 47 after adjustments

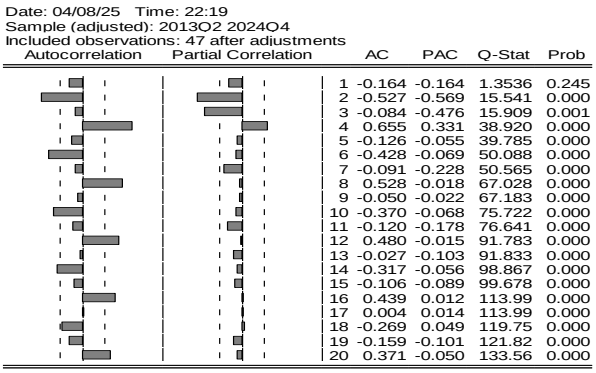
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.092	-0.092	0.4234	0.515
		2	-0.417	-0.429	9.3034	0.010
		3	-0.116	-0.260	10.012	0.018
		4	0.568	0.428	27.306	0.000
		5	0.044	0.101	27.412	0.000
		6	-0.461	-0.219	39.338	0.000
		7	-0.126	-0.085	40.254	0.000
		8	0.425	0.052	50.911	0.000
		9	0.069	-0.064	51.198	0.000
		10	-0.376	-0.049	59.994	0.000
		11	-0.290	-0.296	65.393	0.000
		12	0.427	0.050	77.396	0.000
		13	0.155	0.039	79.027	0.000
		14	-0.300	-0.026	85.309	0.000
		15	-0.228	0.059	89.055	0.000
		16	0.228	-0.175	92.924	0.000
		17	0.219	-0.084	96.608	0.000
		18	-0.211	0.042	100.13	0.000
		19	-0.190	0.021	103.09	0.000
		20	0.154	-0.052	105.11	0.000

(a) **(b)**

Figure 9: ACF and PACF plots of the List series at the level value and its first differenced series.


Date: 04/08/25 Time: 22:19
Sample: 2013Q1 2024Q4
Included observations: 48

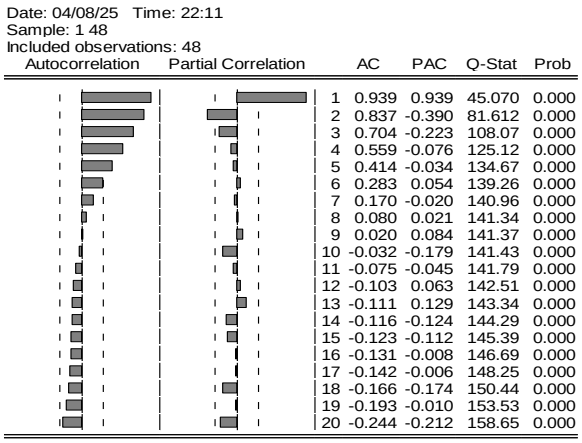
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.340	0.340	5.9022	0.015
		2	-0.106	-0.251	6.4940	0.039
		3	0.153	0.343	7.7450	0.052
		4	0.518	0.385	22.404	0.000
		5	0.014	-0.418	22.416	0.000
		6	-0.328	-0.092	28.553	0.000
		7	-0.088	-0.069	29.010	0.000
		8	0.262	0.104	33.125	0.000
		9	-0.092	-0.091	33.650	0.000
		10	-0.385	-0.106	43.022	0.000
		11	-0.165	-0.047	44.781	0.000
		12	0.204	0.073	47.550	0.000
		13	-0.070	-0.075	47.883	0.000
		14	-0.319	-0.005	55.045	0.000
		15	-0.099	0.007	55.760	0.000
		16	0.245	0.026	60.277	0.000
		17	-0.000	-0.049	60.277	0.000
		18	-0.246	-0.033	65.104	0.000
		19	-0.062	-0.015	65.424	0.000
		20	0.255	0.035	70.999	0.000


Date: 04/08/25 Time: 22:19
Sample (adjusted): 2013Q2 2024Q4
Included observations: 47 after adjustments

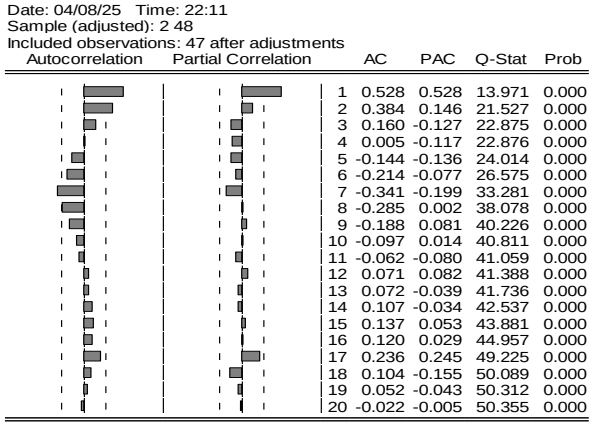
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.164	-0.164	1.3536	0.245
		2	-0.527	-0.569	15.541	0.000
		3	-0.084	-0.476	15.909	0.001
		4	0.655	0.331	38.920	0.000
		5	-0.126	-0.055	39.785	0.000
		6	-0.428	-0.069	50.088	0.000
		7	-0.091	-0.228	50.565	0.000
		8	0.528	-0.018	67.028	0.000
		9	-0.050	-0.022	67.183	0.000
		10	-0.370	-0.068	75.722	0.000
		11	-0.120	-0.178	76.641	0.000
		12	0.480	-0.015	91.783	0.000
		13	-0.027	-0.103	91.833	0.000
		14	-0.317	-0.056	98.867	0.000
		15	-0.106	-0.089	99.678	0.000
		16	0.439	0.012	113.99	0.000
		17	0.004	0.014	113.99	0.000
		18	-0.269	0.049	119.75	0.000
		19	-0.159	-0.101	121.82	0.000
		20	0.371	-0.050	133.56	0.000

(a) **(b)**

Figure 10: ACF and PACF plots of the LTGeliri series at the level value and its first differenced series.


Date: 04/08/25 Time: 22:11
Sample: 1 48
Included observations: 48

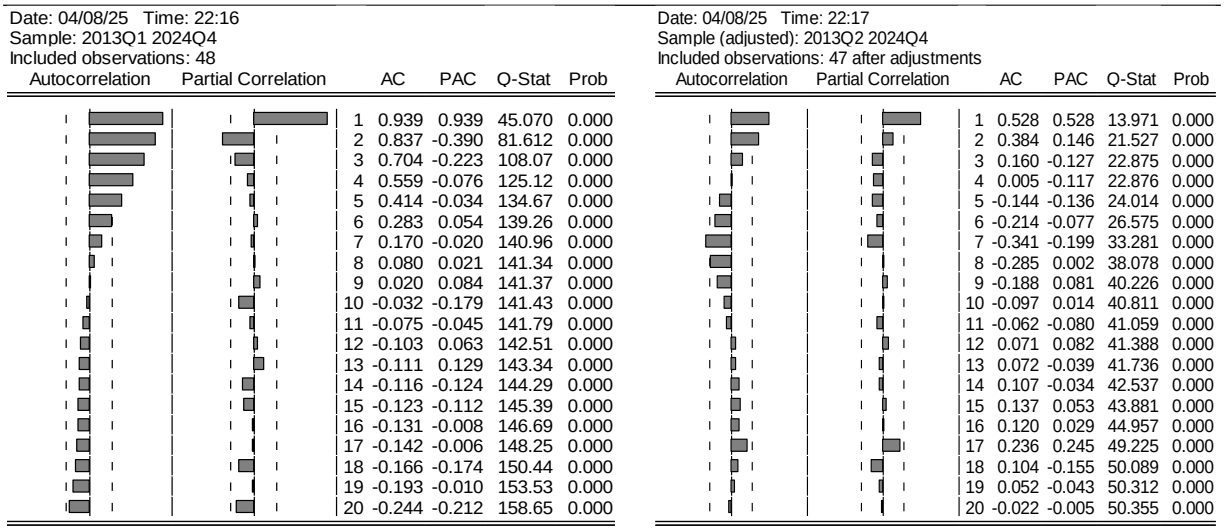
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.939	0.939	45.070	0.000
		2	0.837	-0.390	81.612	0.000
		3	0.704	-0.223	108.07	0.000
		4	0.559	-0.076	125.12	0.000
		5	0.414	-0.034	134.67	0.000
		6	0.283	0.054	139.26	0.000
		7	0.170	-0.020	140.96	0.000
		8	0.080	0.021	141.34	0.000
		9	0.020	0.084	141.37	0.000
		10	-0.032	-0.179	141.43	0.000
		11	-0.075	-0.045	141.79	0.000
		12	-0.103	0.063	142.51	0.000
		13	-0.111	0.129	143.34	0.000
		14	-0.116	-0.124	144.29	0.000
		15	-0.123	-0.112	145.39	0.000
		16	-0.131	-0.008	146.69	0.000
		17	-0.142	-0.006	148.25	0.000
		18	-0.166	-0.174	150.44	0.000
		19	-0.193	-0.010	153.53	0.000
		20	-0.244	-0.212	158.65	0.000


Date: 04/08/25 Time: 22:11
Sample (adjusted): 2 48
Included observations: 47 after adjustments

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.528	0.528	13.971	0.000
		2	0.384	0.146	21.527	0.000
		3	0.160	-0.127	22.875	0.000
		4	0.005	-0.117	22.876	0.000
		5	-0.144	-0.136	24.014	0.000
		6	-0.214	-0.077	26.575	0.000
		7	-0.341	-0.199	33.281	0.000
		8	-0.285	0.002	38.078	0.000
		9	-0.188	0.081	40.226	0.000
		10	-0.097	0.014	40.811	0.000
		11	-0.062	-0.080	41.059	0.000
		12	0.071	0.082	41.388	0.000
		13	0.072	-0.039	41.736	0.000
		14	0.107	-0.034	42.537	0.000
		15	0.137	0.053	43.881	0.000
		16	0.120	0.029	44.957	0.000
		17	0.236	0.245	49.225	0.000
		18	0.104	-0.155	50.089	0.000
		19	0.052	-0.043	50.312	0.000
		20	-0.022	-0.005	50.355	0.000

(a) **(b)**

Figure 11: ACF and PACF plots of the LihBDE series at the level value and its first differenced series.



Date: 04/08/25 Time: 22:16
Sample: 2013Q1 2024Q4
Included observations: 48

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.939	0.939	45.070	0.000
		2	0.837	-0.390	81.612	0.000
		3	0.704	-0.223	108.07	0.000
		4	0.559	-0.076	125.12	0.000
		5	0.414	-0.034	134.67	0.000
		6	0.283	0.054	139.26	0.000
		7	0.170	-0.020	140.96	0.000
		8	0.080	0.021	141.34	0.000
		9	0.020	0.084	141.37	0.000
		10	-0.032	-0.179	141.43	0.000
		11	-0.075	-0.045	141.79	0.000
		12	-0.103	0.063	142.51	0.000
		13	-0.111	0.129	143.34	0.000
		14	-0.116	-0.124	144.29	0.000
		15	-0.123	-0.112	145.39	0.000
		16	-0.131	-0.008	146.69	0.000
		17	-0.142	-0.006	148.25	0.000
		18	-0.166	-0.174	150.44	0.000
		19	-0.193	-0.010	153.53	0.000
		20	-0.244	-0.212	158.65	0.000

(a)

Date: 04/08/25 Time: 22:17
Sample (adjusted): 2013Q2 2024Q4
Included observations: 47 after adjustments

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.528	0.528	13.971	0.000
		2	0.384	0.146	21.527	0.000
		3	0.160	-0.127	22.875	0.000
		4	0.005	-0.117	22.876	0.000
		5	-0.144	-0.136	24.014	0.000
		6	-0.214	-0.077	26.575	0.000
		7	-0.341	-0.199	33.281	0.000
		8	-0.285	0.002	38.078	0.000
		9	-0.188	0.081	40.226	0.000
		10	-0.097	0.014	40.811	0.000
		11	-0.062	-0.080	41.059	0.000
		12	0.071	0.082	41.388	0.000
		13	0.072	-0.039	41.736	0.000
		14	0.107	-0.034	42.537	0.000
		15	0.137	0.053	43.881	0.000
		16	0.120	0.029	44.957	0.000
		17	0.236	0.245	49.225	0.000
		18	0.104	-0.155	50.089	0.000
		19	0.052	-0.043	50.312	0.000
		20	-0.022	-0.005	50.355	0.000

(b)

Figure 12: ACF and PACF plots of the LitBDE series at the level value and its first differenced series.

Upon examining the time series plots of all variables in the study, from Figures 1 to 6, it is observed that the series exhibits a trend at the level value. However, after the differencing process is applied to the series, this trend disappears, and the series display dispersion around a certain mean. When reviewing the correlograms in Figures 7 to 12, it is not entirely clear whether some variables become stationary after differencing or not.

Therefore, to determine whether the variables in the study are stationary, and if not, to ascertain the order of stationarity, unit root tests must be applied. In this study, the ADF unit root test will be employed, and the results obtained are as follows.

Table 3: Unit root test results for the variables

Variables	**ADF Unit Root Test**						
	Critical Level	**Level Value**	**ADF Test Value**	**P-Value**	**First Differenced Value**	**ADF Test Value**	**P-Value**
Lgsyh	1%	-3,5885	-0,0560	0,9478	-3,5885	-19,678	0,0001*
	5%	-2,9293			-2,9293		
	10%	-2,6031			-2,6031		
Lenf	1%	-3,5811	-1,2169	0,6593	-3,5811	-4,8377	0,0003*
	5%	-2,9260			-2,9260		
	10%	-2,6014			-2,6014		
LihBDE	1%	-3,5885	-2,1694	0,2169	-3,5847	-4,4464	0,0001*
	5%	-2,9293			-2,9281		
	10%	-2,6031			-2,6022		
LitBDE	1%	-3,5885	-1,6814	0,4338	-3,5811	-3,7018	0,0072*
	5%	-2,9293			-2,9260		
	10%	-2,6031			-2,6015		
List	1%	-3,5924	-1,9365	0,3139	-3,5847	-7,6979	0,0001*
	5%	-2,9314			-2,9281		
	10%	-2,6039			-2,6022		
LTGeliri	1%	-3,5924	-2,0758	0,2551	-3,5924	-3,4931	0,0129*
	5%	-2,9314			-2,9314		

10%	-2,6039	-2,6039

***** The values indicate the p-value and the level at which the variable is stationary (a significance level of 0.05 has been considered).

When examining the results of the ADF unit root test in Table 3, it is observed that all variables contain a unit root at their level values. However, after applying the differencing process to the series, when the ADF unit root test is re-applied, the series becomes stationary. The hypotheses to be tested here are:

H_0: The variables contain a unit root (Non-stationarity).

H_1: The variables do not contain a unit root (Stationarity).

Looking at the p-values obtained for the level values of the series, it can be concluded that the null hypothesis cannot be rejected, meaning that the series contain a unit root and are not stationary at their level values. After taking the first difference of the series and reapplying the same test, it is found that the series no longer contain a unit root and have become stationary. Thus, all series will exhibit an I(1) stationary process, and the regression analysis will be conducted using the first difference of the series, i.e., their stationary forms.

Table 4: Multicollinearity problem: VIF test results.

Variables	**Variance Inflation Factor**		
	Coefficient Variance	**Uncentered VIF**	**Centered VIF**
ΔLenf	0,002741	1,423603	1,378219
ΔLihBDE	0,261424	2,024767	2,024037
ΔList	0,210328	2,070792	2,070205
ΔLitBDE	0,110172	2,502051	2,501743
ΔLTGeliri	0,000311	2,061741	2,059656
C	1,70E-05	1,048632	NA

According to the test results in Table 4, all VIF values are found to be below 10, indicating that there is no multicollinearity problem among the explanatory variables in the model.

Table 5: Model error terms normality check: Jarque-Bera test results

Variable	**Jarque-Bera Test**		
	Test Statistic	**Number of Observations**	**P-Value**
Residual	0,3122	47	0,8554

The hypothesis regarding whether the model's error terms follow a normal distribution is tested using the Jarque-Bera test, with the hypotheses stated as follows:

H_0: The model's error terms follow a normal distribution.

H_1: The model's error terms do not follow a normal distribution.

According to the results in the table, the Jarque-Bera test statistic is 0,3122, and the corresponding p-value is 0,08554. Therefore, at the 0,05 significance level, the null hypothesis cannot be rejected, indicating that the model's error terms follow a normal distribution.

Table 6: Autocorrelation Test of Model Error Terms

Test	**Breusch-Godfrey Serial Correlation LM Test**			
	F-statistic	**Obs*R-squared**	**Prob. F(2,39)**	**Prob. Chi-squared(2)**
Breusch-Godfrey LM Test	1,300773	2,939137	0,2839	0,2300

The Breusch-Godfrey LM test has been applied to detect the issue of autocorrelation among model error terms. The hypotheses are formulated as follows:

H_0: There is no autocorrelation problem among the model error terms

H_1: There is an autocorrelation problem among the model error terms.

According to the results obtained in the table, the null hypothesis cannot be rejected at the 0,05 significance level, meaning that there is no autocorrelation problem among the error terms.

Table 7: The test for changing variance of model error terms.

Test	**Heteroskedasticity Test: Harvey**			
	F-statistic	**Obs*R-squared**	**Prob. F(5,41)**	**Prob. Chi-squared(5)**
Harvey Test	1,8957	8,8255	0,1161	0,1162

The issue of changing variance in the model error terms has been tested using the Harvey test. The hypotheses set for the test are as follows:

H_0: There is no changing variance among the model error terms.

H_1: There is changing variance among the model error terms.

According to the test results presented in the table, the corresponding p-value for the test statistic is 0.1162. Therefore, based on this outcome, the null hypothesis cannot be rejected, indicating that there is no issue of changing variance among the model error terms.

Table 8: Results of multiple linear regression analysis

Variable	**Dependent Variable** : ΔLgsyh **Method** : Least Squares **Sample (adjusted)** : 2013Q2-2024Q4 **Included observations** : 47 after adjustments			
	Coefficient	**Std. Error**	**t-Statistic**	**Prob.**
ΔLenf	-0,112846	0,052351	-2,155585	0,0370
ΔLihBDE	-2,64301	0,511296	-5,169237	0,0000
ΔList	1,153843	0,458615	-2,706506	0,0159
ΔLitBDE	0,898348	0,331922	2,706506	0,0099
ΔLTGeliri	0,079213	0,017631	4,492835	0,0001

C	0,007215	0,004123	1,750034	0,0876

Tablo 8. Continuation

Findings	Values	Findings	Values
R-squared	0,826000	Mean dependent var	0,006809
Adjusted R-squared	0,683000	S.D. Dependent var	0,047599
S.E. of regression	0,027600	Akaike infomation criterion	-4,223250
Sum squared resid	0,031232	Schwarz information criterion	-3,987061
Log likelihood	105,2464	Hannan-Quinn criterion	-4,134371
F-statstic	19,16307	Durbin-Watson stat	2,341605
Prob(F-statistic)	0,000000		

Table 8 presents the results of the multiple linear regression analysis conducted on the variables included in the study. The main model analyzed is as follows:

$$gsyh_t = \beta_0 + \beta_1 enf + \beta_2 ist + \beta_3 ihBDE + \beta_4 itBDE + \beta_5 TGeliri + \varepsilon_t$$

After applying natural logarithmic transformation to the variables and differencing them to achieve stationarity, the model takes the following form:

$$\Delta\text{Lgsyh}_t = \beta_0 + \beta_1 \Delta\text{L}enf + \beta_2 \Delta\text{L}ist + \beta_3 \Delta\text{L}ihBDE + \beta_4 \Delta\text{L}itBDE + \beta_5 \Delta\text{L}TGeliri + \varepsilon_t$$

The regression output based on this transformed model is provided in Table 8. The hypotheses to be tested for the statistical significance of the model coefficients can be generally stated as follows:

$H_0: \beta_i = 0\ , i = 0,1,2,3,4,5$

$H_1: \beta_i \neq 0\ , i = 0,1,2,3,4,5$

The test statistic used for this purpose is the *t*-statistic. The decision criterion is based on the probability value (p-value) associated with the test statistic at the 0.05 significance level.

Upon examining the results presented in Table 8, it is observed that all explanatory variables are statistically significant, except for the intercept term, which is found to be statistically insignificant. This indicates that inflation, employment, tourism revenues, exports, and imports are influential factors on economic growth within the analyzed period. Moreover, based on the estimated coefficients, inflation and exports have a negative impact on economic growth, whereas employment, tourism revenues, and imports exert a positive effect. The model's R-squared (R^2) value of 0,8260 demonstrates a strong explanatory power, indicating that the selected independent variables effectively capture the majority of the variation in economic growth over the analyzed period.

DISCUSSION AND CONCLUSION

In this study, the effects of key macroeconomic indicators of the Türkiye economy—namely inflation, employment, tourism revenues, exports, and imports—on economic growth were examined. The chained volume index of GDP was used as the indicator of economic growth. The series were transformed into their natural logarithms and their stationarity was assessed through both graphical analysis and unit root tests. It was determined that the series were non-stationary at level but became stationary after taking the first differences.

Prior to the analysis, classical regression assumptions were tested. Issues such as multicollinearity, autocorrelation, heteroskedasticity, and the normal distribution of residuals were considered, and it was found that the model statistically met the necessary assumptions. Accordingly, a multiple linear regression analysis was conducted.

The estimation results revealed that inflation and exports had a negative impact on economic growth. This finding suggests that high inflation may create macroeconomic instability that suppresses economic activity, and that exports, depending on external economic conditions, may not always directly contribute to growth. On the other hand, tourism revenues, imports, and employment were found to have a positive impact on economic growth. These results imply that imports may enhance production capacity through intermediate goods, tourism revenues serve as an important source of foreign exchange, and employment positively affects domestic demand and production.

In this context, in order to support sustainable economic growth in Türkiye, it is recommended to continue the fight against inflation, improve the structural quality of exports, focus on the export of high value-added goods and services, support the tourism sector, and prioritize employment-enhancing policies. Additionally, considering the growth-supporting nature of imports, policies aimed at increasing the efficiency of intermediate goods imports could be developed.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

The authors have made equal contributions.

Assessment of Ankara University ERASMUS+ Programme Outgoing Students by Using Statistical Methodologies for the 2023-2024 Period (1283)

Cafer Yıldırım[1,2*], Özlem Türkşen[3], Necdet Ünüvar[4] , İlker Astarcı[5]

[1]Ankara University, European Union Education Programs Coordinatorship and Project Office, Türkiye
[2]Ankara University, Faculty of DenTistry, Department of Basic Medical Sciences, Türkiye
[3]Ankara University, Faculty of Science, Department of Statistics, Türkiye
[4]Rectorate of Ankara University, Türkiye
[5]Turkish National Agency, Türkiye

*Corresponding author e-mail: cfryildirim@ankara.edu.tr

Abstract

Data analysis has a crucial importance to realize student profiles within universities. The data should be analyzed with proper statistical methods and obtained results could be taken into consideration to make decision for supporting student activities and success. This study aims to realize the profile of Ankara University ERASMUS+ Programme Outgoing Students for the 2023-2024 period. Data set is obtained from the ERASMUS+ Office database and data preprocessing is achieved to organize the data. The well-organized data is analysed according to the categorical and numerical data type by using statistical explaratory data analysis, e.g. descriptive statistics, data visualization. Several parametric and non-parametric statistical tests are applied to the data set. In this context, chi square analysis and correspondence analysis are performed to understand whether there is a relationship between categorical data. One-way analysis of variance (ANOVA) is performed to understand whether there is a difference between groups containing numerical data. Additionaly, classification models were created to predict students' success and failure through machine learning algorithms, called Logistic Regression Analysis, k-Nearest Neighbor, Support Vector Machine and Random Forest Algorithm. It can be said from the results that statistical methodologies help to identify patterns of the data to get knowledge for administrative processes at higher education in European level.

Keywords: *Statistical Explaratory Data Analysis, Data Visualization, Parametric and Non-Parametric Statistical Tests, Machine Learning Classification Algorithms, Higher Education, ERASMUS+.*

INTRODUCTION

In today's data-driven world, effective data analysis is crucial for understanding trends, optimizing processes, and predicting future outcomes in the field of education as in many areas. Just as universities are important in higher education, ERASMUS offices that provide international connections at universities are also important. The ERASMUS Office holds a central position within the university structure as a key unit responsible for coordinating international academic mobility programs, particularly under the ERASMUS+ framework. Data-driven decision-making allows to optimize operations, manage budgets, and plan for future growth for administrative duties. Moreover, the statistical data analysis plays a critical role by enabling evidence-based decision-making across academic, administrative, and research activities. It also helps to assess student performance, improve teaching methods, allocate resources efficiently, and enhance effectiveness (Ünüvar et al., 2023).

The statistical data analysis process typically includes several stages: data collection, data cleaning, data exploration, data modeling or transformation, and interpretation of results to make clear decision. Each step plays a vital role in ensuring the accuracy and reliability of the insights generated. Statistical methodologies such as descriptive statistics, data visualization, classification analysis, regression analysis, and clustering are frequently applied to explore and present data effectively (Türkşen, 2024). Researchers rely on analytical tools to test hypotheses, validate findings, and draw meaningful conclusions from large and complex datasets. While data analysis focuses on interpreting existing data to generate insights, data science involves building predictive models and algorithms using advanced mathematics and programming (Geron, 2017). Data scientists often use machine learning methods and data analysts mainly work with descriptive analytics whereas statisticians use both machine learning methods and descriptive analytics. Common tools used in data analysis include Excel, SQL, Python, R, Tableau, and Power BI.

In this study, it is aimed to assess the profile of Ankara University ERASMUS+ Programme Outgoing Students by using statistical methodologies for the 2023-2024 period since the ERASMUS Office plays an important role in supporting international academic mobility and fostering global connections within universities. The main aim of this study is to reveal the current profile of the Ankara University ERASMUS Office and to strength administrative performance with data-based decision making in several parts e.g. collaboration with partner universities across Europe, promoting academic cooperation, joint research projects, and best practices in education. The rest of the paper is organized as follows. Applied statistical methodologies are given in Materials and Methods section. The analysis results are presented in Results section. Finally, conclusion is given in the last section.

MATERIAL AND METHODS

Material

Data set, in which undergraduate students are taken into consideration, is obtained from the ERASMUS Office database. Data preprocessing stage is achieved with checking missing data, data cleaning and feature selection. Features, namely variables, are defined as Faculty, Research Field (Science, Health and Social), Department, Visited University, Country, Zone (West Europe, East Europe and South Europe), Accomation Time, Visiting Time Period and Success Status. The Success Status, which has categorical data, considered as dependent variable while the others are considered as independent variables which are composed with categorical and continuous data. In addition, a new continuous variable, called Success (%), is calculated according to the achieved ECTS score from the raw data. Data exploration is done by calculating summary statistics (descriptive statistics - central tendency and spread) and data visualization (histogram, barplot, pieplot, boxplot etc.). Some non-parametric statistical tests, (i) Chi-Square Analysis, (ii) Correspondence Analysis, and (iii) Kruskal-Wallis test, are applied to the data set. In this study, the problem is considered as classification problem. Then, Machine Learning Classification Algorithms, called Logistic Regression (LR), k-Nearest Neighbors (KNN), Support Vector Machine (SVM) and Random Forest (RF), are applied to predict students' success status. All the calculations are done by using Python libraries, called numpy, pandas, matplotlib, seaborn, scikit-learn.

Methods

Exploratory Data Analysis

Exploratory Data Analysis (EDA) is an approach to analyze datasets for summarizing their main characteristics, often with visual methods. The main goal of the EDA is to understand the data better before applying any formal statistical models or machine learning algorithms. By using plots like

histograms, scatter plots, and boxplots, as well as summary statistics such as mean, median, and standard deviation, analysts can gain insights into the underlying structure of the data, identify important variables, and detect outliers or missing values that may affect further analysis.

Non-parametric Statistical Analysis

i. Chi-Square Analysis

Chi-Square Analysis, also known as Chi-Squared Test, is a statistical method used to determine if there is a significant association between two categorical variables. It compares the observed frequencies in each category of a contingency table with the expected frequencies if the variables were independent. The main purpose of the Chi-Square Test is to assess whether there is a statistically significant difference between the observed and expected frequencies in one or more categories. The Chi-Square Statistic is calculated using the formula

$$\chi^2 = \sum_{i=1}^{r} \frac{\left(O_i - E_i\right)^2}{E_i}$$

where *r* is the number of category, O_i is observed frequency and E_i is expected frequency. A higher Chi-Square value indicates a greater difference between observed and expected values. The result is compared to a critical value from the chi-square distribution table or evaluated using a p-value to determine significance (Karagöz, 2019). In this study, Chi-Square Test is used to determine if there is a significant association between two categorical variables, called Chi-Square Test of Independence.

ii. Correspondence Analysis

Correspondence Analysis is a multivariate statistical method used to visualize and interpret relationships between categorical variables in a contingency table. It reduces the dimensions of the data, allowing researchers to explore associations between rows and columns by projecting them into a lower-dimensional space, typically a 2D plot. The main goal of the Correspondence Analysis is to summarize and visualize patterns in large categorical datasets (Alpar, 2025).

iii. Kruskal-Wallis Test

The Kruskal-Wallis test is a non-parametric statistical method used to determine if there are statistically significant differences between the medians of three or more independent groups. It is considered the non-parametric alternative to the one-way ANOVA, and it does not assume that the data follows a normal distribution (Karagöz, 2019).

Machine Learning Classification Algorithms

Classification algorithms in machine learning are supervised learning methods used to predict categorical dependent variables based on indenpendent variables. These algorithms learn patterns from labeled training data and apply that knowledge to classify new, unseen instances into one of the predefined classes (Müller and Guido, 2017).

LR is a classification method. It models the probability of a binary outcome using a logistic function. It works well when the relationship between features and the target is approximately linear.

KNN is a simple instance-based classification algorithm that classifies a new data point based on the majority class among its k nearest neighbors in the feature space.

SVM finds the optimal boundary (hyperplane) that best separates different classes. It works well in high-dimensional spaces and with small datasets.

RF is an ensemble method that builds multiple decision trees and combines their outputs to improve accuracy and reduce overfitting. It's robust and effective for many real-world problems.

The tuning parameters of these algorithms are defined by using expert knowledge. The performance comparison of the classification algorithms is achieved by using several metrics, e.g. Accuracy, Precision, Recall, F1-Score and AUC (Ulu Metin, 2024). If the data set is imbalanced the F1-Score metric can be considered as the most preferable one.

RESULTS

The raw data of the Ankara University ERASMUS Office is well-organized according to the defined data preprocessing process. Exploratory data analysis is applied and obtained results are presented in Figure 1.(a)-(c) for the ERASMUS+ Programme outgoing students.

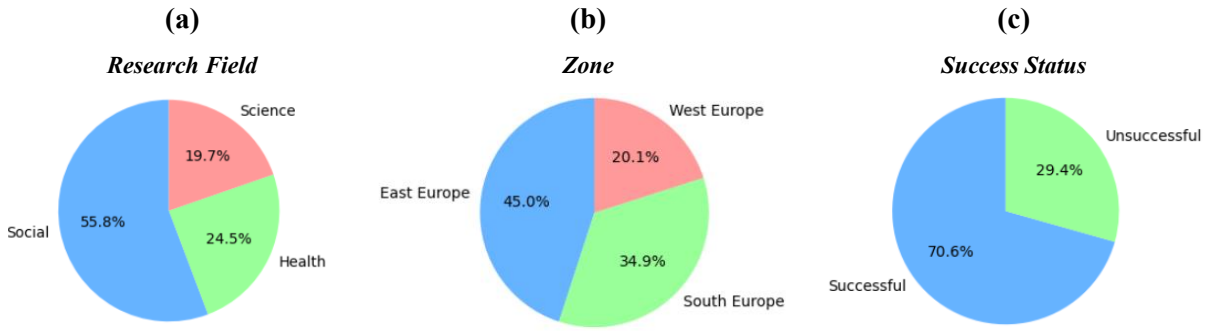
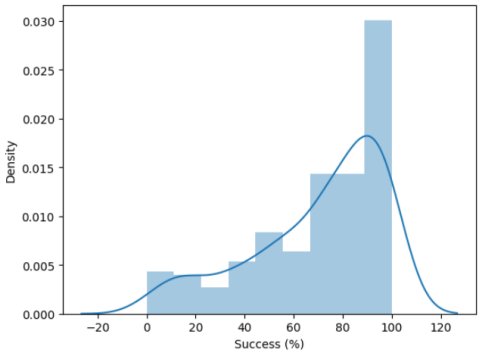


Figure 1. Pie plots of Research Field, Zone and Success Status variables

The distribution of Research Fields can be seen in Figure 1.(a). It can be said from Figure 1.a that the majority of the students are from the social research field. From Figure 1.(b), it is possible to say that the students prefer East Europe. And also, the majority of the students are successful, as presented in Figure 1.(c).

The Success (%) and summary statistics of success quantity can be seen in Figure 2. It is seen from the Figure 2 that the distribution of the Success (%) is left-skewed which means that high-achieving students are the majority. It can be easily said from the summary statistics that number of students, minimum, maximum, mean, median, standart deviation, range and inter quartile range values of the Success (%) are 269, 0, 100, 69.86, 75.76, 27.13, 100 and 36.36, respectively.



Summary Statistics

count	269.000000
mean	69.862566
std	27.132149
min	0.000000
25%	54.545455
50%	75.757576
75%	90.909091
max	100.000000

Figure 2. The distribution of the Success (%) and the summary statistics

The distribution of data about students Visiting Time Period has a two-peaked distribution, presented in Figure 3. It is seen from Figure 3 that the most preferred visiting time periods are 5 and 10 months for the students.

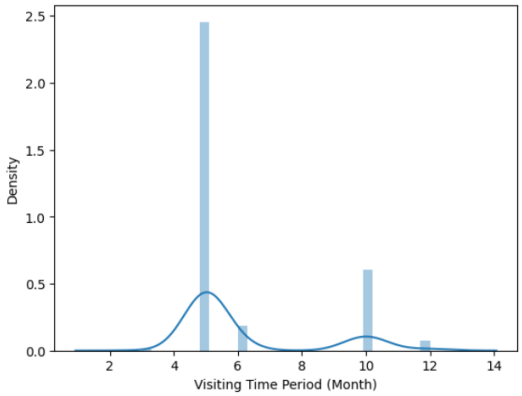


Figure 3. The distribution of the students visiting time period

Chi-Square Analysis is applied to understand whether there is a dependency between success status and Research Field, Success Status and Zone. Contingency tables of Success Status-Research Field and Success Status-Zone can be seen in Table 1 and Table 2, respectively. In Tables 1-2, the frequencies of the categories and expected values, given in parentheses, are presented. Chi-Square statistics is obtained as 6.04 (p-value=0.048) for Success Status and Research Field and the Chi-Square statistics value is equal to 22.98 (p-value=0.0000102) for Success Status and Zone. According to this statistics, Success Status has dependency with Research Field and Zone with %95 confidence.

Table 1. Contingency table of Success Status and Research Field

	Social	Science	Health	Total
Successful	98 (105.95)	38 (37.43)	54 (46.62)	190
Unsuccessful	52 (44.05)	15 (15.57)	12 (19.38)	79
Total	150	53	66	269

Table 2. Contingency table of Success Status and Zone

	East Europe	West Europe	South Europe	Total
Successful	103 (85.46)	34 (38.14)	53 (66.39)	190
Unsuccessful	18 (35.54)	20 (15.86)	41 (27.61)	79
Total	121	54	94	269

The results of Correspondence Analysis are presented in Figure 4. It can be said from Figure 4 that the ERASMUS+ Programme outgoing students from Science, Health and Social research fields are preferred to go to the West Europe, the East Europe and the South Europe, respectively.

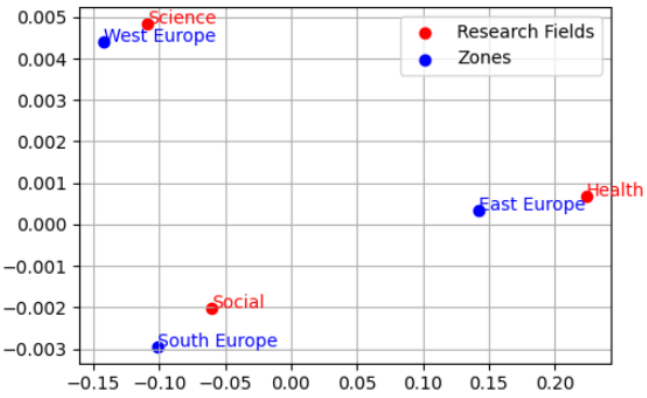


Figure 4. The Correspondence Analysis plot for Research Fields and Zones

It is seen from Figure 5 that the normality assumption is not provided according to the Success (%) in the research fields. Therefore, Kruskal-Wallis test is applied to determine whether there is a difference in Success (%) between the the Research Fields. The Kruskal-Wallis statistics is calculated as 1.6034 and *p*-value is equal to 0.4486. Thus, it is possible to say that there is no difference between the research fields according to the Success (%) with %95 confidence. In addition, this result can be seen from the boxplots clearly as given in Figure 6.

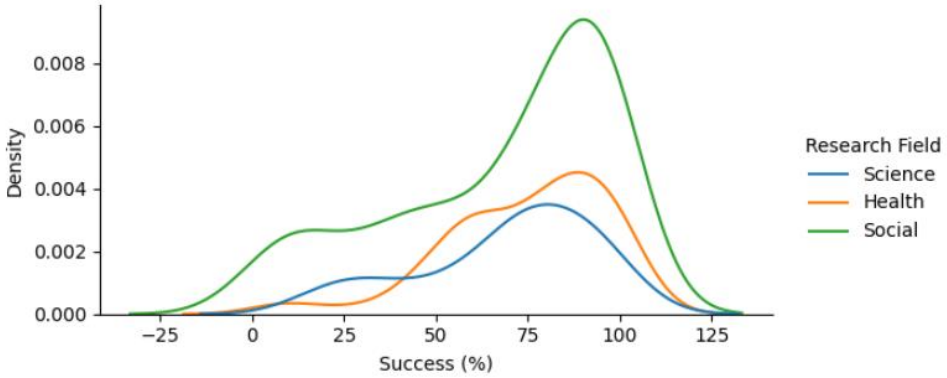


Figure 5. Distribution of the Success (%) for the Research Fields

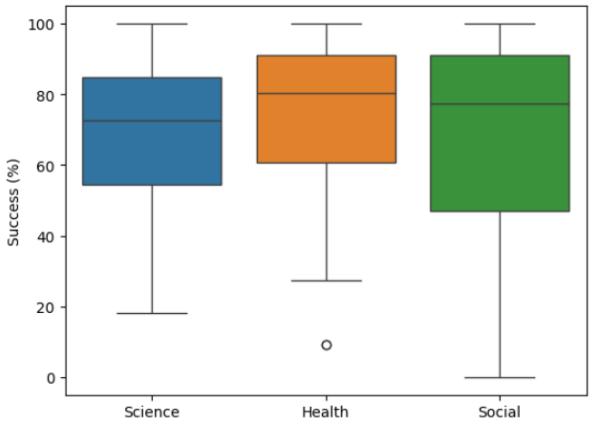


Figure 6. Box-plots of the Success (%) for the research fields

In this study, Success Status is considered as dependent variable with two categories, Successful and Unsuccessful. The values of the dependent variable has imbalanced distribution as can be seen from the pie plot in Figure1.(c). Machine Learning Classification Algorithms are applied to the data set. The obtained performance metrics of the algorithms are given in Table 3. It can be said from the Table 3 that the RF and the LR algorithms have better performance than the KNN and the SVM algorithms for the classification of ERASMUS+ Programme outgoing students according to the their Success Status.

Table 3. Performance metric values of Machine Learning Classification Algorithms

	Accuracy	Precision	Recall	F1-Score	AUC
LR	0.703704	0.720000	0.947368	0.818182	0.657895
KNN	0.666667	0.717391	0.868421	0.785714	0.648849
SVM	0.648148	0.720930	0.815789	0.765432	0.532895
RF	0.703704	0.720000	0.947368	0.818182	0.536184

DISCUSSION AND CONCLUSION

In higher education, the ERASMUS Office, as a key administrative unit within universities, plays a crucial role in facilitating international mobility and cooperation under the ERASMUS+ program, which is one of the most significant educational initiatives of the European Union. Accordingly, it would be appropriate to say that the statistical analysis of the data belonging to the ERASMUS Office is important to track student progress, identify at-risk learners, personalize educational experiences, and data-driven decision making from an administrative perspective.

This study presents the statistical data analysis results of Ankara University ERASMUS Office for Outgoing Students during the 2023-2024 period for the first time. It is seen from the results that majority of the ERASMUS+ Programme outgoing students prefer to go East Europe, majority of them are from social research field and also successful. It is seen from the results that the most preferred visiting time periods are 5 and 10 months for the students. Chi-Squared statistics showed that the success status has dependency with Research Fields and Zones with %95 confidence. The Correspondence Analysis plot helps to realize closeness relationship of the Research Fields and Zones. According to the performance metrics of Machine Learning Classification Algorithms, the LR and the RF can be used for forecasting of Success Status of ERASMUS+ Programme Outgoing Students. It can be also said from the results that the LR is slightly better than the RF for all performance metrics considering the imbalanced data.

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Ünüvar N, Apaydın A, Kutlu Ö, Vural MR, Türkşen Ö, Arıca Akkök E, 2023. Ankara Üniversitesi'nde Eğitim: Düşünce Atölyesi. Ankara Üniversitesi Yayınevi, Ankara.

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Conflict of Interest

If there is no conflict of interest of the authors, it should be written as "The authors have declared that there is no conflict of interest".

Author Contributions

Cafer Yıldırım: Coordinator, Obtaining data from database, Interpretation, Investigation, Writing – review & editing

Özlem Türkşen: Writing – Original draft, review & editing, Methodology, Statistical Analysis, Data visualization, Interpretation, Investigation

Necdet Ünüvar: Resources, Legal Representative of European Union Projects, Policy maker

İlker Astarcı: Resources, Funding, Conceptualization

Assessment of FCM, PCM, and UFPC Algorithms Through Internal and Fuzzy Cluster Validity Indices on Multidisciplinary Benchmark Datasets (1185)

Berna Özbaşaran [1], Gözde Ulutagay[1*]

[1]Ege University, Faculty of Science, Department of Statistics, Turkey

*Corresponding author e-mail: gozde.ulutagay@ege.edu.tr

Abstract

In the contemporary context, due to the high volume of unlabelled data in fields such as medicine, agriculture, chemistry, and many more; unsupervised machine learning models have been topics of interest and of investment. One of the computationally inexpensive and fast models investigated in this paper will be the fuzzy form of K-Means known as Fuzzy C-Means (FCM). Since FCM like K-Means requires the cluster number beforehand it is also vital that the cluster validity indices be fuzzy. In this paper, the evolutionary steps of FCM will be compared by evaluating the models suggested to overcome the pitfalls of the FCM algorithm. As there are many other algorithms created for this purpose, the algorithms analyzed in this article will be Possibilistic C-Means (PCM) and Unsupervised Fuzzy Possibilistic C-Means (UFPC). The comparison of these models is crucial since the new parameters introduced affect the cluster number chosen as seen in the internal validity indices. For applying the algorithms 4 benchmark datasets will be studied in R that belong to fields from biology, chemistry, and demography with. The researcher expects that the UFPC algorithm will surpass the others since, the algorithm uses parameters from both FCM and PCM, however, as real-life datasets are rather complex, it is significant that the analysis be compared to benchmark datasets as proposed in this article. The performance will be evaluated on 12 fuzzy clustering validity indices and 3 internal validity indices that being silhouette, gap, and WSS. Custom R libraries will be used to ease the process of applying the algorithms and validity indices.

Keywords: *FCM, K-Means, Fuzzy Clustering, Unsupervised Machine Learning, Internal Validity Index*

INTRODUCTION

As real-life datasets tend to be more complex in terms of their structure and volume, bettering and adapting an inexpensive model to be more robust seems as a viable option than using more complex and expensive models. The FCM *(*Dunn, J. C. 1973) algorithm (which is the fuzzy version of the K-Means) have been a topic of interest in this aspect, owing its simple but elegant nature. With its pitfalls came some suggestions from numerous academics as to better the algorithm and preserve its effectiveness. In this paper, two of those alternatives will be compared based on benchmark datasets, with reference to crisp and fuzzy indices. For this purpose, the algorithms evaluated will be FCM (Fuzzy C-Means), PCM (Possibilistic C-Means) (Krishnapuram, R., & Keller, J. M. 1993) and lastly UFPC (Unsupervised Fuzzy Possibilistic Clustering) (Yang, M., & Wu, K. 2005). The real-world benchmark datasets will be Glass, Seeds, German Credit and Wine Quality, respectively. Lastly the crisp indices considered will be the elbow method, silhouette score and gap statistics, alongside 12 fuzzy indices which will be named in the methods section. Hence, a comparison of evolutionary steps of an algorithm will be evaluated, with reference to both fuzzy and crisp cluster validity indices.

The investigation is significant in comprehending the evolution of an algorithm, since the latter algorithms have been suggested to better the previous approach via additional parameters. For instance, the pitfalls of the FCM (algorithm which is a fuzzy version of the K-Means) have been altered to overcome its vulnerability towards outliers and noise. In the PCM instance, two new parameters called typicality degree and typicality exponent were produced as it was expected to be a more flexible parameter than the membership degree which will be mentioned in the methods part.

As for the performance metrics, the R libraries fcvalid and ppclust will be utilized alongside others to extract the relevant cluster validity values. The datasets that have been chosen for the comparison are multidisciplinary samples that are known as benchmark datasets. These real-life datasets were chosen for two primary reasons, the first being their versatility in nature and that there is a ground truth meaning the classes are fixed in nature. This means that the datasets tend to have more noise and outliers than synthetic datasets, but also that the cluster number found could be compared with the ground truth to see if it is a match.

In this light, firstly the descriptive statistics will be shown to provide a sense of the data, and to investigate if there are peculiarities in terms of features. Secondly, the crisp validity indices will be shown to see if the classical cluster validity indices are able to spot the ground truth for the designated dataset. Thirdly, the fuzzy indices will be anlayzed for three of the algorithms which are FCM, PCM and UFPC, and the scatter plots will be colored according to cluster numbers for visualization. Finally, the comparison will be made in order to spot some sort of pattern between dataset characters, algorithms and indices. As an educated guess, it can be imagined that UFPC will perform better in rather compelx datasets, since it combines the new parameters introduced by the FCM and PCM algorithms.

METHODS

Statistical Analysis

The main aim of this paper is to compare the three algorithms FCM, PCM and UFPC on the basis of their ability to predict the ground truth with help from crisp or fuzzy validity indices.The algorithms used for this purpose can be introduced as the following.

*FCM (Fuzzy C-Means) (*Dunn, J. C. 1973)

Fuzzy C-Means mathematically works by minimizing the overall weighted distance between data points and cluster centers, where the weights reflect how strongly each point belongs to each cluster. Its aim is to group similar data together while allowing partial membership, so that each point can belong to multiple clusters to varying degrees. The latter algorithms have been suggested since it has a vulnerability towards outliers and noise, as these values tend to receive higher membership degrees that affect the clustering result.

$$J = \sum_{i=1}^{k} \sum_{j=1}^{n} u_{ij}^{m} |\, x_j - \mu_i|^2 \tag{1}$$

where

- k shows the cluster number
- n shows the sample size
- u_{ij}^{m} shows the memberhsip degree
- m is known as the fuzzifier parameter or the fuziness exponent. The algorithm becomes more fuzzy as m increases

- $|x_j - \mu_i|^2$ shows the Squared Euclidean distance

PCM (Possibilistic C-Means) (Krishnapuram, R., & Keller, J. M. 1993)

PCM (Possibilistic C-Means) works by minimizing a cost that measures how typical each data point is to clusters without forcing memberships to sum to one like in the membership degree parameter. Its goal is to group data while allowing points to have low typicality if they do not clearly belong to any cluster, making it robust to noise and outliers. A pitfall to this algorithm is that the initalization of the typicality degree is vital and that the algorithm tends to make coincidental clusters (Cebeci, Z. (2020)).

$$J = \sum_{i=1}^{k}\sum_{j=1}^{n} t_{ij}^{m}|x_j - \mu_i|^2 + \sum_{i=1}^{k}\eta_i \sum_{j=1}^{n}\left(1 - t_{ij}\right)^m \quad (2)$$

where

- t_{ij} shows the typicality degree. Unlike FCM's membership degree, the typicality degree is not constrained to sum to 1. This parameter reflects how typical or compatible the point is with cluster, independently of other clusters.
- ηi shows the regularization parameter for a cluster. This parameter controls how wide or tight the cluster is and influences how quickly the typicality drops with distance from the center.
- $\sum_{j=1}^{n}\left(1 - t_{ij}\right)^m$ balances the influence of values on the clusters by penalizing nontypical values (like outliers). As the typicality degree is lower the equations result will be higher since it is substracted and multiplied by the typicality parameter. Since the cost function ought to be minimal, nontypical values will have a higher cost function and be penalized.

UFPC (Unsupervised Fuzzy Possibilistic C-Means) (Yang, M., & Wu, K. 2005)

UFPC combines fuzzy memberships and possibilistic typicalities to cluster data by balancing soft assignments with typicality measures. It aims to improve clustering flexibility and robustness by integrating both approaches to overcome the pitfalls of FCM and PCM. PCA (Possibilistic Clustering Algorithm) is another algorithm that has been built on top of PCM (Krishnapuram, R., & Keller, J. M. 1996).

$$J_{UFPC}(X;U,V) = \sum_{j=1}^{n}\sum_{i=1}^{c}\left(a \cdot u_{ij,FCM}^{m} + b \cdot u_{ij,PCA}^{\eta}\right) d^2\left(x_j, v_i\right) + \frac{\beta}{n^2\sqrt{c}}\sum_{j=1}^{n}\sum_{i=1}^{c}\left(u_{ij,PCA}^{\eta}\log u_{ij,PCA}^{\eta} - u_{ij,PCA}^{\eta}\right) \quad (3)$$

where

- A and b show the weighting coefficients balancing the influence for FCM and PCA (possibilistic clustering algorithm), respectively. When b is zero, the cost function turns to the FCM algorithm

- β shows the regularization parameter that balances the entropy term, helping control membership distribution
- $\frac{\beta}{n^2\sqrt{c}}\sum u_{ij,\mathrm{PCA}}^{\eta}\left(\log u_{ij,\mathrm{PCA}}^{\eta} - u_{ij,\mathrm{PCA}}^{\eta}\right)$ shows the penalty term, inspired by entropy. The equation encourages diversity, PCA (prevents all being 0 or 1) so asd to promote meaningful typicality distributions.
- While the first term combines FCM and PCA, the second term controls the shape of the possibilistic membership distribution and encourages a balanced spread of memberships.

The fuzzy cluster validity indices mentioned in Table 1 will be used with their abbrevations given below. The rationale behind every fuzzy index is intrinsict, hence, the selection of the lowest or highest value should be selected accordingly. For instance, while PC (Partition Coefficient) presents the best value with the maximum value, in PE (Partition Entropy) the minimum value must be chosen. One can analyze Table 1 to learn which value gives the best result from the different clustering values.

Table 1. Fuzzy Internal Validity Indices for FCM Algorithm for the Glass Dataset

Fuzzy Index	Full name	Optimum Cluster Value
PC	Partition Coefficient	Maximum
MPC	Modified Partition Coefficient	Maximum
PE	Partition Entropy	Minimum
XB	Xie-Beni Index	Minimum
K	Kwon Index	Minimum
TSS	Tang, Sun & Sun Index	Minimum
CL	Chen-Linkens Index	Maximum
FS	Fukuyama Sugeno Index	Minimum
PBMF	Pakhira-Bandyopadyang-Maulik	Maximum
FSIL	Fuzzy Silhouette Index	Minimum
FHV	Fuzzy Hyper Volume	Minimum
APD	Average Partition Density	Maximum

For this purpose, one ought to first pick the relevant sample, which in this case will be the real-world benchmark datasets Glass, Seeds, German Credit and Wine Quality.

*The Glass Dataset (*German, B., 1987).

The dataset belongs to the field of chemistry and consists of **214 instances** of glass samples described by **9 numerical features**, including **refractive index (RI)** and elemental compositions like **Na, Mg, Al, Si, K, Ca, Ba, and Fe**. The ground truth for this dataset is considered as **seven glass types**, varying from materials used so as to build windows or vehicle headlamps.

*The Seeds Dataset (*Charytanowicz, M., et al. 2010)

This dataset which is from biology and agriculture contains **210 instances** of wheat kernels described by **7 numerical features**, including area, perimeter, compactness, kernel length, kernel width, asymmetry coefficient, and groove length. The ground truth for this dataset consist of three wheets which are the following: Kama, Rosa, and Canadian.

*The German Credit Dataset (*Hofmann, H. 1994).

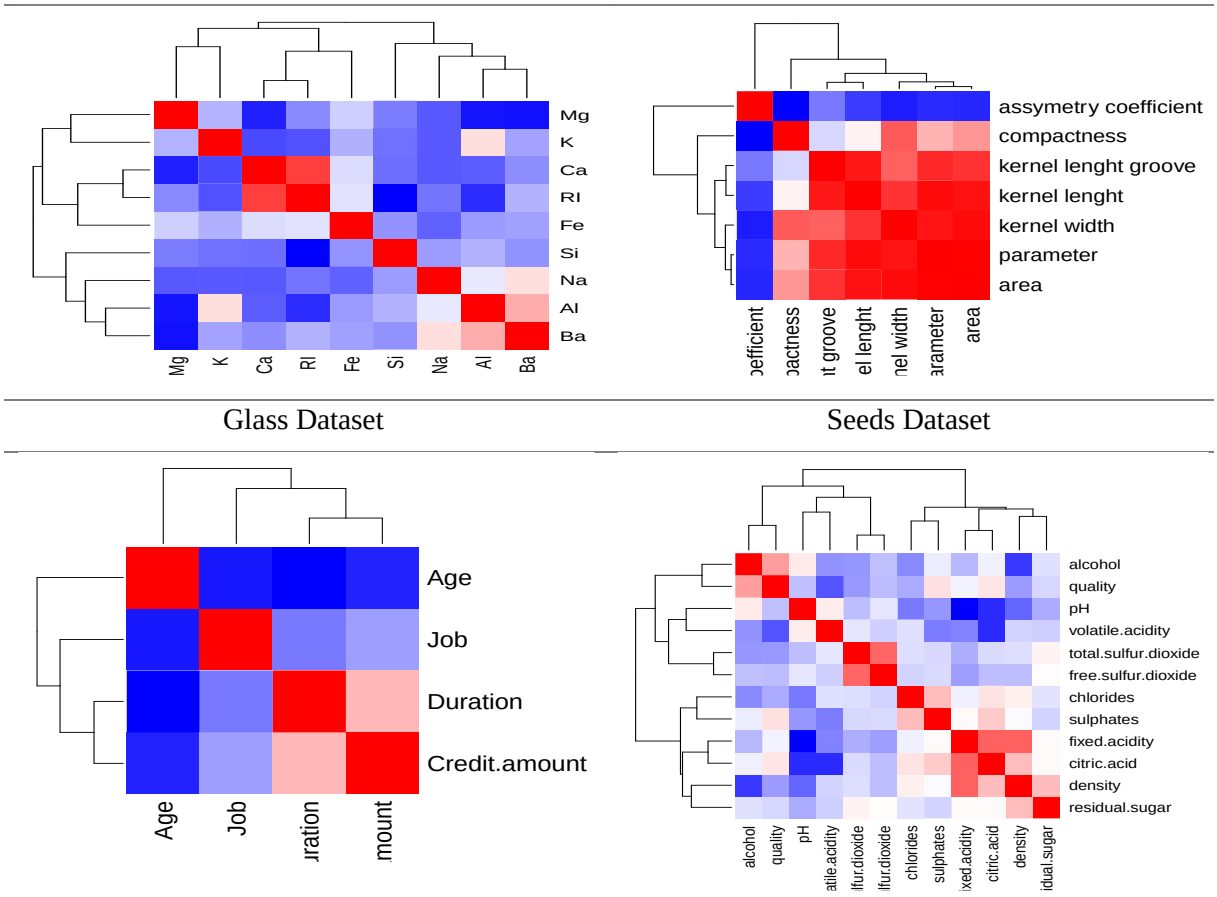
The German Credit dataset is a dataset from the field of demography and economy that contains **1,000 instances** of loan applicants described by **20 attributes**, including credit amount, duration, age,

employment status, and housing type. The labels that are mentioned in the liteature are binary, that is good loan chance and bad loan chance.

*The Wine Quality Dataset (*Cortez, P., et al. 2009)

The last dataset from the field of chemistry contains 1,143 red wine samples described by 11 physicochemical attributes, such as acidity, chlorides, and alcohol content; and the label. Wine quality is rated on a scale from 0 to 10 and is grouped into three classes as a benchmark dataset: low (≤4), medium (5–6), and high (≥7) quality.

For the descriptive analysis, the following numeric features for every variables was selected. In Figure 1, one can see the heatmap for the correlation matrix for all datasets. One can see that the diagonal is red for all the intercept with the variable itself, so this would mean the correlation is 1. The blues represent negative correlation. In the Glass dataset, we can see a strong negative correlation between the variable pairs: Ca, Mg; AI, MG; Ba, Mg; RI, Si; RI, Al, alongside strong positive correlation with RI and Ca. Thus, negative correlation has dominated in the Glass dataset case. For Seeds, one can see that except for the relationship with the assymetry coefficient, the table consist of predominantly strong positive correlation which might suggest multicollinerity. In the German Credit dataset, one can see a stronger negative correlation between age and the other variables. Lastly, there does not seem to be quite as much correlation between the Wine Quality dataset, but one can spot a strong positive and negative correlation chunk between variables. This information will be useful as we are comparing the dataset in the summary section.



Glass Dataset | Seeds Dataset

German Credit Dataset | Wine Quality Dataset

Figure 1. The correlation is respresented as positive with red subtones, and negative with blue subtones. The stronger the color, the more strenght in correlation

RESULTS

The analysis for this paper has been done in R, with libraries such as pastecs, e1071, gridextra, factorextra, ppclust and lastly fcvalid. In Figure 2, one can view the ground truth and the crisp/ classical clustering validity indices. While all internal validity indices have failed in finding the ground truth in the datasets Glass and Wine, in the Seeds dataset, only Gap statistics could find the actual number of clusters. However, in the German Credit dataset, Gap statistics was the only index which failed in finding the accurate number of clusters according to the label variable.

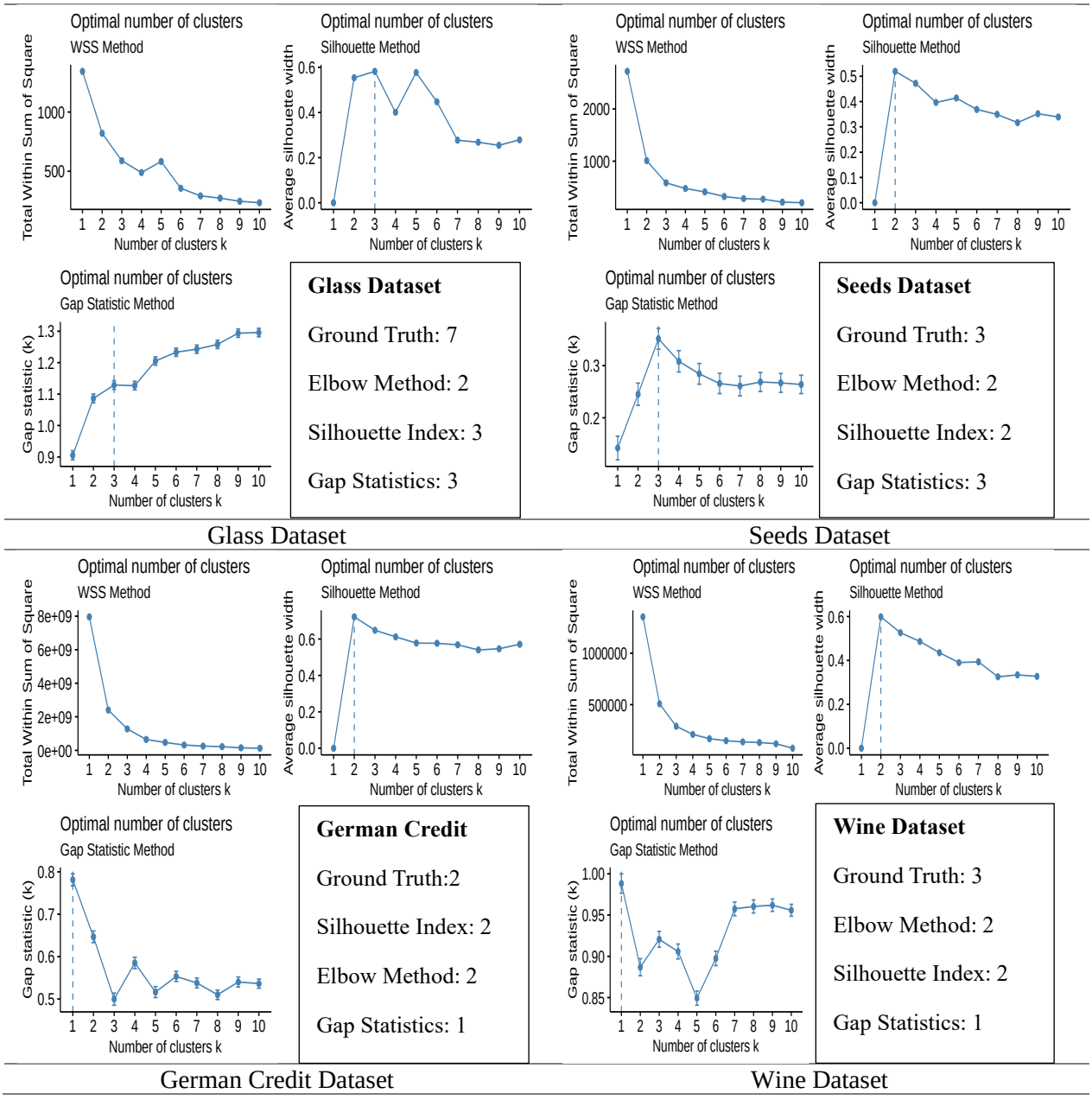


Figure 2. Results for Crisp Internal Validity Methods

As seen in Table 2, 3, 4 and 5 the relevant value according to the previous table (Table 1) was extracted for each fuzzy index for the FCM algorithm. In the Glass dataset, as the ground truth is 7, one can see that TSS (Tang, Sun & Sun Index) which searches the minimal value has matched the ground truth while neither the crisp indices or the other fuzzy indices could. In the seeds dataset it seems that PBMF and APD could find the ground truth 3. Looking at the German Credit dataset, one can see that all except for TSS, PBMF, FSIL and APD could find the ground truth, however, in the later algorithms we see that

these indies also manage to find the accurate cluster number. Lastly, in the Wine Quality dataset FHV and APD have managed to find the actual cluster number which was 3.

Table 2. Fuzzy Internal Validity Indices for FCM Algorithm for the Glass Dataset

FCM	**c = 3**	**c = 4**	**c = 5**	**c = 6**	**c = 7**
PC	**0.756**	0.634	0.499	0.493	0.499
MPC	**0.635**	0.513	0.374	0.392	0.415
PE	**0.461**	0.690	0.948	0.996	1.008
XB	**0.129**	0.591	2.989	2.358	1.972
K	**27.920**	129.532	659.322	531.306	466.117
TSS	25.907	111.814	11.988	8.628	**1.377**
CL	**0.708**	0.564	0.390	0.395	0.408
FS	-714772.886	-599838.578	-472174.360	**-466378.017**	-471225.827
PBMF	23.935	33.632	**9.300**	13.757	23.556
FSIL	0.818	0.622	0.414	**0.401**	0.463
FHV	0.000	0.000	**0.000**	0.000	0.000
APD	**85442062 6.495**	66162612 8.419	323505171.749	148062214.934	103475658.018

Table 3. Fuzzy Internal Validity Indexes for FCM Algorithm for Seeds Dataset

FCM	**c = 2**	**c = 3**	**c = 4**	**c = 5**
PC	**0.805**	0.726	0.639	0.575
MPC	**0.610**	0.589	0.519	0.469
PE	**0.322**	0.500	0.691	0.841
XB	**0.102**	0.151	0.164	0.265
K	**21.694**	32.615	35.570	58.111
TSS	21.159	31.005	**11.396**	29.573
CL	**0.735**	0.668	0.577	0.508
FS	**-31937.055**	-29588.269	-25970.056	-23820.749
PBMF	47.012	**109.262**	45.939	95.275
FSIL	0.791	0.744	0.675	**0.623**
FHV	**0.00000171**	0.00000177	0.00000262	0.00000028
APD	111,363,680.158	**130,082,900.134**	81,572,202.790	89,941,575.179

Table 4. Fuzzy Internal Validity Indexes for FCM Algorithm for German Credit

FCM	**c=2**	**c=3**	**c=4**	**c=5**
PC	**0.900**	0.829	0.813	0.773
MPC	**0.800**	0.743	0.750	0.716
PE	**0.177**	0.311	0.354	0.444
XB	**0.053**	0.083	0.109	0.173
K	**52.718**	83.807	112.806	181.378
TSS	52.468	26.446	3.711	**3.029**

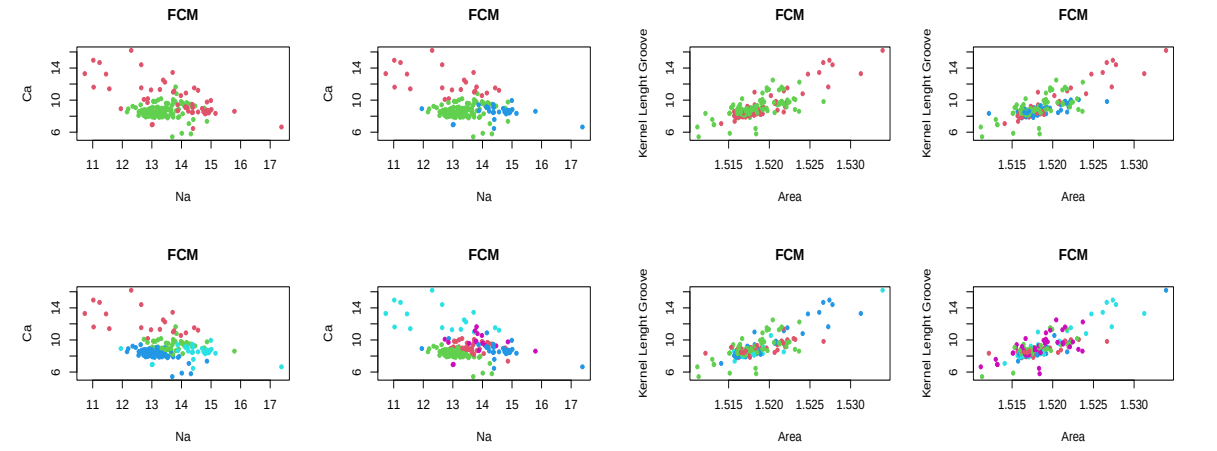
Table 5. Fuzzy Internal Validity Indexes for FCM Algorithm for **The Wine Dataset**

FCM	c = 2	c = 3	c = 4	c = 5
PC	**0.843**	0.771	0.713	0.666
MPC	**0.687**	0.656	0.617	0.582
PE	**0.262**	0.418	0.547	0.655
XB	**0.088**	0.131	0.148	0.186
K	**100.987**	150.567	171.123	216.905

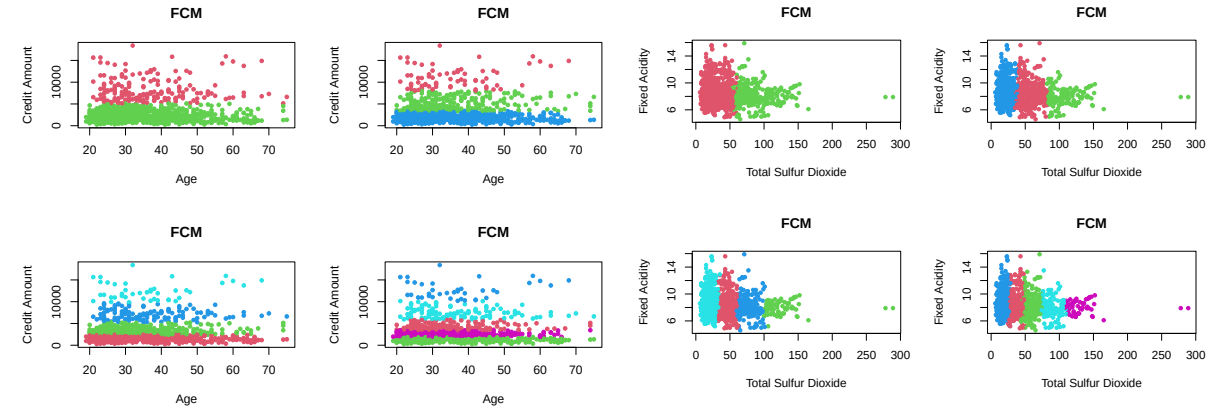
CL	**0.871**	0.787	0.776	0.732
FS	-98889882 48.183	-11621 09615 0.883	**-12951 30571 0.689**	-120786 01698.260
PBMF	4486 3314 .899	10828 0617.789	13466 2779.782	**208025 590.231**
FSIL	0.899	0.856	0.842	**0.816**
FHV	**3822 29.742**	54464 5.189	64488 9.918	696087.279
APD	0.002	0.003	**0.006**	0.001

TSS	100.724	66.412	**20.299**	32.581
CL	**0.787**	0.719	0.661	0.609
FS	-1930 172. 422	**-20664 30.093**	-20151 91.039	-19744 03.848
PBMF	4309 .187	9469. 070	**14165. 947**	13527 .127
FSIL	0.834	0.789	0.751	**0.715**
FHV	0.00 0008	0.000 008	0.0000 09	0.000 010
APD	1655 0662 0.897	27560 7986. 305	75829 566.98 7	**28873 8803. 942**

One can view the different clustering results as the cluster number changes. The x and y values have been chosen as to visualize the sepeation in the best way. One can remind themselves that the ground truth for the datasets were 7, 3, 2, 3 for Glass, Seeds, German Credit and Seeds respectively.



Glass Dataset (x= Na, y= Ca) | Seeds Dataset (x= Area, y= KLG)

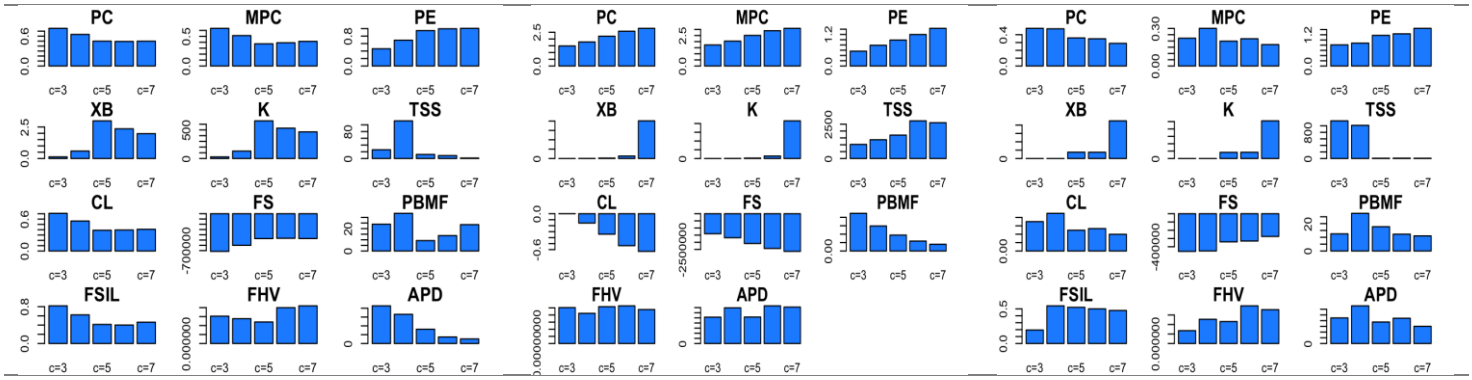


German Credit Dataset (x= Age, y= Credit Amount) | Seeds Dataset (x= TSD, y= Fixed Acidity)

Figure 6. Fuzzy C-Means Clustering C=2, 3, 4, 5 for all dataset

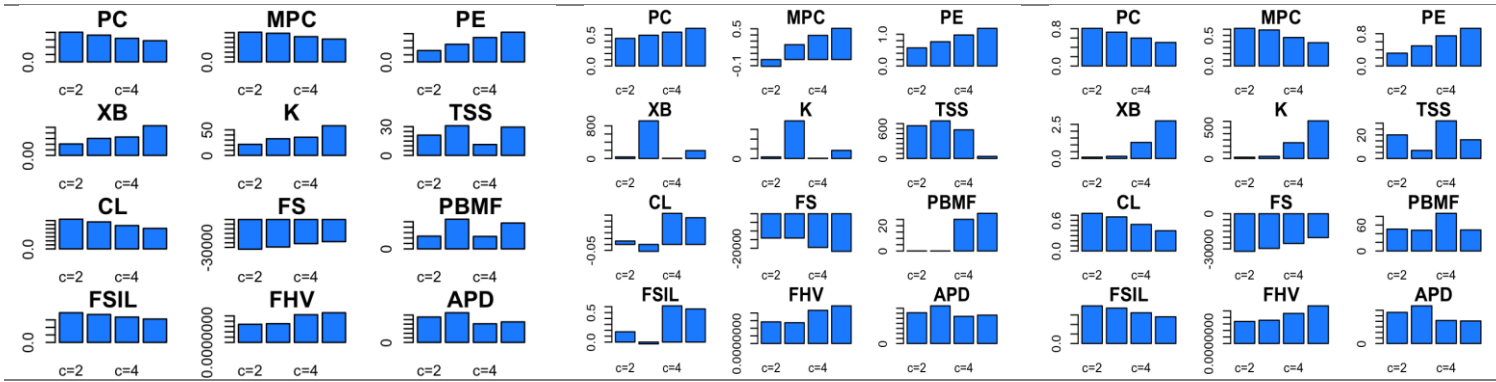
The below Figures represent the bar plots for the Tables 2,3,4 and 5 which were the values for cluster numbers. From these figures, one can make the following deductions for the datasets at hand:

- PC and MPC tend to overestimate with the PCM algorithm (in cases of Seeds, German Credit, and Wine Quality) but they succeeded in estimating the Glass label which is 7.
- FSIL tends to act best with the PCM algorithm (according to the Seeds and German Credit datasets) but has underestimated the cluster number in Wine Quality and has printed the value NA in the Glass dataset. Furthermore, it can be seen that it has tended to overestimate the cluster number with FCM and UFPC algorithms.
- PE has managed to remain stable in its cluster estimates in all datasets, regardless of the algorithm change.
- PBMF worked best with FCM in the Seeds dataset, with UFPC in the German Credit dataset and in PCM in the Wine Quality dataset. Hence, it could be wise to also take into consideration the features of the dataset.
- APD managed to hit the mark with atleast one algorithm in all datasets, meaning it had correctly matched the ground truth 4 miss 12 thus with a 66,7 % accuracy rate. It has aced the datasets Seeds and Wine Quality. This is especially significant since the most succesful crisp index were unable to find these datasets as shown in Figure 2 (only gap statistics have correctly estimated the label for the Seeds dataset, others failed).
- Lastly FHV was succesful in all cluster estimations except for the Glass dataset.



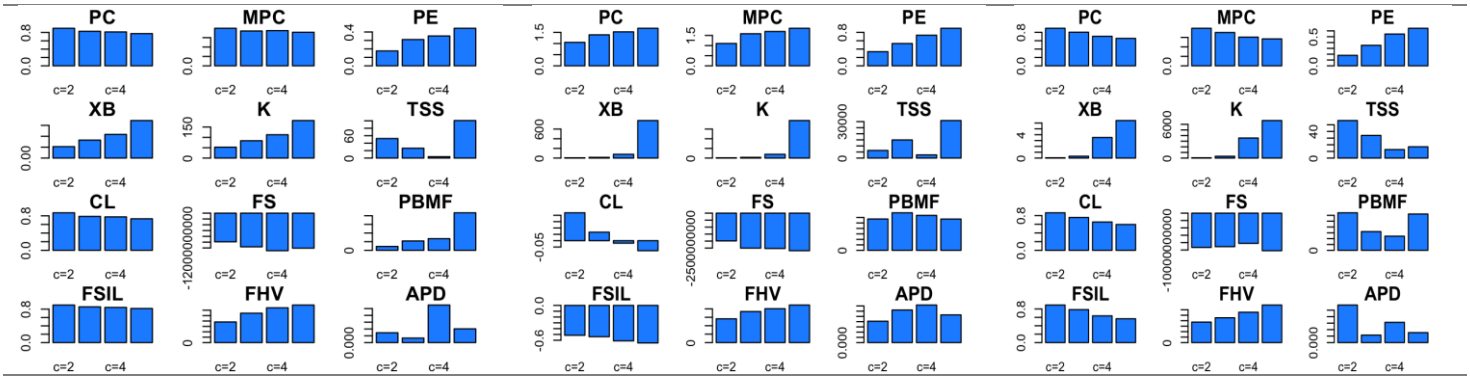
FCM PCM UFPC

Figure 3. Values of Fuzzy Validity Indices for FCM, PCM and UFPC of the Glass Dataset



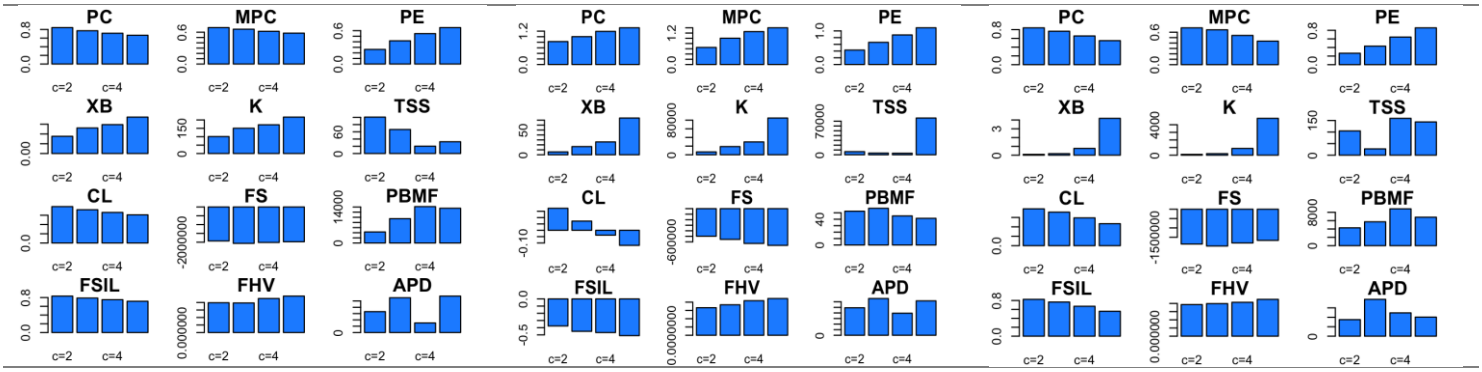
FCM PCM UFPC

Figure 4. Values of Fuzzy Validity Indices for FCM, PCM and UFPC of the Seeds Dataset



FCM PCM UFPC

Figure 5. Values of Fuzzy Validity Indices for FCM, PCM and UFPC of the German Credit Dataset



FCM PCM UFPC

Figure 6. Values of Fuzzy Validity Indices for FCM, PCM and UFPC of the Wine Dataset

The above Figures have been summarized in the below tables for the comparison. The cluster numbers for the optimal values have been noted and those who have hit the mark of the ground truth have been written in bold, alongside the fuzzy clustering indices which have succeded with atleast one algorithm. It could be noted that APD (Average Partition Density) have performed best in all datasets by successeding to find the ground truth in numerous algorithms, showing its stability throughout algorithms. On the other hand, it could be seen that some indices performed better with particular algorithms. Such as Fuzzy Silhouette Index (FSIL) that had manage to select the accurate cluster number

mostly with the PCM algorithm however, it could not be calculated with the PCM objective function in the Glass dataset (yet again it has accurately stated the cluster number with the UFPC algorithm).

Table 7. Comparison of Internal Validity Indices for Glass

Cluster Number	**FCM**	**PCM**	**UFPC**
PC	3	**7**	3
MPC	3	**7**	4
PE	3	3	3
XB	3	3	3
K	3	3	3
TSS	**7**	3	**7**
CL	3	**7**	4
FS	6	3	**7**
PBMF	4	3	4
FSIL	5	NA	3
FHV	5	4	3
APD	**7**	5	4

Table 8. Comparison of Internal Validity Indices for Seeds

Cluster Number	**FCM**	**PCM**	**UFPC**
PC	2	5	2
MPC	2	5	2
PE	2	2	2
XB	2	4	2
K	2	4	2
TSS	4	5	**3**
CL	2	4	2
FS	5	2	5
PBMF	**3**	5	4
FSIL	5	**3**	5
FHV	2	**3**	2
APD	**3**	**3**	**3**

Table 9. Comparison of Internal Validity Indices for German Credit

Cluster Number	**FCM**	**PCM**	**UFPC**
PC	**2**	5	**2**
MPC	**2**	5	**2**
PE	**2**	**2**	**2**
XB	**2**	**2**	**2**
K	**2**	**2**	**2**
TSS	4	4	4
CL	**2**	**2**	**2**
FS	**2**	**2**	4
PBMF	5	3	**2**
FSIL	5	**2**	5
FHV	**2**	**2**	**2**
APD	4	4	**2**

Table 10. Comparison of Internal Validity Indices for Wine Quality

Cluster Number	**FCM**	**PCM**	**UFPC**
PC	2	5	2
MPC	2	5	2
PE	2	2	2
XB	2	2	2
K	2	2	2
TSS	4	4	**3**
CL	2	2	2
FS	2	2	5
PBMF	4	**3**	4
FSIL	5	2	5
FHV	**3**	2	2
APD	**3**	**3**	**3**

DISCUSSION AND CONCLUSION

Before moving forward with the conclusion, it could be useful to view the summary of the evaluation of this paper, so as to conclude in objective remarks. One can benefit from Table 11 and Table 12 to recall the essential information concerning the algorithms and the datasets analysis results.

Table 11. Comparison of Alternative FCM Algorithms

Method	**FCM**	**PCM**	**UFPC**
Parameters	Number of clusters (c), fuzzifier (m)	c, fuzzifier (m), typicality degree (t), typicality exponent (η)	c, fuzzifier (m), typicality degree (t), typicality exponent (η), weights (α, β)

Novelties	Introduced fuzzy membership	Introduced typicality degree (t) and typicality exponent (η) to model noise and outliers	Combined membership and typicality parameters in one objective function
Strengths	Simple, efficient, interpretable	Robust to noise and outliers via typicality	Handles both overlapping clusters and noise
Weaknesses	Sensitive to noise and outliers	Prone to coincident clusters, sensitive to setting of t and η	Needs careful tuning of multiple parameters: m, t, η, α, β
Optimal Dataset	Well-separated, compact clusters	Datasets with overlap or noise, where fuzzy membership is insufficient	Complex datasets involving both overlapand noise simultaneously

Table 12. Comparison of Datasets

Dataset	Glass	Seeds	German Credit	Wine Quality
Sample Size	214	210	1000	1143
Variable Count	9	7	4	12
Silhouette Score	0.58	0.52	0.65	0.6
Optimal Cluster	3	2	2	2
Ground Truth	7	3	2	3
Successful Indices	FCM: TSS, APD PCM: PC, MPC, CL UFPC: TSS, FS	FCM: PBMF, APD PCM: FSIL, FHV, APD UFPC: TSS, APD	FCM: all except TSS, FS, PBMF, FSIL, APD PCM: all except PC, MPC, TSS, PBMF, APD UFPC: all except TSS, FS, FSIL	FCM: FHV, APD PCM: PBMF, APD UFPC: TSS, APD
Peculiarities	The fuzzy indices had lots of variation, Mostly negative correlation	High positive correlation and one variable has hight negative correlation. FHV was very close in FCM and UFPC while FS estimated pretty closely in PCM but they were incorrect in the end	Large dataset with few dimensions; best quality clusters, mostly negative and slightly positive correlation	Mostly negative and nearly no correlation between chosen variables
Best Method	FCM or UPFC with TSS	PCM with APD	UFPC with PE	FCM, PCM or UFPC with TSS

In conclusion, it has been investigated that some indices behave better with particular algorithms. This alligns with the argument of Krisphapuram, R., and Keller, M. (1996) mentioning an algorithm not giving the correct result does not necessarily mean that the algorithm is poorly designed, but rather that it has been missapplied in that case. Thus by this reference one can infer that as diverse algorithms have different assumptions to be fulfilled, this also means that they have pecular instances that enable them to perform their best. Therefore, just as datasets have different features that are not known beforehand,

it might be suggested to apply not only multiple methods, but also multiple typologies of an algorithm that performs better in different instances, like in the case of FCM, PCM and UFPC. With domain knowledge, these complex datasets can be interpreted better. Finally, for further research, analysts and researchers can apply the algorithms in further contexts and maybe in some challenging datatypes like audio or image to test their limits.

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Conflict of Interest

The authors declare no conflict of interest

Author Contributions

Berna ÖZBAŞARAN: writing- original draft, methodology, application

Gözde ULUTAGAY: conceptualization, writing- review & editing, supervision

Determination of Optimal Threshold for Pairs Trading Using Hierarchical Machine Learning Methods (1187)

Bahar Uçar[1], Pınar Kaya Soylu[1], Mahmut Bağcı[1]*

[1] Marmara University, Faculty of Business Administration, Department of Management Information Systems, Türkiye

*Corresponding author e-mail: mahmut.bagci@marmara.edu.tr

Abstract

Pairs trading is a strategy that aims to make a profit by taking advantage of the different price movements that occur between two correlated or comoving assets. This study proposes a novel hierarchical machine learning framework to classify optimal trading threshold levels. Historical price data is used to identify potential asset pairs and compute their value differences over time. When these differences exceed a stepwise threshold (ranging from 1% to 30%), a mean reversion trade is executed, shorting the relatively overvalued asset and buying the undervalued asset. These optimal threshold values are used as the target (dependent) variable for machine learning applications. The input variables (features) are calculated as portfolio mean, variance, skewness, kurtosis, value at risk, and correlation coefficient of the pairs. Unlike conventional models, the proposed approach utilizes hierarchical supervised learning to improve classification accuracy and robustness. The obtained results will be verified by testing with an unused data set in the machine learning phase. The approach enhances trade efficiency, improves profitability, and reduces downside risk, particularly under volatile market conditions. Since real stock market data will be used in the research, the obtained results can be used directly by individual and institutional investors. Moreover, the methodology is designed to be market-agnostic, facilitating its application across global financial markets.

Keywords: *Pairs trading, Machine learning, Threshold optimization, Hierarchical classification.*

INTRODUCTION

Pairs trading is a quantitative arbitrage strategy created by mathematicians, physicists, and computer scientists, working at Morgan Stanley in the mid-1980's (Vidyamurthy, 2004). This strategy seeks to earn returns based on the relative price movements of pairs of associated assets. Given the prominence of algorithmic trading in financial markets today, the creation of new methods in statistical arbitrage and risk management has become essential. In the last twenty years, the development of algorithmic trading (AT) has exponentially grown due to technological advancements, and now accounts for up to 70% of the trading volume in the foreign exchange markets (Bank, 2019).

AT usually provides automatic execution of orders with predefined mathematical rules based on factors such as price, time, and quantity, with minimal human intervention (Cartea et al., 2015; Haboud et al., 2014). In this context, determining the optimal threshold (OT) levels to increase the effectiveness of the strategies is of critical importance for pairs trading strategies. In (Zilbering et al., 2015), it was aimed to determine the OT levels by analyzing periodic and threshold rebalancing strategies. When applying the periodic rebalancing strategy, the portfolio manager adjusts to 20 the target weights at a specific time (e.g., monthly, quarterly). The disadvantage of this approach is that the trading rules are decoupled from market performance. Consequently, rebalancing can take place even when the portfolio is close to optimal (Bağcı and Kaya Soylu, 2024a; Sun et al., 2006).

On the other hand, threshold rebalancing helps investors optimize portfolio risk by ensuring that portfolio weights are rebalanced when they deviate from the target asset allocation (Moallemi and Saglam, 2017). In this case, rebalancing occurs if market movements cause large deviations, but it may not be triggered if there are small fluctuations. These approaches, aimed at increasing the effectiveness of portfolio rebalancing strategies, can provide advantages to investors, especially in volatile market conditions. In addition, in recent years, the application of advanced computational methods in portfolio rebalancing algorithms have offered new opportunities to reduce uncertainties and increase profitability (Lim et al., 2022). It has been suggested that the symmetric threshold level determination method, called the 'New Optimal Rule', increases the expected return in the long run (Zeng and Lee, 2014). However, in their study, the model parameters were assumed to be fixed. Although model parameters can remain unchanged in a short period of time, it is not realistic to apply strategies in the long term, since these parameters are subject to various factors such as general market conditions and internal governance problems in individual firms. At this point, the dynamic rebalancing approach offers a more flexible alternative.

Some studies adopted a structure where portfolio weights vary according to market sentiment based on sentiment analysis. In this way, investors can respond to changes in market conditions more quickly and effectively (Renault, 2017). Recently, in a study on stablecoins, threshold levels were classified using machine learning methods and the effect of these levels on returns was revealed (Bağcı et al., 2024). More recently, a study has suggested a procedure to classify OT ranges for a high-frequency pair trading (HFPT) algorithm by supervised machine learning (ML) techniques (Bağcı and Kaya Soylu, 2025).

In this study, it is aimed to determine the OT for the HFPT algorithm by creating equally weighted portfolios without any restrictions between crypto asset pairs. The unique value of this research is that, unlike the applications in the literature, supervised machine learning methods are applied hierarchically to the data set created with dependent and independent variables. Hierarchical learning is a strategy for creating a more effective and efficient model by using feature extraction and classifiers at different levels of detail in machine learning. This method uses multi-layered levels of features and classifiers, inspired by human learning processes. By integrating various levels of representation from the finest level of detail (pixel-based) to the coarsest level (global visual features), it reduces computational complexity and increases model performance. In this way, the learning process is strengthened and the accuracy rate is increased by the combination of representations at different levels. As emphasized in (Zhang and Zhang, 2006), the multi-layered structure of this method has the potential to improve the learning process by using the combination of representations of features at different levels.

With this approach, it is anticipated that the complexity of the data set will be better managed and more effective results will be obtained in determining the OT levels. In particular, most of the existing research is limited to the assumption of fixed model parameters; however, this study aims to provide investors with a more accurate and flexible strategy by taking into account the dynamic model parameters and the variability in market conditions. The research question aims to determine which threshold levels are optimal for pairs trading and to investigate how these thresholds can improve risk-return performance by increasing the accuracy rates. This study is designed to develop new approaches to reduce uncertainty, which is an important problem in financial markets, and to increase the efficiency of existing strategies. In particular, it has the potential to analyze the data set in more detail by using the hierarchical method and to develop a more flexible and effective pairs trading strategy with combinations of different representations. This will provide a significant advantage for investors and portfolio managers, while also contributing to a better understanding and more effective management of financial markets.

The aim of this research is to determine the OT values to be used in pairs trading strategies with higher accuracy. Hierarchical machine learning techniques will be used to classify the threshold values. The research examines existing portfolio rebalancing techniques, focusing on periodic and threshold rebalancing procedures. It determines the advantages and disadvantages of traditional methods in order to identify potential areas for development and to provide the basis for a more adaptable approach to rebalancing.

First, a comprehensive sample dataset is created with real price series obtained from the Binance crypto exchange. Real market data is used to test machine learning-based rebalancing algorithms. The dataset contains historical prices of fifty crypto-assets with the highest trading volume on the Binance exchange in 2022 and 2023. We consider the price series of these crypto-assets on the Bitcoin (BTC) market, that is, the quote currency is BTC e.g., AAVE/BTC or ADA/BTC markets. There are 43,200 closing price information for each crypto-asset in the dataset.

In the second stage, preprocessing steps are performed for the obtained dataset. In this context, the dataset is cleaned, missing data is completed, outlier data are cleaned, noise data is corrected, and the dataset is scaled (normalized).

The next step is to use hierarchical machine learning algorithms to predict when portfolio rebalancing is most effective. Naive Bayes, K-Nearest Neighbors, Logistic Regression, Decision Trees, Random Forest, and Support Vector Machine are the primary algorithms to be used in this study. In order to find the most effective methods for managing portfolio pairs, these models are applied to datasets containing both independent and dependent factors. In order to determine which algorithm is the best for rebalancing the portfolio, the performance of each algorithm is evaluated according to its success in predicting the market movement.

The performances of the methods is compared to determine which machine learning method is most effective in predicting the ideal rebalancing threshold. The hierarchical machine learning model that shows the best classification performance is applied to a dataset created from real market data that was not used in the previous stages, and the obtained results are verified. This verification also reveals the applicability of the proposed methodology to real markets. In addition, the results obtained in the last stage of the study are compared with the results of previous studies.

As a result, our research aims to determine the OT value for pairwise rebalancing algorithms using data-driven hierarchical machine learning techniques. It is anticipated that the research results will make significant contributions to more effective and efficient portfolio management, especially in dynamic and volatile markets. In addition, this study provides a useful framework for the application of machine learning in portfolio management.

In the rest of the study, the dataset, algorithmic and statistical approach, and machine learning methods used are presented in detail in the materials and methods section. In the results section, the classification performance results for each correlation group are given comparatively. Finally, in the discussion and conclusion section, the findings obtained are compared with the literature and suggestions are presented.

MATERIAL AND METHODS

Material

This study uses supervised machine learning (ML) techniques to determine the most appropriate threshold range for use in the High Frequency Pair Trading (HFPT) strategy. HFPT has a hybrid structure that combines the classical pair trading strategy with a portfolio balancing approach based on specific threshold values. The analysis will be carried out using historical price data of crypto assets. With this data, a structured data set containing both dependent and independent variables will be created.

The Collection of the Data

In this study, historical data of 50 crypto assets listed on the Binance exchange throughout 2022 and 2023 is utilized. The list of these asstes is presented in **Table 1.**

The study focuses on the price movements of each asset against Bitcoin (BTC). Since these assets are traded in Bitcoin markets, their price series are expressed in BTC terms; for instance, AAVE/BTC or ADA/BTC. The dataset consists of minute-level price data collected over a 24-month period, aiming to provide 43,200 closing price points per crypto asset. This figure is calculated based on an average of 30 days per month ($30 \times 24 \times 60 = 43{,}200$). This comprehensive dataset enables an in-depth analysis of price movements and trends in the crypto markets during this period.

The study reveals that there is no universal OT value by calculating threshold values ranging from 1% to 30% with 1% increments for different portfolios. In the same framework, the performance of different threshold levels is also analyzed by taking into account a transaction cost of 0.1%.

Table 1. The list of 50 cryto-assets.

Number	Crypto-asset	Symbol	Number	Crypto-asset	Symbol
1	Aave	AAVE	26	Kava	KAVA
2	Cardano	ADA	27	Chainlink	LINK
3	Algorand	ALGO	28	Litecoin	LTC
4	Cosmos	ATOM	29	Decentraland	MANA
5	Avalanche	AVAX	30	Polygon	MATIC
6	Axie Infinity	AXS	31	Mina	MINA
7	Bitcoin Cash	BCH	32	Maker	MKR
8	BNB	BNB	33	NEAR Protocol	NEAR
9	Conflux	CFX	34	NEO	NEO
10	Chiliz	CHZ	35	Quant	QNT
11	Dash	DASH	36	Render	RNDR
12	Dogecoin	DOGE	37	Oasis Network	ROSE
13	Polkadot	DOT	38	THORChain	RUNE
14	MultiversX (Elrond)	EGLD	39	The Sandbox	SAND
15	EOS	EOS	40	Synthetix Network Token	SNX
16	Ethereum Classic	ETC	41	Solana	SOL
17	Ethereum	ETH	42	Stacks	STX
18	Filecoin	FIL	43	Theta Token	THETA
19	Flow	FLOW	44	Tron	TRX
20	Fantom	FTM	45	Uniswap	UNI
21	The Graph	GRT	46	VeChain	VET
22	Hedera Hashgraph	HBAR	47	Stellar Lumens	XLM
23	Internet Computer	ICP	48	Ripple	XRP
24	Injective	INJ	49	Tezos	XTZ
25	MIOTA	IOTA	50	Zcash	ZEC

[a] The listed assets were selected based on the criteria of having high trading volume and liquidity in BTC pairs during the 2022–2023 period.

[b] Prices are expressed in BTC (e.g. ADA/BTC), thus ensuring comparability across all asset pairs.

[c] Symbols represent short names commonly used on crypto-asset exchanges.

Methods

High Frequency Pairs Trading (HFPT) Algorithm

In this study, an algorithmic trading (AT) strategy applied to portfolios containing pairs will be used. Since its first academic publication, pairs trading is a frequently applied and developed procedure for trading securities in financial markets (Gatev et al., 2006).

While pairs trading is a widely used method in financial markets, the concept of high-frequency trading has also emerged due to the large number of transactions taking place within milliseconds (Aldridge, 2009). This strategy has been named as "high-frequency pairs trading" (HFPT) to emphasize the pairs traded in a high-frequency environment. HFPT is a method that combines pairs trading and threshold-based rebalancing strategies (Bağcı and Kaya Soylu, 2024b).

For HFPT, we assume that we have a portfolio of two equally weighted assets at the starting point of the transaction. Therefore, the two assets in the portfolio are of equal value (current price × quantity). For example, there are 50 of the first asset with a price of 0.02 (50 × 0.02 = 1) and 25 of the second asset with a price of 0.04 (25 × 0.04 = 1). After the portfolio is created, the HFPT will be applied as follows: If the parity (or percentage difference) of the weights of the two assets is greater than a threshold, the overvalued asset will be sold and the undervalued asset will be bought to equalize the weight of them to the average value. In their study, it is demonstrated that there is no universal OT value applicable for the high-frequency rebalancing algorithm (Bağcı and Kaya Soylu, 2024b). This research will apply the HFPT algorithm to the actual price series of 50 crypto-assets. To determine the threshold that maximizes the profitability of HFPT, the threshold value will be increased gradually from 1% to 30% in steps of 1%. For a realistic assessment, a transaction fee of 0.1%—corresponding to the taker's fee on the Binance crypto-asset exchange—will be incorporated. The HFPT algorithm will be utilized to compute OT values for various portfolios (or pairs), which will serve as the target (dependent) variables in the supervised machine learning models. The input (independent) variables will include portfolio characteristics such as mean, variance, skewness, kurtosis, Value at Risk (VaR), and the correlation coefficient between pairs. These variables will be calculated on a monthly basis and analyzed across three portfolio categories: positively correlated, weakly correlated, and negatively correlated pairs.

Algorithm 1 states the pseudo code that implements the HFPT for 2 assets (P1 and P2). The following definitions are used in the algorithm. Current price of an asset: Pi[t], Quantity of an asset: Qi, Total value of an asset: Vi, Threshold for the ratio: T, Trading fee: TF.

Algorithm 1. Pseudo Code of HFPT.

```
while (t < N) do
  V1 = P1[t] × Q1
  V2 = P2[t] × Q2
  V_total = V1 + V2
  W1 = V1 / V_total
  W2 = V2 / V_total
  if |W1 - W2| > T then
    target_value = V_total / 2
    if W1 > W2 then
          delta = V1 - target_value
      Q1 = Q1 - (delta / P1[t])
      Q2 = Q2 + (delta / P2[t])
        else
      delta = V2 - target_value
      Q2 = Q2 - (delta / P2[t])
```

```
      Q1 = Q1 + (delta / P1[t])
    end if
    V_total = V_total - (delta × TF)
  end if
  t=t+ 1
end while
V_final = Q1 × P1[N] + Q2 × P2[N]
Return profit = (V_final - V0) / V0
```

Machine Learning Methods

Various supervised ML methods, such as Naive Bayes (NB), K-Nearest Neighbors (KNN), Logistic Regression (LR), Decision Trees (DT), Random Forest (RF) and Support Vector Machine (SVM), have been used to classify the OT range in different correlation classes for the HFPT algorithm described above.

The NB is one of the simplest probabilistic classifiers. It is based on a strong assumption that all features are conditionally independent given the class, but it performs well in most real applications. In the learning process of this classifier, class probabilities and conditional probabilities are calculated using training data, and these probability values are used to classify new observations (Taheri & Mammadov, 2013).

The K-NN is a traditional nonparametric method used for classification. K-NN is a type of example-based learning (lazy learning); therefore, only data is stored in the training process and all computational cost is incurred in the testing phase to find the k nearest neighbors (Shi, Yang, Yang, & Zhou, 2021). In this context, the K-NN algorithm can be effectively used in similarity-based classification problems such as the stock estimation problem, since the stock estimation problem is based on an approach that requires the classification of historical data by representing them as vectors. The historical data of stock and test data are transformed into a set of vectors. Each vector consists of N dimensions representing the stock features. Then, a similarity metric such as Euclidean distance will be used to make a decision (Alkhatib, Najadat, Hmeidi, & Shatnawi, 2013).

The LR is generally used in analyses for classification and prediction purposes (Gong and Sun, 2009). It has been stated that the model is particularly suitable for dependent variables for binomial or multinomial classification problems. In the prediction of stock prices, the price trend of the next month can be divided into two classes: "1" or "0". "1" indicates that the average price of the next month will be higher or equal to the current month, while "0" indicates that the price will decrease.

The DT is a type of tree structure that can determine the classification of input data. Each non-leaf node in the decision tree represents a test, while each leaf node holds a class value. The classification process starts from the root node and ends when the leaf node is reached; thus, the class in the leaf node is considered as the result (Xu, 2023). In algorithmic trading strategies applied to calculate the OT value for pairs, decision trees play an important role in determining the most appropriate trading signals by analyzing the relationships between each asset pair. This method makes investment decisions more systematic.

The SVM is a discriminative learning algorithm known for providing a way to control the decision function using kernel functions and sparse solutions. Based on the principle of structural risk minimization, it aims to minimize the upper bound of the generalization error and is therefore highly robust against overfitting. In addition, training an SVM involves solving a linearly constrained quadratic programming problem, which guarantees a unique and globally optimal solution, unlike neural networks that use nonlinear optimization and run the risk of getting stuck in local minima (Gong and Sun, 2009).

SVM produces more accurate trading signals by effectively modeling the complex relationships between asset pairs.

The RFs are ensemble classifiers that repeatedly use randomly selected training examples as well as randomly selected subsets of variables to produce multiple, heterogeneous decision trees. They have become popular due to their ability to capture complex dependency patterns between outcome and input features (Li et al., 2024). RFs provide a better understanding of the relationships between different asset pairs and guide trading decisions more accurately. By combining results from multiple decision trees, this method increases the accuracy and generalizability of the model.

The RF algorithm has some basic advantages. It is resistant to overfitting due to the combination of many decision trees. It also works effectively with large and multidimensional data sets. This feature makes the algorithm advantageous in multivariate environments such as financial markets. In addition, RF calculates the contribution of each feature to the model performance, presents variable importance levels and facilitates interpretation (Breiman, 2001). However, the algorithm also has some limitations. The interpretability of the model is lower than a single decision tree. Because its structure, which consists of the combination of many trees, does not provide a transparent decision process to the user. In addition, the training time may be longer when a large number of trees are produced. This may increase computational costs, especially in very large data sets (Li et al ., 2024).

Statistical Analysis

This study includes the process of feature construction, threshold modeling, and performance evaluation in the context of OT estimation for crypto asset pairs. By leveraging statistical characteristics of return series and applying a rule-based portfolio rebalancing algorithm, the relationship between portfolio features and OT levels is modeled. The analysis includes correlation-based asset classification, simulation of trades with rebalancing triggers, and performance metrics. Furthermore, hierarchical learning methods are employed to improve the accuracy of threshold prediction.

The methodology consists of nine stages:

1.Correlation Matrix and Asset Pair Selection

Initially, the correlation matrix of returns among 50 selected crypto assets is computed to classify asset pairs according to their degree of association. The Pearson correlation coefficient between asset i and asset j is given by

$$\rho_{j,i} = \frac{Cov(r_i, r_j)}{\sigma_i \, . \, \sigma_j}$$

ρ_{ij}*:* Pearson correlation coefficient between asset i and asset j

$Cov(r_i, r_j)$*:* Covariance of the returns r_i and r_j

σ_i, σ_j*:* Standard deviations of the returns of asset i and asset j, respectively

The asset pairs are classified based on the following correlation coefficient thresholds:

Positive Correlation*:* $\rho > 0.3$

Weak Correlation: $-0.3 \leq \rho \leq 0.3$

Negative Correlation*:* $\rho < -0.3$

2. Initial Portfolio Construction

An equally weighted initial portfolio is created from each asset pair. The initial value of the two assets in the portfolio:

$$\text{Initial Value} = P_i \cdot Q_i + P_j \cdot Q_j$$

$$P_i, P_j \text{ and } V_{j,0} = P_j \cdot Q_j$$

P_i, P_j*:* Prices of the assets

Q_i, Q_j*:* Quantities of the assets

For an equally weighted portfolio*:* $V_{i,0} = V_{j,0}$

3. Calculation of Weight Parity and Threshold Value (OT)

The weight difference between the two assets in the portfolio is compared with the trading threshold value (OT).

$$W_i = \frac{P_i \cdot Q_i}{V_{\text{total}}}, W_j = \frac{P_j \cdot Q_j}{V_{\text{total}}},$$

$$\text{Weight Difference} = |W_i - W_j|$$

Comparison with the Threshold Value (OT):

$$|W_i - W_j| > OT$$

If this difference exceeds the threshold value (OT), rebalancing is performed.

4. Rebalancing Process

During rebalancing, the asset with the higher weight is sold, and the asset with the lower weight is bought.

$$\Delta Q_i = \frac{(W_i - W_j) \cdot V_{\text{total}}}{P_i}$$

Where:

ΔQ*:* Quantity to be bought or sold

P_i: Current price of the asset

For each trade, the Binance transaction fee (0.1%) is included in the cost:

$$\text{Transaction Cost} = 0.001\, V_{\text{Transaction}}$$

5.Performance Measurement (Profit and Loss Calculation)

For each threshold value, the total return of the portfolio is calculated using the following formula:

$$\text{Return} = \frac{V_{\text{final}} - V_{\text{initial}}}{V_{\text{initial}}}$$

V_{final}: Final value of the portfolio

V_{initial}: Initial value of the portfolio

6.Threshold Value Optimization (OT Value)

For each correlation group, threshold values ranging from 1% to 30% are tested. The threshold value that provides the maximum return is:

$$\mathrm{OT}* = \underset{OT\in[1\%,30\%]}{\arg\max}\,\mathrm{Return}(OT)$$

7. Calculation of Portfolio Characteristics (Independent Variables)

For machine learning models, the statistical characteristics (independent variables) of each portfolio are calculated:

$$Mean = R(w) = W^T\ \overline{R} = \sum_{i=1}^{n} w_i\, R_i$$

$$Variance = V(w) = W^T V\, W = \sum_{i=1}^{n} w_i^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} w_i\, w_i\sigma_{ij}\,,\quad (i \neq j)$$

$$Skewness = S(w) = E\left(W^T\left(R-\overline{R}\right)\right)^3 = \sum_{i=1}^{n} w_i^3\, s_i^3\ + 3\sum_{i=1}^{n}(\sum_{j=1}^{n} w_i^2 w_j\, s_{iij} + \sum_{j=1}^{n} w_i w_j^2 s_{iij,})(\,i \neq j)$$

$$Kurtosis = K(w) = E\left(W^T\left(R-\overline{R}\right)\right)^4$$
$$= \sum_{i=1}^{n} w_i^4 k_i^4 + 4\sum_{i=1}^{n}(\sum_{j=1}^{n} w_i^3 w_j k_{iiij} + \sum_{j=1}^{n} w_j w_i^3 k_{ijjj}) + 6\sum_{i=1}^{n}\sum_{j=1}^{n} w_i^2 w_j^2 k_{iijj,}\ \ (i \neq j)$$

$$VaR_{\boldsymbol{i}} = \mu\ + \sigma\ \cdot\ \mathrm{Z}_{\alpha}$$

Here, Z_{α} is the *Z*-score corresponding to the selected confidence level.

Correlation Coefficient

$$\rho_{j,i} = \frac{Cov(r_i\,, r_j)}{\sigma_i\,.\,\sigma_j}$$

8. Prediction of OT Values Using Machine Learning Model

Using the independent variables calculated for each portfolio, the OT values are predicted with a machine learning model. The model's dependent variable is:

$$OT = \ f(\mu, \sigma^2, Skewness, Kurtosis, VaR, \rho)$$

9.Performance Analysis of Hierarchical Methods

To evaluate hierarchical methods, hierarchical metrics proposed by Kiritchenko et al. (2005) are critically important for understanding the performance of such methods. In this context, the hierarchical F-measure (hF), hierarchical precision (hP), and hierarchical recall (hR) are used.

$$hP = \frac{\sum i\ |Z_i\ \cap\ C_i\,|}{\sum i\ |Z_i|}$$

$$hR = \frac{\sum i\ |Z_i\ \cap\ C_i\,|}{\sum i\ |C_i|}$$

$$hF = \frac{2\ *\ hP\ *hR}{hP\ +\ hR}$$

EXPERIMENTAL RESULTS

In this study, a hierarchical classification model is developed that can predict the most appropriate threshold level in a pair-to-pair trading strategy based on the correlation of crypto asset pairs. The model has a three-stage structure: first, the main label is determined (e.g. low or high threshold), then a middle decision is made, and finally a structure is formed with a total of eight different classes with sub-labels (2×2×2). The threshold value is varied between 0.01 and 0.30. Different machine learning methods such as LR, SVM, KNN, DT, RF and NB are applied to sample data to test whether the model could predict these threshold values correctly.

Model training is performed using a layered cross-validation method. The success of the classification models is tested at three different classification levels in three correlation groups: positive, negative, and weak — 2-class, 4-class and 8-class. Statistical features, variance, skewness, kurtosis, value at risk (VaR) and correlation coefficient, are used in the input of the model. The optimal threshold (OT) value is determined as the target variable. A sample list of these variables according to the correlation groups is given in **Table 2.**

Table 2. A sample list of the input and target variables for ML applications.

Correlation Label	Sample List of the Input and Target Variables						
	Mean	**Variance**	**Skewness**	**Kurtosis**	**VaR**	**Correlation**	**OT**
Negative	0.4647	0.0040	0.2902	-0.0423	-0.0316	-0.5128	0.1865
Positive	-0.1910	0.0060	-0.1143	0.0387	0.0172	0.7106	0.0816
Weak	0.2648	-0.0172	0.1527	-0.0786	-0.0288	0.0465	0.1276

[a] The table shows the statistical attributes and optimal threshold (OT) values obtained from randomly selected sample asset pairs belonging to three different correlation groups.

[b] Input variables include mean, variance, skewness, kurtosis, risk of non-valuation (VaR) and correlation coefficient.

[c] OT was used as the target variable of machine learning models and was filtered between 1% and 30%.

Figure 1 shows the general process of the experimental design used in this study. First, information about past financial markets is collected. Return-based correlation coefficients are then calculated for selected crypto asset pairs. Based on the correlation values, the asset pairs are categorized as either positively correlated, negatively correlated, or weakly correlated. The statistical properties of each portfolio are then computed, including correlation coefficient, value at risk (VaR), skewness, kurtosis, variance, and mean. In parallel, OT values are found using the High-Frequency Pairs Trading (HFPT) algorithm. These are target variables in the context of supervised learning. After scaling the input features using the StandardScaler method, hierarchical labels (main, mid, sub) are assigned based on OT ranges. The dataset is subsequently split into training and testing sets. Various machine learning algorithms—such LR, SVM, KNN, DT, RF, and NB —are employed to classify the OT values. Finally, model performance is evaluated to determine the most accurate classification structure across different correlation groups.

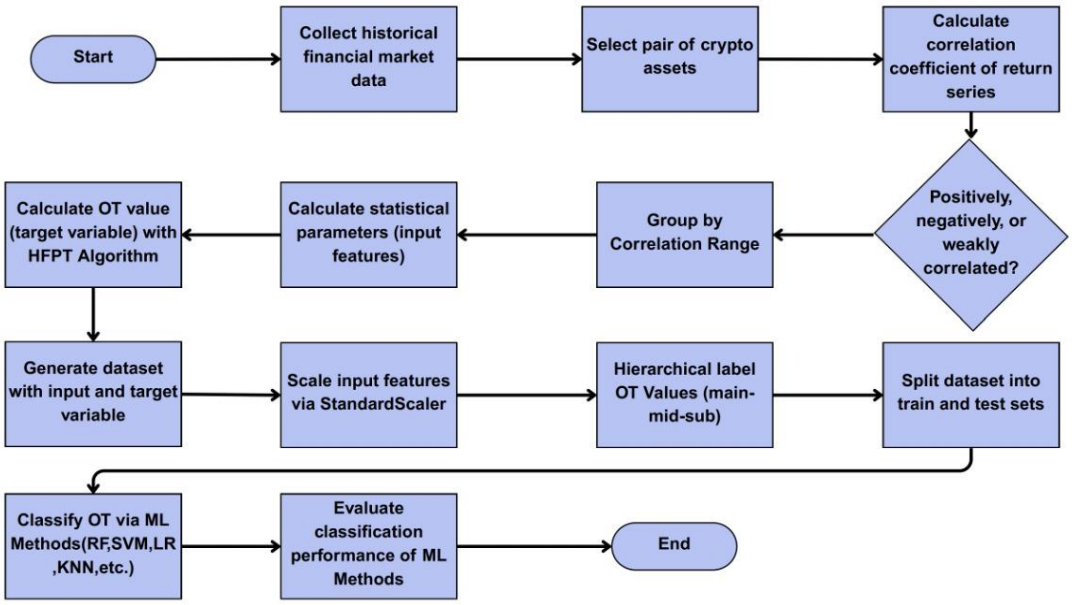

Figure 1. Experimental design/workflow.

Figure 2 shows the distribution of OT values found for each portfolio pair in the dataset. The histogram clearly shows that OT values are concentrated below 4%. This shows that even small changes in the weight balance are sufficient to initiate profitable transactions for many asset pairs. The main classes determined according to OT values (main=0 and main=1) are separated at approximately 16% level. The average OT value shows the general trend of the distribution and reveals that the majority of threshold values are concentrated at low levels. In addition, thresholds at 12%, 20% and 27% levels are used to define sub-classes (sub-labels) for more detailed classification. The shift of the distribution towards low threshold values shows that short-term mean reversion opportunities are at the forefront in high-frequency trading strategies.

Figure 3 shows the relationship between profit (%) and the OT level (%). The graph presents asset pairs in different correlation categories in a single visual. In asset pairs with positive correlation, high profits are observed at very low threshold levels (around 1%), then a decline occurs and a tendency for increasing profits again occurs at threshold levels above 20%. In pairs with weak correlation, profits are more concentrated at medium threshold levels and the variability between returns is lower. In pairs with negative correlation, profits are generally concentrated below the 16% threshold level, and high-return situations are observed more rarely and scattered.

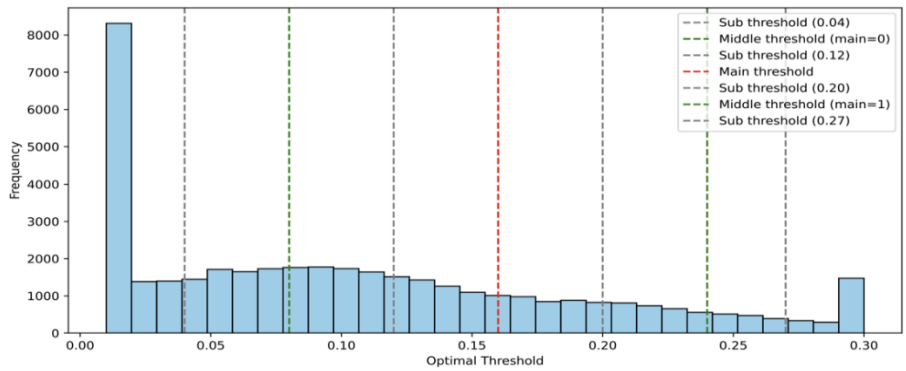

Figure 2. Frequency distribution of optimal threshold (OT) values.

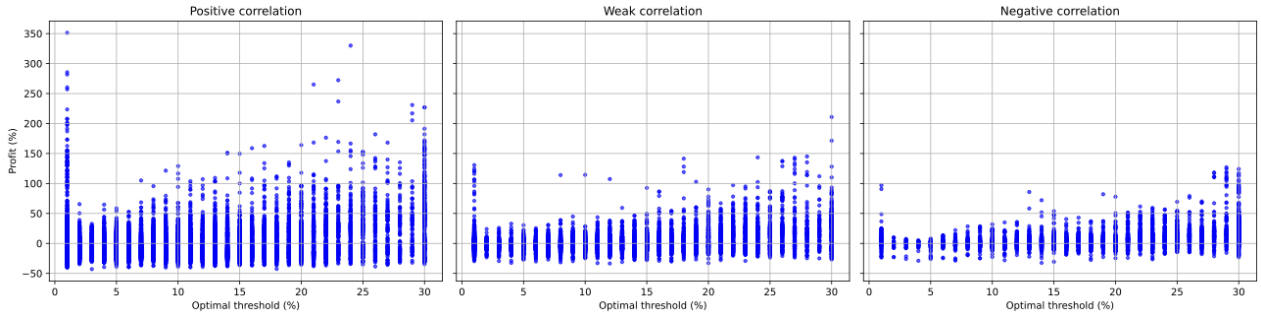

Figure 3. Profit (%) vs. optimal threshold (%) for positively correlated asset pairs

Figure 4 shows the relationship between the OT and profit for three correlation groups. For positively correlated pairs, the data is mostly concentrated in the low threshold values and high profits are obtained in this region. This range corresponds to the C_1 class. For weakly correlated pairs, there is a data density in both the C_1 (0–4%) and C_2 (4–12%) thresholds. For negatively correlated pairs, the data is more evenly distributed, with some high profit examples especially in the C_4 class corresponding to high threshold values. This shows that each correlation group is concentrated in different threshold classes and that the threshold class is related to the return structure.

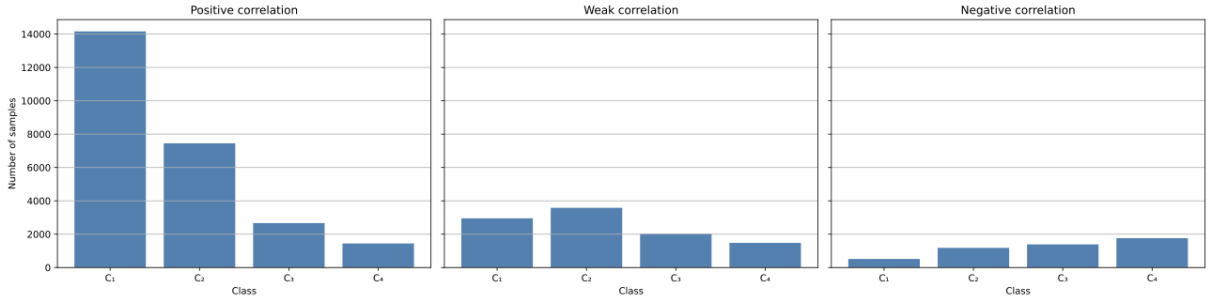

Figure 4. Frequency distribution of threshold classifications for positive correlation group.

Model performance results are given in Tables 3, 4 and 5. In Table 3, the negative correlation group is examined, it is seen that the Random Forest algorithm provides the highest accuracy at all classification levels; 83% accuracy was achieved in 2-class tests and 59% accuracy is achieved in 8-class tests. For the positive correlation group, as clearly seen in Table 4, Random Forest is the most successful model; it provided 90% accuracy in 2-class structure and 68% accuracy in 4-class structure. The K-Nearest Neighbor (KNN) algorithm also shows a remarkable performance in this group. For the weak correlation group, as can be seen in Table 5**,** the Random Forest algorithm produces both high and consistent accuracy values at all classification levels.

Table 3. Multi-Level Accuracy Evaluation Table for Negative Correlation Group

Method	**Negative Group Accuracy Table**								
	2 classes (Avg)	**2 classes (Best)**	**2 classes (Test)**	**4 classes (Avg)**	**4 classes (Best)**	**4 classes (Test)**	**8 classes (Avg)**	**8 classes (Best)**	**8 classes (Test)**
LR	0.68	0.7	0.57	0.56	0.58	0.63	0.54	0.57	0.51
SVM	0.75	0.76	0.68	0.59	0.62	0.52	0.57	0.6	0.47
KNN	0.78	0.8	0.76	0.61	0.63	0.55	0.59	0.61	0.55
DT	0.75	0.76	0.73	0.62	0.64	0.58	0.57	0.59	0.54

RF	0.81	0.83	0.83	0.66	0.69	0.58	0.6	0.62	0.59
NB	0.64	0.68	0.42	0.56	0.57	0.76	0.55	0.58	0.66

[a] The table shows the accuracy values of the classification models created using the entity pairs belonging to the negative correlation group.

[b] Three different accuracy values are reported for each method: average accuracy (Avg), best accuracy (Best) and test set accuracy (Test).

[c] Performance metrics are given separately for three different classification levels: 2-class (binary), 4-class (medium level) and 8-class (sub level).

Table 4. Multi-Level Accuracy Evaluation Table for Positive Correlation Group

Method	**Positive Group Accuracy Table**								
	2 classes (Avg)	**2 classes (Best)**	**2 classes (Test)**	**4 classes (Avg)**	**4 classes (Best)**	**4 classes (Test)**	**8 classes (Avg)**	**8 classes (Best)**	**8 classes (Test)**
LR	0.68	0.69	0.85	0.55	0.56	0.65	0.52	0.52	0.58
SVM	0.78	0.8	0.65	0.61	0.62	0.42	0.56	0.57	0.43
KNN	0.87	0.88	0.87	0.68	0.7	0.63	0.6	0.6	0.55
DT	0.85	0.86	0.8	0.65	0.66	0.61	0.57	0.57	0.54
RF	0.89	0.89	0.9	0.71	0.73	0.68	0.64	0.65	0.6
NB	0.84	0.84	0.86	0.67	0.68	0.6	0.61	0.62	0.58

[a] The tables show the accuracy performances of the classification models applied to positively correlated crypto asset pairs.

[b] Three different accuracy values are presented for each classification algorithm: average accuracy (Avg), best accuracy (Best) and accuracy for the test set (Test).

[c] Performance metrics are given separately for three different classification levels: 2-class (binary), 4-class (medium level) and 8-class (sub level).

Table 5. Multi-Level Accuracy Evaluation Table for Weak Correlation Group

Method	**Weak Group Accuracy Table**								
	2 classes (Avg)	**2 classes (Best)**	**2 classes (Test)**	**4 classes (Avg)**	**4 classes (Best)**	**4 classes (Test)**	**8 classes (Avg)**	**8 classes (Best)**	**8 classes (Test)**
LR	0.67	0.7	0.63	0.53	0.54	0.65	0.52	0.54	0.53
SVM	0.78	0.79	0.67	0.57	0.59	0.54	0.58	0.59	0.53
KNN	0.81	0.81	0.64	0.61	0.62	0.55	0.56	0.58	0.53
DT	0.77	0.78	0.59	0.57	0.58	0.53	0.54	0.56	0.51
RF	0.83	0.84	0.65	0.63	0.65	0.56	0.57	0.58	0.54
NB	0.74	0.76	0.51	0.55	0.56	0.52	0.57	0.58	0.53

[a] The tables show the accuracy performances of the classification models applied to weakly correlated crypto asset pairs.

[b] Three different accuracy values are presented for each classification algorithm: average accuracy (Avg), best accuracy (Best) and accuracy for the test set (Test).

[c] Performance metrics are given separately for three different classification levels: 2-class (binary), 4-class (medium level) and 8-class (sub level).

The hierarchical classification results obtained for each correlation group are summarized in Table 6. In this table, three basic metrics, hierarchical precision (hP), recall (hR) and F1 score (hF), are presented at 2-class, 4-class and 8-class classification levels, respectively. The results show that the model shows its best performance for the positive correlation group and 2-class classification (hF = 0.9039). As the

number of classes increases, it is observed that the performance decreases, especially for the weak correlation group (hF = 0.532 for 8 classes). However, a significant distinction can be made at each classification level thanks to the hierarchical structure. This shows that the model can make more accurate predictions when the relationship between the entities is strong and directional; on the other hand, classification becomes more difficult when the correlation is weak. In general, Table 6 clearly shows how the hierarchical classification approach offers an advantage in multi-level class predictions.

Table 6. Hierarchical Classification Performance for Correlation Groups

Correlation	**Hierarchical Metric Performance**								
	2 classes (hP)	**2 classes (hR)**	**2 classes (hF)**	**4 classes (hP)**	**4 classes (hR)**	**4 classes (hF)**	**8 classes (hP)**	**8 classes (hR)**	**8 classes (hF)**
Positive	0.9128	0.8956	0.9039	0.6346	0.5902	0.6082	0.6103	0.6088	0.609
Weak	0.6676	0.6690	0.6618	0.601	0.5954	0.5971	0.5333	0.5325	0.532
Negative	0.8039	0.8031	0.8035	0.6622	0.6106	0.6144	0.6301	0.6299	0.63

[a] The table shows the hierarchical classification results according to different correlation groups. hP, hR and hF represent hierarchical accuracy (precision), recall and F1 score, respectively.

[b] Model performance is presented separately for each correlation group (Positive, Weak, Negative) at 2, 4 and 8-class classification levels.

[c] The values are based on the outputs of the Random Forest model trained and tested according to the hierarchical label structure.

Table 7 shows the forecast results for the portfolio pairs that offered the highest returns in January and February 2024 in order to show the success of the model in practice and to show the effect of threshold value (OT) classification on trading profitability. Besides in this table are the actual OT value for every trading pair, class labels projected by the model (at two, four, and eight class levels), and the profit percentage acquired from the transaction. Particularly, the 41.2% return acquired on the STX-THETA pair in February 2024 together with the appropriate classification of the 24% OT value calculated for this pair, so amply illustrating the importance of the choice of the ideal threshold in terms of trading strategies.

The results reveal that the probability of correctly predicting asset pairs is very high in two-class classification, partial success is achieved in the four-class structure, and at the eight-class level, the model still achieves good accuracy even when more detailed OT intervals are classified. These findings indicate that despite the difficulty of estimating OT values with regression, interval estimations can be performed reliably with supervised classification algorithms. In particular, the Random Forest algorithm shows a prominent performance for two- and four-class structures in terms of both accuracy and profitability.

Table 7. The actual and predicted OT, and profits of the best practices

		Predicted class					
Correlation	**OT**	**Two-class**	**Four-class**	**Eight-class**	**Pairs**	**Month**	**Profit**
Positive	24%	C2	C4	C8	STX – THETA	Feb. 2024	41.2%
Positive	21%	C1	C2	C3	CHZ – ETC	Jan. 2024	18.1%
Weak	30%	C2	C4	C6	MANA – THETA	Feb. 2024	26.0%
Weak	12%	C2	C3	C5	MKR – STX	Jan. 2024	9.4%
Negative	29%	C2	C4	C8	DOGE – STX	Feb. 2024	25.2%

Negative	21%	C2	C3	C6	CHZ-STX	Jan. 2024	12.4%

[a] The table shows the actual OT value, the class labels predicted by the model (two, four, eight classes) and the profit rates obtained as a result of the transaction for the transaction pairs that provided the highest returns in January and February 2024.

[b] Class labels such as C1, C2, C3, etc. represent the OT ranges predicted by the hierarchical classification model.

[c] The results obtained show that the OT value is limited to estimating with regression; however, the OT range can be successfully estimated with classification methods and this directly affects transaction success.

Figures 5, 6 and 7 show the classification results for each correlation group. As seen in Figure 5, the two-class model is quite successful for positively correlated pairs; most of the data are placed in the correct class. The results are not bad in the four-class model either, but there is some confusion in the C_3 and C_4 classes. In the eight-class model, the predictions are more scattered and the error rate is increased.

Figure 6 includes negatively correlated pairs. The two-class model also performs well here. However, C_3 and C_4 are mixed in the four classes. Although there are correctly predicted classes in the eight classes, the overall success decreases.

As seen in Figure 7, classification for the weakly correlated group is more difficult than in the other groups. The error rate is high in the two-class model. In the four classes, the predictions are generally concentrated in C_2 and C_3, but the other classes are mixed. In the eight class, the confusion is the highest; it becomes more difficult for the model to distinguish the correct classes. This shows that classifying threshold levels for weakly correlated pairs is more complicated.

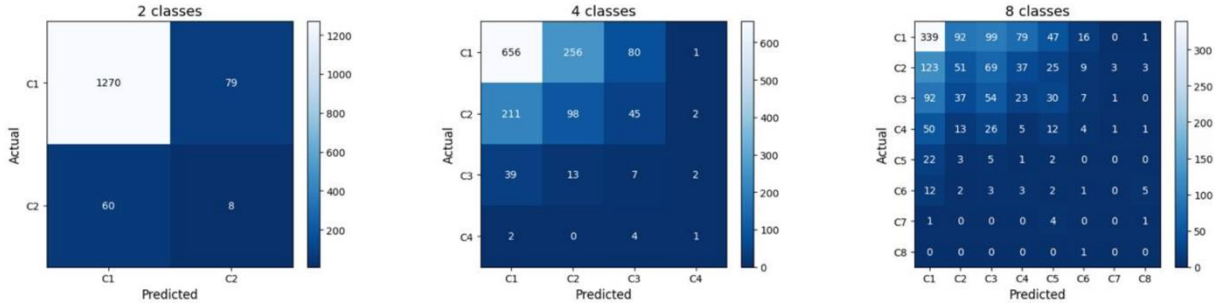

Figure 5. Confusion matrices for hierarchical classification for positive correlation group.

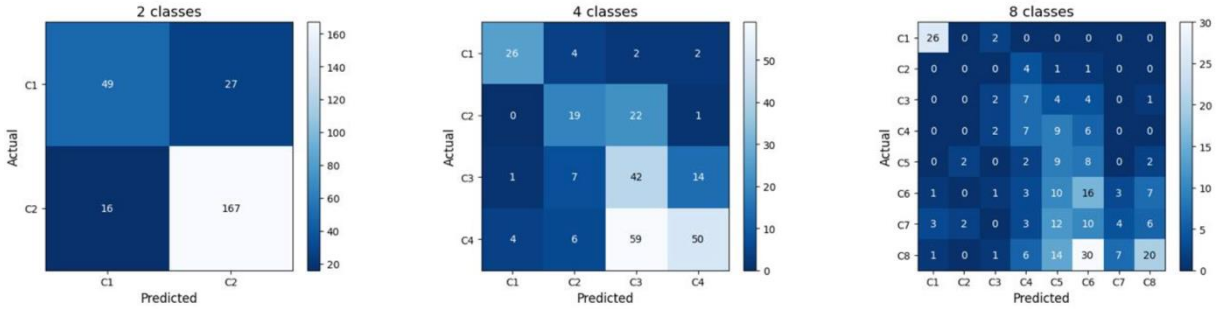

Figure 6. Confusion matrices for hierarchical classification for negative correlation group.

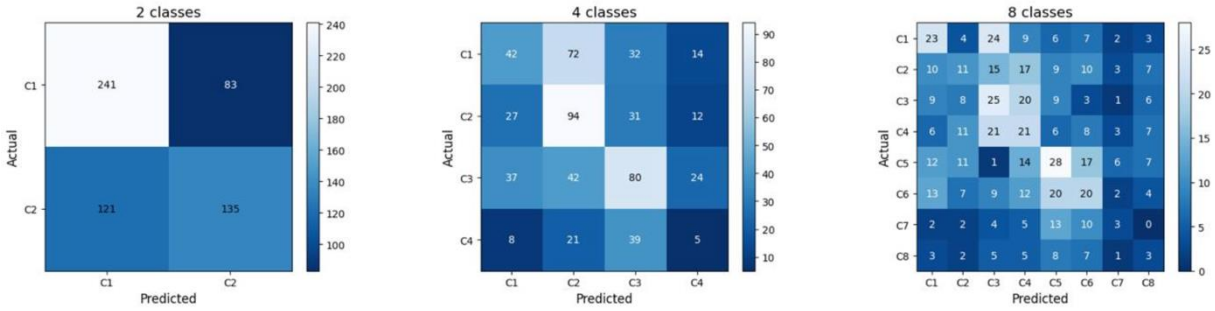

Figure 7. Confusion matrices for hierarchical classification for weak correlation group.

DISCUSSION AND CONCLUSION

In this study, the HFPT algorithm, which rebalances the assets' allocation when a certain threshold value is exceeded, was applied to portfolios consisting of two equally weighted crypto assets. Then, it was examined whether the OT value ranges obtained with this algorithm could be classified by hierarchical machine learning methods. For this purpose, the HFPT algorithm was run on the real price data of 50 crypto assets for the years 2022–2023 and the target variable OT value was calculated for each portfolio. Statistical features such as mean, variance, skewness, kurtosis, Value at Risk (VaR) and correlation coefficient were obtained as input variables.

Asset pairs were divided into three groups as positively, weakly and negatively correlated. OT values for each group were tested between 1% and 30% and it was aimed to classify these values. Different machine learning algorithms such as LR, SVM, KNN, DT, RF and NB were used in the classification process and model performances were compared. It was observed that as the number of classes increases, the model success decreases; the highest accuracy, precision and F1 scores were obtained in two-class classifications and especially in portfolios with positive and negative correlations. These results are also consistent with studies in the literature emphasizing that binary classification is more effective in correlation-based asset groupings (Zhang and Zhang, 2006; Gong and Sun, 2009).

In order to test the accuracy and practical applicability of the model, OT estimates were made in portfolios created with the price series of 50 crypto assets for January and February 2024. It was observed that the correct classification rates obtained for this test dataset were consistent with the success in the training process and that the RF algorithm stood out with its high accuracy rate, especially in two-class classification. In addition, it was determined that there is a harmony between the OT intervals estimated by the model and the realized returns in the trading pairs that provided the highest profit during this period. This clearly shows that the correct classification of the threshold value has a direct impact on transaction success and profitability.

The results of the study revealed that OT intervals can be successfully estimated with hierarchical ML models using the four statistical moments of the portfolio (mean, variance, skewness, kurtosis), VaR and correlation coefficient. In addition, it was shown that when the HFPT algorithm and classification are applied together, a significant profit can be achieved when the correct threshold interval is selected. An important advantage of this method is that both the HFPT algorithm and the OT classification have a market-agnostic structure. By calculating the necessary input and target variables, this approach can be easily applied in different stock markets.

In the study, each portfolio was created from only two assets and the portfolio grouping was done only according to the correlation coefficient. This methodological choice was made in order to increase the success of the classification models. However, in practical applications, it is important to trade on exchanges with high trading volumes and limit the capital size to reduce market impact (Renault, 2017).

In future studies, this proposed HFPT algorithm along with machine learning classification method can be adapted to portfolios of multiple assets. Moreover, advanced machine learning techniques such as ensemble methods or deep learning can be used to improve the classification success.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Statistics As Measure of The Reality – Philosophical Approach (1191)

Silvana Shehu[1*]

[1]University of Tirana, Faculty of Political Sciences, Political Science Department, Albania

*Corresponding author e-mail: shehu.silvana@hotmail.com

Abstract

This study aims to present the value of statistics as a measure of reality by making an approach to the way in which the human being's conceptual apparatus perceives reality. The methodology used is a qualitative method by analysing the value of statistics as a measure of reality in a philosophical approach to how human beings perceive reality and how statistics constitute a facilitating instrument in this process. From this analysis a similarity results between the way in which the conceptual apparatus functions and the way in which the process of statistical production functions. We are dealing with a similar methodology of what human beings use to measure what surrounds them and what statisticians use to measure the reality. The value of statistics lies initially in this ontological process of the human being and then in the benefits that statistical indicators brings in the function of undertaking the appropriate policies for the effective and efficient management of various areas of life. Statistics need to be promoted not only for statistical products but also for the way they are produced representing a pragmatic form of reflecting macro reality in representative usable and valuable data for the person who represents a micro reality.

Keywords: *Ontology, Statistics, Measurement, Recognition.*

INTRODUCTION

The human being by nature cannot fully perceive the reality due to the limitation of being a partial being that is part of something greater than himself. In these conditions, human beings work by applying the rules and mechanisms that enable them to expand as much as possible the angle of view of the reality, aiming to bring this angle towards the 360-degree measure. In this ontological and physical effort, a human being perceives his externality as a partial reality and a plurality of partial entities.

The philosophers pre-Socratic have transmitted the idea that the form of perception of the external is through opposites that is through the ratios of different measures (The University of British Columbia, 2025). Therefore, absolute values of the dimensions of the external cannot be absorbed and cannot be part of the human mind and perception. What creates the idea of the absolute in this limitation is the instrument of comparing everything with the parameters that we have socially agreed to use as a measuring box. So, the concept of size is a concept that in the human mind stands as a product of comparing the entity and the measuring box or entities with each other in comparison with the measuring box.

By staying on the line of comparison ontologically, man moves towards perceiving and creating identifying notions of the reality. These notions constitute the conceptual network of the human being, a network that connects everything with everything in the conditions of the unified conceptual referential system in man (Agamben, 2005). Seen from this perspective, it seems as if the products perceived and cognitively known by man are dedicated to this apparatus that enables the mechanism of comparison and measurement. What measurement accomplishes is not just a process of operationalization but rather

the creation of identities through operationalization. Reality takes on the colours of identity in this process. The creation of concepts, notions, and identities of the external world enable man to see the picture of the reality. In order to move on to the creation of the value, human beings begin to compare identities (Hersh, 2013). From this comparison start the measurement of values for which there is a prior socially agreed-upon framework. The load that the external world takes on in this direction represents the load that the human being has previously internalized. So, the values perceived as such by man are both a product of the human being and not a product of their being in themselves. This is the point of measuring the entities distinguished as different parts, loaded with different values, in the reality that is the sum of them and the interconnected processes between them.

MATERIAL AND METHOD

The methodology used is a qualitative method by analysing the value of statistics as a measure of reality in a philosophical approach to how human beings perceive reality and how statistics constitute a facilitating instrument in this process. The philosophical literatures has been as a platform for this study.

RESULTS

The human beings endowed with a "conceptual apparatus" through which they perceive reality, manage to create this perception through putting correlations the various manifestations of the reality, as this is the only way of understanding the reality in the conditions of being a "limited being". Thus, in the conditions of the impossibility of the recognition of the absolute fullness of the reality, the human beings understand it by putting in comparison the parts of the reality. The same function is performed by the statistical production process and from the statistics as a product of this process.

Step 1. *Identifying of the possible entities of the reality*. Statistics focuses on reality and makes it comparable to other similar realities, creating the most likely picture of the whole truth of reality. This way, putting the "borders" of this reality gives identity to it and this constitutes *the first and most important step* of the process of cognition which is *the identifying of the possible entities of the reality*.

Step 2: *Comparing the identities.* Creating identities creates the opportunity to compare other realities that have already an identity that have the same base to be comparable, so the realities are put in a correlation between them.

Step 3: *Loading the "values"* of the data. The main product of comparing realities through their statistical measurement is the loading of the values for the representative data of the reality being measured. So, the data is already not just a fact but is a fact which is "dressed" with a positive, or neutral, or negative "presence", and negative, or neutral, or positive connotation as well as.

Step 4: *The values analyzing*. The values in this context are not just values in themselves but are values that affect society in various ways. This process signs the start of the analyze of the reality that is measured in terms of right and wrong, in terms of fullness and gaps and in terms of the positive and negative manifestation related to their impact in society. After this analyze need arises for the undertaking of the actions for the further/ new improvements.

The crucial result: *The positive progress of human beings in the terms of understanding their reality and striving for the harmonization of their reality towards the progress of self-actualization in harmony with their external reality that affects them.* Through the measurement that statistics make into the reality, the human being begins to see himself connected with these realities as a subject that conceives what surrounds him in relation to himself and as an object that "weighs the burden" of this reality. This way we can undertake the proper initiatives to improve the reality. Measuring it gives to us opportunity to define the current state and strategic objectives that create the opportunity for actualizing of them.

CONCLUSION

The statistic is a very important tool that offers to the users the possibility of conceiving the reality in the same "essential" way that the man conceptual apparatus works. The creation of identities, their comparison, the undertaking of actions for the improvement of realities is in the essence of man's duty as present and potential existential beings as the first step of the whole development begins with the understanding of what is needed to be developed to actualize the potential willpower that this existence contains. This existence is the human being, society, reality and The NATURE that is "fused" into each of its entities. In this context the actions that arise from statistics are all in function of this major process, in function of the continuous development of being in terms of the actualization of the potency that every being has in its essence and they became a catalyst for actualizing of each entity conform to what it is destined to be cause of its inner power.

In terms of social well-being this measuring process enable us to understand the strengths and weakness of the social reality in order to determine the best actions and efforts that create better conditions for us as social being and in the same time for as human being. The statistics are crucial for raising the national strategies. Evidence-based strategies asses in the best way the current situation of the reality and foreseen the best action for improving it in the future.

So, thank you to all statistical producers and everyone who contributes into analyzing and distribution process of them in this way facilitating the process of conceiving the reality through the representative data as well as serving as the main mechanism for taking the actions in order to reach the multidimensional well-being, since in the end of day this is the crucial goal for a social sustainable development.

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Impact of the COVID-19 Outbreak on Health Indicators: The Case of OECD Countries (1195)

Nuray Tezcan[1*]

[1]Haliç University, Faculty of Business Admnistration, Department of Management Information Systems, 345060, Istanbul, Türkiye

Corresponding author e-mail: nuraytezcan@halic.edu.tr

Abstract

The Sustainable Development Agenda 2030 includes 17 Sustainable Development Goals (SDGs) announced by United Nations (UN) in 2015 and this Agenda has several different targets to be fulfilled by 2030. Although some targets have been achieved so far, the COVID-19 pandemic experienced worldwide in 2020 and 2021 negatively affected the health indicators of countries. According to the Sustainable Development Goals 2023 Report, there has been the largest decline in childhood immunisations in the last thirty years, and tuberculosis and malaria deaths have increased compared to pre-pandemic levels.

From this point of view, the aim of the study is to investigate whether health indicators have changed over time. Considering 2019 and 2022, health indicators of The Organisation for Economic Co-operation and Development (OECD) countries were included in the analysis and permutational multivariate analysis of variance (PERMANOVA) was used as method in the study. PERMANOVA is the multivariate nonparametric counterpart of multivariate analysis of variance (MANOVA) and it is based on distance or similarity measures. In addition, it does not require strong assumptions such as conformity to normal distribution and equality of variance-covariance matrices. According to the results obtained, it was found that there was no statistically significant difference between the years in terms of health indicators.

Keywords: *Sustainable Development Goals, Health Indicators, PERMANOVA*

INTRODUCTION

The COVID-19 outbreak, which started in Wuhan province of China in 2019 and spread from there to the whole world, has greatly damaged the health systems of countries as well as their economies. According to the Sustainable Development Goals Report 2020, in the course of the COVID-19 pandemic countries were suffered from inadequacies or shortcomings regarding the density of healthcare workers, medical equipment and immunization were revealed. Moreover, people avoided going to hospitals or healthcare facilities even to get basic health care services. Childhood immunization programs have been disrupted, which has an impact on maternal and child health. Fearing exposure to the epidemic, thousands of women delayed medical exams or were unable to obtain contraceptive services. Thousands of unintended pregnancies resulted from this situation (UN, 2020).

The negative effects of the pandemic period continue today. According to the Sustainable Development Goals 2023 Report, there has been the largest decline in childhood immunizations in the last thirty years, tuberculosis and malaria deaths have rised compared to pre-pandemic levels (UN, 2023). Additionally, as of 2025 April, the number of COVID-19 deaths reported to World Health Organization (WHO) has reached almost 7.1 million as cumulative total throughout the world. (WHO, 2025)

Another negative effect of the outbreak is related to the life expectancy at birth indicator. While this value is 73.1 years before the pandemic, it decreaased to 71.4 years in 2021 and this level is the value of ten years ago. Similar situation can be observed for communicable diseases. Following a decreasing trend, the mortality rate from communicable diseases, which decreased to 18.2 in 2019, increased to 28.1 in 2021 As a result, COVID-19 can be considered as one of the top three leading causes of death in the world in 2020 and 2021 with variations across regions (UN, 2024). With 5 years to 2030, it is known that only 16 per cent of the SDGs targets have been achieved, therefore it gains great importance to monitor the indicators and know how much progress has been made (Sachs et al., 2024).

From this point of view, the purpose of the study is to investigate whether there is a difference between the pre- and post- Covid-19 periods in terms of health indicators in The Organisation for Economic Co-operation and Development (OECD) countries using Permutational Multivariate Analysis of Variance (PERMANOVA) analysis.

MATERIAL AND METHODS

Material

OECD Countries located in various parts of the world, mostly in Europe were used in the study. One of the main advantages of OECD countries is the absence of missing data. There are 38 countries in the sample and 2019 and 2022 are analyzed as pre- and post- Covid-19 periods.

OECD Countries have approxmately 1.38 billion people, 16% of the world population, and have more than 35 million km square land, roughly 27% of the world's land surface. Additionally, average GDP per capita is quite high that is 56267,85 as of 2022 since most countries are high-income (World Bank, 2025).

Methods

The Collection of the Data

Considering the indicators stated in Sustainable Development Goals Reports four variables were determined and, World Development Indicators Database from World Bank was used to collect data (World Bank, 2025). Variables used in the analyses are given in Table 1.

Table 1. Variables Used and Their Abbreviations

Variables	Abbreviation
Life expectancy at birth, total (years)	LEAB
Incidene of tuberculosis (per 100,000 people)	IOT
Mortality rate, under-5 (per 1,000 live births)	MRU5
Maternal mortality ratio (modeled estimate, per 100,000 live births)	MMR

Statistical Analysis

To determine whether there is a difference between the 2019 and 2022, permutational multivariate analysis of variance (PERMANOVA) analysis was used in the study. PERMANOVA is the multivariate nonparametric counterpart of multivariate analysis of variance (MANOVA) and it is based on distance or similarity measures. In addition, it does not require strong assumptions such as conformity to normal distribution and equality of variance-covariance matrices. The only required assumption is that the observations are interchangeable.

The key insight behind PERMANOVA is that the sum of squared distances between points and their centroid is equal to (and can be calculated directly from) the sum of squared interpoint distances divided

by the number of points (Anderson, 2001) In this analysis, variation within groups (SSW - sum of squares within groups) and variation between groups (SSB - sum of squares between groups) are found as well as total variation (SST – total sum of squares). Based on these measures, "*Pseudo-F* statistic" calculated to test the multivariate hypothesis. It assumes 0 or positive values and larger *F* values indicate greater between-group variability (Anderson, 2001a; Anderson, 2001b).

$$F = \frac{SSB/(a-1)}{SSW/(N-a)}$$

$$SSB = SST - SSW$$

where, a is the number of groups, N is the total number of observations.

RESULTS

To conduct analysis, R Studio (vegan, MVN, biotools packages) and SPSS are used. First, data set is examined regarding descriptive statistics, multicollinearity and ouliers using boxplot. Descriptive statistics for the variables are presented in Table 2.

Table 2. Descriptive Statistics for the Variables

	Mean	Median	Std. Dev.	Range	Minimum	Maximum
2019						
LEAB	80,87	81,79	2,63	9,83	74,53	84,36
IOT	11,50	6,50	11,51	55,10	2,90	58,00
MMR	10,37	6,00	12,21	64,00	2,00	66,00
MRU5	4,64	3,85	2,86	11,90	2,30	14,20
2022						
LEAB	80,36	81,29	2,76	10,03	73,97	84,00
IOT	9,98	5,70	10,31	44,40	2,60	47,00
MMR	12,47	7,50	13,58	72,00	2,00	74,00
MRU5	4,43	3,70	2,66	10,70	2,20	12,90

As can be seen in Table 3, Although it is not very high, it is revealed that all variables are correlated to each other. Additionally, when the required assumptions for the MANOVA are checked it has been seen that variance-covariance matrices equality is met based on BoxM test but multivariate normality assumption is not met based on Mardia test.

Table 3. Pearson Correlation Coefficients for the Variables

2019	LEAB	IOT	MMR	MRU5	2022	LEAB	IOT	MMR	MRU5
LEAB	1	-,349*	-,570**	-,595**	**LEAB**	1	-,442**	-,537**	-,551**
IOT	-,349*	1	,408*	,302	**IOT**	-,442**	1	,654**	,494**
MMR	-,570**	,408*	1	,847**	**MMR**	-,537**	,654**	1	,795**
MRU5	-,595**	,302	,847**	1	**MRU5**	-,551**	,494**	,795**	1

*, ** are significant levels at 5% and 1%, respectively.

To explore outliers boxplot is used and it has been seen that presence of extreme values causes the variables to be skewed. Life expectancy at birth variable has no outlier. Box-plots for both 2019 and 2022 are provided in Figure 1.

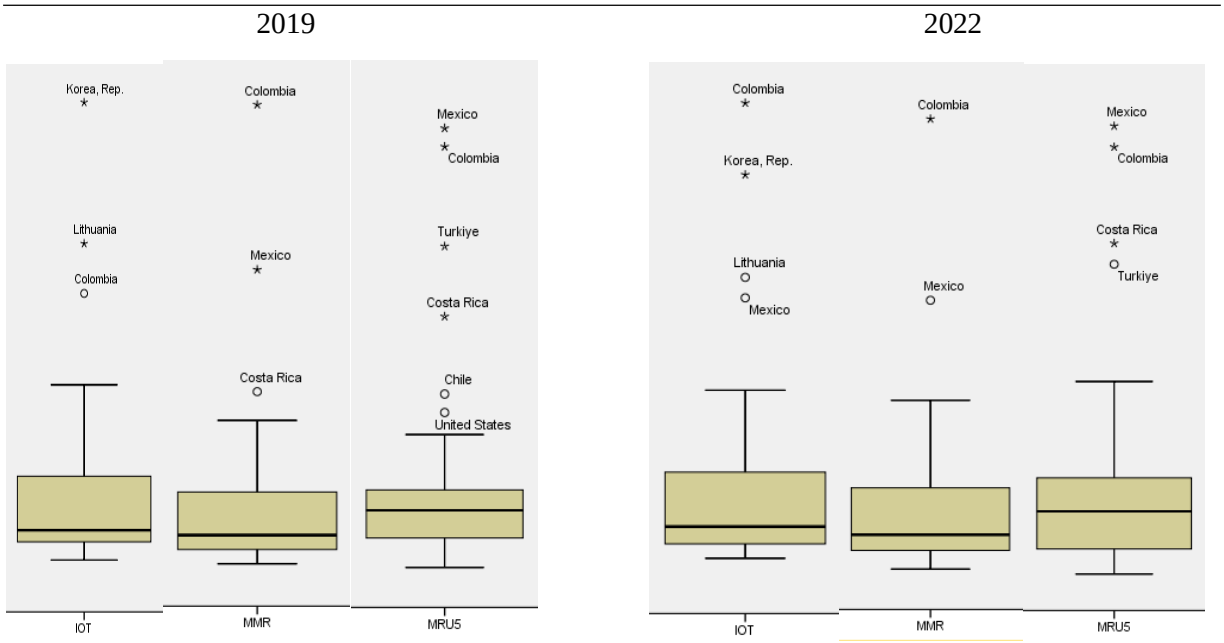

Figure 1. Outliers for IOT, MMRand MRU5 Variables in 2019 and 2022

According to the boxplots, Korea, Colombia, Mexico, Costa Rica, Turkiye, Chile, Lithuania and USA are found to be outliers, especially Colombia and Mexico are extreme countries in terms of these variables.

As a result, presence of outliers and multicollinearity and because the assumption of multivariate normality is not met, PERMANOVA analysis is conducted. Euclidean distance is used as distance measure. With respect to results, there is no statistically significant difference between the years in terms of health indicators. Results of the analysis is provided in Table 4.

Table 4. Results of PERMANOVA

Number of Permutations: 10000					
	df	**Sum of Sqs**	**R^2**	**F**	**Pr(>F)**
Model	1	133,8	0,00597	0,4442	0,671
Residual	74	22283,2	0,99403		
Total	75	22417,0	1,00000		

DISCUSSION AND CONCLUSION

COVID-19 pandemic experienced in 2020 and 2021 affected whole world and especially health systems of the countries have been damaged. In addition, countries have lost some of their gains obtained during the past years. in health area. Considering that we have only 5 years left until 2030, the determination of the current situation gains great importance.

Depending on this, the aim of the study is to investigate whether there is a difference between the pre- and post- COVID-19 periods in terms of health indicators in the OECD countries. In this study PERMANOVA is used because strong assumptions such as multivariate normality and equality of variance-covariance matrices of the MANOVA are not met. Results obtained showed that there was no statistically significant difference between the years in terms of health indicators.

One of the main reasons of this result is the income level of the countries. Most OECD members are high-income level and also they have very high scores in Human Developent Index that is one of the

most important indices for sustainable development. Moreover, these countries have well- educated people, low fertility rate, high urbanization and internet usage rates. All these situations lead positive impact in recovering health indicators after COVID-19 pandemic.

Although the negative effects of COVID-19 continue in different parts of the world, it is understood that OECD countries have eliminated these effects. Significant improvement in health indicators before the pandemic can be considered as one of the reasons for this result. However, in the light of the experiences gained during the pandemic, it is inevitable for OECD countries to take measures to strengthen their health systems until 2030.

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Regional Distribution of Handled Containers at Turkish Ports (1199)

Elvan Deniz[1*]
[1]Çanakkale Onsekiz Mart Üniversitesi, Çanakkale, Turkiye

Corresponding Author e-mail: elvandeniz@comu.edu.tr

Abstract

Ports, as one of the vital arteries of global trade, constitute the most critical link in maritime transportation. Ensuring the continuity of goods flow, enhancing the efficiency of supply chains, and contributing to economic development make port logistics a matter of great importance. As centers of intense logistics activity, the use of containers has increased in recent years, especially due to the convenience they provide in handling and storage operations. Containers are large, durable, and standardized transport units used across all modes of transportation. These units, which allow for the safe, economical, and rapid movement of goods and products, have become fundamental components of modern logistics systems. Today, containerization accounts for a significant portion of international trade and plays a decisive role in planning logistics processes at ports. In this context, the effective management of port logistics assumes a strategic role in reducing costs, shortening delivery times, and gaining a competitive advantage. This study analyzes the quantity of containers handled at Turkish ports between 2012 and 2024 on a regional basis and provides future-oriented forecasts using time series analysis methods. Through methods such as Exponential Smoothing, Trend Analysis, and Simple Moving Average, different structural trends across regions have been identified. Accordingly, the most suitable forecasting models were determined using Minitab 17 software. Model performances were evaluated using key error metrics, namely Mean Absolute Percentage Error (MAPE), Mean Squared Deviation (MSD), and Mean Absolute Deviation (MAD). The study contributes significantly to understanding the future direction of container transportation in Turkey and supports the development of scientifically grounded port policies. The findings offer insights into the structural trends observed in Turkey's foreign trade and provide forward-looking projections for investors. Ultimately, the study underlines the value of time series analysis in economic decision-making and demonstrates its potential to forecast trade data effectively.

Keywords: *Time series analysis, Container, Turkish Ports, Foreign trade, Port logistics*

INTRODUCTION

Ports, as strategic components of global logistics networks, play a vital role in ensuring the uninterrupted continuation of international trade. Benefiting from its unique geopolitical position at the intersection of Asia, Europe, and Africa, Türkiye has become a significant maritime transportation hub. The country hosts numerous ports distributed across the Marmara, Aegean, Mediterranean, and Black Sea regions. Container handling activities at these ports not only drive regional economic dynamism but also directly influence Türkiye's position in the global logistics chain. In this regard, analyzing the container traffic of ports in different geographical regions is crucial for determining the efficient use of port capacities and for restructuring national logistics strategies.

Türkiye's foreign trade volume and logistics capabilities are directly associated with the effectiveness of its regional port infrastructure. Accordingly, the Marmara, Aegean, Mediterranean, and Black Sea regions serve as Türkiye's primary maritime gateways in container transportation. The Marmara Region, with its dense industrial zones, strategic port infrastructure, and extensive hinterland, is positioned at the

core of both domestic and international trade. Ports located in cities such as Istanbul, Kocaeli, and Tekirdağ handle a significant portion of the country's total container volume.

The Aegean Region, through key ports like İzmir Alsancak and Aliağa, facilitates the integration of export-oriented production into maritime trade and functions as a critical logistics connection point between inland regions and the sea. In the Mediterranean Region, Mersin and İskenderun ports stand out as key gateways to the Eastern Mediterranean basin. In particular, the Port of Mersin is one of Türkiye's busiest container ports due to its capacity and strategic location.

The Black Sea Region serves as a strategic corridor connecting Central Asia and Eastern Europe via maritime routes. Although ports such as Samsun and Trabzon support the region's trade potential, infrastructure constraints limit the full utilization of their capacities. This study provides a comparative assessment of container handling performance across Türkiye's four coastal regions and offers insights to guide regional logistics strategies.

When reviewing academic studies on port performance in Türkiye, recent years have witnessed a notable increase in quantitative analyses, especially focusing on container terminals. One of the pioneering studies in this field, conducted by Çelik and Yorulmaz (2023), evaluated 13 container terminals across Türkiye using the Entropy and TOPSIS methods, comparing the relative efficiencies of ports in the Marmara, Aegean, and Mediterranean regions. This approach enabled the measurement of regional performance disparities.

Similarly, in a study by Kılınç, Karaatlı, and Ömürbek (2022), the volumes of containers and cargo handled at Turkish ports were forecasted using the NARX neural network model. Focusing particularly on the Marmara, Aegean, and Mediterranean regions, the study provided future-oriented predictions and highlighted the potential of data-driven logistics planning at the regional level.

On the other hand, Açık (2020) examined how economic uncertainty affects foreign trade operations at Turkish container ports. Using an asymmetric causality test, the study analyzed the impact of uncertainty levels on export and import container traffic and emphasized the structural divergences resulting from these effects. However, the limited number of systematic studies at the regional level in the literature enhances the originality and contribution of this research.

Within the scope of this study, annual container handling data from ports located in Türkiye's various geographical regions were utilized to identify historical trends and generate regional forecasts for the 2025–2027 period. The analyses focused on the Marmara, Aegean, Mediterranean, and Black Sea regions, where container traffic at each port was evaluated statistically. The container handling data were obtained from institutional and reliable sources such as the Turkish Port Operators Association (TÜRKLİM, 2025), which ensures consistency and reliability in modeling. The continuity and consistency of the datasets acquired from these official sources have increased the validity of the forecasting models and enabled a more accurate assessment of regional logistics performance.

MATERIAL AND METHODS

Methods

The forecasting process is a systematic approach aimed at predicting future developments within a probabilistic framework by analytically evaluating current data and historical trends. The statistical techniques and quantitative models employed in this process enable decision-makers to develop more accurate, predictable, and strategically sound plans. As emphasized by Armstrong (2001), forecasting should not be confined solely to mathematical algorithms; it must also be supported by expert judgment, intuitive assessments, and experience-based insights. This holistic approach offers significant contributions, particularly in dynamic sectors such as logistics, transportation, and foreign trade.

In this context, the regional analysis and forward-looking forecasting of container volumes handled at Turkish ports are of critical importance for shaping transport policies, port management strategies, and external trade planning. One of the most frequently used methods in this field is time series modeling, which allows for the systematic evaluation and decomposition of components such as trend, seasonal variation, random fluctuations, and structural shifts in the data. By identifying patterns observed in the past, this method makes it possible to generate future projections and predict regional port activity with greater reliability.

Trend Analysis and Error Metrics

Trend analysis is one of the fundamental statistical techniques used to identify long-term patterns in time series data. This method plays a critical role in enhancing strategic planning and decision support mechanisms, especially in areas where time-dependent changes are prominent, such as logistics, transportation, and port operations (Lütkepohl, 2005). The trend component reflects the structural movement of a variable over time, free from short-term fluctuations and random effects.

In time series analysis, trends can be modeled using various functional forms. While simple linear regression models are applied for linear trends, second-degree (quadratic), logarithmic, or exponential functions are preferred in cases involving curvature or structural change (Gujarati & Porter, 2009). In practice, moving averages, least squares approaches, and exponential smoothing methods are widely employed to estimate trend patterns. These analyses not only provide a retrospective evaluation of the data but also serve as a foundation for generating future forecasts. Particularly in areas such as capacity planning, infrastructure investments, and service level improvement, trend analysis offers significant analytical support (Hyndman & Athanasopoulos, 2021).

The accuracy of the selected trend model largely depends on the structural characteristics of the dataset. For instance, if container handling volumes exhibit a consistent upward trajectory, linear models may yield accurate results. However, in the presence of abrupt changes, seasonal effects, or structural breaks in the series, more complex component-based models may be required (Wei, 2006). Therefore, trend analysis should be considered not merely as a data exploration technique but as a powerful tool for reducing future uncertainties.

The forecasting performance of time series models is typically assessed using specific error metrics. Among the most widely used indicators are the Mean Absolute Percentage Error (MAPE), Mean Absolute Deviation (MAD), and Mean Squared Deviation (MSD) (Diebold, 2012). MAPE facilitates comparisons across series of different scales, while MAD provides direct insight into the average magnitude of deviations. MSD, on the other hand, assigns greater weight to large errors, making it more sensitive in identifying the influence of outliers (Stock & Watson, 2015).

Evaluating these error metrics in conjunction enables a more balanced and comprehensive assessment of model performance. Therefore, selecting an appropriate trend model along with the integration of the correct error criteria is essential for enhancing the accuracy and reliability of time series-based forecasting models.

RESEARCH METHODOLOGY

The primary objective of this study is to examine the structural trends in the regional container handling volumes (TEU) at Turkish ports between 2012 and 2024 and to produce demand forecasts for the period of 2025–2027. Ports in Türkiye, particularly concentrated in the Marmara, Aegean, Mediterranean, and Black Sea regions, constitute critical infrastructure points that shape the country's foreign trade capacity (Akar & Esmer, 2017). Port handling capacities not only reflect current trade volumes but also serve as key indicators influencing regional development dynamics (Aydın & Kaptan, 2022).

Analyzing port operations on a regional scale is essential for understanding spatial disparities and planning resource allocation more efficiently. For instance, ports in the Marmara Region, supported by a strong industrial hinterland, handle significantly higher cargo volumes, whereas in regions like Eastern Black Sea or Southeastern Anatolia, the volume remains relatively limited (Yeşilbayrak & Ertem, 2023). This highlights the structural differences and planning needs of ports across different regions.

Through the analyses conducted, the TEU-based performance of ports in various regions has been compared over time. It has been observed that ports in the Marmara Region exhibit a steadily increasing trend, while ports in the Aegean and Mediterranean regions show periodic fluctuations (Deniz & Aydın, 2021). These findings aim to provide data-driven strategic guidance for infrastructure investments, capacity expansion decisions, and logistics planning by public authorities and private stakeholders.

In the analytical process, annual container handling volumes at ports located in different geographical regions of Türkiye were used to conduct modeling using six different time series forecasting methods. The applied methods include: linear trend analysis, quadratic (second-degree polynomial) trend analysis, exponential trend analysis, simple moving average, single exponential smoothing, and double exponential smoothing techniques.

The forecasting performance of each model was evaluated using commonly accepted error metrics: Mean Absolute Percentage Error (MAPE), Mean Absolute Deviation (MAD), and Mean Squared Deviation (MSD). Based on these evaluations, the most appropriate forecasting method for each region was determined, and regional projections of container traffic at Turkish ports for the 2025–2027 period were generated.

This approach contributes to the effective planning of port capacities and offers scientifically grounded projections for regional logistics decision-makers. Moreover, by comparing the error performance of different modeling techniques, the study aims to provide highly reliable forecasting results for both the public and private sectors.

RESULTS

The regional distribution of container volumes handled at Turkish ports was analyzed based on annual data covering the period from 2012 to 2024. According to the statistical data presented in Table 1 and the time series trends illustrated in Figure 1, a clear upward trend has been observed across all regions over the years, with the Marmara Region showing the most significant growth.

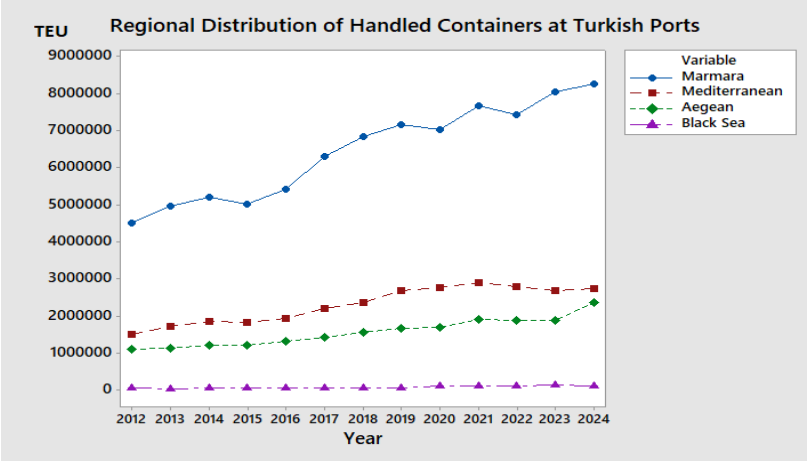

Figure 1. Time series of regional container handling at Turkish ports

In the Marmara Region, the container handling volume increased significantly from approximately 4.5 million TEUs in 2012 to 8.3 million TEUs by the end of the analysis period in 2024. This notable growth reflects the region's continued central role in Türkiye's foreign trade and indicates a gradual strengthening of its port infrastructure. Similarly, the Mediterranean and Aegean regions have demonstrated a steady and sustainable growth trend over the years. In particular, the sudden and substantial increase observed in the Aegean Region as of 2024 highlights the rising impact of regional port investments and the growing logistical potential of its hinterland.

On the other hand, although the Black Sea Region holds a limited share in the national total, it has exhibited a relative upward trend in recent years. This suggests that ports in the region primarily serve regional transportation functions and possess a development potential that is limited yet stable. When all regions are evaluated together, it becomes evident that regional capacity differences persist within Türkiye's port network, although an overall upward trend in container handling volumes is clearly observed (see Table 1).

Table 1. Regional distribution of handled containers at Turkish ports

Years	Marmara	Mediterranean	Aegean	Black Sea
2012	4 510 561	1 513 431	1 109 372	59 034
2013	4 963 615	1 731 399	1 149 618	55 305
2014	5 210 325	1 858 236	1 215 273	67 289
2015	5 007 726	1 834 986	1 230 025	73 663
2016	5 419 372	1 953 696	1 322 746	66 160
2017	6 299 111	2 201 827	1 432 255	77 346
2018	6 843 524	2 365 581	1 555 613	79 282
2019	7 159 361	2 685 110	1 674 159	73 209
2020	7 034 053	2 768 691	1 711 906	112 000
2021	7 670 831	2 902 350	1 905 742	112 545
2022	7 414 284	2 788 227	1 894 085	118 673
2023	8 048 454	2 695 136	1 883 136	141 126
2024	8 267 909	2 747 347	2 381 136	133 338

Before proceeding with the time series analysis, the statistical characteristics of the container handling data used in the study were examined, and the distributional properties that would form the basis of the modeling process were evaluated. Accordingly, the normality of the data for the Marmara, Mediterranean, Aegean, and Black Sea regions was tested using the Anderson-Darling test, which is recommended for small samples and is known for its sensitivity to extreme values (Anderson & Darling, 1954).

According to the test results, the p-values obtained for all regions were found to be above the 5% significance level, statistically confirming that the data sets follow a normal distribution. Specifically, the results were as follows: Marmara Region AD = 0.284 (p = 0.572), Mediterranean Region AD = 0.317 (p = 0.463), Aegean Region AD = 0.246 (p = 0.602), and Black Sea Region AD = 0.298 (p = 0.514). These findings indicate that parametric time series modeling techniques can be appropriately applied in line with the structure of the data (see Figure 2).

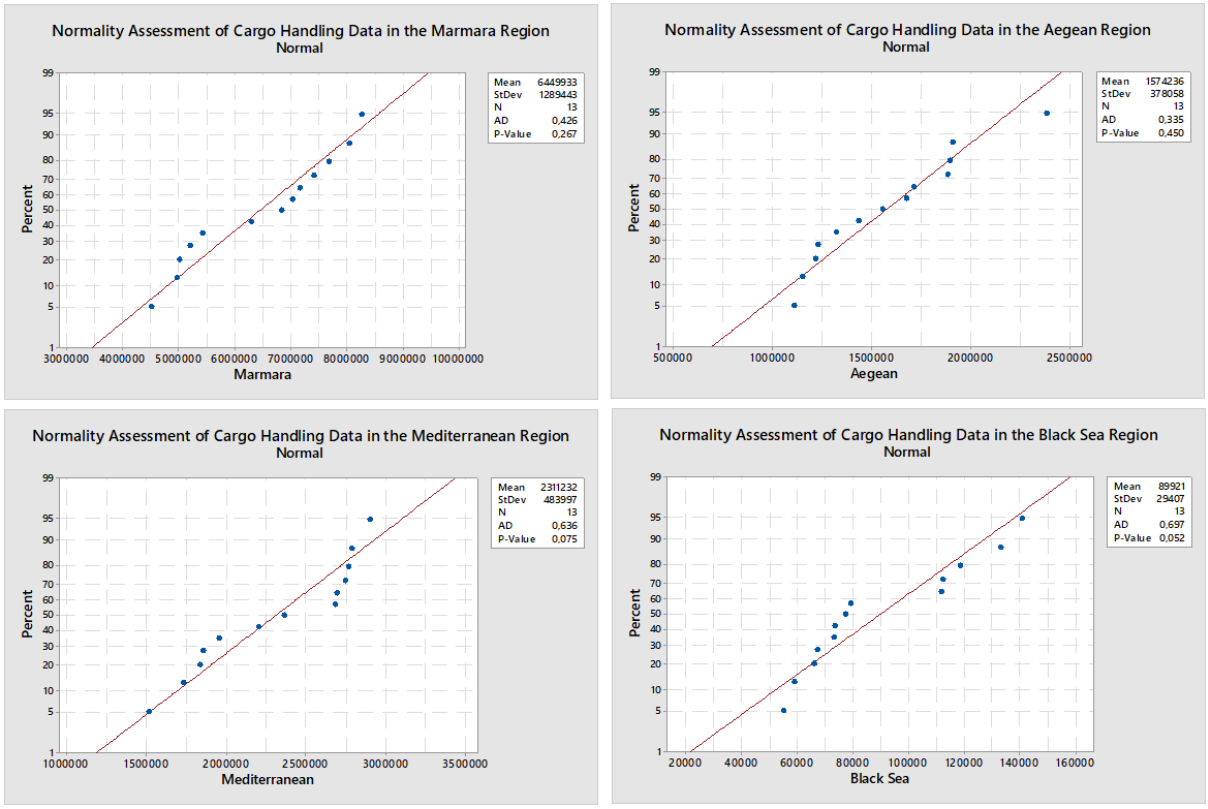

Figure 2. Normality test results based on the Anderson-Darling method

In order to model the trend component of the time series, linear, quadratic (second-degree), and exponential trend structures were evaluated, and the suitability of each model to the data set was analyzed comparatively. The time variable was transformed and used in the form of t = Year – 2011.

Time Series Forecasting Results for the Marmara Region

In the time series modeling applied to the annual container handling data of the Marmara Region, the most successful results were obtained through linear trend analysis. This model yielded the lowest error rates, with a MAPE value of 3.42%, a MAD of 213.67, and an MSD of 7.03×10^{10}. The resulting regression equation is as follows:

$Y_t = 4185969 + 323423 \times t$

Another method that demonstrated a performance close to the linear model was the quadratic trend analysis, with a MAPE of 3.51%, a MAD of 211.52, and an MSD of 6.71×10^{10}. Moreover, the double exponential smoothing technique also provided a significant level of accuracy with a MAPE of 4.93%. On the other hand, the moving average method (MAPE of 8.74%) and simple exponential smoothing (MAPE of 7.08%) produced higher error rates, indicating insufficient forecasting performance for this region. These results suggest that the container handling volume in the Marmara Region follows a relatively stable and linear trend over time, making linear models highly compatible for predictive analysis.

Time Series Forecasting Results for the Mediterranean Region

The findings obtained for the Mediterranean Region indicate that the most successful model is the double exponential smoothing method. This approach yielded a Mean Absolute Percentage Error (MAPE) of 4.56%, a Mean Absolute Deviation (MAD) of 104.66, and a Mean Squared Deviation (MSD) of 1.75×10^{10}. These results demonstrate that the data series in the region contains a pronounced

trend component, which can be more effectively captured through the adjustment parameters inherent in this method.

Alternatively, the quadratic trend analysis (MAPE: 4.95%) and linear trend analysis (MAPE: 5.12%) also provided reasonably accurate forecasting performances with relatively close error rates. However, the moving average method (MAPE: 9.32%) and simple exponential smoothing (MAPE: 7.46%) exhibited higher error levels and performed poorly in terms of forecast accuracy. This suggests that the data structure in the Mediterranean Region exhibits non-linear and time-dependent changes, and therefore, methods incorporating correction mechanisms tend to yield more consistent predictions.

Time Series Forecasting Results for the Aegean Region

In the model comparisons conducted for the Aegean Region, the most successful results were obtained using the quadratic trend analysis and the exponential trend model. Both models achieved the lowest error rate with a MAPE value of 3.00%. The forecasting accuracy of the quadratic model, in particular, indicates the presence of a nonlinear upward trend in the series. The regression equation is as follows:

$Y_t = 1061108 + 36518 \times t + 4087 \times t^2$

In addition, the linear trend model (MAPE: 4.00%) and the double exponential smoothing method (MAPE: 4.36%) also provided forecasts with acceptable accuracy levels. On the other hand, the simple exponential smoothing and especially the moving average method (MAPE: 6.71% and 10.89%, respectively) demonstrated weaker performance for this region. These results reveal that the cargo movements in the Aegean Region follow a curvilinear growth pattern, suggesting that nonlinear models are more effective for forecasting in this context.

Time Series Forecasting Results for the Black Sea Region

In the analyses conducted for the Black Sea Region, the quadratic trend model yielded the lowest error rates, with a MAPE of 7.00%, MAD of 6.28, and MSD of 6.34×10^7. This model was found to best represent the relatively low-volume and variable data structure of the region. The regression equation is as follows:

$Y_t = 57732 + 170 \times t + 492 \times t^2$

The exponential trend model also achieved a similar level of accuracy with a MAPE of 8.00%. In contrast, the moving average method performed the weakest, with a MAPE of 14%. The simple and double exponential smoothing methods demonstrated moderate performance, each with a MAPE of 10%. These findings indicate that the Black Sea Region's data exhibit a more volatile and challenging-to-predict structure, suggesting that classical methods provide limited accuracy and that polynomial approaches may be more appropriate.

The results also reveal that the performance of time series forecasting methods applied across different regions varies significantly depending on the regional characteristics of the data sets. As shown in Table 2, container handling volumes in the Marmara Region demonstrate a distinct and consistent upward trend over the years. Therefore, the linear trend model stands out as the most suitable model, yielding the lowest error rate (MAPE: 3.42%).

Table 2. Model performance metrics and comparison

Method	Region	MAPE	MAD	MSD
Linear	Marmara	3.41547	213 674	$7.033\ 10^{10}$
Quadratic	Marmara	3.50708	211 519	$6.709\ 10^{10}$
Exponential	Marmara	4.09365	264 408	$9.515\ 10^{10}$
Moving Average	Marmara	8.74371	608 205	$5.162\ 10^{11}$

Simple Exponential Smoothing	Marmara	7.08377	436 371	$4.277\ 10^{10}$
Double Exponential Smoothing	Marmara	4.92875	297 485	$1.270\ 10^{11}$
Linear	Mediterranean	5.12407	124 856	$2.524\ 10^{10}$
Quadratic	Mediterranean	4.95497	111 386	$1.561\ 10^{10}$
Exponential	Mediterranean	6.03829	148 381	$3.552\ 10^{10}$
Moving Average	Mediterranean	9.3241	230 156	$7.674\ 10^{10}$
Simple Exponential Smoothing	Mediterranean	7.45509	158 331	$3.401\ 10^{10}$
Double Exponential Smoothing	Mediterranean	4.55701	104 656	$1.754\ 10^{10}$
Linear	Aegean	4	70 552	$8.911\ 10^{09}$
Quadratic	Aegean	3	52 854	$6.338\ 10^{09}$
Exponential	Aegean	3	56 615	$6.601\ 10^{09}$
Moving Average	Aegean	10.8886	191 645	$5.075\ 10^{10}$
Simple Exponential Smoothing	Aegean	6.71103	112 701	$2.815\ 10^{10}$
Double Exponential Smoothing	Aegean	4.36264	72 182	$1.001\ 10^{10}$
Linear	Black Sea	10	7 626	$1.007\ 10^{08}$
Quadratic	Black Sea	7	6 278	$6.343\ 10^{07}$
Exponential	Black Sea	8	6 871	$7.352\ 10^{07}$
Moving Average	Black Sea	14	14 525	$3.313\ 10^{08}$
Simple Exponential Smoothing	Black Sea	10	9 768	$1.915\ 10^{08}$
Double Exponential Smoothing	Black Sea	10	8 605	$1.283\ 10^{08}$

Given that the Mediterranean Region's data exhibit a trend component but follow a nonlinear structure, the double exponential smoothing method provided the most accurate forecasting performance for this region (MAPE: 4.56%). This model stood out due to its ability to effectively capture the trend and its low MAD and MSD values.

In the Aegean Region, the evaluation revealed a nonlinear pattern in the time series, whereby both the quadratic trend model and the exponential model achieved equally low error rates with a MAPE of 3.00%. These findings suggest that more complex (curvilinear) models are required for forecasting in this region.

For the Black Sea Region, the relatively low and volatile container volumes constrained the overall forecasting performance. Nevertheless, under these conditions, the quadratic trend model once again outperformed the others, achieving the lowest error rate with a MAPE of 7.00%. The high variance in this region indicates that more complex or parameter-adjusted models fail to deliver sufficient accuracy.

In conclusion, the comparative performance metrics presented in Table 2 highlight the importance of carefully analyzing region-specific data structures and selecting the most appropriate forecasting model accordingly. Applying a single model type uniformly across all regions may lead to high error rates. Therefore, it is recommended to examine regional characteristics in detail and tailor model selection to the structural attributes of each dataset.

DISCUSSION AND CONCLUSION

This study analyzed container handling activities at ports located in the Marmara, Mediterranean, Aegean, and Black Sea regions of Türkiye during the period 2012–2024 and generated projections for the 2025–2027 period through forecasting models. Based on the time series analyses conducted, regional container traffic trends were identified, and various statistical modeling approaches were compared. The modeling process incorporated linear, exponential, and second-degree (quadratic) trend analyses, along with moving average and exponential smoothing techniques. According to the evaluations based on error metrics (MAPE, MAD, MSD), the quadratic trend model was determined to produce the most accurate forecasts.

The projected container volumes for the period 2025–2027, based on the model with the highest forecasting accuracy, are presented in Table 3.

Table 3. Projected container handling volumes by region based on forecasting results

Years	Marmara	Mediterranean	Aegean	Black Sea
2025 F	8 713 896	2 842 219	2 373 477	156 552
2026 F	9 037 319	2 930 554	2 528 528	170 991
2027 F	9 360 743	3 018 889	2 691 754	186 414

According to the forecasting results, the Marmara Region is expected to continue its upward trend, reaching 8.7 million TEUs in 2025, 9.0 million TEUs in 2026, and 9.36 million TEUs in 2027. This increase reflects the region's dominance in foreign trade volume and its port infrastructure's capacity to respond to rising demand. Similarly, the Mediterranean Region is projected to maintain a steady growth trend, with its container volume expected to surpass 3 million TEUs by 2027. The Aegean Region, following a sharp increase observed in 2024, is forecasted to maintain its momentum, reaching 2.69 million TEUs by 2027. Although the Black Sea Region has a relatively limited total volume, it is exhibiting a stable upward trajectory, projected to reach 186,000 TEUs in 2027.

These findings highlight the differentiated dynamics of container transportation across regions and indicate that each region is undergoing a distinct process of logistic development. The projections obtained are graphically illustrated in Figure 3, with regional trends visualized separately over time.

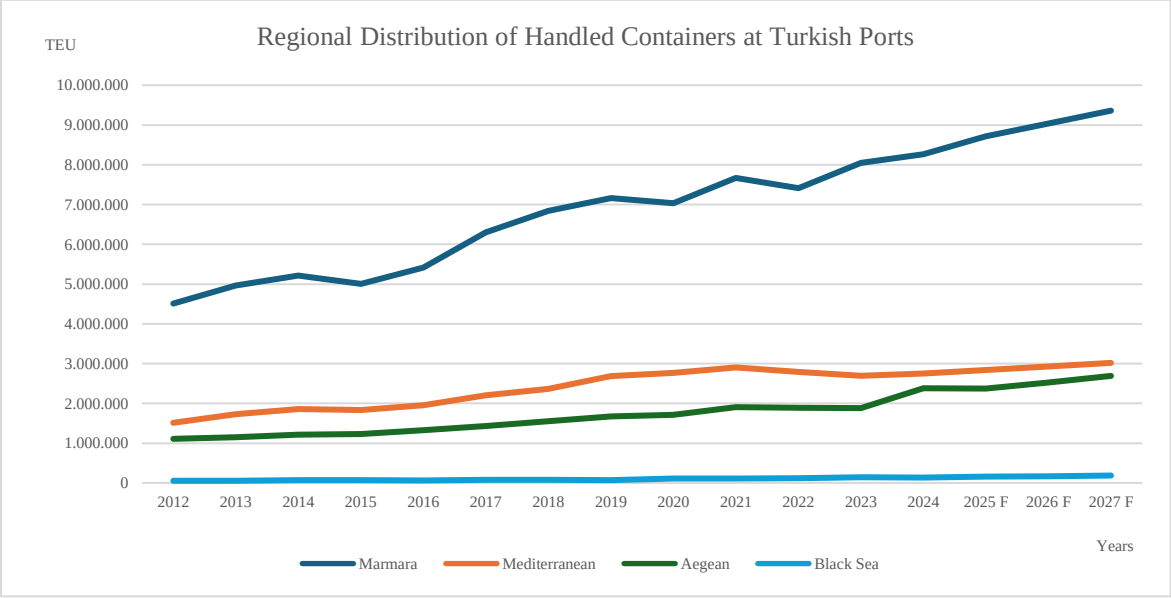

Figure 3. Projected time series trends of regional container handling (2012–2027)

The analysis results reveal that container transportation in Türkiye exhibits a nonlinear structure. This indicates that fluctuations in foreign trade volumes, global economic trends, supply chain crises, and changes in infrastructure capacity significantly impact cargo volumes. Therefore, logistics planning processes should not only rely on historical trends but also incorporate approaches that account for structural breaks and periodic volatility.

In this context, the following recommendations are proposed:

The Marmara Region currently has the highest capacity utilization due to its increasing container volumes. Consequently, expanding port infrastructure, strengthening intermodal connections, and accelerating digitalization investments are critically important for managing regional cargo flows efficiently.

The Aegean, Mediterranean, and Black Sea Regions, while operating below their potential capacity, have promising hinterlands. Infrastructure investments, port modernization efforts, and operational efficiency enhancements in these regions are strategically important for ensuring the balanced development of Türkiye's port system.

The presence of nonlinear trends in Türkiye's container transportation data necessitates the use of more advanced forecasting methods in future research. In particular, models such as LSTM (Long Short-Term Memory), Prophet, and hybrid artificial intelligence algorithms may offer more reliable forecasts by better capturing seasonal fluctuations and external shocks.

In conclusion, this study demonstrates that container transportation at Turkish ports varies significantly across regions, with the Marmara Region maintaining its leading position. The rising potential of the Aegean and Mediterranean Regions should be considered in future planning. Data-driven planning approaches contribute to the efficient use of logistics infrastructure and support evidence-based strategy development for policymakers.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Customs Regime Distribution of Handled Containers at Turkish Ports (1200)

Elvan Deniz[1*]

[1]Çanakkale Onsekiz Mart Üniversitesi, Çanakkale, Turkiye

Corresponding Author e-mail: elvandeniz@comu.edu.tr

Abstract

Turkey has emerged as a significant transit and logistics hub in maritime transportation, primarily due to its strategic geographical location. Container handling activities at Turkish ports not only reflect the volume and structure of the country's foreign trade, but they are also categorized under various customs regimes. This classification provides valuable insights into the type of trade or transaction—whether entry or exit—under which the containers, measured in TEUs (Twenty-foot Equivalent Units), are processed. A customs regime refers to the legal framework that determines the customs procedures applied to goods entering or leaving a country, as well as the obligations that arise from these procedures.In this study, a time series-based forecasting model has been developed using container-based data on the major customs regimes in Turkey, specifically focusing on imports, exports, cabotage, and transit operations. The use of containers in Turkey has significantly increased in recent years due to the growth in foreign trade, the modernization of logistics infrastructure, and greater integration into global supply chains. Using official annual data from 2012 to 2024 on container throughput at Turkish ports classified by customs regime, a national-level analysis was conducted to identify the most suitable forecasting approach. Within this scope, a total of six different time series modeling techniques—including Trend Analysis, Exponential Smoothing, and Simple Moving Average—were applied using Minitab 17 statistical software. Model performances were evaluated based on Mean Absolute Percentage Error (MAPE), Mean Squared Deviation (MSD), and Mean Absolute Deviation (MAD) criteria. The findings indicate that while cabotage and transit operations exhibit a fluctuating pattern, both imports and exports demonstrate an upward trend. These results underline Turkey's increasing prominence as a transshipment and distribution hub in global maritime transportation, further strengthening its potential to become a regional logistics center.

Keywords: *Time series analysis, Container, Customs regimes, Turkish Ports, Foreign trade*

INTRODUCTION

The increasing volume of global trade has rendered maritime transportation and port operations indispensable components of modern logistics systems. In particular, container transportation has gained significant momentum in parallel with the growth in commercial volume. This development has necessitated not only the enhancement of operational efficiency in ports but also the analysis of container traffic in terms of customs regimes. Customs regimes are fundamental instruments that define the legal status of international trade transactions and are categorized into types such as import, export, transit, bonded warehousing, inward processing, and outward processing. Analyzing these regimes at the port level plays a critical role in understanding the nature of foreign trade structures, port governance policies, and the overall functioning of the logistics system.

One of the early notable studies in this context is by Notteboom & Winkelmans (2001), which addresses the transformation of port structures and the influence of trade regimes on port authorities. Rodrigue,

Comtois, & Slack (2017) emphasize that categorizing container flows according to regime types is an effective tool in shaping trade policies on a global scale.

In the context of Turkey, most studies have focused on general handling volumes and port efficiency, while research on regime-based container distribution remains limited. For instance, Kuru & Bayraktutan (2017) conducted a performance analysis of container ports in Turkey and demonstrated the impact of import and export regimes on port productivity. Çetinkaya & Tuna (2020) analyzed the influence of Turkey's geostrategic location on its foreign trade and discussed how regime-based differences reflect on logistics strategies. Ertem & Deniz (2020) examined regional and annual variations in container traffic at Turkish ports but did not delve into regime-specific distributions. Öztürk & Acar (2021) investigated the relationship between trade regimes and transportation modes in Turkey, although they did not provide empirical data at the port level. The study by Kose, Çakır & Arı (2021), also published in the same year, explored the evolution of Turkey's foreign trade regimes since 2000 and their impact on port utilization. Additionally, Yılmaz & Genç (2019) analyzed the effects of customs regime-based logistics processes in terms of time and cost, highlighting the functional significance of the cabotage regime in regional trade.

Recent international reports have also emphasized the importance of this topic. The World Bank (2022) noted that port efficiency is not only dependent on physical infrastructure but also influenced by regime diversity, customs procedures, and documentation systems. Similarly, the UNCTAD Review of Maritime Transport (2023) underlined the strong connection between ports' integration into global supply chains and the structure of customs regime distributions.

At the sectoral level, the 2022 Turkish Port Sector Report published by TÜRKLİM presents the port-level distribution of import and export regimes, although it lacks detailed data on specific regimes such as transit and bonded warehousing. Data from the Ministry of Customs and Trade (2023) underscore the regional concentration of different regime types.

The main objective of this study is to analyze the quantitative distribution of containers handled at Turkish ports according to customs regimes over a defined period. By evaluating the regime-based components of the current foreign trade structure, the study aims to provide a scientific foundation for port management, logistics strategy development, and customs policy planning. The limited number of systematic studies on this topic in the existing literature enhances the originality and contribution of this research. Furthermore, based on historical data, the study will forecast regime-based container traffic for the period 2025–2027, thereby offering forward-looking insights for policymakers. In this way, the study contributes not only to the analysis of the current situation but also to the formulation of data-driven strategies for future logistics planning.

MATERIAL AND METHODS

Methods

Making forward-looking projections has become a strategic necessity not only in academic research but also in practical domains such as public policy, logistics planning, and infrastructure investment. Forecasting is a systematic process that involves analyzing patterns in historical data to reduce uncertainty and support the development of sound decision-making mechanisms (Makridakis, Spiliotis & Assimakopoulos, 2020). This process is not limited to quantitative modeling alone; when integrated with expert judgment, sectoral experience, and decision support systems, it yields significantly more effective results (Gorr, 2009).

In dynamic and uncertainty-prone sectors such as port operations, forecasting plays a critical role—particularly in projecting container movements. In this context, time series analyses offer a robust

framework by modeling regular structures and patterns in historical data, thereby enabling systematic projections of future developments (Hyndman & Athanasopoulos, 2021). Time series are analyzed through their fundamental components—trend, seasonality, cyclicity, and random variation—facilitating reliable interpretation of fluctuating and volume-intensive variables such as cargo flows (Chatfield, 2016).

Within the scope of this study, annual data on container volumes handled at Turkish ports were utilized to identify historical trends and generate forecasts for the 2025–2027 period. The data were obtained from official sources such as the Turkish Port Operators Association (TÜRKLİM, 2025). The continuity and consistency of the data from these sources have strengthened the statistical validity of the applied forecasting models.

Trend Analysis and Error Metrics

Trend analysis is one of the fundamental methods used to identify long-term tendencies within time series data. The primary objective of these analyses is to isolate structural changes in the data by filtering out random fluctuations and short-term effects (Cryer & Chan, 2008). Among the most frequently applied techniques for identifying trend structures are the Ordinary Least Squares (OLS) method, moving averages, and exponential smoothing models (Gardner, 2006). These methods serve as effective tools not only for analyzing past tendencies but also for generating forward-looking forecasts.

Within trend analysis, several functional model structures are commonly utilized. Linear models are typically used to test for the presence of constant rates of increase or decrease over time. Quadratic (second-degree polynomial) models, on the other hand, offer higher accuracy in capturing nonlinear patterns or accelerations within the series. Logarithmic models are particularly effective for series that exhibit rapid initial growth followed by a plateau. Moving average methods help smooth short-term fluctuations, thereby revealing the underlying trend more clearly, whereas exponential models and exponential smoothing techniques provide flexible structures that respond more sensitively to recent changes by assigning greater weights to the most recent observations. Each model type may outperform others depending on the characteristics and structure of the dataset.

However, in cases where the series exhibits seasonal effects, structural breaks, or cyclical fluctuations, more advanced models such as Holt-Winters, ARIMA, or SARIMA should be employed (Wei, 2006). These models are capable of producing more robust forecasts by simultaneously accounting for both trend and seasonal components.

In model selection, not only the structural characteristics of the time series but also factors such as the length of the forecast horizon, model simplicity, number of parameters, and the accuracy of the dataset play a critical role. These elements directly influence the reliability and applicability of the forecasting model. To determine the most appropriate model, various error metrics are commonly employed to assess model performance. Among these, MAPE (Mean Absolute Percentage Error), which evaluates forecast errors in percentage terms; MAE (Mean Absolute Error), which indicates the average magnitude of the errors; and RMSE (Root Mean Squared Error), which assigns greater weight to larger deviations, are widely recognized as fundamental criteria.

These error measures are used to objectively assess forecasting accuracy and to compare the performance of alternative models (Armstrong, 2001; Hyndman & Koehler, 2006). However, model selection should not rely solely on error magnitudes. Consideration must also be given to model interpretability, statistical validity, and contextual relevance in practical applications.

RESEARCH METHODOLOGY

The primary objective of this study is to analyze the distribution of container volumes (TEU) handled at Turkish ports between 2012 and 2024 by customs regimes and to generate regime-based forecasts for the 2025–2027 period based on historical data. Owing to its strategic geographical location, Türkiye serves as a critical logistics transit hub between Europe and Asia, as well as within the hinterlands of the Black Sea, the Mediterranean, and the Middle East. In this context, classifying the containers handled at ports by customs regimes—such as import, export, transit, bonded warehousing, and other special regimes—is of significant importance for understanding the structural features of foreign trade, guiding investments in logistics infrastructure, and evaluating the effectiveness of customs policies.

The dataset utilized in this study was obtained from the Turkish Port Operators Association (TÜRKLİM) and includes annual container handling volumes and their distribution by customs regime types for the period from 2012 to 2024. The data are presented in a complete and consistent annual format, making them statistically suitable for time series analyses and trend modeling. The findings of this study are expected to provide a scientific foundation for decision-makers in areas such as port capacity planning, regional logistics strategies, and the effective management of foreign trade policies.

RESULTS

The quantities of containers handled at Turkish ports by customs regime were analyzed based on annual data from the 2012–2024 period. According to the statistical data presented in Table 1 and the time series trends illustrated in Figure 1, a general upward trend is observed across all customs regime types over the years, particularly in import and export regimes.

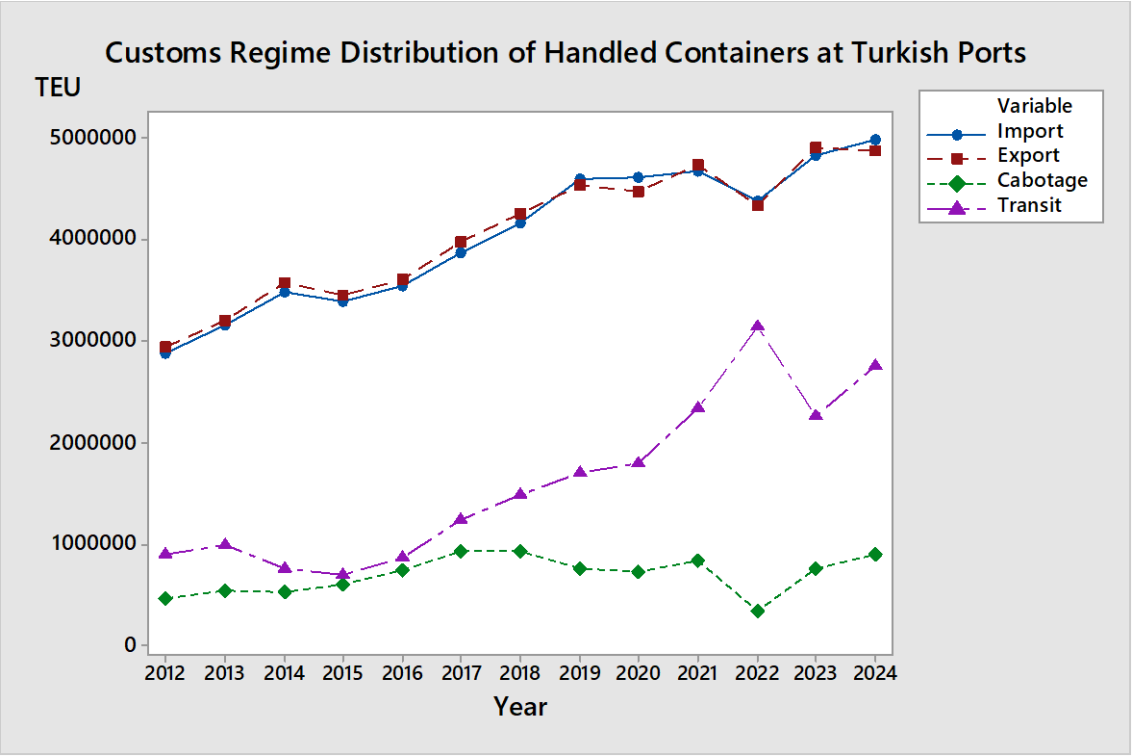

Figure 1. Customs regime-based time series of handled containers at Turkish ports

In particular, the container handling volume under the import regime, which was approximately 2.9 million TEUs in 2012, approached 5 million TEUs by 2024. Similarly, under the export regime, the volume increased from 2.9 million to 4.8 million TEUs. This indicates a significant volumetric growth in foreign trade and suggests that port capacities have been able to respond to the rising demand.

In contrast, the cabotage regime has exhibited certain fluctuations and followed an unstable trend. Transit container transportation showed a generally increasing pattern until 2022; however, it has displayed a periodic and volatile structure over the years.

Table 1. Customs regime distribution of handled containers at Turkish ports

Years	Import	Export	Cabotage	Transit
2012	2 879 123	2 942 562	472 345	898 368
2013	3 165 656	3 199 970	544 496	989 815
2014	3 488 008	3 581 811	527 066	754 238
2015	3 394 508	3 454 346	606 065	691 481
2016	3 543 805	3 607 087	738 311	872 772
2017	3 866 876	3 975 207	935 520	1 232 936
2018	4 160 125	4 259 030	935 661	1 489 184
2019	4 594 648	4 540 202	753 268	1 703 721
2020	4 618 226	4 480 472	731 352	1 796 601
2021	4 677 413	4 744 226	831 986	2 337 843
2022	4 386 890	4 337 176	341 544	3 149 659
2023	4 830 826	4 910 525	759 611	2 266 972
2024	4 987 903	4 875 265	903 194	2 763 368

Prior to the time series analysis, the statistical structure of the container handling data used in the study was examined, and the distributional characteristics forming the basis of the modeling process were evaluated. In this context, the normality of the annual container data for import, export, cabotage, and transit regimes was assessed using the Anderson-Darling test, which is recommended for small samples and is known for its sensitivity to tail deviations in the distribution (Anderson & Darling, 1954).

According to the test results, the obtained p-values for all regimes were found to be above the 5% significance level, statistically confirming that the datasets follow a normal distribution. Specifically, the Anderson-Darling statistics were AD = 0.377 ($p = 0.355$) for the import regime, AD = 0.321 ($p = 0.487$) for the export regime, AD = 0.307 ($p = 0.517$) for the cabotage regime, and AD = 0.408 ($p = 0.296$) for the transit regime.

These findings indicate that the data structure is suitable for analysis using parametric time series modeling techniques. Furthermore, this normality assessment conducted prior to the modeling phase represents an essential step in enhancing the reliability of the statistical methods applied in the forecasting process. The graphical representation of the normality test results is provided in Figure 2.

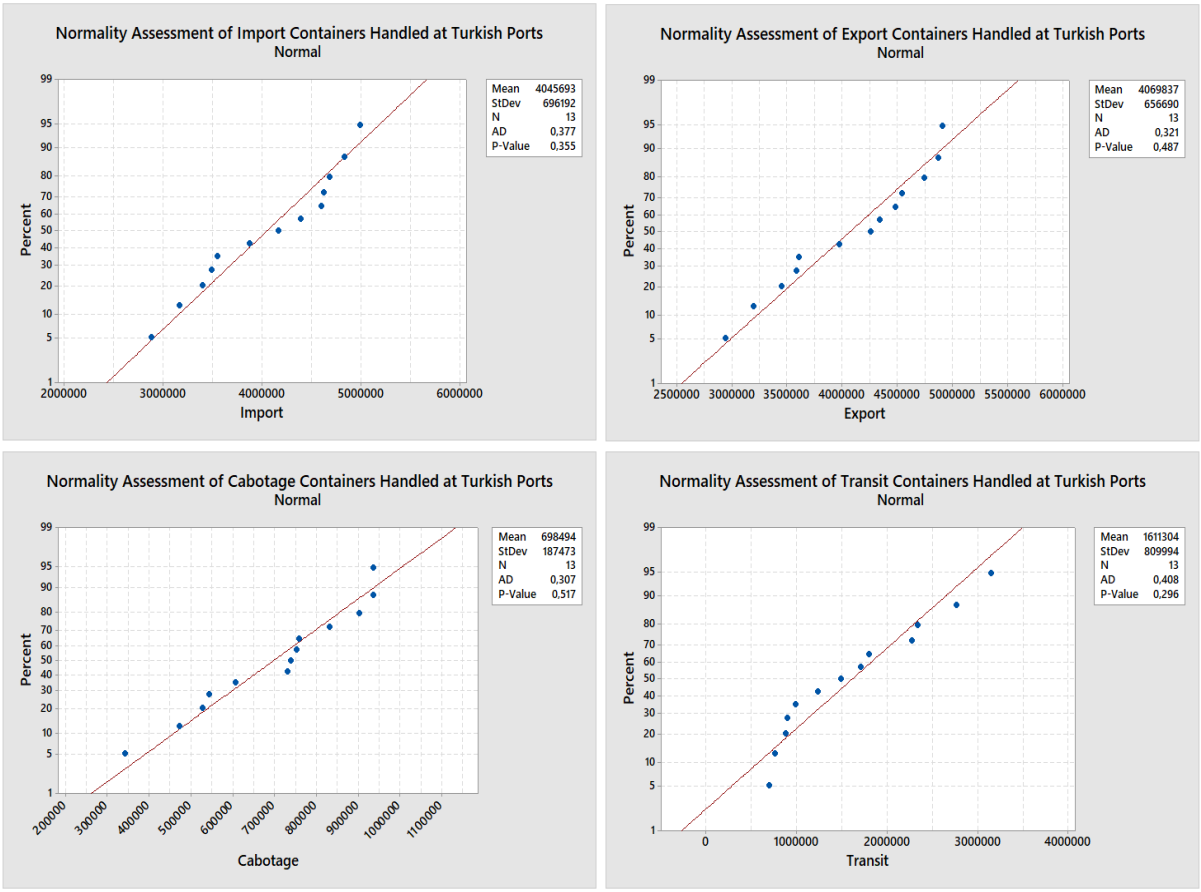

Figure 2. Normality test results based on the Anderson-Darling method

In order to model the trend component of the time series, linear, quadratic (second-degree), and exponential trend structures were evaluated, and the suitability of each model to the dataset was comparatively analyzed. The time variable was transformed and used in the form of t = Year – 2011.

Time Series Forecasting Results for the Import Regime

Based on the time series analysis applied to the annual container handling data under the import regime, the most accurate forecasting performance was achieved using the quadratic trend model. This model outperformed the others by yielding the lowest error metrics, with a Mean Absolute Percentage Error (MAPE) of 3.14%, a Mean Absolute Deviation (MAD) of 126,485 units, and a Mean Squared Deviation (MSD) of 2.641×10^{10}. The regression equation derived from the quadratic model is as follows:

$$Yt=2.606.979+265.931\times t-6.711\times t^2$$

Other methods that demonstrated forecasting accuracy close to that of the quadratic model include the linear trend analysis and the double exponential smoothing method, which yielded MAPE values of 3.64% and 4.97%, MAD values of 148,616 and 198,371 units, and MSD values of 3.34×10^{10} and 5.11×10^{10}, respectively. In contrast, the moving average method (8.33% MAPE) and the single exponential smoothing method (6.85% MAPE) exhibited higher error rates and lower forecasting accuracy, rendering them less effective.

These findings indicate that the container traffic under the import regime follows a non-linear yet consistent upward trend. The strong fit of the quadratic model particularly suggests a decelerating growth rate over time, which may imply that the import volume is approaching a saturation point or that port capacity constraints are becoming more pronounced.

Time Series Forecasting Results for the Export Regime

The time series analysis conducted on annual container handling data under the export regime revealed that the quadratic trend model exhibited the highest level of forecasting accuracy. This model provided the lowest error metrics, with a Mean Absolute Percentage Error (MAPE) of 2.94%, a Mean Absolute Deviation (MAD) of 119,959 units, and a Mean Squared Deviation (MSD) of 2.512×10^{10}. The corresponding regression equation is as follows:

$$Yt=2.676.067+267.215\times t-7.567\times t^2$$

Following the quadratic model, the linear trend model (MAPE = 3.93%) and the double exponential smoothing method (MAPE = 4.46%) achieved relatively strong performances, yielding MAD values of 159,226 and 177,475 units and MSD values of 3.394×10^{10} and 4.998×10^{10}, respectively. On the other hand, the moving average method (MAPE = 7.87%) and the single exponential smoothing model (MAPE = 5.90%) recorded higher error levels and lower forecasting accuracy.

These results indicate that the export regime data follow a non-linear but robust upward trend. The strong fit and low error statistics of the quadratic model suggest the continuation of sustainable growth in export volumes, despite occasional fluctuations. Furthermore, the presence of accelerated growth during specific periods supports the use of parabolic models over classical linear models for more accurate forecasting.

Time Series Forecasting Results for the Cabotage Regime

Time series analyses conducted on the annual container handling data for the cabotage regime identified the quadratic trend model as the most accurate forecasting method. This model yielded the lowest error metrics compared to alternative approaches, with a Mean Absolute Percentage Error (MAPE) of 18.19%, a Mean Absolute Deviation (MAD) of 101,933 units, and a Mean Squared Deviation (MSD) of 2.324×10^{10}. The derived regression equation is as follows:

$$Yt=382.471+92.781\times t-5.293\times t^2$$

Among the other evaluated models, the exponential trend analysis (MAPE = 21.33%) slightly outperformed the linear trend model (MAPE = 21.41%). Meanwhile, the single exponential smoothing (MAPE = 23.72%), double exponential smoothing (MAPE = 25.97%), and moving average method (MAPE = 30.07%) produced the highest error rates and thus were deemed inadequate for modeling the cabotage regime data.

These findings suggest that the volume of containers transported under the cabotage regime has exhibited irregular fluctuations over the years and follows a non-linear, volatile trend. This behavior likely stems from the fact that cabotage shipping primarily serves domestic markets, involving regional and short-distance transport operations that differ significantly from international trade dynamics. Consequently, traditional linear forecasting models fall short in capturing these patterns, while curvilinear models offer more realistic and reliable estimations.

Time Series Forecasting Results for the Transit Regime

Time series analyses applied to the annual container handling data under the transit regime revealed that the best forecasting performance was achieved using the quadratic (second-degree polynomial) trend model. This model recorded the lowest error metrics with a Mean Absolute Percentage Error (MAPE) of 15.30%, a Mean Absolute Deviation (MAD) of 220,475 units, and a Mean Squared Deviation (MSD) of 8.413×10^{10}. The derived parabolic regression equation is as follows:

$$Yt=695.433+25.980\times t+11.651\times t^2$$

In the model comparison, the exponential trend analysis (MAPE = 15.98%) achieved a level of accuracy close to that of the quadratic model, whereas the linear trend model performed less effectively with a MAPE of 20.79%. Other models, including the moving average (MAPE = 23.72%), single exponential smoothing (MAPE = 19.03%), and double exponential smoothing (MAPE = 22.57%), produced relatively higher error rates and thus exhibited weaker forecasting performance.

The results indicate that transit container volumes have demonstrated a growing trend over the years, with a distinct nonlinear pattern. This can be attributed to Turkey's strengthening position in international transit corridors and the increasing impact of logistics integration investments. Given that transit transportation is highly sensitive to external factors (e.g., shifts in global supply chains, political stability, development of new railway lines), the data exhibit significant variability, making accurate forecasting more complex. Therefore, curvilinear models provide a more appropriate and precise approach for modeling transit transportation patterns.

Following the confirmation of data normality, forecasting models were constructed for each customs regime using linear, quadratic, and exponential trend models, as well as moving average and exponential smoothing methods. The summary of error metrics (MAPE, MAD, MSD) and the corresponding regression equations is presented in Table 2.

Table 2. Model performance metrics and comparison

Method	Customs Regime	MAPE	MAD	MSD
Linear	Import	3,6382	148 616	$3{,}335\ 10^{10}$
Quadratic	Import	3,1424	126 485	$2{,}641\ 10^{10}$
Exponential	Import	4,3194	178 567	$4{,}409\ 10^{10}$
Moving Average	Import	8,3349	359 901	$1{,}570\ 10^{11}$
Simple Exponential Smoothing	Import	6,8522	263 261	$9{,}083\ 10^{10}$
Double Exponential Smoothing	Import	4,9720	198 371	$5{,}110\ 10^{10}$
Linear	Export	3,9337	159 226	$3{,}394\ 10^{10}$
Quadratic	Export	2,9448	119 959	$2{,}512\ 10^{10}$
Exponential	Export	4,4367	180 960	$4{,}398\ 10^{10}$
Moving Average	Export	7,8730	340 094	$1{,}364\ 10^{11}$
Simple Exponential Smoothing	Export	5,9020	242 254	$8{,}516\ 10^{10}$
Double Exponential Smoothing	Export	4,4577	177 475	$4{,}998\ 10^{10}$
Linear	Cabotage	21,4130	119 416	$2{,}756\ 10^{10}$
Quadratic	Cabotage	18,1933	101 933	$2{,}324\ 10^{10}$
Exponential	Cabotage	21,3389	124 539	$2{,}850\ 10^{10}$
Moving Average	Cabotage	30,0720	185 731	$4{,}713\ 10^{10}$
Simple Exponential Smoothing	Cabotage	23,7184	141 594	$3{,}565\ 10^{10}$
Double Exponential Smoothing	Cabotage	25,9719	148 113	$4{,}068\ 10^{10}$
Linear	Transit	20,7926	262 207	$1{,}050\ 10^{11}$
Quadratic	Transit	15,2983	220 475	$8{,}413\ 10^{10}$
Exponential	Transit	15,9822	227 677	$9{,}214\ 10^{10}$
Moving Average	Transit	23,7257	431 178	$2{,}882\ 10^{11}$
Simple Exponential Smoothing	Transit	19,0307	328540	$1{,}732\ 10^{11}$
Double Exponential Smoothing	Transit	22,5735	318 845	$1{,}518\ 10^{11}$

According to the analysis results, the examination of container handling activities based on customs regimes reveals that each regime exhibits a unique data structure. In the evaluation conducted for the import regime, the lowest error rate was obtained with the quadratic trend model, which yielded a MAPE of 3.14%. Similarly, for the export regime, the highest forecasting accuracy was also achieved by the quadratic model, with a MAPE of 2.94%. For the cabotage regime, the quadratic trend analysis produced the lowest error rate of 18.19%, while in transit container transportation, the lowest MAPE value of

15.30% was again associated with the quadratic trend model. These results indicate that the quadratic model provides the highest level of accuracy across the examined customs regimes.

These findings suggest that container transportation data categorized by customs regimes largely exhibit nonlinear characteristics, and that parabolic models, which reflect curvilinear patterns, stand out in terms of predictive performance. Therefore, careful analysis of the time series dynamics for each regime and the selection of an appropriate model structure accordingly are of critical importance to enhance forecasting accuracy.

DISCUSSION AND CONCLUSION

This study analyzes container handling activities at Turkish ports between 2012 and 2024 based on customs regimes (import, export, cabotage, and transit), and generates projections for the 2025–2027 period using forecasting models. Through time series analyses, container traffic trends for each regime type were identified, and various statistical modeling approaches were compared. During the modeling process, linear, exponential, and quadratic (second-degree polynomial) trend analyses were applied alongside moving average and exponential smoothing techniques. Based on performance evaluations using error metrics such as MAPE, MAD, and MSD, the quadratic trend model was found to provide the most accurate forecasts across all regimes.

The forecasted container volumes for the 2025–2027 period, derived from the best-performing model, are presented in Table 3. Based on the findings, the following conclusions have been drawn:

Import and export regimes exhibit high volumes and stable upward trends, indicating that port capacities may be reaching critical levels.

Cabotage transportation demonstrates a more irregular and fluctuating structure, leading to weaker model fit.

Transit transportation has gained momentum in recent years, revealing an emerging potential that requires more effective management.

A key result of the modeling process is the inadequacy of linear models across all customs regimes, whereas parabolic and exponential models yielded more accurate forecasts. This highlights the presence of structural dynamics in transport volumes that extend beyond historical trends, suggesting the need for more advanced modeling approaches.

Table 3. Projected container handling volumes by customs regime based on forecasting results

Years	Import	Export	Cabotage	Transit
2025 F	5 014 624	4 933 902	644 023	3 342 741
2026 F	5 085 930	4 981 667	583 313	3 706 598
2027 F	5 143 815	5 014 299	512 019	4 093 758

According to the forecasting results, the container volume handled under the import regime is projected to reach approximately 5.01 million TEUs in 2025, 5.08 million TEUs in 2026, and 5.14 million TEUs in 2027. This steady growth reflects the continuity of import operations and the effectiveness of Turkish port infrastructure in meeting the increasing demands of foreign trade.

Projections for the export regime indicate container volumes of 4.93 million TEUs in 2025, 4.98 million TEUs in 2026, and 5.01 million TEUs in 2027. This trend suggests a stable horizontal growth in export transportation and indicates that Turkey is maintaining its position in international markets.

For the cabotage regime, it is estimated that 644 thousand TEUs will be handled in 2025, 583 thousand TEUs in 2026, and 512 thousand TEUs in 2027. This downward trend may signal a contraction in domestic maritime transport capacity or a shift toward alternative transportation modes.

In the case of the transit regime, forecasts suggest that container volumes will reach 3.34 million TEUs in 2025, 3.71 million TEUs in 2026, and 4.09 million TEUs in 2027. This significant upward trajectory highlights Turkey's growing potential to become a regional logistics hub in transit transportation.

These findings reveal that container transportation by customs regime exhibits distinct trends, with each regime evolving according to its own set of dynamics. The projections are presented graphically in Figure 3, where the annual variations are visualized separately for each regime.

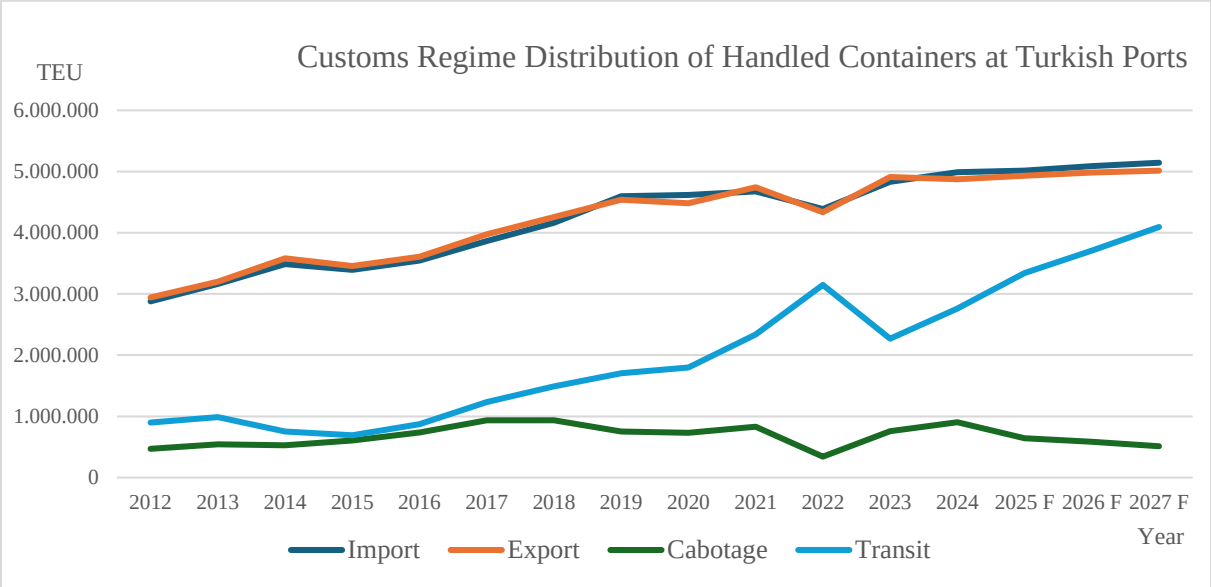

Figure 3. Projected time series trends of customs regime container handling (2012–2027)

Based on the findings, both the import and export regimes demonstrate a consistent growth trend, indicating a sustainable increase in container handling volumes in parallel with Turkey's expanding foreign trade volume. In contrast, the cabotage and transit regimes exhibit more volatile and irregular patterns due to periodic fluctuations and the influence of external factors. The observed decline in cabotage volumes may reflect shifting domestic transport demands and the growing use of alternative modes, whereas the upward trend in transit transportation suggests a strengthening of Turkey's role as a logistics hub within the global supply chain.

The analysis also reveals that the nonlinear nature of container transportation data, segmented by customs regimes, is shaped by numerous external variables such as supply chain disruptions, global economic fluctuations, trade policies, and the level of development of port infrastructure. Therefore, it is crucial to move beyond models based solely on historical trends and develop forecasting approaches that account for seasonality and structural breaks.

In this context, the following recommendations are proposed:

The increasing volume of cargo under import and export regimes necessitates the prioritization of strategies such as optimizing existing port infrastructure, accelerating digital transformation initiatives, and supporting intermodal transportation systems.

Despite having capacity, cabotage transportation has shown a downward trend. Regional incentive mechanisms and national strategies aimed at revitalizing maritime transport are of strategic importance.

The positive trajectory observed in the transit regime enhances Turkey's potential within the Eurasian logistics corridor. In this regard, reducing transit times, enhancing customs integration, and improving port–hinterland connectivity should be considered strategic priorities.

In conclusion, this study demonstrates that container handling volumes at Turkish ports exhibit dynamic patterns depending on the customs regime, and that distinct planning and management strategies must be developed for each regime. The forecasts obtained offer a scientific basis for decision-makers in port capacity planning, infrastructure investment prioritization, and logistics policy formulation. These findings underscore the importance of data-driven and regime-specific planning. Furthermore, future research could benefit from the application of advanced methods such as LSTM, Prophet, or hybrid AI models, which can offer more accurate predictions and contribute to strategic sectoral planning.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Parameter Estimation For The Xlindley Distribution Based On Different Set Sampling Methods (1207)

Tenzile Erbayram[1*], Yunus Akdoğan[1]

[1]Selçuk University, Faculty of Science, Department of Statistics, Konya, Türkiye

*Corresponding author e-mail: tenzile.erbayram@selcuk.edu.tr

Abstract

Ranked set sampling has emerged as an efficient and practical method for parameter estimation, particularly in cases where direct measurement of all observations is expensive or difficult, but ranking is relatively easy and inexpensive. This study investigates the estimation of the parameters of the XLindley distribution using the maximum likelihood method based on ranked set sampling techniques, including rank, extreme, and median ranks. A comprehensive simulation study is conducted to assess the performance of the proposed estimation methods. In this simulation, the estimators are evaluated using criteria such as bias, mean square error, and mean relative error under various sample sizes and set configurations. The results indicate that ranked set sampling and its variations produce more reliable and efficient estimators compared to traditional simple random sampling. Additionally, the practical applicability of the proposed methodology is demonstrated using real datasets. The findings of this study suggest that ranked set sampling offers a viable and effective alternative for parameter estimation and introduces a new area of application for the XLindley distribution in the statistical literature.

Keywords: *Rank set sampling, extreme rank set sampling, median rank set sampling, estimation, Monte-Carlo simulation*

INTRODUCTION

Statistical sampling methods are fundamental tools in data collection and analysis processes. One of the most commonly used techniques is simple random sampling (SRS), which is based on the principle that each unit in the population has an equal probability of being selected. However, SRS may be inefficient in populations where measurements are difficult or costly to obtain, and it may yield low precision, particularly in cases where extreme values are of primary interest.

To overcome these limitations, alternative methods such as ranked set sampling (RSS) have been developed. RSS aims to improve sampling efficiency by selecting observations based on their ranking. In this method, small sets of units are randomly selected and ranked according to an auxiliary variable; then, specific ranked units are chosen to form the final sample. This approach is particularly advantageous when measurement costs are high, as it allows for more accurate estimates with fewer measurements. The main advantage of RSS lies in its ability to provide more representative samples from the population, thereby increasing sampling efficiency.

Although the initial study on RSS by McIntyre (1952) lacked a solid mathematical foundation, Takahashi and Wakimoto (1968) provided the first theoretical results. They demonstrated that, under perfect ranking, the RSS mean is an unbiased estimator of the population mean and has a smaller variance compared to the SRS mean. Additionally, Dell and Clutter (1972) obtained similar results even without the assumption of perfect ranking.

Numerous studies in the literature have focused on estimating distributional parameters using RSS and its variants. Stokes (1995) investigated the estimation of parameters for location-scale families using a modified RSS. Shaibu and Muttlak (1997) compared maximum likelihood (ML) estimates of these parameters under various sampling schemes, including RSS, median ranked set sampling (MRSS), extreme ranked set sampling (ERSS), and percentile ranked set sampling (PRSS). Dey et al. (2017) examined the estimation of the Rayleigh distribution using both ML and bayesian methods under SRS, RSS, MRSS, and modified RSS designs. The following works illustrate the growing body of research in this area, showcasing the evolution of RSS and ERSS applications from earlier to more recent studies. For instance, compared the efficiency of RSS and SRS in parameter estimation, showing that RSS outperformed SRS in terms of precision, particularly in cases involving skewed distributions (Al-Nasser et al., 2013). Esemen and Gürler (2018) investigated the parameter estimation of the generalized Rayleigh distribution using the RSS method and demonstrated its effectiveness in improving estimation accuracy. Extended the application of RSS to extreme value theory, demonstrating its effectiveness in modeling rare events (Kumar et. Al., 2024). Erbayram et al. (2025) carried out a study employing the RSS method for estimating stress-strength reliability, under the assumption that the underlying variables follow a Poisson-Lindley distribution.

Despite its ease of implementation, SRS may be inadequate for parameter estimation when extreme values are significant or sample sizes are small. To address these shortcomings, sequential sampling methods such as RSS, ERSS, and MRSS have been proposed. These methods enhance sampling efficiency and yield more consistent estimates with lower error rates. In recent years, the RSS method has shown notable advantages in various applications. For instance, in reliability testing, both RSS and ERSS have demonstrated superior sensitivity compared to SRS. Similarly, RSS has been found to be more effective than traditional methods in estimating population means from skewed distributions. In this context, the XLindley distribution has emerged in the literature as a flexible and robust probability distribution suitable for modeling such data.

This study utilizes the XLindley distribution to further improve sampling efficiency. This distribution is appropriate for modeling heavy-tailed and skewed data and is commonly used in extreme value analysis. The probability density function (PDF) and cumulative distribution function (CDF) of the XLindley distribution are given as follows:

$$f_{XL}(x;\alpha) = \frac{\alpha^2(2+\alpha+x)e^{-\alpha x}}{(1+\alpha)^2} \quad x,\alpha > , \tag{1}$$

$$F_{XL}(x;\alpha) = 1 - \left(1 + \frac{\alpha x}{(1+\alpha)^2}\right) e^{-\alpha x} \quad x,\alpha > 0 . \tag{2}$$

The primary objective of this study is to provide ML estimation results for the parameter of the XLindley distribution and to compare these results across different sampling schemes, namely SRS, RSS, ERSS, and MRSS. Through Monte Carlo simulations, the mean squared errors (MSE) and mean relative errors (MRE) for each method are calculated to evaluate the accuracy and consistency of the estimates. The findings reveal that RSS methods particularly RSS, ERSS, and MRSS provide more reliable estimates with lower error rates compared to the traditional SRS method. These results highlight the significant advantages of RSS techniques in parameter estimation for flexible distributions such as the XLindley distribution.

The structure of the paper is as follows: Section 2 derives the ML estimation equations under the SRS, RSS, ERSS, and MRSS schemes. Section 3 presents a Monte Carlo simulation study comparing the methods in terms of MSE, bias, and MRE for various sample sizes and parameter values. Sections 4 and 5 provide a real data application and concluding remarks, respectively.

ESTIMATION

This section discusses the ML estimation of the parameter of the XLindley distribution based on the SRS, RSS, ERSS, and MRSS sampling methods.

Estimation of the parameter in SRS

This section presents the ML estimation of XLindley(α) given by Chouia and Zeghdoudi (2021) when the parameter is unknown. Let $X_1, X_2, \dots, X_n$ is a random sample of size n from XLindley(α), then likelihood function $L(\alpha)$ and log-likelihood function $\ell(\alpha)$ is as follows

$L(\alpha; x) = \prod_{i=1}^{n} f(x_i; \alpha)$,

$L(\alpha; x) = \left(\frac{\alpha^2}{(1+\alpha)^2}\right)^n \prod_{i=1}^{n}(2 + \alpha + x_i)e^{-\alpha \sum_{i=1}^{n} x_i}$.

Log-likelihood function is:

$\ell(\alpha) = 2n[log(\alpha) - log(1 + \alpha)] + \sum_{i=1}^{n} log(2 + \alpha + x_i) - \alpha \sum_{i=1}^{n} x_i$

The derivative of $\ell(\alpha)$ with respect to α is:

$$\frac{d\ell(\alpha)}{d\alpha} = \frac{2n}{\alpha} - \frac{2n}{1+\alpha} + \sum_{i=1}^{n}\left(\frac{1}{2+\alpha+x_i}\right) - \sum_{i=1}^{n} x_i = 0. \quad (3)$$

To obtain the ML estimation of α can maximize Equation (3) directly with respect to α, or we can solve the non-linear equation. Notethat the ML estimation cannot solved analytically; numerical iteration techniques, such as the Newton-Raphson algorithm, are thus adopted to solve the logarithm of likelihood equation for which Equation (3) is maximized.

Estimation of the parameter in RSS

In this section, the RSS method, first proposed by McIntyre (1952), is mentioned. To select a sample of size $n = mr$ from a population using the RSS method, the implementation steps are as follows.

Step 1: A sample of m^2 size is randomly selected from the population. The selected sample is randomly allocated to m clusters of m size.

Step 2: Units in each cluster are ordered from smallest to largest. This ranking can be done without precise visual measurement or by utilizing an auxiliary variable that is easy to masure and highly correlated with the variable of interest.

Step 3: Of the units listed; the unit in the first row from the first cluster, the unit in the second row from the second cluster, and so on, the unit in the $m - th$ row from the $m - th$ cluster is selected anda the selected units are measurd in terms of the variable of interest.

Step 4: Step 1 to 3 are repeated r ties until the sample size is $n = mr$. Here m and r correspond to the set size or the number of cycles. It should be noted that the set size m plays an important role in the RSS method.

Let $X_{(i)ik}$, $(i = 1, \dots, m; k = 1, \dots, r)$ and indicate the RSS from XLindley(α) with sample size $n = mr$ where m is the set size and r is the cycle size. For simplifying the notations, $X_{(i)ik}$ by X_{ij}, then Y_{ij} are independent with density equal to the same density of the $i - th$ order statistic from a sample of size m and is given by

$$g_i(y_{ij}; \alpha) = \frac{m!}{(i-1)!\,(m-i)!} f(y_{ij}; \alpha)\left[F(y_{ij}; \alpha)\right]^{i-1}\left[1 - F(y_{ij}; \alpha)\right]^{m-i} \quad (4)$$

where $f(y_{ij}; \alpha)$ is the pdf and $F(y_{ij}; \alpha)$ is the cdf of X.

$$g_i(y_{ij};\alpha) = K\alpha^2(2+\alpha+y_{ij})e^{-\alpha y_{ij}}(1+\alpha)^{-2m-2}$$
$$\times\left((-1-\alpha^2+(-2-y_{ij})\alpha)e^{-\alpha y_{ij}}+(1+\alpha)^2\right)^i\left((1+\alpha^2+(2+y_{ij})\alpha)e^{-\alpha y_{ij}}\right)^{m-i} \quad (5)$$

where $K=\frac{m!}{(i-1)!(m-i)!}$.

Using the RSS algorithm and Equation (5), the likelihood function can be written as

$$L(\alpha;y)=\prod_{j=1}^{r}\prod_{i=1}^{m}g_i(y_{ij};\alpha)$$
$$=\prod_{j=1}^{r}\prod_{i=1}^{m}K\alpha^2(2+\alpha+y_{ij})e^{-\alpha y_{ij}}(1+\alpha)^{-2m-2}$$
$$\times\prod_{j=1}^{r}\prod_{i=1}^{m}\left((-1-\alpha^2+(-2-y_{ij})\alpha)e^{-\alpha y_{ij}}+(1+\alpha)^2\right)^i\left((1+\alpha^2+(2+\ y_{ij})\alpha)e^{-\alpha y_{ij}}\right)^{m-i}$$
$$=K^{mr}\alpha^{2mr}(1+\alpha)^{-2m^2r}\ \prod_{j=1}^{r}\prod_{i=1}^{m}(2+\alpha+y_{ij})e^{-\alpha y_{ij}{}^i}$$
$$\times\prod_{j=1}^{r}\prod_{i=1}^{m}\ \left((-1-\alpha^2+(-2-y_{ij})\alpha)e^{-\alpha y_{ij}}+(1+\alpha)^2\right)\left((1+\alpha^2+(2+y_{ij})\alpha)e^{-\alpha y_{ij}}\right)^{m-i},$$

nd the log-likelihood function is

$$\ell(\alpha)\ = mrlogK+2mrlog(\alpha)-2m^2rlog(1+\alpha)m$$
$$-\sum_{j=1}^{r}\sum_{i=1}^{m}log(2+\alpha+y_{ij})-\alpha\sum_{j=1}^{r}\sum_{i=1}^{m}y_{ij}$$
$$+i\sum_{j=1}^{r}\sum_{i=1}^{m}log\left((-1-\alpha^2+(-2-y_{ij})\alpha)e^{-\alpha y_{ij}}+(1+\alpha)^2\right)$$
$$+(m-i)\sum_{j=1}^{r}\sum_{i=1}^{m}log\left((1+\alpha^2+(2+y_{ij})\alpha)e^{-\alpha y_{ij}}\right).$$

The derivative of $\ell(\alpha)$ with respect to α is:

$$\frac{d\ell(\alpha)}{d\alpha}=\frac{2mr}{\alpha}-\frac{2m^2r}{1+\alpha}+\sum_{j=1}^{r}\sum_{i=1}^{m}\left(\frac{1}{2+\alpha+y_{ij}}\right)-\sum_{j=1}^{r}\sum_{i=1}^{m}y_{ij}$$
$$-\sum_{j=1}^{r}\sum_{i=1}^{m}\left(\frac{\left((-2+y_{ij}\alpha^2+(-2+2y_{ij}+{y_{ij}}^2)\alpha)e^{-\alpha y_{ij}}+2+2\alpha\right)i}{(1+\alpha^2+(2+y_{ij})\alpha)e^{-\alpha y_{ij}}-(1+\alpha)^2}\right)$$
$$-\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{(-m+i)\left((2\alpha+2+y_{ij})e^{-\alpha y_{ij}}-(1+\alpha^2+(2+y_{ij})\alpha)e^{-\alpha y_{ij}}y_{ij}\right)}{(1+\alpha^2+(2+y_{ij})\alpha)e^{-\alpha y_{ij}}}=0$$

Since equation has a nonlinear form, it is solved numerically to obtain the ML estimator of α and $\hat{\alpha}_{RSS}$.

Estimation of the parameter in ERSS

In this section, the ML estimation for the parameter of the XLindley distribution will be derived based on the ERSS. This represents the first modification of RSS proposed by Samawi et al. (1996) where only the maximum or minimum ranked unit from each set is used to estimate the population mean. The estimation procedure based on ERSS is carried out as follows. A total of m clusters, each consisting of m randomly selected units from the population, are ranked within each cluster according to a variable of interest, using visual inspection or any cost-free method. The selection strategy varies depending on whether the cluster size m is even or odd. If mmm is even, the smallest-ranked unit is selected from each of the first $m/2$ clusters, and the largest-ranked unit is selected from each of the remaining $m/2$ clusters. If mmm is odd, the smallest-ranked unit is selected from the first $(m-1)/2$ clusters, the largest-ranked unit from the last $(m-1)/2$ clusters, and the median-ranked unit from the remaining cluster. The ERSS sample size can be increased by repeating this procedure r times, resulting in a total sample size of $n=mr$.

Let $U = \{X_{i(1:m)j}, i = 1, \dots, m/2\ , j = 1, \dots, r\} \cup \{X_{i(m:m)j}, i = m/2 + 1, \dots, m\ , j = 1, \dots, r\}$. Here $X_{i(1:m)j}$ is distributed as the first-order statistic and $X_{i(i:m)j}$ as the maximum order statistic in a random sample of size m from the original distribution. Then the densities of U_{ij} are given by

$$g_1(u_{ij}; \alpha) = mf(u_{ij}; \alpha)[1 - F(u_{ij}; \alpha)]^{m-1} \tag{6}$$

and

$$g_m(u_{ij}; \alpha) = mf(u_{ij}; \alpha)[F(u_{ij}; \alpha)]^{m-1}. \tag{7}$$

Combining the ERSS procedure and densities, for even set size m, the likelihood function of parameters given $U = u$ is as follows

$$\begin{aligned} L_{ERSS_e} &= \prod_{j=1}^{r} \prod_{i=1}^{m/2} g_1(u_{ij}; \alpha) \prod_{j=1}^{r} \prod_{i=\frac{m}{2}+1}^{m/2} g_m(u_{ij}; \alpha) \\ &= E^{mr} \alpha^{2mr} (1+\alpha)^{-2mr} \prod_{j=1}^{r} \prod_{i=1}^{m} (2 + \alpha + u_{ij}) e^{-\alpha u_{ij}} \\ &\times \prod_{j=1}^{r} \prod_{i=1}^{\frac{m}{2}} \left(1 - \left(1 - \left(1 + \frac{\alpha u_{ij}}{(1+\alpha)^2}\right) e^{-\alpha u_{ij}}\right)\right)^{m-1} \\ &\times \prod_{j=1}^{r} \prod_{i=\frac{m}{2}+1}^{m/2} \left(1 - \left(1 + \frac{\alpha u_{ij}}{(1+\alpha)^2}\right) e^{-\alpha u_{ij}}\right)^{m-1}. \end{aligned} \tag{8}$$

Also let $V = \{X_{i(1:m)j}, i = 1, \dots, (m-1)/2\ , j = 1, \dots, r\} \cup \{X_{i(m:m)j}, i = (m+1)/2\ , \dots, m-1\ , j = 1, \dots, r\} \cup \{X_{i((m+1)/2:m)j}, i = m, j = 1, \dots, r\}$ Then the likelihood function of parameters given $V = v$, for odd set sze can be written as

$$\begin{aligned} L_{ERSS_o} &= \prod_{j=1}^{r} \prod_{i=1}^{(m-1)/2} g_1(v_{ij}; \alpha) \prod_{j=1}^{r} \prod_{i=\frac{m+1}{2}}^{m-1} g_m(v_{ij}; \alpha) \prod_{j=1}^{r} \prod_{i=m}^{m} g_{(m+1)/2}(v_{ij}; \alpha) \\ &= E^{mr} \alpha^{2mr} (1+\alpha)^{-2mr} \prod_{j=1}^{r} \prod_{i=1}^{m} (2 + \alpha + v_{ij}) e^{-\alpha v_{ij}} \\ &\times \prod_{j=1}^{r} \prod_{i=1}^{(m-1)/2} \left(1 - \left(1 - \left(1 + \frac{\alpha v_{ij}}{(1+\alpha)^2}\right) e^{-\alpha v_{ij}}\right)\right)^{m-1} \\ &\times \prod_{j=1}^{r} \prod_{i=\frac{m+1}{2}}^{m-1} \left(\left(1 - \left(1 + \frac{\alpha v_{ij}}{(1+\alpha)^2}\right) e^{-\alpha v_{ij}}\right)\right)^{m-1} \\ &\times \prod_{j=1}^{r} \prod_{i=m}^{m} \left(\left(1 - \left(1 + \frac{\alpha v_{ij}}{(1+\alpha)^2}\right) e^{-\alpha v_{ij}}\right)\right)^{(m-1)/2} \end{aligned} \tag{9}$$

Where $E = m!/\left((m-1)/2\right)!^2$, $g_i(;\alpha)$, $g_1(;\alpha)$ and $g_m(;\alpha)$, are given by Equations (6), (7) and (4), respectively. As there is no closed form of the ML estimator of he parameter under odd and even sample size m, $\hat{\alpha}_{ERSS}$ is obtained numerically by maximizing the likelihood functions (8) and (9), respectively.

Estimation of the parameter in MRSS

The aim of this section is to estimate the parameters of the XLindley distribution using the ML estimation method based on the MRSS scheme, which is a modification of the RSS method. The MRSS method is introduced by Muttlak (1997) to improve the efficiency of population mean estimation. Muttlak demonstrated that when the underlying distribution is symmetric, MRSS yields an unbiased estimator for the mean and achieves higher efficiency compared to the SRS. The MRSS procedure,

similar to the conventional RSS design, involves ranking and random selection within clusters, each consisting of m units, based on a variable of interest. If the cluster size m is odd, the median-ranked unit is selected from each cluster; if m is even, the $m/2$ th order statistic is selected from the first $m/2$ clusters, and the $(m+2)/2$ th order statistic is selected from the remaining $m/2$ clusters. This procedure may be repeated r times, if necessary, to enhance estimation accuracy.

Let $W = \{X_{i((m+1)/2:m)j}, i = 1, \dots, m/2, j = 1, \dots, r\}$ Then the likelihood function of parameter given $W = w$, for odd set size can be written as

$$\begin{aligned} L_{MRSS_o(\alpha;w)} &= \prod_{j=1}^{r}\prod_{i=1}^{m} g_{(m+1)/2}(w_{ij};\alpha) \\ &= M^{mr}\alpha^{2mr}(1+\alpha)^{-2mr}\prod_{j=1}^{r}\prod_{i=1}^{m}(2+\alpha+w_{ij})e^{-\alpha w_{ij}} \\ &\times \prod_{j=1}^{r}\prod_{i=1}^{m}\left(1-\left(1+\frac{\alpha w_{ij}}{(1+\alpha)^2}\right)e^{-\alpha w_{ij}}\right)^{\frac{(m+1)}{2}-1} \\ &\times \left(1-\left(1-\left(1\frac{\alpha w_{ij}}{(1+\alpha)^2}\right)e^{-\alpha w_{ij}}\right)^{\frac{(m-1)}{2}}\right), \end{aligned} \tag{10}$$

where $M = m!/\big((m-1)/2\big)!^2$ and $g_i(;\alpha)$ is given by Equations (4).

Also let $T = \{X_{i(m/2:m)j}, i = 1, \dots, m/2\ , j = 1, \dots, r\} \cup \{X_{i((m+2)/2:m)j}, i = (m+2)/2\ , \dots, m\ , j = 1, \dots, r\}$. Then the likelihood function of parameters given $T = t$, for odd set size can be written as

$$\begin{aligned} L_{MRSS_e(\alpha;w)} &= \prod_{j=1}^{r}\prod_{i=1}^{m/2} g_{(m)/2}(t_{ij};\alpha)\prod_{j=1}^{r}\prod_{i=(m+2)/2}^{m} g_{(m+2)/2}(t_{ij};\alpha) \\ &= P^{mr}\alpha^{2mr}(1+\alpha)^{-2mr}\prod_{j=1}^{r}\prod_{i=1}^{m}(2+\alpha+t_{ij})e^{-\alpha t_{ij}} \\ &\times \prod_{j=1}^{r}\prod_{i=1}^{m/2}\left(1-\left(1+\frac{\alpha t_{ij}}{(1+\alpha)^2}\right)e^{-\alpha t_{ij}}\right)^{(m-2)/2} \\ &\times \prod_{j=1}^{r}\prod_{i=(m+2)/2}^{m}\left(1-\left(1+\frac{\alpha t_{ij}}{(1+\alpha)^2}\right)e^{-\alpha t_{ij}}\right)^{m/2} \end{aligned} \tag{11}$$

where $P = m!/\big((m-1)/2\big)!\,(m/2)!$. Also, ML estimator of parameter says, $\hat{\alpha}_{MRSS}$ is obtained numerically method because of complex forms of (8) and (9).

SIMULATION STUDY

This study evaluates the performance of ML estimators based on RSS, ERSS, and MRSS by comparing them with the ML estimator obtained through the conventional SRS method. To achieve this, a comprehensive Monte Carlo simulation study is carried out using the R programming software. In the simulation study, MSE, MRE, and biases of estimators based on estimation methods are calculated with 5000 (N) repetitions for the sample sizes such as $n = 12, 24, 36, 48$. The comparison, in terms of point estimation accuracy, is conducted for parameter settings $\alpha = 0.5, 0.75, 1.5$. In all simulations, the samples are generated from $XLindley(\alpha)$ distribution. Estimaton performance is assessed using averages, incorporating metrics such as Bias, MSEs, and MREs for all methods. These criteria are computed as follows:

$$\widehat{Bias}_{\alpha} = \frac{1}{N}\sum_{i=1}^{N}(\hat{\alpha}_i - \alpha),$$

$$\widehat{MSE}_{\alpha} = \frac{1}{N}\sum_{i=1}^{N}(\hat{\alpha}_i - \alpha)^2,$$

and

$$\widehat{MRE}_{\alpha} = \frac{1}{N}\sum_{i=1}^{N}\frac{|\hat{\alpha}_i - \alpha|}{\alpha}.$$

The results have been reported in Tables 1-4.

Table 1. Estimate of the parameter of the XLindley distribution

n	m;r	SRS	RSS	ERSS	MRSS
$\alpha = 0.5$					
12	3;4	0.5547	0.5510	0.5520	0.5648
	4;3	0.5555	0.5131	0.5159	0.5110
	6;2	0.5513	0.5129	0.5185	0.5118
24	3;8	0.5369	0.5339	0.5342	0.5439
	4;6	0.5395	0.5062	0.5069	0.5055
	6;4	0.5369	0.5049	0.5072	0.5054
36	3;12	0.5318	0.5267	0.5269	0.5355
	4;9	0.5298	0.5038	0.5051	0.5041
	6;6	0.5324	0.5024	0.5040	0.5022
48	3;16	0.5239	0.5229	0.5233	0.5304
	4;12	0.5223	0.5030	0.5031	0.5026
	6;8	0.5227	0.5024	0.5039	0.5022
$\alpha = 0.75$					
12	3;4	0.8332	0.8247	0.8248	0.8432
	4;3	0.8328	0.7681	0.7679	0.7633
	6;2	0.8313	0.7664	0.7705	0.7641
24	3;8	0.7872	0.7989	0.8001	0.8139
	4;6	0.7881	0.7615	0.7623	0.7612
	6;4	0.7904	0.7595	0.7618	0.7595
36	3;12	0.7789	0.7898	0.7894	0.8019
	4;9	0.7793	0.7568	0.7587	0.7566
	6;6	0.7768	0.7548	0.7581	0.7554
48	3;16	0.7742	0.7852	0.7850	0.7955
	4;12	0.7733	0.7543	0.7557	0.7546
	6;8	0.7742	0.7529	0.7567	0.7538
$\alpha = 1.5$					
12	3;4	1.6167	1.6191	1.6195	1.6446
	4;3	1.6105	1.5433	1.5508	1.5410
	6;2	1.6125	1.5313	1.5392	1.5285
24	3;8	1.5691	1.5855	1.5856	1.6076
	4;6	1.5611	1.5180	1.5244	1.5213
	6;4	1.5617	1.5193	1.5214	1.5154
36	3;12	1.5439	1.5713	1.5699	1.5909
	4;9	1.5513	1.5161	1.5195	1.5157
	6;6	1.5444	1.5106	1.5131	1.5103
48	3;16	1.5407	1.5612	1.5633	1.5824
	4;12	1.5433	1.5126	1.5136	1.5122
	6;8	1.5384	1.5073	1.5101	1.5084

Table 2. MSE of the parameter of the XLindley distribution

n	m;r	SRS	RSS	ERSS	MRSS
$\alpha = 0.5$					
12	3;4	0.0160	0.0084	0.0085	0.0077
	4;3	0.0166	0.0054	0.0060	0.0048
	6;2	0.0153	0.0039	0.0048	0.0033
24	3;8	0.0065	0.0038	0.0037	0.0033
	4;6	0.0068	0.0026	0.0029	0.0023
	6;4	0.0065	0.0018	0.0023	0.0016
36	3;12	0.0041	0.0023	0.0024	0.0022
	4;9	0.0040	0.0017	0.0019	0.0015
	6;6	0.0041	0.0013	0.0015	0.0011
48	3;16	0.0032	0.0018	0.0017	0.0016
	4;12	0.0031	0.0013	0.0013	0.0011
	6;8	0.0031	0.0009	0.0011	0.0008
$\alpha = 0.75$					
12	3;4	0.0375	0.0188	0.0193	0.0173
	4;3	0.0365	0.0124	0.0136	0.0111
	6;2	0.0379	0.0089	0.0116	0.0078
24	3;8	0.0160	0.0086	0.0086	0.0077
	4;6	0.0158	0.0058	0.0065	0.0056
	6;4	0.0168	0.0043	0.0051	0.0037
36	3;12	0.0099	0.0056	0.0054	0.0049
	4;9	0.0100	0.0040	0.0042	0.0036
	6;6	0.0100	0.0027	0.0033	0.0025
48	3;16	0.0072	0.0043	0.0041	0.0037
	4;12	0.0071	0.0029	0.0031	0.0026
	6;8	0.0073	0.0021	0.0027	0.0018
$\alpha = 1.5$					
12	3;4	0.1604	0.0753	0.0758	0.0674
	4;3	0.1510	0.0587	0.0633	0.0531
	6;2	0.1607	0.0397	0.0491	0.0353
24	3;8	0.0686	0.0365	0.0345	0.0292
	4;6	0.0685	0.0275	0.0300	0.0248
	6;4	0.0685	0.0206	0.0234	0.0173
36	3;12	0.0420	0.0227	0.0227	0.0193
	4;9	0.0426	0.0170	0.0188	0.0162
	6;6	0.0411	0.0131	0.0157	0.0112
48	3;16	0.0306	0.0170	0.0170	0.0149
	4;12	0.0308	0.0128	0.0140	0.0118
	6;8	0.0317	0.0096	0.0115	0.0086

Table 3. Bias of the parameter of the XLindley distribution

n	m;r	SRS	RSS	ERSS	MRSS
$\alpha = 0.5$					
12	3;4	0.0547	0.0510	0.0520	0.0648
	4;3	0.0555	0.0131	0.0159	0.0110
	6;2	0.0513	0.0129	0.0185	0.0118
24	3;8	0.0369	0.0339	0.0342	0.0439
	4;6	0.0395	0.0062	0.0069	0.0055
	6;4	0.0369	0.0049	0.0072	0.0054

36	3;12	0.0318	0.0267	0.0269	0.0355
	4;9	0.0298	0.0038	0.0051	0.0041
	6;6	0.0324	0.0024	0.0040	0.0022
48	3;16	0.0239	0.0229	0.0233	0.0304
	4;12	0.0223	0.0030	0.0031	0.0026
	6;8	0.0227	0.0024	0.0039	0.0022
$\alpha = 0.75$					
12	3;4	0.0832	0.0747	0.0748	0.0932
	4;3	0.0828	0.0181	0.0179	0.0133
	6;2	0.0813	0.0164	0.0205	0.0141
24	3;8	0.0372	0.0489	0.0501	0.0639
	4;6	0.0381	0.0115	0.0123	0.0112
	6;4	0.0404	0.0095	0.0118	0.0095
36	3;12	0.0289	0.0398	0.0394	0.0519
	4;9	0.0293	0.0068	0.0087	0.0066
	6;6	0.0268	0.0048	0.0081	0.0054
48	3;16	0.0242	0.0352	0.0350	0.0455
	4;12	0.0233	0.0043	0.0057	0.0046
	6;8	0.0242	0.0029	0.0067	0.0038
$\alpha = 1.5$					
12	3;4	0.1167	0.1191	0.1195	0.1446
	4;3	0.1105	0.0433	0.0508	0.0410
	6;2	0.1125	0.0313	0.0392	0.0285
24	3;8	0.0691	0.0855	0.0856	0.1076
	4;6	0.0611	0.0180	0.0244	0.0213
	6;4	0.0617	0.0193	0.0214	0.0154
36	3;12	0.0439	0.0713	0.0699	0.0909
	4;9	0.0513	0.0161	0.0195	0.0157
	6;6	0.0444	0.0106	0.0131	0.0103
48	3;16	0.0407	0.0612	0.0633	0.0824
	4;12	0.0433	0.0126	0.0136	0.0122
	6;8	0.0384	0.0073	0.0101	0.0084

Table 4. MRE of the parameter of the XLindley distribution

n	m;r	SRS	RSS	ERSS	MRSS
$\alpha = 0.5$					
12	3;4	0.1873	0.1381	0.1405	0.1308
	4;3	0.1901	0.1149	0.1202	0.1084
	6;2	0.1845	0.0970	0.1068	0.0898
24	3;8	0.1226	0.0943	0.0937	0.0882
	4;6	0.1255	0.0802	0.0841	0.0764
	6;4	0.1228	0.0669	0.0760	0.0640
36	3;12	0.0981	0.0750	0.0757	0.0713
	4;9	0.0954	0.0651	0.0681	0.0609
	6;6	0.0973	0.0560	0.0616	0.0517
48	3;16	0.0858	0.0653	0.0644	0.0610
	4;12	0.0852	0.0564	0.0573	0.0522
	6;8	0.0857	0.0471	0.0531	0.0437
$\alpha = 0.75$					
12	3;4	0.1862	0.1386	0.1397	0.1278
	4;3	0.1838	0.1155	0.1212	0.1101
	6;2	0.1854	0.0989	0.1128	0.0930

24	3;8	0.1291	0.0946	0.0951	0.0865
	4;6	0.1289	0.0800	0.0843	0.0779
	6;4	0.1316	0.0685	0.0744	0.0640
36	3;12	0.1030	0.0772	0.0759	0.0700
	4;9	0.1037	0.0660	0.0685	0.0633
	6;6	0.1036	0.0549	0.0607	0.0527
48	3;16	0.0887	0.0681	0.0665	0.0612
	4;12	0.0895	0.0573	0.0584	0.0546
	6;8	0.0884	0.0479	0.0547	0.0452
$\alpha = 1.5$					
12	3;4	0.1969	0.1367	0.1375	0.1211
	4;3	0.1915	0.1255	0.1298	0.1191
	6;2	0.1957	0.1034	0.1149	0.0971
24	3;8	0.1318	0.0971	0.0952	0.0824
	4;6	0.1338	0.0868	0.0900	0.0820
	6;4	0.1328	0.0752	0.0805	0.0692
36	3;12	0.1060	0.0775	0.0769	0.0670
	4;9	0.1069	0.0687	0.0718	0.0665
	6;6	0.1051	0.0602	0.0659	0.0559
48	3;16	0.0903	0.0669	0.0668	0.0595
	4;12	0.0907	0.0598	0.0628	0.0573
	6;8	0.0919	0.0518	0.0564	0.0487

Simulation results show that among the methods used in parameter estimation of XLindley distribution, RSS techniques exhibit superior performance compared to traditional SRS method. In particular, RSS, ERSS, and MRSS methods provide more consistent and accurate estimations with lower MSE and MRE values. This situation shows the potential of RSS methods to increase estimation accuracy. In addition, a decrease in MSE values is observed in all methods with the increase in sample size. However, RSS methods reflect this decrease more clearly and provide more effective estimations in large samples. These findings show that RSS techniques can be preferred to obtain more reliable results especially in large data sets. In conclusion, this study conducted for XLindley distribution emphasizes the effectiveness of RSS methods in statistical estimations and their advantages over traditional methods. It is recommended that these methods be preferred especially in applications requiring low error rates and high estimation accuracy.

REAL DATA

First data set: The first dataset consists of 20 observations on the time between failures for repairable items, discussed by Murthy et.al. (2004). The first data set is 1.43, 0.11, 0.71, 0.77, 2.63, 1.49, 3.46, 2.46, 0.59, 0.74, 1.23, 0.94, 4.36, 0.40, 1.74, 4.73, 2.23, 0.45, 0.70, 1.06, 1.46, 0.30, 1.82, 2.37, 0.63, 1.23, 1.24, 1.97, 1.86, 1.17.

Before comparing the performance of RSS, MRSS, ERSS, and SRS estimators, it is critical to evaluate the extent to which the data fits the XLindley distribution. Kolmogorov-Smirnov (KS) test and its corresponding p-value are used to evaluate the effectiveness of the model. For the time between failures for repairable items, the KS statistic is calculated as 0.1669 and the p-value is 0.3778. Also, the model parameter is defined as $\alpha = 0.8365$. These results show that the data set fits the XLindley distribution well.

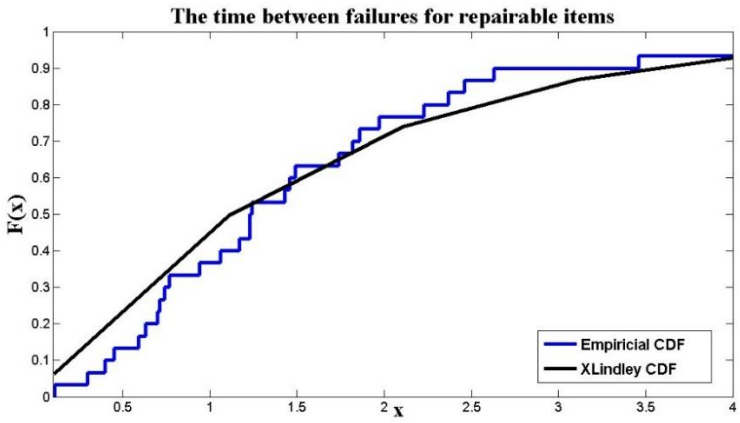

Figure 1. Empirical and XLindley distributions based on the first data set.

Figure 1 the calculated CDF exhibit a high level of agreement when compared to the theoretical distribution. This indicates that the model fits the data successfully and that the distribution parameter is estimated accurately.

For the analysis, a random sample of size 12 is drawn without replacement with different sampling scheme; SRS, RSS, MRSS, and ERSS. In RSS and its modifications, we determined m = 4 and r = 3. The result of the analysis presented in Table 5. We see from the result, estimates based on ERSS are closer to the given value of the parameter α. Thus, ERSS gives us the most efficient results.

Table 5. The ML estimations of parameters for XLindley distribution

SRS	RSS	ERSS	MRSS
0.7069	0.8605	0.8601	0.8022

Second dataset: The second data discussed in (Lee and Wang, 2003) is the survival times of 121 patients with breast cancer obtained from a large hospital in a period from 1929 to 1938. The observations are 0.3, 0.3, 4, 5, 5.6, 6.2, 6.3, 6.6, 6.8, 7.4, 7.5, 8.4, 8.4, 10.3, 11, 11.8, 12.2, 12.3, 13.5, 14.4, 14.4, 14.8, 15.5, 15.7, 16.2, 16.3, 16.5, 16.8, 17.2, 17.3, 17.5, 17.9, 19.8, 20.4, 20.9, 21, 21, 21.1, 23, 23.4, 23.6, 24, 24, 27.9, 28.2, 29.1, 30, 31, 31, 32, 35, 35, 37, 37, 37, 38, 38, 38, 39, 39, 40, 40, 40, 41, 41, 41, 42, 43, 43, 43, 44, 45, 45, 46, 46, 47, 48, 49, 51, 51, 51, 52, 54, 55, 56, 57, 58, 59, 60, 60, 60, 61, 62, 65, 65, 67, 67, 68, 69, 78, 80, 83, 88, 89, 90, 93, 96, 103, 105, 109, 109, 111, 115, 117, 125, 126, 127, 129, 129, 139 and 154.

Before comparing the performance of RSS, MRSS, ERSS, and SRS estimators, it is critical to evaluate the extent to which the data fits the XLindley distribution. KS test and its corresponding p-value are used to evaluate the effectiveness of the model. For the time between failures for repairable items, the KS statistic is calculated as 0.0670and the p-value is 0.6794. Also, the model parameter is defined as $\alpha = 0.0415$. These results show that the data set fits the XLindley distribution well.

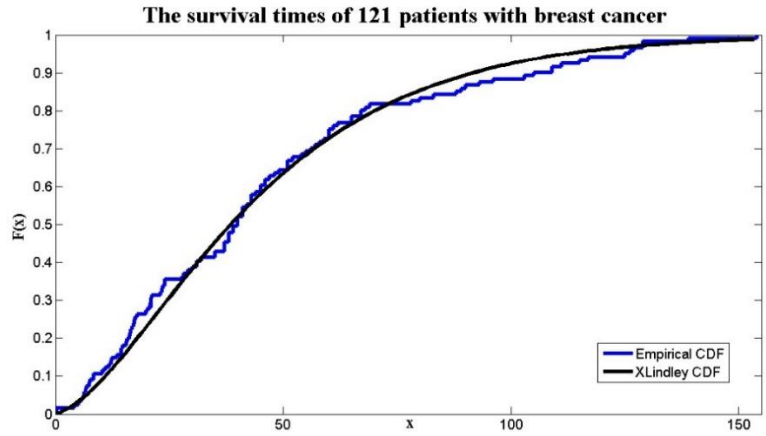

Figure 2. Empirical and XLindley distributions based on the second data set.

Figure 2 the calculated CDF exhibit a high level of agreement when compared to the theoretical distribution. This indicates that the model fits the data successfully and that the distribution parameter is estimated accurately.

For the analysis, a random sample of size 44 is drawn without replacement with different sampling scheme; SRS, RSS, MRSS and ERSS. In RSS and its modifications, we determined m = 11 and r = 4. The result of the analysis presented in Table 6. We see from the result, estimates based on ERSS are closer to the given value of the parameter α. Thus, ERSS gives us the most efficient results.

Table 6. The ML estimations of parameters for XLindley distribution

SRS	RSS	ERSS	MRSS
0.0486	0.0407	0.0413	0.0416

CONCLUSION

In this study, the parameters of the XLindley distribution are estimated using the ML method, and the estimation performances of different sampling schemes are compared. Monte Carlo simulations are employed to compute the MSE, bias, and MRE for each sampling method, thereby enabling a comprehensive assessment of the accuracy and consistency of the estimators. The findings revealed that ranked sampling techniques particularly RSS, ERSS, and MRSS outperform the traditional SRS method in terms of lower error rates and improved estimation reliability. These advantages are especially pronounced in situations involving high measurement costs, skewed distributions, and the presence of extreme values. For flexible distributions such as the XLindley, the use of these ranked sampling methods enhances statistical efficiency and improves the accuracy of parameter estimation. Accordingly, the results of this study suggest that RSS, ERSS, and MRSS offer strong alternatives to conventional sampling approaches in applied statistical analysis. Future research may focus on extending these methods to other distributions and exploring their integration with non-parametric estimation techniques. Moreover, increasing applications to real datasets will contribute to a deeper understanding of the practical utility of these sampling designs.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

These authors contributed equally to this work.

Artificial Intelligence Success in Content Marketing: A Systematic Literature Review and Bibliometric Analysis (1212)

İbrahim Halil Efendioğlu[1*], Gaye Aydemir[2]

[1] Gaziantep University, Faculty of Economics and Administrative Sciences, Department of Business Administration, Turkey
[2] Gaziantep University, Faculty of Economics and Administrative Sciences, Department of Business Administration, Turkey

*Corresponding author e-mail: efendioglu@gantep.edu.tr

Abstract

This study integrates a systematic literature review (SLR) with bibliometric analysis to comprehensively evaluate recent academic developments in AI-assisted content marketing. Based on the PRISMA protocol, 21 high-quality articles (SSCI and SCI-E –indexed) published between 2021 and 2025 were analyzed. Thematic coding was used for content analysis, while bibliometric techniques examined publication trends, author collaborations, citation networks, and keyword co-occurrence clusters. Findings reveal that Generative AI (GenAI) plays a complementary and substitutive role in content marketing practices. While hybrid content creation models show promise for optimizing user experience, fully automated content generation may pose risks to brand authenticity and consumer trust. Consumer perceptions are primarily shaped by psychological variables such as perceived creepiness, lack of empathy, and artificiality. In the context of chatbots and virtual assistants, anthropomorphic design and localized content strategies significantly enhanced user satisfaction. Bibliometric insights indicate a sharp increase in scholarly interest after 2023, with studies predominantly employing experimental, qualitative, and mixed methods. Key research themes include digital ethics, user responses to AI-generated content, audiovisual content production, and business model innovation. As one of the few studies to holistically assess the multidimensional impact of AI integration into content marketing, this research provides a robust conceptual framework for scholars and actionable insights for marketing practitioners. The paper concludes with a discussion of theoretical contributions, managerial implications, and directions for future research.

Keywords: *Generative AI, Content Marketing, Systematic Literature Review, Bibliometric Analysis, Consumer Perception*

INTRODUCTION

The rapid advancement of digitalization has triggered a radical transformation in the field of marketing (Helmer et al., 2025). With the growing volume of data, diverse channels, and changing customer expectations, traditional methods are increasingly being replaced by AI-powered solutions (Wang et al., 2025). In this new dynamic environment, marketers are embracing more effective, faster, and scalable solutions. Artificial intelligence, in this context, has emerged as a strategic actor, not only in data analysis but also in content creation, promising a future of more personalized and targeted marketing (Lund et al., 2025).

AI-powered content creation enables the automatic or semi-automatic generation of multimedia formats, including text, images, audio, and video, making marketing strategies more personalized and targeted (Münster et al., 2024). In this process, technologies such as natural language processing, machine

learning, and visual generation are redefining content production not only in quantitative terms but also in terms of quality. Thus, brands can establish more meaningful, consistent, and data-driven relationships with consumers.

However, these technological advancements necessitate addressing marketing communication not only from a technical perspective but also from ethical, cultural, and societal dimensions. Complex issues such as content originality, consumer trust, algorithmic bias, and privacy violations require a reconsideration of the language and methods of marketing (Karami et al., 2024). AI-generated content affects not only the speed of production but also the responsibility for the content and its impact on consumer perception.

In this context, AI-powered content marketing has become a rapidly growing field of research in the literature. However, most existing studies either remain limited at a conceptual level or focus on specific contexts. This underscores the urgent need for a more systematic, multidimensional, and comparative examination of the field to understand its implications and potential fully.

The primary goal of this study is to comprehensively evaluate the effects of AI-powered content generation and generative AI on content marketing, assess their position and challenges in the marketing domain, systematically map the existing literature, and identify research gaps. The objectives of this study are of utmost importance as they pave the way for a deeper understanding of the evolving landscape of content marketing in the AI era.

The study seeks to answer the following research questions:

RQ1: Around which themes are AI-powered content generation addressed in the marketing literature?

RQ2: Which theoretical approaches and research methods are predominant in the literature on AI-powered content generation?

RQ3: What are the effects of AI-generated content on consumer behavior, brand perception, and personalization strategies?

RQ4: In what ways are the ethical, cultural, and social aspects of AI-generated content examined in the context of marketing?

RQ5: What are the current research gaps in the field of AI-powered content generation, and what future directions are anticipated?

The structure of the paper is as follows: The second section reviews the literature on AI-powered content marketing. The third section explains the methodology and the bibliometric analysis process. The fourth section presents the findings. The fifth section discusses these findings, and the final section concludes the study with its results, limitations, suggestions for future research, and theoretical and practical contributions.

LITERATURE REVIEW

The Intersection of Artificial Intelligence and Content Marketing

Artificial intelligence (AI) refers to computer systems capable of mimicking human-like thinking, learning, problem-solving, and decision-making abilities. These systems are developed to automate specific tasks or support human intelligence (Sreenivasan & Suresh, 2024). AI is generally categorized into three types based on its capacity: Artificial Narrow Intelligence (ANI) refers to systems specialized in specific tasks (e.g., chatbots like ChatGPT or Siri); Artificial General Intelligence (AGI) is envisioned to possess human-like, versatile learning and reasoning abilities, though it is still under development;

and Artificial Super Intelligence (ASI) refers to a hypothetical form of AI that could surpass human intelligence in the future (Andrejkovics, 2019).

Additionally, based on technical features and usage purposes, AI is classified into several types: rule-based systems operate according to predefined rules; machine learning improves itself by learning from data; deep learning processes complex data by mimicking neural networks in the human brain; natural language processing (NLP) focuses on understanding and generating language in written and spoken forms; and computer vision performs tasks related to recognizing and interpreting visual data (Kumar & Renuka, 2023). These categories determine the functional capabilities of AI across various application areas.

Content marketing is a strategic approach that aims to establish sustainable relationships between brands and their target audiences through valuable, informative, and engaging content. However, as the need for speed and volume in content creation has grown in digital environments, managing this process through traditional methods has become increasingly challenging (Jami Pour et al., 2024). At this point, AI transforms content marketing into a more efficient and data-driven process. AI not only generates content but also analyzes consumer data and delivers the right content to the right person at the right time (Vashishth et al., 2025). Through NLP algorithms, blog posts, email texts, and advertising content can be automatically generated, while visual content creation tools are widely used for everything from social media posts to product promotions (Roshne et al., 2024).

Advantages such as personalization, speed, and efficiency make AI-powered content creation highly attractive for brands and transform content marketing into a more dynamic, scalable, and measurable process (Adepoju et al., 2024). This transformation not only saves time for marketing professionals but also enhances the strategic impact of the content.

AI-Based Content Creation Technologies

AI-powered content creation, a marvel of efficiency, enables the rapid, automated, and personalized production of various multimedia formats commonly used in marketing. These technologies, rooted in natural language processing (NLP), machine learning, and generative AI models (Bansal et al., 2024), are a powerful ally for content creators.

For text generation, large language models such as GPT, Claude, and Gemini are notable for their capabilities in content writing, summarization, rewriting, and text suggestions (Zhao et al., 2025). In visual content creation, tools such as DALL•E, Midjourney, and Canva AI enable the generation of original visuals. On the video and audio front, platforms like Synthesia, Pictory, and Murf AI facilitate the creation of marketing materials using talking avatars or automated voiceovers (Gupta et al., 2025; Renaldo et al., 2025).

These tools not only accelerate content production but also serve as a wellspring of creative ideas for marketing teams, providing testable content alternatives and bringing greater flexibility to content strategy, which inspires new directions and approaches.

AI-Supported Content Types in Marketing Communication

Artificial intelligence plays an active role in producing various types of content used in marketing communication. From text-based content to visuals and videos, content can now be generated quickly and efficiently through AI tools (Raut et al., 2025).

Blog posts and email content, crafted in minutes by large language models, can be personalized to resonate with the target audience. AI can optimize social media posts by crafting engaging headlines and interaction-focused captions, thereby fostering a sense of connection and engagement. Advertising

copy is automatically generated with high-click-through-rate alternatives, making it ready for A/B testing (Phillips & Carley, 2024).

Additionally, product descriptions can be tailored to different formats for various customer segments based on user data. Video content scripts can be generated by AI and integrated into automated production workflows, including voice narration, subtitles, and visual editing. These types of content play a critical role in delivering more consistent, targeted, and multichannel communication within marketing strategies.

AI-Powered Content Personalization and Segmentation

Artificial intelligence is transforming content marketing not only in terms of production but also in targeting and personalization. AI-based systems go beyond classical segmentation methods by analyzing user behaviors, interests, demographic characteristics, and past interactions to deliver highly personalized content (Bano et al., 2025).

Thanks to AI-supported algorithms, each user visiting a website can be presented with a different blog post, email content, or product recommendation. Dynamic content is shaped in real-time based on each user's data (Kumar et al., 2024). For example, if a customer has previously shown interest in red shoes, the email content can directly include suggestions focused on that color.

Moreover, chatbots and voice assistants also offer content recommendations using personal data, making the customer experience more seamless and satisfying (Bhuiyan, 2024). These advancements make marketing communication not only more effective but also more human-centered.

Predicting and Optimizing Content Performance with AI

AI enhances not only the production and personalization processes in content marketing but also the ability to predict and improve content performance (Islam et al., 2024). Predicting content performance is a crucial step in enhancing the efficiency of marketing activities.

AI algorithms can analyze metrics such as click-through rate (CTR), conversion rate, reading time, and engagement level to forecast which content will perform better (Sangsawang, 2024). As a result, brands can focus on high-potential content without wasting time on underperforming materials.

AI-driven optimization tools, such as automated A/B testing and SEO compliance, streamline the process of content optimization (Šola et al., 2025; Salem et al., 2025). This efficiency allows **marketers to focus on making strategic decisions, thereby maximizing their content ROI.**

Ethics, Trust, and Artificial Intelligence

While AI-powered content creation offers tremendous marketing opportunities, it also raises significant ethical concerns and trust issues. The originality, accuracy, and traceability of automatically generated content pose serious risks that may undermine consumer trust (Chaurasia, 2025; He & Fang, 2024).

Another primary concern relates to the collection and processing of user data. AI systems may push the boundaries of privacy when delivering content recommendations based on personal data. Particularly in areas such as facial recognition, voice analysis, and emotional AI, the use of consumer data without explicit consent is ethically questionable (Katirai, 2024).

Algorithmic biases, such as favoring certain demographics or promoting stereotypes, can also lead to the production of discriminatory or exclusionary content targeting specific groups, which may conflict with corporate social responsibility and inclusion principles. A lack of transparency can result in users consuming content without knowing who created it or how it was produced (Mergen et al., 2025).

In this context, the ethical use of AI in marketing requires human oversight, clear policy disclosures, and robust safeguards. These ethical principles include respect for user privacy, transparency in content creation, and a commitment to avoiding algorithmic biases. They are essential not only for ensuring legal compliance but also for maintaining a long-term brand reputation and sustainability.

METHODOLOGY

This study was conducted by integrating Systematic Literature Review (SLR) and Bibliometric Analysis methods to examine the effectiveness of AI-supported content creation in the field of marketing. A systematic literature review is a research method that summarizes scientific evidence by examining existing academic studies in a planned, transparent, and replicable manner based on a specific research question (Azarian et al., 2023; Chintalapati & Pandey, 2022).

The research process was structured in accordance with the PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analyses) guidelines. PRISMA is an internationally recognized guideline developed to ensure that systematic reviews and meta-analyses are reported transparently, comprehensively, and in line with established standards (Asar et al., 2016). The study was conducted based on the following steps:

Planning Phase: Defining the research questions, determining the scope, and creating the review protocol.

Execution Phase: Selecting articles in line with the predefined search strategy and inclusion criteria.

Analysis and Synthesis Phase: Conducting content analysis of the selected studies and identifying key themes, findings, theoretical frameworks, methodologies, and research gaps.

During the data collection phase, only the Web of Science (WoS) Core Collection database was used. WoS indexes only peer-reviewed, high-impact, and prestigious journals. This ensures that the study is based on more reliable and academically robust sources. Furthermore, WoS allows for field-specific filtering based on research areas (e.g., Business, Economics, Communication) and indexes (SSCI, SCI-E), which improves the quality of the sample in the SLR.

The literature search was conducted using the following Boolean query to identify relevant studies:

TS = (("artificial intelligence" OR "AI") AND ("marketing content" OR "content marketing" OR "content creation" OR "content generation") AND ("effectiveness" OR "impact" OR "performance" OR "efficacy"))

Based on this query and the PRISMA protocol, the study was structured in four phases:

Identification: A total of 248 publications were retrieved from the Web of Science database using the defined keywords.

Screening: A publication type filter was applied to exclude conference proceedings, book chapters, and editorial pieces, leaving only journal articles (180). Then, 3 articles in Spanish and Ukrainian were excluded, resulting in 177 articles.

Eligibility: Only studies indexed in SSCI and SCI-Expanded were included (105 articles). To narrow the focus, only publications categorized under Business Economics, Public Administration, and Social Sciences were considered (29 articles).

Included: These 29 articles were evaluated at the abstract and full-text level, and 21 articles that were directly related to the research questions were included in the final sample. This structured process ensured that the study rests on a systematic and methodologically sound foundation.

The selected articles were evaluated using thematic and content analysis methods, and the key concepts, methods, sample types, and results were systematically coded. Microsoft Excel was used to perform the thematic and content analyses in a structured manner. In Excel, each article was entered as a row and analysis categories (e.g., research aim, method, theoretical framework, findings, limitations) as columns. This matrix structure enabled efficient coding of the data. Recurrent themes across the articles were identified, frequency analyses were conducted, and literature trends were easily tracked.

Bibliometric analysis refers to the statistical evaluation of academic literature based on quantitative publication data. The aim is to measure, map, and evaluate scientific output within a given field (Passas, 2024). The findings of the bibliometric analysis were used to reveal the structural characteristics of the literature (e.g., collaboration networks, key concepts, research domains). In this way, the study presents a comprehensive overview of the effectiveness of AI-supported content creation in marketing through both in-depth content analysis and large-scale bibliometric mapping.

To support the study, bibliometric analysis was conducted on the 21 selected articles using the Bibliometrix package within RStudio. The analysis process included the following steps:

Data Preparation: Full records were exported from Web of Science in BibTeX format and imported into RStudio.

Types of Analyses:

Scientific Production Analysis: Identification of annual article production, most productive countries, and institutions.

Collaboration Analysis: Analysis of inter-country and inter-institutional collaborations.

Keyword Network Analysis: Creation of keyword networks and conceptual clusters using author keywords.

Thematic Map Analysis: Identification of core themes and mapping of thematic development areas.

Visualization: Generation of co-word network maps, collaboration networks, thematic maps, and scientific production graphs.

The resulting bibliometric outputs were used to complement the systematic findings and strengthen the analytical depth of the study.

RESULTS

As part of this study, 21 articles published between 2021 and 2025 focusing on the effectiveness of AI-powered content creation in the field of marketing were examined using both systematic literature review and bibliometric analysis methods. The findings provide a holistic analysis of both qualitative trends and quantitative patterns.

Findings from the Systematic Literature Review

In recent years, the integration of artificial intelligence technologies into marketing processes has accelerated significantly. Studies published between 2021 and 2025 investigate the impact of AI on content creation at both theoretical and empirical levels. Most of these studies employ a variety of methods, including experimental designs, surveys, systematic literature reviews, and bibliometric analyses.

Especially in 2024 and 2025, a surge in publications has focused on the influence of AI-powered content generation on consumer perceptions, brand attitudes, engagement rates, and content quality in detail

(Brüns & Meißner, 2024; Carlson et al., 2023; Chen & Chan, 2024; De Cicco et al., 2025). Table 1 below presents the authors, publication year, and article titles of the studies included in the analysis.

Table 1. Authors and Titles of Articles

Authors (Year)	**Title of the Article**
Akpan, I. J., Kobara, Y. M., Owolabi, J., Akpan, A. A., & Offodile, O. F. (2025)	Conversational and generative artificial intelligence and human–chatbot interaction in education and research
Brüns, J. D., & Meißner, M. (2024)	Do you create your content yourself? Using generative artificial intelligence for social media content creation diminishes perceived brand authenticity
Carlson, K., Kopalle, P. K., Riddell, A., Rockmore, D., & Vana, P. (2023)	Complementing human effort in online reviews: A deep learning approach to automatic content generation and review synthesis
Chen, Z., & Chan, J. (2024)	Large Language Model in Creative Work: The Role of Collaboration Modality and User Expertise
Chintalapati, S., & Pandey, S. K. (2022)	Artificial intelligence in marketing: A systematic literature review
De Cicco, R., Francioni, B., Curina, I., & Cioppi, M. (2025)	AI, human or a blend? How the educational content creator influences consumer engagement and brand-related outcomes
Del Vecchio, M., Kharlamov, A., Parry, G., & Pogrebna, G. (2021)	Improving productivity in Hollywood with data science: Using emotional arcs of movies to drive product and service innovation in entertainment industries
González-Arias, C., & López-García, X. (2023)	ChatGPT: Stream of opinion in five newspapers in the first 100 days since its launch
Gu, C., Jia, S., Lai, J., Chen, R., & Chang, X. (2024)	Exploring Consumer Acceptance of AI-Generated Advertisements: From the Perspectives of Perceived Eeriness and Perceived Intelligence
Hannigan, T. R., McCarthy, I. P., & Spicer, A. (2024)	Beware of botshit: How to manage the epistemic risks of generative chatbots
Hartmann, J., Exner, Y., & Domdey, S. (2025)	The power of generative marketing: Can generative AI create superhuman visual marketing content?
Hermann, E., & Puntoni, S. (2024)	Generative AI in Marketing and Principles for Ethical Design and Deployment
Jia, M., Zhao, Y. C., & Zhang, X. (2025)	Will potential threats persist? Attitudes and behavioral responses of creative professionals toward GenAI
Mishra, A. N., Sengupta, P., Biswas, B., Kumar, A., & Coussement, K. (2025)	Leveraging Machine learning and generative AI for content Engagement: An Exploration of drivers for the success of YouTube videos
Misra, R., Malik, G., & Singh, P. (2025)	A localized and humanized approach to chatbot banking companions: implications for financial managers
Oc, Y., Plangger, K., Sands, S., Campbell, C. L., & Pitt, L. (2023)	Luxury is what you say: Analyzing electronic word-of-mouth marketing of luxury products using artificial intelligence and machine learning
Perez-Castro, A., Martínez-Torres, M. D. R., & Toral, S. L. (2023)	Efficiency of automatic text generators for online review content generation

Teng, D., Ye, C., & Martinez, V. (2025)	Gen-AI's effects on new value propositions in business model innovation: Evidence from information technology industry
Vaccaro, M., Almaatouq, A., & Malone, T. (2024)	When combinations of humans and AI are useful: A systematic review and meta-analysis
Zhang, Y., Zhu, J., Chen, H., & Jiang, Y. (2025)	Enhancing Trust and Empathy in Marketing: Strategic AI and Human Influencer Selection for Optimized Content Persuasion
Zhou, K. Z., Choudhry, A., Gumusel, E., & Sanfilippo, M. R. (2025)	'Sora is incredible and scary': public perceptions and governance challenges of text-to-video generative AI models

Considering the technological infrastructure, methodological diversity, and application areas highlighted in the literature on AI-powered content marketing, it is evident that the field has a multidimensional and rapidly evolving structure. Among the most frequently encountered core technologies in the reviewed studies are Generative AI (GenAI), Large Language Models (LLMs), Transformer architectures, and advanced generative systems such as DALLE and GPT-4. Generative AI refers to artificial intelligence systems capable of automatically generating content in the form of text, images, audio, and video. LLMs, on the other hand, are models trained on large datasets primarily for tasks related to language generation and understanding. Most of these models are built on Transformer architecture, which has revolutionized language processing and content generation. GPT-4, developed by OpenAI, is a prominent example of an advanced LLM capable of producing human-like text. DALLE is a GenAI model that can generate original visuals based on text prompts.

These transformative AI technologies significantly enhance the speed, efficiency, and personalization capacity of content creation in marketing. They are effectively used in generating textual, visual, and video content, revolutionizing marketing practices across the board. From a methodological standpoint, the studies present a wide array of research approaches. They range from in-depth qualitative methods like interviews and thematic analysis to quantitative and mixed methods such as experimental designs, structural equation modeling (SEM), and case studies. This methodological diversity is a testament to the field's growing theoretical and empirical depth. AI-powered content creation has made a significant impact in a variety of application areas. It is particularly prominent in advertising, social media content management, chatbot applications, educational technologies, the film industry, and luxury consumption. These fields are characterized by high levels of user interaction, making personalized and creative content a key competitive advantage. This triadic structure technology, methodology, and application provides a comprehensive framework that equips the reader with a deep understanding of the rise of Generative AI in the marketing domain. Table 2 below presents the main topics addressed in the reviewed studies and the corresponding conclusions drawn by the authors.

Table 2. Overview of Topics and Key Conclusions

Authors (Year)	**Topic**	**Conclusion**
Akpan et al. (2025)	Conversational and generative AI interactions	CGAI is effective in education; policy is needed for ethics, transparency, and ownership issues.

Brüns & Meißner (2024)	Impact of GenAI on consumer perception	GenAI reduces perceived authenticity; more positive when used as an assistant.
Carlson et al. (2023)	Transformer-based review synthesis	Transformer models can perform expert-level reviews; collaboration model suggested.
Chen & Chan (2024)	LLM collaboration in copywriting	LLMs are helpful for non-experts, harmful for experts; short ads are more effective.
Chintalapati & Pandey (2022)	Use of AI in marketing functions	AI enhances creativity and personalization in marketing; sustainability is crucial.
De Cicco et al. (2025)	AI vs human in educational digital content	AI has lower content quality but hybrid models perform well; class-based effects observed.
Del Vecchio et al. (2021)	Emotion-based content in the film industry	Emotional structures affect profits; integration with data science recommended.
González-Arias & López-García (2023)	Impact of ChatGPT in journalism	ChatGPT faced distrust in media discourse; regulation is a key need.
Gu et al. (2024)	Perception of eeriness and intelligence in AI ads	Eeriness in AI ads is negative; perceived intelligence has a positive impact.
Hannigan et al. (2024)	Chatbot content creation and hallucination risk	Chatbot content carries epistemic risks; four usage types proposed.
Hartmann et al. (2025)	Effectiveness of visual content creation with GenAI	AI-generated visuals are competitive in aesthetics and quality; campaign success demonstrated.
Hermann & Puntoni (2024)	GenAI's role in the marketing cycle	GenAI plays both supportive and substitutive roles; a “TRUST” principle is proposed.
Jia et al. (2025)	Creative workers' perceptions of GenAI	Perceptions of GenAI evolve with experience; individual factors are influential.
Mishra et al. (2025)	Video content analysis and success metrics	GenAI is successful in video analysis; narrative and audio factors affect view count.
Misra et al. (2025)	Anthropomorphism and chatbot user satisfaction	Anthropomorphic design increases satisfaction; trust is a mediating factor.
Oc et al. (2023)	Luxury brand video content and comment analysis	Luxury brand reviews vary; AI-driven sentiment analysis is significant.

Perez-Castro et al. (2023)	Deceptive review generation using GPT-2	GPT-2 makes fake reviews harder to detect; high risk to consumer trust.
Teng et al. (2025)	GenAI's impact on business model innovation	GenAI introduces new value propositions; five effective areas identified.
Vaccaro et al. (2024)	Unpublished media content (empty)	Human-AI collaboration is more effective in creative tasks; meta-analysis gaps identified.
Zhang et al. (2025)	Comparison of AI and human influencers	AI influencers foster trust; humans are stronger in empathy.
Zhou et al. (2025)	Sora and public perception	Public concerns about Sora focus on data privacy, ethics, and governance.

As a result of the systematic literature review conducted in the field of AI-powered content marketing, it was observed that the literature is shaped around six main thematic clusters. These themes encompass a broad spectrum from the use of GenAI technologies in content creation to consumer perceptions and behavioral responses, from human-AI interactions via chatbots to debates on media ethics and digital responsibility. Additionally, strategic marketing practices, sectoral applications, and current discussions on security, ethics, and regulation complete this thematic framework. Each theme not only reveals the opportunities offered by AI in marketing processes but also the associated risks and responsibilities, thus enabling a holistic analysis of the existing knowledge in the literature.

The Use of Generative AI (GenAI) in Content Creation

Studies underscore the transformative potential of GenAI systems to revolutionize productivity and user experience, particularly in text (Brüns & Meißner, 2024; Carlson et al., 2023), visual (Hartmann et al., 2025), and video (Mishra et al., 2025) content creation. The ability to rapidly and cost-effectively generate content heralds a significant shift in marketing workflows, sparking excitement about the future of marketing.

Consumer Perception and Behavior

The perceptual effects of AI-generated content on consumers are a frequent focus (Gu et al., 2024; Zhang et al., 2025; De Cicco et al., 2025). Consumer reactions are evaluated in emotional and cognitive dimensions, such as eeriness, lack of empathy, or concerns over authenticity. This situation presents a challenge that necessitates new approaches, such as AI-human content hybridization, in marketing strategies, inspiring the audience to innovate.

Chatbots and Interaction with AI

Within the context of chatbot systems (Akpan et al., 2025; Misra et al., 2025; Hannigan et al., 2024), the quality of human-AI interaction, safety, and anthropomorphic design elements take center stage. Particularly, the importance of localized content and natural language use as critical factors influencing user satisfaction is underscored, emphasizing the urgency of meeting consumer needs.

Media and Digital Ethics

The impact of AI on journalism (González-Arias & López-García, 2023) and the ethical issues arising in content creation processes (Zhou et al., 2025; Perez-Castro et al., 2023) are frequently discussed in the literature. Key risks include plagiarism, misinformation, and copyright violations. In response, solutions such as legal labeling mechanisms and public awareness initiatives have been proposed.

Strategic Marketing and Sectoral Applications

Several studies (Hermann & Puntoni, 2024; Chintalapati & Pandey, 2022; Del Vecchio et al., 2021) have focused on the influence of GenAI across the marketing cycle and its strategic use cases. Sector-specific application examples in education, film, and luxury brands offer rich insights into AI's adaptation across industries.

Security, Ethics, and Regulation

Finally, issues such as content safety, deceptive reviews, and hallucination risks (Perez-Castro et al., 2023; Hannigan et al., 2024) underscore the growing need for regulation in AI systems. In this context, topics such as transparency, algorithmic accountability, and alignment with ethical principles, which may include considerations of fairness, privacy, and non-discrimination, are considered crucial for future research.

Bibliometric Findings

This bibliometric summary presents the academic output in the field of AI-powered content marketing between 2021 and 2025. The 21 documents analyzed were published across 19 different sources and authored by a total of 76 unique contributors. There were no single-author studies indicating that the field is progressing through a collaborative research structure. On average, each document had 3.62 co-authors.

The international co-authorship rate, at 33.33%, is a significant indicator of the topic's global relevance and its openness to multinational collaborations. This rate highlights the fact that AI-powered content marketing is a field that transcends geographical boundaries, with researchers from diverse parts of the world collaborating to contribute to its growth and development.

The annual publication growth rate, a staggering 77.83%, signals a rapidly emerging research trend in the field of AI-powered content marketing. The average age of documents, at 0.952, indicates that the vast majority of publications are relatively recent, adding to the excitement and dynamism of this field. Each document received an average of 10.43 citations, further demonstrating the substantial scientific impact of this research area.

A total of 103 keywords were used across the documents, reflecting the multidimensional nature of the research domain. Some of the most frequently used keywords include 'AI,' 'content marketing,' 'machine learning,' 'digital marketing,' and 'data analytics,' providing a glimpse into the diverse topics and areas of interest within this field. The overall findings are illustrated in Figure 1.

Figure 1. General Information

Distribution of Publications by Year

The distribution of the reviewed publications by year indicates a growing interest in the research field. While only one article was published in both 2021 and 2022, the number increased to four in 2023, five in 2024, and ten in 2025. This trend demonstrates that the topic is gaining rapid importance in the academic literature. The distribution of publications by year is illustrated in Figure 2.

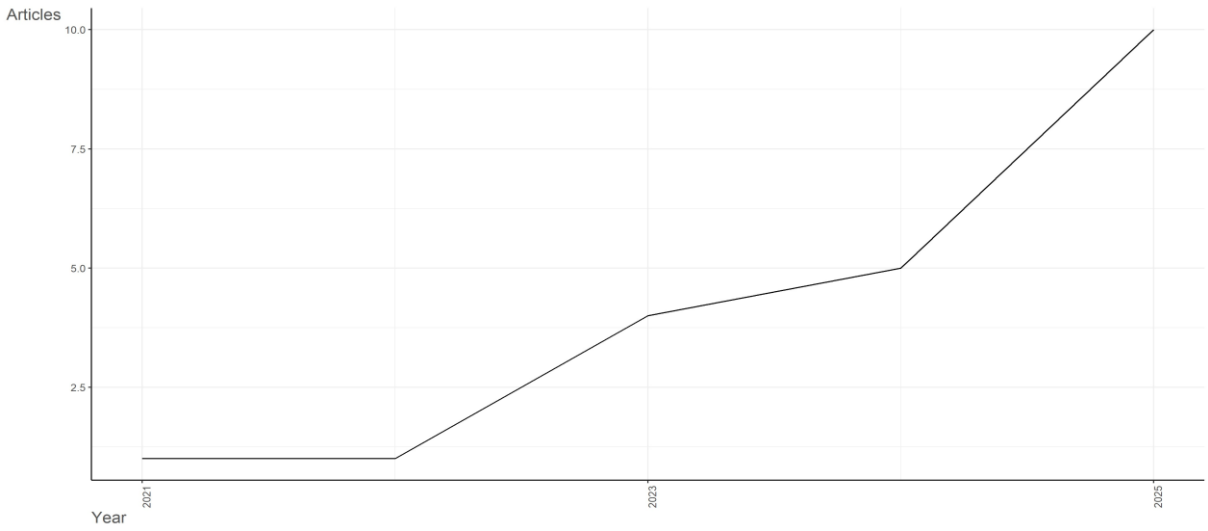

Figure 2. Distribution of publications by year

Publication Production by Country

The United States has the highest number of publications, while countries such as China, the United Kingdom, Germany, India, and Australia have also made significant contributions. The research is particularly concentrated in North America, Europe, and the Asia-Pacific region. The publications, with their multinational character, are a testament to the growing global interest in the topic, connecting researchers and academics from around the world. Country-level publication output is illustrated in

Figure3.

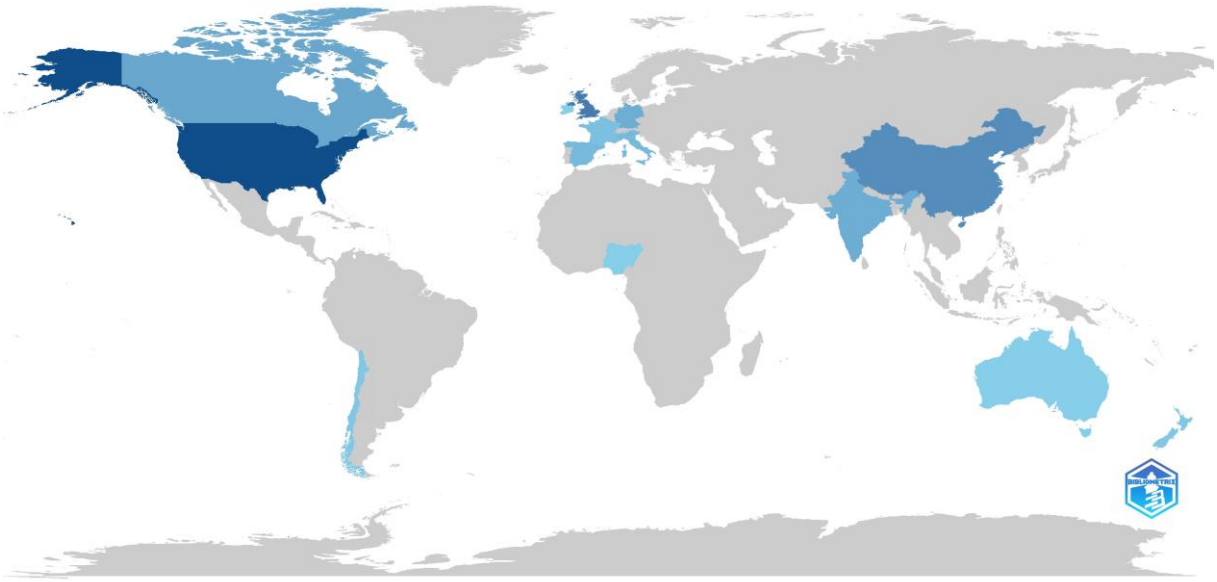

Figure 3. Country-wise Distribution of Academic Publications

Publication Production by Institution

Dartmouth College and the Technical University of Munich stand out as the most productive institutions in this field, each contributing four publications. Their significant contributions underscore their leadership in AI-powered content marketing research. They are followed by institutions such as King's College London, Simon Fraser University, University of Cambridge, University of Illinois, and the University of Seville (Spain) each having produced three publications. In addition, institutions such as Aston Business School, the Center for Collective Intelligence, and the Guangdong University of Technology have contributed two publications each (See Figure 4).

This distribution not only showcases the active exploration of the field by universities in both North America and Europe but also underscores the pivotal role of institutional diversity and academic collaboration. These factors are instrumental in advancing the research landscape, inspiring us with the potential of collective efforts.

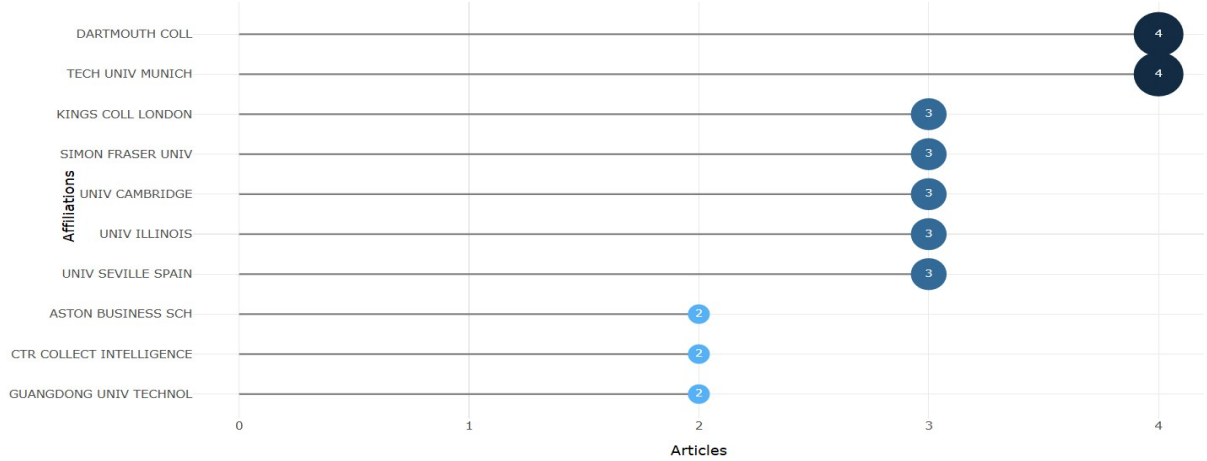

Figure 4. Most Productive Institutions

Publications by Journals

Based on the results of a rigorous bibliometric analysis, it is evident that Information Research – An International Electronic Journal and the International Journal of Research in Marketing have published the highest number of studies on this topic, each featuring two publications related to AI-powered content creation. Other notable sources include Business Horizons, International Journal of Market Research, International Transactions in Operational Research, Journal of Business Research, Journal of Consumer Behaviour, Journal of Public Policy & Marketing, Journal of Retailing and Consumer Services, and the Journal of Services Marketing—each of which contains one relevant publication. This distribution underscores the diverse nature of AI-driven content creation research, which is published across a range of disciplines. This insight broadens your perspective on the interdisciplinary nature of the field, with a significant presence in information systems and marketing-oriented journals. The analysis underscores the burgeoning academic interest at the intersection of marketing and information technologies, as depicted in Figure 5, thereby highlighting the multidisciplinary nature of the field.

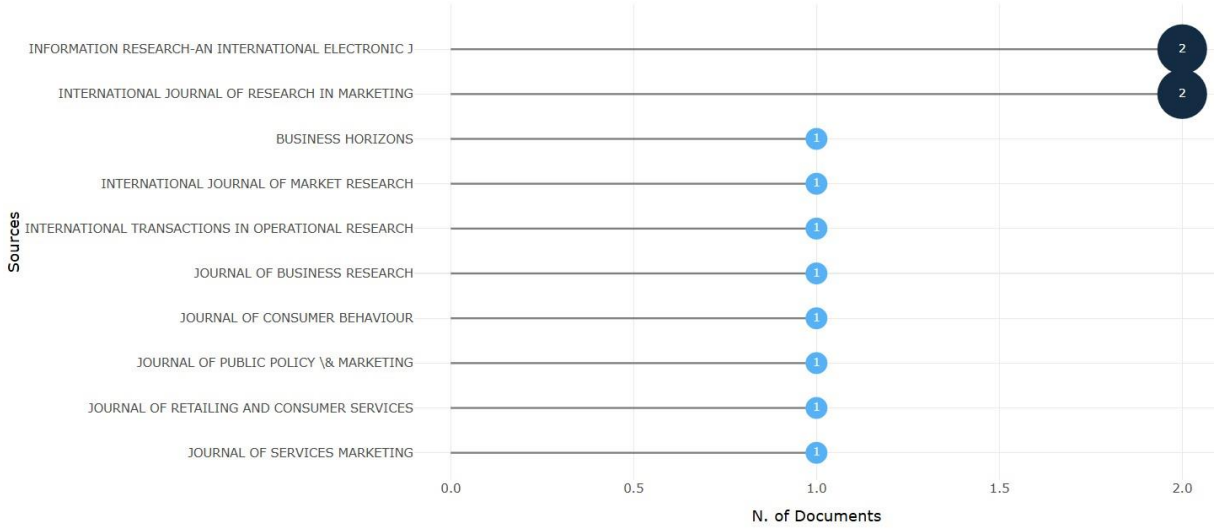

Figure 5. Distribution of Publications by Academic Journal

Most Frequently Used Terms

Based on the keyword analysis, it's evident that the most prevalent concepts in the academic literature are artificial intelligence, impact, performance, knowledge, social media, and content creation. The frequency of these terms underscores the significant role of AI in content creation, keeping you informed about the current trends in the field.

The word cloud, presented in Figure 6, is a visual representation of the core trends in this field. The size and prominence of the words in the cloud correspond to their frequency in the literature, providing a quick and intuitive way to understand the most prevalent concepts in AI-powered content creation research.

The dominance of the terms “artificial” and “intelligence” clearly reveals that AI technology occupies a central position in studies on content production. Meanwhile, the prominence of terms like “impact,” “performance,” and “knowledge” not only suggests that research often explores AI’s role in knowledge generation and marketing effectiveness but also enlightens you about its potential in these areas.

Other notable keywords such as “social media,” “product,” and “image” highlight the significant role of AI in social media content generation, visual design, and the development of digital products, showcasing its versatility and intriguing potential.

Furthermore, terms like “consumer,” “engagement,” “emotions,” and “reactions” underscore the importance of marketing psychology-oriented studies. These studies examine the impact of AI-generated content on consumer behavior, interaction levels, and emotional responses. For example, AI can be used to personalize marketing content based on individual consumer preferences, leading to higher engagement and more positive emotional responses.

Contemporary AI tools directly referenced in the literature include “ChatGPT,” “GPT-4”, “bots,” and “algorithm.” Finally, keywords such as “adoption,” “acceptance,” and “innovation” reflect a strong research focus on the adoption of AI technologies, organizational adaptation processes, and their potential for innovation.

Figure 6. Word Cloud of the Most Frequently Used Terms

Factor Analysis

This analysis visualizes the relationships and thematic clusters among the most frequently co-occurring keywords in studies on AI-powered content creation. At the center of the map, the concepts of artificial intelligence and impact form strong connections with other terms, indicating that the primary research focus in this field is shaped around these two core axes.

Furthermore, the terms performance, social media, knowledge, and product also demonstrate significant levels of connectivity. This underscores the multidimensional nature of AI, which is not confined to technical aspects like algorithms and performance assessments, but also extends to practical application areas such as social media, and functional dimensions, including knowledge generation and product development. This complexity is both intriguing and engaging, offering a rich landscape for exploration.

The overall network structure vividly illustrates that AI-supported content creation is not just a technological domain, but a rich and diverse interdisciplinary research area. It involves not only technical aspects but also behavioral and applied dimensions. This rich tapestry of disciplines coming together in AI content creation is both inspiring and motivating, offering endless possibilities for exploration and innovation. Figure 7 illustrates the keyword co-occurrence network and thematic clusters.

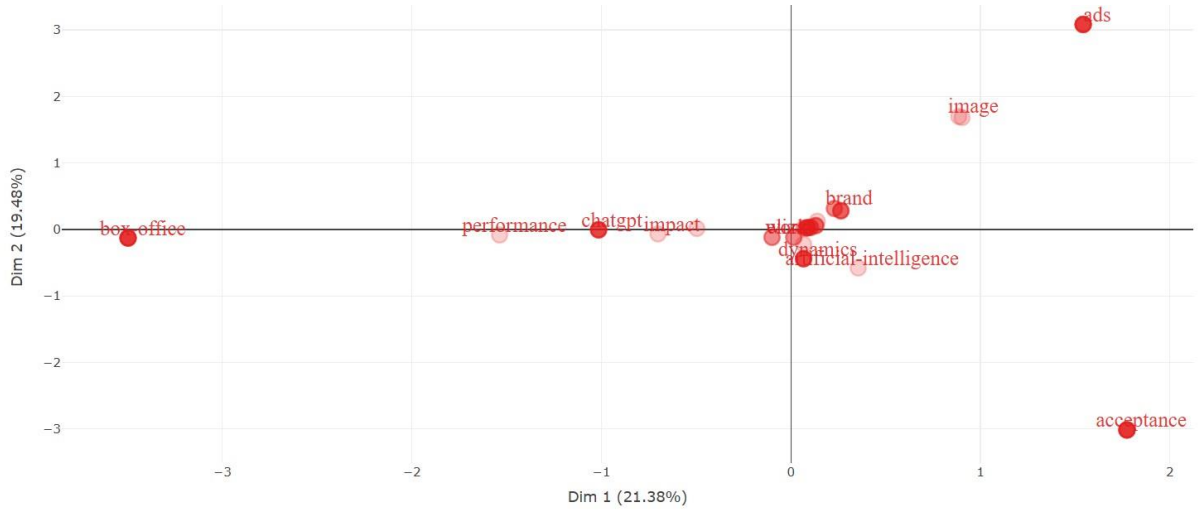

Figure 7. Keyword Co-Occurrence Network and Thematic Clusters

Thematic Network Analysis

The multidimensional scaling (MCA)-based map, presented in Figure 8, positions the keywords found in the literature on AI-powered content creation within a two-dimensional space based on their contextual proximity and co-occurrence frequencies. This visualization reveals the relational distances between concepts, allowing researchers to analyze the degree to which themes are interconnected or distinct.

Keywords located at the center of the map—such as artificial intelligence, impact, performance, brand, ChatGPT, and dynamics—frequently appear together in the literature and are discussed within common thematic frameworks. This tight clustering indicates that these terms typically share similar research questions, theoretical foundations, and application contexts.

In contrast, some keywords placed at the outer regions of the map represent more isolated research themes. For example, box office, located in the lower-left corner, diverges from the other terms and points to a more niche research direction specific to the entertainment industry. Similarly, acceptance, positioned in the lower-right corner, indicates that studies focused on the adoption processes of AI technologies are examined on a relatively independent axis. Terms such as image and ads also suggest that research in visual design and advertising occupies a separate thematic domain.

As illustrated in Figure 8, this multidimensional conceptual map highlights both the strong central structure of the literature and the existence of specialized, narrowly focused subthemes, thereby contributing to the understanding of the field’s diversity and the emergence of evolving topic clusters..

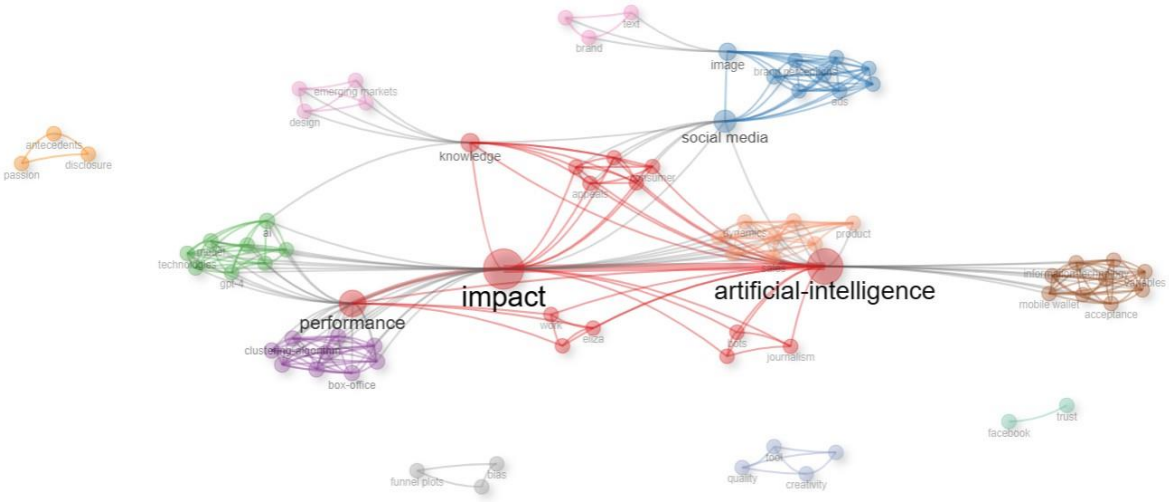

Figure 8. Thematic Concept Map Based on Multidimensional Scaling

International Collaboration Network

An examination of the international collaboration network reveals strong scientific interaction among countries such as the United States, the United Kingdom, Canada, and Italy. The overall rate of internationally co-authored studies across all analyzed articles is calculated to be 33.33%.

The collaboration network map, shown in Figure 9, illustrates the co-authorship relationships among academic researchers in the field of AI-powered content creation. Notably, the cluster formed around Chang and Gu C demonstrates the strongest and most concentrated connections, positioning it as one of the key productive hubs in the field. Similarly, the group led by Akpan and Asuama A. AA stands out for its tight collaboration links.

Authors such as Cioppi and Francioni B have fewer collaborators but are considered high-contributing individuals. Many other authors contribute to the field through dyadic (two-person) collaborations, indicating a limited scope of collaborative engagement.

Overall, while the network is centered around a few strong nodes, it does not show widespread distribution, and collaborations remain relatively limited. This suggests a need for more multi-authored and interdisciplinary studies to strengthen the collaborative structure of the field..

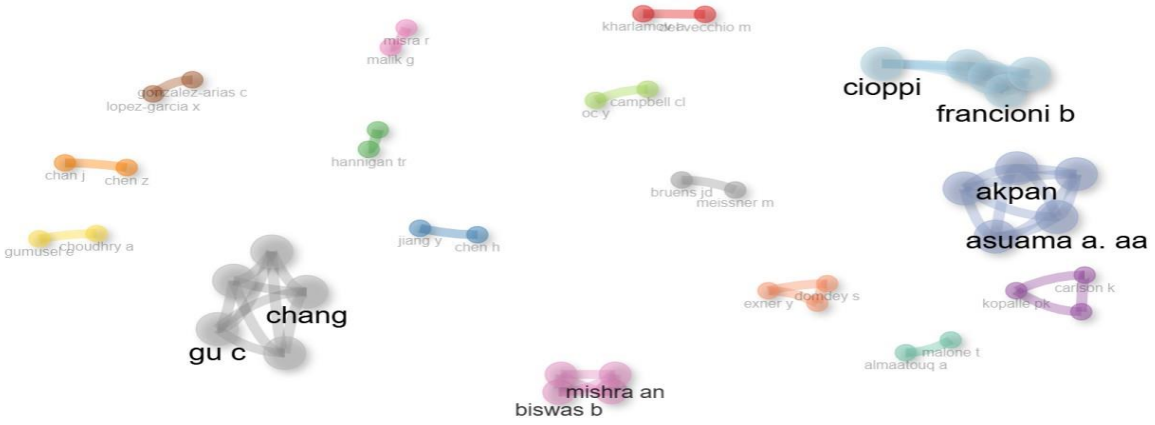

Figure 9. International Co-Authorship Network

DISCUSSION

This study comprehensively examines the effectiveness of AI-powered content creation in marketing through a systematic literature review and bibliometric analysis. The findings reveal that while AI technologies play a transformative role in marketing content production, they also introduce new challenges and ethical responsibilities.

The potential of Generative AI (GenAI) in content creation has been a subject of extensive discussion across numerous studies. Some research underscores the positive effects of using GenAI as a supportive tool in the creative process—especially in enhancing the performance of non-experts (e.g., Chen & Chan, 2024). This optimistic view is balanced by warnings that full automation may weaken consumer trust and brand perception (Brüns & Meißner, 2024). Hybrid approaches, combining human and AI-generated outputs, have been proposed as the ideal solution (De Cicco et al., 2025).

Consumer responses to AI-generated content are found to be highly complex. Studies have shown that perceived artificiality in product advertisements (Gu et al., 2024), a lack of empathy in influencer content (Zhang et al., 2025), and low engagement in educational content (De Cicco et al., 2025) can negatively impact trust and interaction. Therefore, brands must choose content formats that align with their target audience segments, as consumer trust is a key factor in the success of AI-generated content.

In the context of conversation-based systems (e.g., chatbots and digital assistants), epistemic risks, misinformation, and anthropomorphic design have emerged as key concerns (Hannigan et al., 2024; Misra et al., 2025). Epistemic risks refer to threats to information reliability resulting from AI-generated content that is false, fabricated, or misleading, which can seriously undermine consumer trust in the information. The anthropomorphic design aims to give AI systems human-like features (e.g., facial expressions, voice tone, conversational style) to foster a warmer and more trustworthy user experience. However, such designs may distort perceptions of reality and raise ethical concerns. For chatbots to function effectively, it is recommended to incorporate localization, emotional intelligence, and trust mechanisms into their design.

The impact of GenAI on visual (Hartmann et al., 2025) and video content (Mishra et al., 2025) is receiving increasing attention. It has been demonstrated that AI-generated visuals can rival human-made content in both aesthetic appeal and performance. Video studies also found that the social distance between the narrator and the viewer influences content success.

Ethical boundaries and content safety related to GenAI are central to many studies included in this review. Issues such as plagiarism, manipulative content, and the generation of fake reviews pose significant risks in various domains, including media (González-Arias & López-García, 2023), e-commerce (Pérez-Castro et al., 2023), and social media (Zhou et al., 2025). The gravity of these issues underscores the need for practical solutions, including watermarking, labeling, public awareness campaigns, and regulatory frameworks, to mitigate these risks.

GenAI is not limited to content creation alone; it has also been integrated into areas such as business model innovation (Teng et al., 2025), emotional data analytics (Del Vecchio et al., 2021), and luxury consumption strategies (Oc et al., 2023). From a strategic standpoint, it is recommended that firms view GenAI not only as a tool for operational efficiency but also as a source of creative and differentiating competitive advantage.

CONCLUSION

This systematic literature review analyzed 21 SSCI- and SCI-E-indexed studies published between 2021 and 2025, examining the impact of artificial intelligence (AI)—particularly generative AI (GenAI)—on content marketing thematically. The findings reveal that AI has triggered transformations across a broad

spectrum, from content creation to consumer engagement, brand strategies, and ethical and legal challenges.

One of the key findings of the study is that GenAI plays both an assistive and transformative role in marketing processes. GenAI offers significant speed and cost advantages, particularly in the production of advertising copy, product reviews, and visual and video content. However, the unrestricted use of these technologies may lead to a decline in perceived brand authenticity, a lack of trust, and a lack of empathy. This highlights the importance of hybrid content models that leverage human-AI collaboration.

From the perspective of consumer perception, AI-generated content is often questioned in terms of eeriness, artificiality, and authenticity. At the same time, human influencers are found to be more effective in establishing emotional connections. Therefore, content type, target audience characteristics, and perceptual alignment strategies are critical to achieving marketing effectiveness.

Conversational AI applications (e.g., chatbots) present mixed findings regarding user satisfaction and epistemic risks. The accuracy, transparency, and localized language use of chatbot-generated content have a significant influence on user experience and continued usage intention. Anthropomorphic design elements and trust factors play a vital role in sensitive sectors such as banking.

The studies also draw attention to the ethical and regulatory dimensions of AI technologies. Risks such as fake content generation, deceptive reviews, copyright infringement, and misinformation necessitate proactive measures by both brands and policymakers. Proposed regulations include labeling AI-generated content, implementing watermarking techniques, promoting public data literacy education, and utilizing transparent algorithms.

In conclusion, this study makes significant contributions to both academic and practical domains by revealing the multifaceted effects of generative AI in content marketing. Future research is encouraged to explore further psychological consumer responses, long-term brand impacts, and interactive content dynamics and to employ more inclusive datasets. Additionally, comparative studies on the effects of GenAI across different geographic and cultural contexts will be valuable for shaping global marketing strategies.

Theoretical Contributions

This study presents a comprehensive and multi-layered theoretical analysis of the field of AI-powered content marketing, drawing on recent SSCI- and SCI-E-indexed publications to provide a thorough understanding of the topic.

First, this study introduces a novel framework by systematically mapping the ongoing debate in the literature regarding whether GenAI serves as an “assistant” or a “replacement” in marketing contexts. It re-evaluates the interaction between human-centered creative processes and AI-driven production practices in terms of content quality, consumer perception, and brand authenticity. In this context, the study fills a gap in the literature on hybrid content creation models and proposes a classification that could form the basis for future theoretical models.

Second, the research reveals how variables such as trust, empathy, eeriness, and perceived intelligence of AI-generated content function in marketing communication, especially from a consumer behavior perspective. It advances the literature by moving beyond prior studies. It presents new findings that can be linked to emotional-technological interface theories regarding the psychological effects of GenAI on user experience.

Third, by examining the epistemic risks associated with conversational AI tools and the role of anthropomorphic design elements on user satisfaction, the study deepens the marketing communication

literature from the perspectives of digital ethics and algorithmic trust. This contribution enables a reinterpretation of frameworks such as UTAUT, ECM, and anthropomorphism theories within the context of content marketing.

Fourth, the study contributes to visual marketing theories by addressing the performance, aesthetic, and engagement dimensions of AI-supported video and visual content through the lens of sensory and visual persuasion models. It also emphasizes the role of elements such as emotional arcs, semantic flows, and social distance in determining content success in media analysis.

Finally, the study integrates ethical, legal, and societal dimensions of AI-generated content into the marketing literature by introducing perspectives on digital ethics, data governance, and algorithmic accountability. In doing so, it offers a holistic evaluation of the impact of GenAI applications not only at an operational level but also at a normative one.

These theoretical contributions not only promote an interdisciplinary approach in both content marketing and AI-driven digital marketing research but also offer a structured conceptual foundation for future scholarly work, providing practical insights for marketing and AI practitioners.

Practical Contributions

This systematic literature review provides valuable insights for practitioners in the field of AI-powered content marketing. The findings offer a range of recommendations relevant to a broad set of stakeholders, including marketing managers, content strategists, technology developers, and policymakers.

First, the efficiency and speed advantages offered by GenAI in content creation are undeniable. However, the direct and fully automated use of such technologies may erode consumer trust in brand authenticity. Therefore, companies are strongly encouraged to adopt hybrid content strategies supported by human input. This balanced approach reassures the audience that authenticity is preserved while maintaining operational efficiency.

Consumer responses to AI-generated content are not solely influenced by technical factors but also by psychological and emotional dynamics. Feelings of artificiality, lack of empathy, and eeriness can deter less digitally literate users. Therefore, it is crucial to tailor content to target audiences and design it with emotionally appropriate tones for different demographic groups.

In conversational AI applications, such as chatbots and digital assistants, elements like anthropomorphic design and cultural localization are crucial for enhancing user satisfaction. Trust, effort expectancy (the perceived ease of use of the chatbot), and perceived empathy are key factors in chatbot usage. Therefore, companies should not only ensure technical accuracy but also focus on the relational quality of interactions.

Fourth, the benefits of GenAI in visual and video content creation—in terms of aesthetics, speed, and cost-effectiveness—are particularly notable. However, the effectiveness of such content should be monitored regularly through A/B testing, sentiment analysis, and user feedback. It's crucial to maintain a balance between quality standardization and authenticity, which should instill confidence in the audience's content strategies.

Fifth, the growing role of AI in content production introduces ethical and security-related challenges that cannot be overlooked. Risks such as deceptive reviews, copyright violations, and algorithmic bias pose threats to brand reputation and consumer trust. Therefore, companies should develop transparency policies, such as clearly labeling AI-generated content, and implement technological solutions (e.g., watermarking and content labeling) that are aligned with applicable regulations.

Finally, GenAI offers opportunities not only in content creation but also in business model innovation. Firms seeking to develop new value propositions, personalized service models, and creative solutions should position GenAI as a strategic lever, integrating it into a holistic marketing vision that delivers not just tactical benefits but also long-term competitive advantage. This innovation potential should inspire practitioners to explore new possibilities in their field.

Limitations

This study has evaluated the effectiveness of AI-powered content creation in marketing through a comprehensive systematic literature review and bibliometric analysis. However, like all research, it has certain limitations.

First, the analysis was conducted using only the Web of Science (WoS) database, and only articles indexed in SSCI and SCI-E were included. While this approach ensured a high-quality subset of the literature, the exclusion of studies published in other vital sources (e.g., Scopus, Google Scholar) may have limited the scope of the review.

Second, the 21 selected articles cover a relatively recent period (2021–2025). Since the academic literature on AI-powered content creation has not yet reached full maturity, some theoretical frameworks and conceptual models are still under development. As a result, the findings may evolve as the broader literature base matures.

Third, the bibliometric analysis focused on metrics such as collaboration networks, keyword clusters, and publication productivity. However, more in-depth analytical techniques—such as content analysis or text mining—were not employed. This may have limited the depth of the conceptual insights that could have been extracted.

Finally, the review was restricted to articles published in the English language. This may have excluded cultural or regional variations reflected in studies written in languages other than English, thus limiting the global generalizability of the findings.

Recommendations for Future Research

While this systematic literature review identifies key trends and the current state of knowledge in the field of AI-powered content marketing, it also reveals several research gaps. These gaps present exciting opportunities for future studies that could significantly advance our understanding of AI in marketing. Accordingly, the following multifaceted recommendations are proposed, each with the potential to inspire and motivate further research:

First, the debate over whether GenAI plays a supportive or substitutive role in content creation remains theoretically unresolved. Future research should investigate human-AI collaboration models in greater detail, focusing on measuring their direct and indirect effects on content quality, brand perception, and consumer behavior. In particular, the performance of hybrid content production models should be assessed using longitudinal experimental designs.

Second, the psychological mechanisms underlying consumer responses (e.g., eeriness, perceived artificiality, perceived empathy) have mostly been tested through experimental research. However, the generalizability of these findings across different cultural, socio-demographic, and platform-specific contexts remains underexplored. Therefore, multi-country and cross-cultural comparative studies will enhance our understanding of how GenAI content is perceived and accepted.

Third, existing findings on epistemic risks, anthropomorphism, and trust factors in conversational AI applications (e.g., chatbot digital assistants) are primarily based on cross-sectional data. Thus, longitudinal studies are needed to model better how user perceptions evolve and how continuous usage

behavior is shaped. Additionally, real-time interactions and neuromarketing-based methods could be employed to gain deeper insight into the emotional effects of these interactions.

Fourth, the impact of GenAI on video and visual content has not yet been examined within the broader marketing mix (4Ps). Future research should explore the influence of GenAI in visual content production not only through performance metrics but also through behavioral outcomes such as consumer experience, perceived brand value, and purchase intention.

Fifth, ethical, legal, and governance-related issues emerging from the integration of GenAI into marketing strategies should be examined more comprehensively. This examination should not be limited to a single discipline but should instead be approached through interdisciplinary methods. In particular, experimental and conceptual models related to fake reviews, algorithmic bias, and content transparency should be developed to inform future policy frameworks.

Finally, the opportunities offered by GenAI extend far beyond content creation. Future research should develop theoretical frameworks for its use in areas such as business model innovation, synthetic data generation, micro-segmentation, and hyper-personalization. For instance, GenAI can be used to create synthetic data for training AI models or to personalize marketing messages based on micro-segmented consumer groups. Supporting these areas with case studies of innovative applications will further demonstrate the potential of GenAI.

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Conflict of Interest

The authors have declared that there is no conflict of interest

Author Contributions

G.A. contributed to the development of the conceptual framework; İ.H.E. conducted the data analysis and completed the remaining sections of the manuscript.

The Impact of Brand Trust Erosion on Boycott Behavior: A Contemporary Assessment (1213)

Muhammed Hayri Haci İbrahim[1], Filiz Çayırağası [1], İbrahim Halil Efendioğlu[1*]

[1]Gaziantep University, Faculty of Economics and Administrative Sciences, Department of Business Administration, Turkey

*Corresponding author e-mail: efendioglu@gantep.edu.tr

Abstract

Trust is pivotal in shaping consumer decision-making processes and constitutes the foundation of brand-consumer relationships. The erosion of this trust can lead to heightened negative perceptions toward the brand and may trigger reactive consumer behaviors, such as boycotts. This study examines the impact of brand trust erosion on boycott behavior. Previous literature suggests that trust erosion directly fosters negative attitudes and boycott intentions. In this context, the present research adopts a quantitative methodology and investigates this phenomenon within a sample of Turkish and Syrian consumers residing in Gaziantep, Türkiye. Data were collected through an online survey administered to 422 participants and analyzed using SPSS software. The findings reveal that trust erosion has a statistically significant and positive effect on consumer boycott behavior. Notably, subdimensions such as emotional and social pressure, dissenting opinions, and the perceived effectiveness of boycotts were significantly associated with trust erosion. Furthermore, independent samples t-tests and one-way ANOVA results indicate that both gender and ethnicity significantly influence perceptions of trust erosion and tendencies toward boycott behavior. Female participants and Turkish consumers reported higher trust erosion and stronger boycott inclinations than their male and Syrian counterparts. In contrast, no significant differences were found regarding age and income levels. However, educational level demonstrated a meaningful variation: individuals with middle school education and those holding doctoral degrees perceived trust erosion differently. In conclusion, the findings largely align with the extant literature, emphasizing that trust erosion is a critical antecedent of collective consumer actions, particularly boycotts. Based on the observed variations in trust perceptions between Turkish and Syrian consumers, it is recommended that brands design and implement culturally sensitive trust-building strategies to address the expectations of diverse consumer segments

Keywords: *Trust Erosion, Boycott Behavior, Consumer Behavior*

INTRODUCTION

In today's increasingly competitive market conditions, achieving sustainable success for brands is not solely dependent on the quality of products or services; it is also directly related to the level of trust consumers place in the brand (Özdemir & Yaman, 2021). Within the context of consumer-brand relationships, trust is considered a critical factor in fostering long-term loyalty. This loyalty, often referred to as brand commitment, is the extent to which a consumer is dedicated to a particular brand, and is regarded as one of the fundamental building blocks of brand commitment (Delgado-Ballester et al., 2003; Kim et al., 2021).

However, any deterioration in this trust can lead consumers not only to develop negative attitudes toward the brand but also to engage in punitive actions such as boycott behavior (Grégoire, Tripp & Legault, 2009; Lim et al., 2020). Particularly, unethical corporate practices such as misleading advertising or labor exploitation, or brand indifference to social crises like environmental disasters or human rights

violations, accelerate this process by eroding consumer trust, paving the way for widespread boycott movements (Chen & Yeh, 2021; Akhtar et al., 2022).

With the advancement of social media and digital communication technologies, consumer reactions have become more visible, organized, and impactful. This shift has significantly altered the dynamics of consumer-brand relationships. As a result, trust deterioration has evolved from being an individual consumer issue into a collective form of societal response (Zarantonello, Romani, & Grappi, 2022). This transformation demonstrates that boycotting is not merely an individual act but also a social reflex rooted in a collective quest for justice.

Therefore, the primary aim of this study is to investigate the impact of brand-related trust deterioration on consumer boycott behavior, to evaluate this relationship through sub-dimensions such as emotional-social pressure (which refers to the influence of social networks and emotional responses on boycott decisions), oppositional views, and perceived boycott effectiveness, and to examine whether these relationships vary significantly across demographic groups.

It is important to note that this study has certain limitations, which are crucial for a comprehensive understanding of the research. First, the research was conducted solely in the province of Gaziantep, which may limit the generalizability of the findings to other regions. The data collection was conducted between June and December 2024, which may have influenced participant responses due to period-specific events. Additionally, the sample consisted only of Turkish and Syrian consumers; thus, the findings cannot be generalized to other ethnic groups. There is also a potential for response bias due to participants' political sensitivities and the influence of social desirability. These limitations are significant and should be considered when interpreting the results..

THEORETICAL BACKGROUND AND HYPOTHESES

Consumer trust, a pivotal area of research in the marketing and consumer behavior literature, is the focus of our study. Delgado-Ballester et al. (2003) underscore the impact of brand trust on consumer loyalty, while Kim et al. (2021) extend this relationship to digital platforms. The deterioration of consumer trust in a brand can lead to adverse behavioral responses and, in some cases, to active boycott behavior (Grégoire, Tripp & Legault, 2009). Our research aims to explore this phenomenon in greater depth and its implications.

Boycott behavior, the deliberate rejection of a brand or product by consumers based on moral, ethical, or political grounds (Sen, Gürhan-Canli & Morwitz, 2001), has gained prominence in recent years. This is mainly due to the increasing prevalence of consumer boycotts in response to perceived corporate ethical violations or social injustices. The role of social media in amplifying these reactions by enabling their rapid dissemination (Zarantonello, Romani & Grappi, 2022) cannot be overstated.

Akhtar et al. (2022) indicate that brands' inaction in the face of social crises significantly undermines consumer trust and directly triggers boycott tendencies. Similarly, Chen and Yeh (2021) argue that a lack of corporate responsibility reduces brand value in the eyes of consumers, leading to erosion of trust. Lim et al. (2020) suggest that emotional responses play a key role in guiding boycott behavior.

To explain consumer responses, this study adopts the Stimulus–Organism–Response (S-O-R) model (Mehrabian & Russell, 1974) as its theoretical foundation. Additionally, Blau's (1964) Social Exchange Theory and Greenberg's (1990) Justice Theory are employed to provide a more comprehensive interpretation of the individual and societal impacts of trust deterioration.

Despite the growing interest in the topic, research examining the relationship between trust deterioration and boycott behavior through the lens of demographic variables remains limited in the literature. Our

study, with its multidimensional approach, aims to fill this gap and make a significant contribution to the field.

Based on the literature review and theoretical framework, the following hypotheses were developed to explain the effects of trust deterioration on boycott behavior. The S-O-R model (Mehrabian & Russell, 1974), which views the loss of trust as a stimulus triggering emotional and cognitive responses in the consumer (organism), ultimately resulting in behavioral outcomes (response), provides the basis for these hypotheses.

In this context, trust deterioration may lead consumers to question their relationship with the brand and engage in punitive behaviors such as boycotts. The hypotheses proposed to explain this process are as follows:

H1: Deterioration of consumer trust in a brand has a statistically significant and positive effect on boycott behavior.

H1.1: Trust deterioration positively affects the emotional and social pressure sub-dimension of boycott behavior.

H1.2: Trust deterioration has a positive effect on the oppositional views sub-dimension of boycott behavior.

H1.3: Trust deterioration has a positive effect on the perceived effectiveness sub-dimension of boycott behavior.

Additionally, it is predicted that consumer responses to trust deterioration and boycott behavior may vary depending on demographic variables. Accordingly, a second central hypothesis is proposed:

H2. Consumer levels of trust deterioration and their corresponding boycott behaviors differ significantly across socio-demographic variables (gender, age, education level, income level, and ethnicity).

H2.1: Consumer levels of trust deterioration and boycott behaviors differ significantly based on the variable of gender.

H2.2: Consumer levels of trust deterioration and boycott behaviors differ significantly based on the variable of age.

H2.3: Consumer levels of trust deterioration and boycott behaviors differ significantly based on the variable of education level.

H2.4: Consumer levels of trust deterioration and boycott behaviors differ significantly based on the variable of income level.

H2.5: Consumer levels of trust deterioration and boycott behaviors differ significantly based on the variable of ethnicity.

MATERIAL AND METHODS

This study was conducted within the framework of a quantitative research approach using a correlational survey model. The primary objective of this research is to investigate the impact of deteriorating consumer trust in brands on boycott behavior and to examine how this relationship varies across different demographic variables.

The study population comprises Turkish citizens and Syrian immigrant consumers residing in the province of Gaziantep who are in direct or indirect contact with brands. The sample was determined using a simple random sampling method. Based on similar studies in the literature, a sample size of 300

was projected as a minimum. In this study, an online questionnaire was distributed to 470 individuals, yielding 422 valid responses with a response rate of 89.8%.

The data collection process was carried out in three stages. In the first stage, preliminary face-to-face interviews were conducted with 50 participants to identify product categories where boycott behavior is more prevalent. Based on these interviews, a brief pre-survey was prepared, and the most commonly boycotted categories were identified as beverages, food, cosmetics, skincare, clothing, and health products.

The questionnaire form consists of three main sections:

Questions related to socio-demographic characteristics such as gender, age, education level, income level, and ethnicity;

The boycott behavior scale developed by Klein et al. (2004), adapted in this study into three sub-dimensions (emotional-social pressure, oppositional views, and perceived boycott effectiveness), consisting of 11 items;

A 7-item trust deterioration scale was adapted from Garbarino and Johnson (1999).

Both scales were translated into Turkish and Arabic, and back-translation and expert consultation were used to ensure cultural validity and linguistic equivalence. All items were rated on a 5-point Likert scale (1 = Strongly Disagree, 5 = Strongly Agree).

The collected data were analyzed using Microsoft Excel and IBM SPSS Statistics 26.0. Descriptive statistics and frequency analyses were performed first. Subsequently, exploratory factor analysis (EFA) and Cronbach's alpha coefficients were calculated to assess construct validity and reliability.

For hypothesis testing, simple linear regression analysis was used to assess the explanatory power of variance, and differences across socio-demographic variables were evaluated using one-way ANOVA and independent samples t-tests. The assumption of normal distribution was checked using skewness and kurtosis values.

RESULTS

Among the 422 participants in the study, 52.9% were Turkish citizens and 46.8% were Syrian refugees. The gender distribution was balanced, with 49.5% male and 50.5% female participants. The largest age group was those aged between 31–36, accounting for 24.6% of the sample. In terms of education level, the highest proportion was observed among university graduates, comprising 43.7% of the participants. Additionally, 46% of the respondents reported a monthly income between 22,100 and 27,000 Turkish Liras.

The construct validity and internal consistency of the scales used were evaluated using Exploratory Factor Analysis (EFA) and Cronbach's Alpha coefficients. For the trust deterioration scale, the Kaiser-Meyer-Olkin (KMO) value was 0.815, and Bartlett's Test was significant ($p < 0.001$). The Cronbach's Alpha value was calculated as 0.823. For the boycott behavior scale, the KMO value was 0.751, Bartlett's Test was significant ($p < 0.001$), and the Cronbach's Alpha was 0.852. The total variance explained was 67.94% for trust deterioration and 64.13% for boycott behavior. Factor loadings ranged between 0.669 and 0.783 for both scales.

A positive and moderately significant correlation was found between trust deterioration and boycott behavior ($r = 0.483$, $p < 0.01$). The mean score for trust deterioration was $\bar{X} = 3.845$, while the mean score for boycott behavior was $\bar{X} = 3.962$. Skewness and kurtosis values were within ±2, confirming the assumption of normal distribution.

According to simple linear regression analysis:

H1 was supported: Trust deterioration had a statistically significant and positive effect on boycott behavior ($\beta = 0.483$, $R^2 = 0.234$, $p < 0.001$).

H1.1 was supported: Trust deterioration positively affected the emotional and social pressure sub-dimension of boycott behavior ($\beta = 0.402$, $R^2 = 0.162$, $p < 0.001$).

H1.2 was supported: Trust deterioration positively affected the oppositional views sub-dimension ($\beta = 0.331$, $R^2 = 0.109$, $p < 0.001$).

H1.3 was supported: Trust deterioration positively affected the perceived effectiveness of boycott behavior ($\beta = 0.323$, $R^2 = 0.104$, $p < 0.001$).

Although no statistically significant differences were found based on gender and ethnicity, female participants and Turkish participants had higher average scores. No significant differences were observed in trust deterioration or boycott behavior based on age and income levels.

However, a statistically significant difference was found in trust deterioration based on education level ($p = 0.042$), particularly between participants with middle school education and those with doctoral degrees. No significant differences were detected in boycott behavior in terms of education level.

DISCUSSION AND CONCLUSION

In this study, the effect of consumer trust deterioration on boycott behavior was examined based on a sample of 422 Turkish and Syrian individuals residing in the province of Gaziantep. The findings revealed that trust deterioration has a significant and positive effect on boycott behavior. In particular, it was determined that the loss of trust leads consumers to exhibit punitive responses both individually and collectively. These responses manifest in various sub-dimensions such as emotional-social pressure, oppositional views, and perceived boycott effectiveness.

According to the regression analysis, trust deterioration explains 23.4% of the variance in boycott behavior, which is consistent with previous studies by Kim et al. (2021) and Akhtar et al. (2022). This indicates that brand trust is a strategic asset, and its erosion may result not only in customer loss but also in reputational damage.

The results also show that female consumers are more sensitive to trust deterioration and more inclined to engage in boycott behavior. This supports the idea that women are more attentive to brand behavior and may respond more emotionally (Alhouti et al., 2016). In terms of ethnicity, Turkish participants were found to experience greater trust loss and exhibit a higher tendency to boycott compared to Syrian participants. This difference may be attributed to cultural context, social media usage, and political sensitivity.

On the other hand, no significant differences were found in trust deterioration or boycott behavior based on age and income levels. However, education level did reveal a significant difference in trust deterioration. Specifically, individuals with higher levels of education were found to detect ethical violations more easily and react more strongly to such incidents (Erdoğan & Gümüş, 2022).

Brands should prioritize transparency, accountability, and ethical principles in order to sustain consumer trust. In times of crisis, trust must be rebuilt through effective communication strategies, and potential boycott risks should be anticipated in advance. Corporate social responsibility initiatives and actions addressing societal sensitivity should be used as tools to restore trust. Value-based communication strategies that address the concerns of female consumers and highly educated individuals should be developed.

From an academic perspective, future research should analyze other factors affecting boycott behavior—such as media influence, group norms, and ideological orientations—using multivariate models. Ultimately, trust deterioration should be recognized not only as a loss of individual loyalty but also as a critical factor that triggers broader societal reactions. Therefore, brands must invest in trust management at a strategic level.

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Conflict of Interest

The authors have declared that there is no conflict of interest

Author Contributions

This study is derived from the doctoral dissertation of Muhammed Hayri Haci İbrahim conducted at the Graduate School of Social Sciences, Gaziantep University. M.H.H.İ.: Data collection, execution of analyses, literature review, and initial draft writing. F.Ç.: Guidance and supervision on conceptual framework, hypothesis development, methodology, findings, and conclusion. İ.H.E.: Methodological consultancy and guidance during the analysis process.

Local and Global Interpretations in Psychological Modeling (1214)

Fatih Sağlam[1*], Filiz Koçoğlu Sağlam[2], Yeliz Kındap Tepe[3]

[1]Ondokuz Mayıs University, Faculty of Science, Department of Statistics, Türkiye
[2]Ondokuz Mayıs University, Vezirkoptu Vocational School, Child Development Program, Türkiye
[3]Ondokuz Mayıs University, Faculty of Humanities and Social Sciences, Department of Psychology, Türkiye

*Corresponding author e-mail: fatih.saglam@omu.edu.tr

Abstract

This study investigates how different machine learning methods can be used to identify and interpret the most influential features in a binary classification problem, with a particular focus on applications within psychology. Several well-established algorithms were applied to construct highly accurate classification models. Following model development, a comparative analysis of feature importance rankings was conducted using multiple approaches, including ensemble-based methods, statistical relevance measures, and model-agnostic techniques. These global importance metrics were interpreted not only from a computational perspective but also in light of their potential psychological significance. Beyond aggregated feature rankings, the study emphasizes the value of individualized explanations. Using a game-theoretic approach based on Shapley values, the contribution of each feature was assessed separately for every observation in the dataset. This localized analysis revealed how the influence of specific variables—such as screen time, perceived social support, or emotional regulation—varies across individuals. For example, while one participant's classification may be primarily driven by behavioral factors, another's outcome may hinge on social or emotional variables. By combining global and local interpretation techniques, this work offers a more transparent and psychologically informed understanding of machine learning outputs. The results highlight how explainable artificial intelligence (XAI) can be used not only to validate predictive models but also to generate person-centered insights that support clinical decision-making, individualized assessment, and theory-driven research in psychology. This dual-layered interpretation framework serves as a bridge between statistical modeling and applied psychological evaluation.

Keywords: *Feature Importance, Feature Rank, Explainable Artificial Intelligence, Addiction, Shapley Values*

INTRODUCTION

Machine learning methods are now widely used in various fields such as healthcare, finance, education, and industry (Jordan & Mitchell, 2015). These methods are mostly applied to predict the value of a specific target variable, and their performance is largely evaluated based on prediction accuracy. However, the potential of machine learning algorithms is not limited to high-accuracy predictions; they also offer powerful tools for making meaningful data-driven inferences (Murdoch et al., 2019).

Traditionally, inference-based analyses have relied on statistical methods, through which hypothesis tests are conducted on parameters and the effects of variables are interpreted. However, thanks to the flexibility and modeling capacity offered by machine learning methods, it is possible not only to make predictions but also to perform deeper inference (Ij, 2018). In this context, machine learning models have the potential to go beyond classical statistical methods by uncovering both linear and nonlinear

effects of variables (Molnar, 2022). In doing so, they can better handle more realistic and complex data structures and allow for the explanation of intricate interactions.

Inferences drawn from machine learning methods not only contribute to theoretical knowledge but also enable the development of more advanced and interactive learning frameworks. For instance, in active learning approaches, the model can guide data collection by inferring which examples would be most beneficial to label (Settles, 2009). Similarly, in the context of selective learning, the model can be designed to abstain from making predictions in cases of high uncertainty, offering predictions only when confident (Geifman & El-Yaniv, 2017). Such approaches strengthen collaboration between humans and machines, facilitating the development of systems that are more reliable and high-performing.

An important application area of inference-based approaches is the assessment of variable effects. This allows for identifying the degree to which variables influence the dependent outcome and for ranking the importance of these variables. Such insights are particularly useful in dimensionality reduction and model simplification, guiding decisions about which variables to retain (Guyon & Elisseeff, 2003). Moreover, it is possible to determine not only general model behavior but also the variables that are influential for a specific individual or subgroup. This enables the development of interpretable and personalized models at the individual level (Lundberg & Lee, 2017).

In this study, the application of certain inference approaches used in machine learning methods will be examined on a mobile addiction dataset. In recent years, the growing popularity of mobile phones due to their convenience has facilitated the integration of technological innovations into individuals' lives, while also leading to various problems (Yang et al., 2023). Smartphone addiction, characterized by excessive or uncontrolled preoccupation, urges, or behaviors related to smartphone use, often results in the neglect of other areas of an individual's life (Ching et al., 2015). Excessive mobile phone use has been closely associated with psychological health problems such as sleep disturbances, stress, low self-esteem, smoking, social isolation, and depression (Ching et al., 2015; Yang et al., 2023). Due to these adverse outcomes, it has been suggested that the condition resembles internet gaming disorder and may be considered a form of behavioral addiction (Ivanova et al., 2020; Yang et al., 2020).

Although various terms are used to describe the phenomenon, such as problematic cell phone/smartphone use, cell phone overuse, mobile phone dependence, or mobile/smartphone addiction, it is generally conceptualized as a form of behavioral addiction characterized by the core symptoms of addictive behaviors (Ivanova et al., 2020).

In the literature, the prevalence of smartphone addiction has been reported to range between 53% and 72% (Buctot et al., 2020; Machado et al.; Mokhtarinia et al., 2022). According to the 2024 Household Information Technologies Use Survey Report published by the Turkish Statistical Institute (TÜİK), 86.2% of individuals in Turkey use WhatsApp, 71.3% use YouTube, and 65.4% use Instagram. This trend is seen as a new form of socialization through digital identity. The prevalence of smartphone addiction in Turkey is reported to be between 25.9% and 34.8%, and it is noted that 77.3% of individuals use their smartphones for more than four hours a day (Korkmazer et al., 2022; Yağcı Şentürk et al., 2024). Therefore, understanding the variables that initiate and sustain smartphone addiction, and developing effective prevention and intervention programs targeting these variables, is considered essential.

To understand the variables that initiate and sustain smartphone addiction, machine learning models will first be developed on the dataset, and variable effects will be evaluated using classical statistical methods. Subsequently, these effects will be re-examined and compared using alternative approaches based on machine learning. In addition, variable effects will be explored at the individual level, with illustrative examples drawn from both addicted and non-addicted individuals. Finally, a selective

learning application will be presented within a Bayesian modeling framework, and the potential for establishing human–machine collaboration through this application will be explained. In this context, the study will discuss how the proposed methods can be applied in real-life scenarios and made functional at the project level, offering practical recommendations.

MATERIAL AND METHOD

This study presents a framework aimed at exploring the inferential potential of machine learning methods from multiple perspectives. The analysis will be conducted under four main headings: (1) general variable importance ranking, (2) instance-specific variable importance levels, (3) prediction and uncertainty analysis using Bayesian learning, and (4) selective learning applications.

Feature Selection Methods

Feature selection is a fundamental preprocessing step used in machine learning and statistical modeling processes to enhance model performance, facilitate interpretability, and reduce the risk of overfitting. This process aims to identify the independent variables most associated with the target variable (dependent variable). In high-dimensional datasets, especially in applications involving a limited number of samples, feature selection plays a critical role. Additionally, dimensionality reduction helps lower computational costs and increase the model's generalization capacity (Guyon & Elisseeff, 2003). Many methods have been developed for feature selection. In this study, four methods representing different approaches are introduced: importance measures derived from tree-based models, permutation-based importance scores, the BORUTA algorithm, and contribution metrics based on MARS regression.

Tree-Based and Knot-Based Methods

Decision trees are methods that perform classification or regression based on how variables split the data. Random Forest and XGBoost are ensemble methods that strengthen this tree structure through multiple models. Such methods can relatively score the contribution of each variable by recording how often and effectively each variable is used in the model's decision-making process (Breiman, 2001; Chen & Guestrin, 2016). Although these contributions cannot be confirmed through statistical significance tests, they provide strong intuition about the relative importance ranking of variables.

Multivariate Adaptive Regression Splines (MARS) is a flexible generalization of linear regression that captures nonlinear relationships by identifying optimal cut points (knots) on independent variables. This method allows for the calculation of variable importance levels by measuring the contributions of piecewise linear components included in the model (Friedman, 1991). MARS is particularly prominent in datasets that involve interactive and nonlinear structures.

Permutation-Based Importance Score

Permutation-based methods offer a flexible approach that directly evaluates the contribution of independent variables in a model. In this method, the values of a selected variable are randomly shuffled (permuted), disrupting its structure in the dataset. The model is then retrained, and the change in performance is observed. By repeating this process multiple times (e.g., 30, 50, or 100 iterations), variables that cause the largest drop in performance when permuted are considered more important (Altmann et al., 2010). This approach allows for the assessment of variable importance not only in tree-based models but across all machine learning algorithms.

BORUTA Algoritması

BORUTA is a Random Forest–based but more advanced feature selection method. It generates random noise variables (shadow features) and statistically tests the importance of each original variable against this noise. In doing so, it determines not only the contribution to the model but also the significance

level (Kursa & Rudnicki, 2010). With its ability to detect both linear and nonlinear effects, BORUTA offers a reliable advantage in feature selection, particularly in complex data structures.

Shapley values

Shapley values are a powerful interpretability method based on cooperative game theory, used to explain the decisions of machine learning models. Originally developed by Lloyd Shapley (Shapley, 1953), the method aims to fairly distribute the total payoff among players based on their individual contributions. In the context of machine learning, the "game" refers to the model's prediction process, while the "players" are the model inputs—i.e., the features.

The main advantage of this method is that it makes the model interpretable by quantifying the contribution of each feature to a given prediction. These contributions help in understanding both the general behavior of the model and the rationale behind individual predictions (Lundberg & Lee, 2017).

For a given observation, the Shapley value is calculated as the average difference between the model's prediction using a subset of features and the prediction when an additional feature is added to that subset. More formally:

$$\phi_j(f,x) = \sum_{S \subseteq N\{j\}} \frac{|S|!\,(|N| - |S| - 1)!}{|N|!} \left[f_{S\cup\{j\}}\left(x_{S\cup\{j\}}\right) - f_S(x_S) \right]$$

Here, $\phi_j(f,x)$ is the Shapley value of the j-th variable for observation x, S is a subset of variables that does not include variable j, f_S is the model trained using only the subset S, and $|N|$ is the total number of variables.

Since this calculation requires evaluating all possible subsets, it is theoretically expensive (requiring 2^n model evaluations). Therefore, in practical applications, various approximation methods—such as sampling-based SHAP algorithms—are used (Lundberg et al., 2020).

Shapley values represent the marginal contribution of each feature to the model output for a specific instance. Positive values indicate a contribution to an increase in the prediction, while negative values indicate a contribution to a decrease. In total, the Shapley values of all features sum to the model's output (e.g., predicted probability):

$$f(x) = \phi_0 + \sum_{j=1}^{p} \phi_j(f,x)$$

Here, ϕ_0 is the model's baseline prediction (e.g., the average prediction over all observations). This provides local interpretability for the model.

- Explanation of black-box models (e.g., XGBoost, Neural Networks)
- Instance-level decision analysis
- Assessment of model reliability
- Individual-level interpretations, especially in clinical decision support systems

In this study, with applications of SHAP values, both global and individual-level feature contributions will be interpreted on the mobile addiction dataset. In this way, the model's behavior will be made more transparent, and its inferential power will be enhanced.

Bayesci learning

Bayesian learning is a machine learning approach grounded in probability theory. Its aim is to make inferences about a model's parameters in light of observed data. At the core of Bayesian methods lies Bayes' theorem, which updates the probability of a hypothesis based on prior beliefs and new evidence (Gelman et al., 2013).

Bayes' theorem is expressed as:

$$P(\theta|D) = \frac{P(D|\theta) \cdot P(\theta)}{P(D)}$$

Here, $P(\theta)$ is the prior distribution, reflecting what is known about parameter θ before observing data. $P(D|\theta)$ is the likelihood, the probability of the observed data under parameter θ. $P(D)$ is the marginal likelihood or normalization constant. $P(\theta|D)$ is the posterior distribution, representing updated beliefs about θ after observing the data.

In Bayesian learning, the model does not simply produce a single "best" estimate of the parameters, but instead calculates the full probability distribution over them. This posterior distribution allows for assessing how confident the model is in its predictions (Murphy, 2012). While classical methods often provide a single point estimate, Bayesian models reflect the uncertainty around that estimate.

Uncertainty estimation in machine learning is critical for assessing model reliability. Uncertainty is typically divided into two types. Aleatoric uncertainty arises from inherent randomness in the data (such as measurement error), and cannot be reduced by collecting more data. Epistemic uncertainty results from a lack of knowledge or limited data, and can be reduced with more information (Kendall & Gal, 2017). Classical machine learning methods generally address epistemic uncertainty only indirectly. In contrast, Bayesian methods can explicitly model both types of uncertainty.

Another key advantage of Bayesian models is their capacity to say "I'm not sure" in high-uncertainty cases. This is particularly important in systems designed to operate alongside human oversight. For instance, the model may issue a warning or abstain from making a prediction when confidence is low, deferring the decision to a human. Thus, Bayesian learning provides not only accuracy but also interpretability and reliability, making it a powerful framework.

Selective Learning

Selective learning is an approach that allows a classifier to make predictions only for instances where it has sufficient confidence. While traditional classification models are expected to make predictions for all instances, in selective learning, the model may abstain from predicting certain inputs. This mechanism requires the definition of a decision function along with a selection function (El-Yaniv & Wiener, 2010).

Formally, for a predictor $f: X \to Y$ defined over an input space X and output space Y, a selective system is defined together with a selection function $g: X \to \{0,1\}$. If $g(x) = 1$, the model makes a prediction for input x; if $g(x) = 0$, the input is skipped. In this way, the system operates only on a subset of instances where predictions are considered reliable.

The goal of selective learning is to minimize the error rate while maintaining the coverage rate as high as possible. This trade-off is formally expressed as:

$$Risk(f, g) = \frac{E[\ell(f(x), y) \cdot g(x)]}{E[g(x)]}$$

where ℓ is a loss function (e.g., 0-1 loss), and $E[g(x)]$ represents the proportion of instances for which the model makes a prediction. The selection function can be defined based on uncertainty, for example:

$$g(x) = \begin{cases} 1 & \text{If } Unc(x) \leq \tau \\ 0 & \text{otherwise} \end{cases}$$

Here, $Unc(x)$ is an uncertainty measure for instance x, and τ is the confidence threshold. The uncertainty metric can include both epistemic and aleatoric components. Since Bayesian models output a full posterior distribution, they are well-suited for such selection mechanisms. For instance, the variance or entropy of class probabilities in a classification task can be used as $Unc(x)$:

$$Unc(x) = -\sum_{k=1}^{K} p(y = k|x)\, log\, p(y = k|x)$$

Because the output of a Bayesian model is a posterior distribution, this entropy quantitatively reflects the model's confidence in its prediction. Thus, selective learning makes it possible to establish a controlled balance between prediction confidence and coverage within a Bayesian framework.

APPLICATION

Dataset

The dataset used in this study is the Mobile Addiction Data set, which is publicly available on the Kaggle platform at https://www.kaggle.com/datasets/cloudymts/mobile-addiction-data. The dataset contains a total of 13,589 observations. Of these, 6,846 correspond to individuals classified as mobile-addicted, while 6,743 are non-addicted individuals. Since the class distribution is balanced, there is no class imbalance problem in the analyses.

Table 1. Information about explanatory variables in the mobile addiction dataset.

Variable	Description	$\bar{X}$	*SS*
X_1	Daily hours spent on the smartphone	3.77	1.9
X_2	Number of app sessions per day	30.04	7.41
X_3	Hours spent on social media per day	1.55	1.2
X_4	Hours spent gaming on the smartphone per day	1.03	0.99
X_5	Number of notifications received per day	60.02	12.73
X_6	Hours of smartphone usage at night (e.g., after bedtime)	0.99	0.95
X_7	Age of the individual	33.06	10.12
X_8	Hours spent on work or study per day	5.98	2.07
X_9	Self-reported stress level (on a scale)	4.27	2.29
X_{10}	Number of apps installed on the smartphone	27.53	5.89

Experimental Design

In the first stage of the study, the distribution of variables by the dependent variable was visualized using boxplots, and group differences were assessed using statistical tests. Then, classification models were built using Logistic Regression (LogReg), Linear Discriminant Analysis (LDA), k-Nearest Neighbors (KNN), Random Forest (RF), Neural Network (NNET), and Gaussian Naive Bayes (GNB) algorithms, with hyperparameters optimized. Model performance was evaluated using Accuracy (ACC), F1 score, Matthews Correlation Coefficient (MCC), and Area Under the ROC Curve (AUC) metrics.

After comparing model performances, the statistical significance of the variables was tested using LogReg. Next, variable importance rankings were obtained using Random Forest, permutation methods, BORUTA, and MARS, and the results were compared in a joint table. In the following stage, individual-level feature effects were analyzed using SHAP values. Finally, posterior distributions of predictions were computed using the Bayesian Additive Regression Trees (BART) model, and the applicability of selective learning was demonstrated.

All statistical tests in the study were evaluated at a 0.05 significance level.

Findings

Figure 1 presents the distributions of the explanatory variables according to the addicted and non-addicted groups. Since the groups did not follow a normal distribution, all variables were evaluated using the independent group Wilcoxon test, and a statistically significant difference was found in each variable ($p < .05$). This finding suggests that the variables may be distinctive in explaining addiction.

As part of the machine learning applications, classifiers were trained, and the classification performances of the selected methods are presented in Table 2. All models made predictions with similarly high accuracy and demonstrated strong performance in distinguishing addiction status.

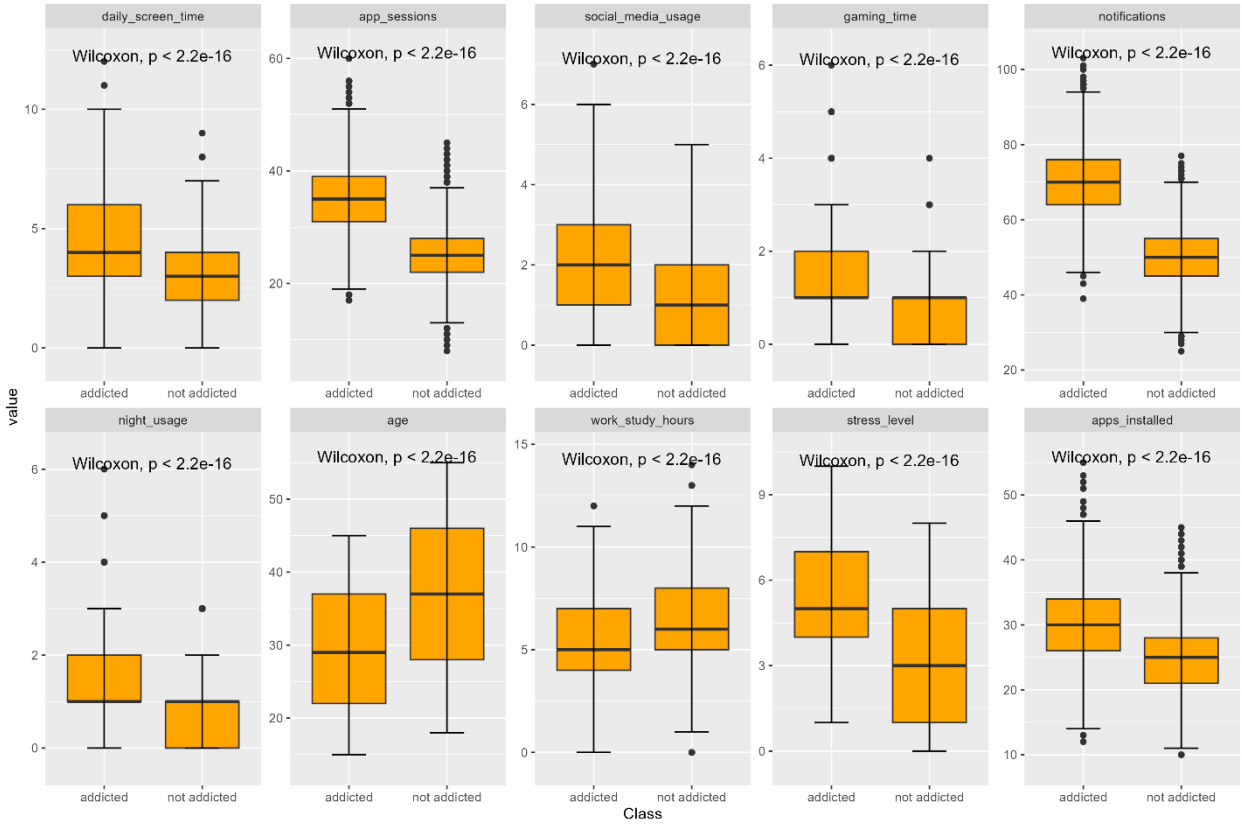

Figure 1. Distributions of Explanatory Variables by Addiction Group and Wilcoxon Test Results

Table 2. Performance of the Best Models Trained with Different Machine Learning Methods

Classifier	ACC	F1	MCC	AUC
LogReg	0.979	0.979	0.957	0.998
LDA	0.979	0.979	0.957	0.998
RF	0.979	0.979	0.958	0.997
NNET	0.977	0.977	0.955	0.998
KNN	0.976	0.976	0.952	0.994
GNB	0.980	0.980	0.960	0.998

High prediction accuracy of machine learning models does not directly reveal the contribution of variables in explaining addiction status. Therefore, as a statistical approach, the logistic regression model was used in the first step to examine the significance of the effects of the variables on the dependent variable. The results are presented in Table 3. The findings are consistent with the distributions shown in Figure 1; all variables were found to significantly affect addiction status ($p < .05$).

Table 3. Logistic Regression Coefficient Results

Variable	Estimate	Standart error	*z*	*p*
Intercept	39.222	1.297	30.241	<.001*
X_1	-0.466	0.041	-11.447	<.001*
X_2	-0.328	0.015	-22.098	<.001*
X_3	-0.813	0.065	-12.425	<.001*
X_4	-0.987	0.080	-12.406	<.001*
X_5	-0.345	0.012	-27.712	<.001*
X_6	-0.979	0.085	-11.512	<.001*
X_7	0.074	0.008	9.790	<.001*
X_8	0.267	0.034	7.890	<.001*
X_9	-0.659	0.040	-16.564	<.001*
X_{10}	-0.195	0.014	-14.071	<.001*

*$p < .05$

The logistic regression results presented in Table 3 demonstrate the linear discriminative power of the variables in relation to addiction status. However, to evaluate variable effects beyond their linear components, nonlinear effects and relative importance magnitudes were examined using various machine learning methods.

Figures 2, 3, 4, and 5 respectively present the variable importance rankings based on Random Forest, the permutation method (applied in conjunction with Random Forest), MARS, and the BORUTA algorithm. These values reflect the contribution magnitude of each variable to the model, but they do not guarantee statistical significance. In this context, Random Forest and MARS naturally produce variable importance measures within their modeling processes, while the permutation method offers flexibility by being model-agnostic.

The BORUTA algorithm, unlike the others, has the ability to statistically test whether a variable's effect is significant. It does so by comparing each variable against randomized shadow variables using a t-test. In this study, BORUTA results indicated that the effects of all variables were statistically significant ($p < .05$).

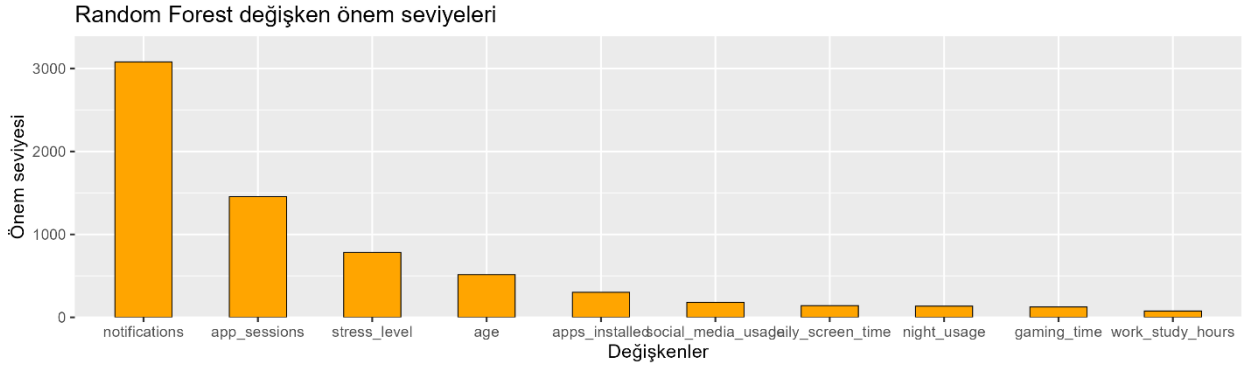

Figure 2. Variable Importance Levels According to Random Forest

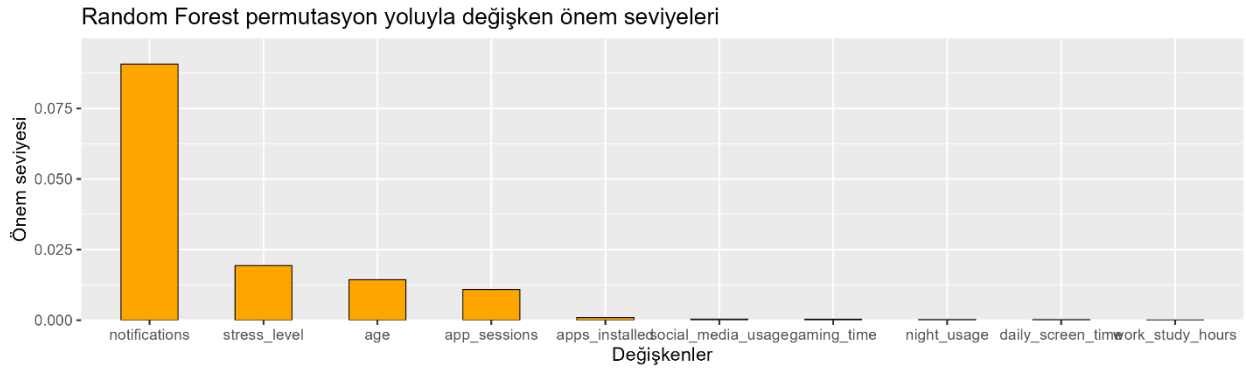

Figure 3. Variable Importance Levels from the Permutation Method Applied with Random Forest

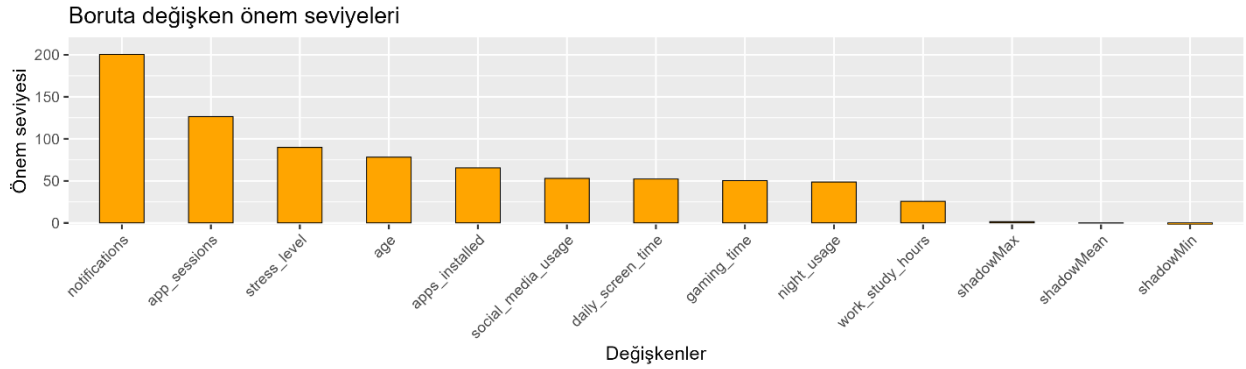

Figure 4. Variable Importance Levels According to the BORUTA Method

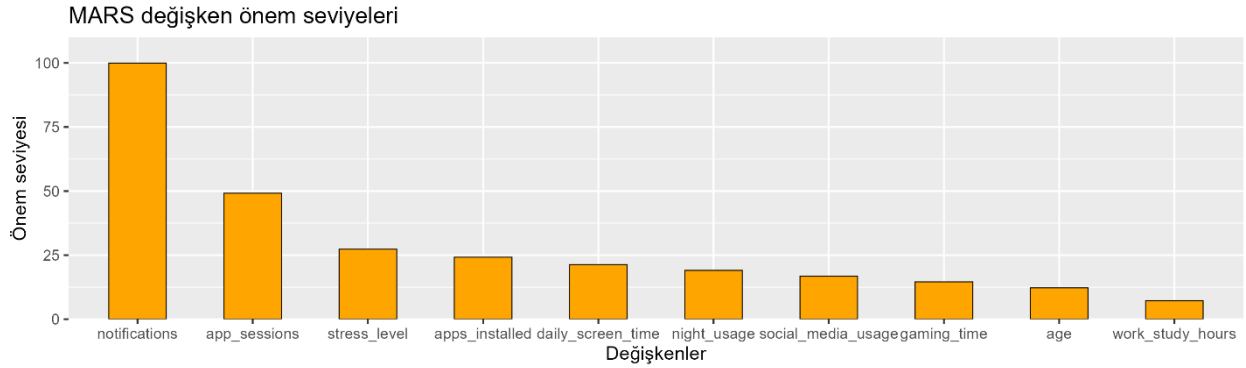

Figure 5. Variable Importance Levels According to the MARS Method

The variable importance rankings presented in Figures 2, 3, 4, and 5 are consolidated in Table 4. All methods consistently indicate that the number of daily phone notifications has the highest impact in explaining addiction, while the amount of time spent daily on work or study has the lowest impact. Rankings of the other variables may vary across methods. In such cases, jointly considering the results from different methods can lead to more reliable conclusions. In Table 4, the average ranks of each variable across methods are calculated and presented for this purpose.

In some applications, using all significant variables may not be practical or economically feasible. Considering data collection costs, it may be necessary to build models using a limited number of variables. The obtained rankings can serve as a guide in determining which variables should be prioritized for inclusion in the model.

Table 4. Variable Importance Rankings Across Different Methods

Description	RF	Perm. (RF-based)	BORUTA	MARS	Mean Rank
Daily hours spent on the smartphone	7	9	7	5	7
Number of app sessions per day	2	4	2	2	2.5
Hours spent on social media per day	6	6	6	7	6.25
Hours spent gaming on the smartphone per day	9	7	8	8	8
Number of notifications received per day	1	1	1	1	1
Hours of smartphone usage at night (e.g., after bedtime)	8	8	9	6	7.75
Age of the individual	4	3	4	9	5
Hours spent on work or study per day	10	10	10	10	10
Self-reported stress level (on a scale)	3	2	3	3	2.75
Number of apps installed on the smartphone	5	5	5	4	4.75

In addition to overall model classification performance, individual-level feature effects were assessed using SHAP analysis. Figure 6 presents instance-based feature contributions for three individuals with mobile addiction, while Figure 7 shows the same for three non-addicted individuals. In real-world applications, where the primary goal is often to reduce addiction, the findings in Figure 6 are particularly valuable for clinical decision-making processes.

For the first individual in Figure 6, the most influential variable contributing to addiction is the reported stress level. In the second individual, the number of notifications, nighttime smartphone usage, screen time, and the number of installed apps stand out. For the third individual, while the number of notifications remains a key factor, the number of app sessions and age appear to be more dominant. These individual effects highlight that the key drivers of addiction differ across users, offering important insights for developing personalized intervention strategies.

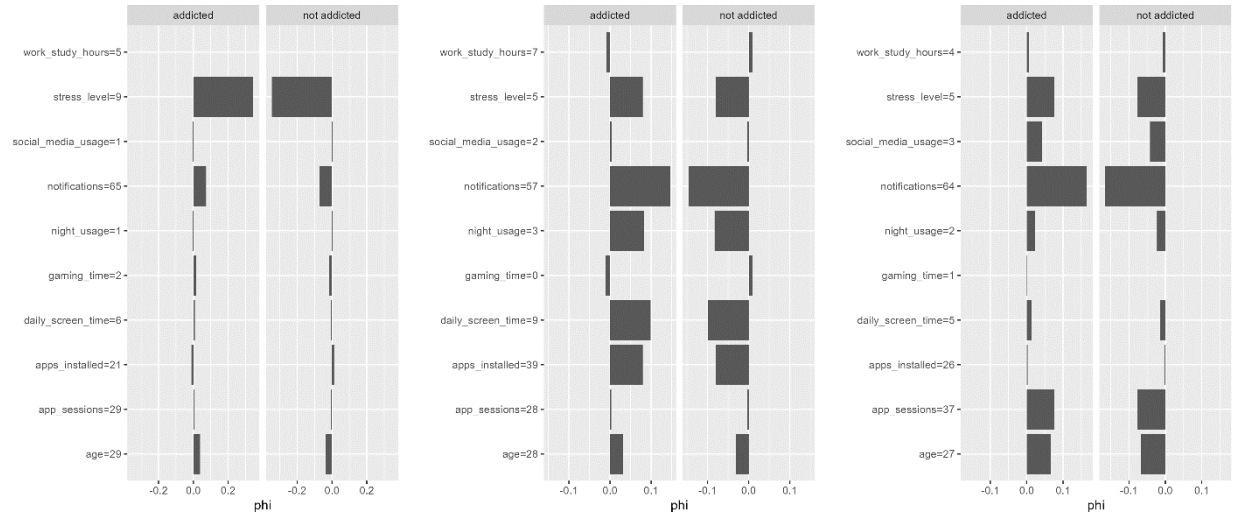

Figure 6. SHAP-Based Feature Importance Levels for Three Individuals with Mobile Addiction

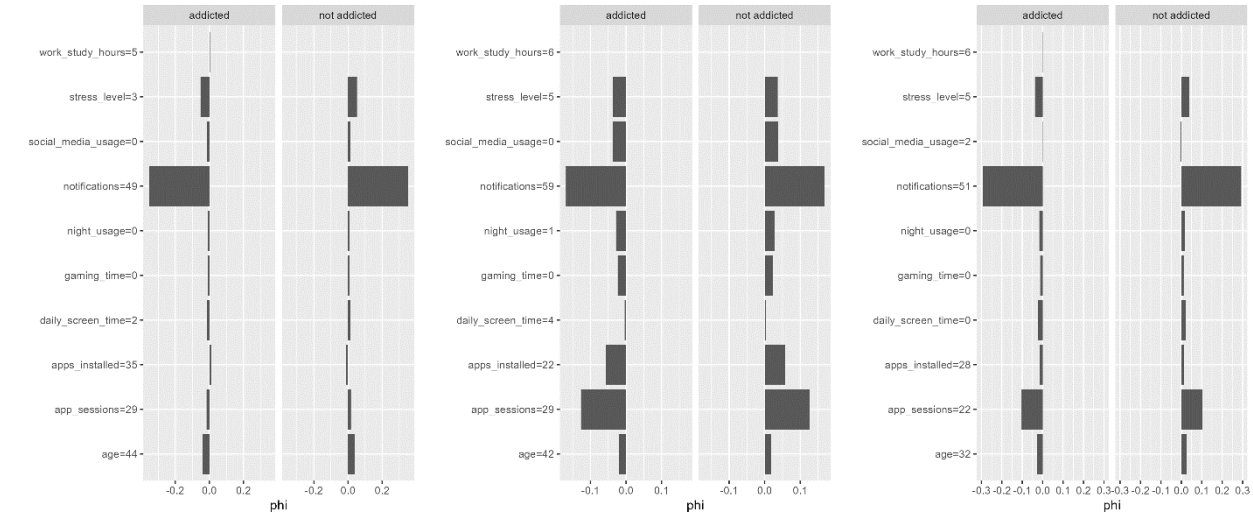

Figure 7. SHAP-Based Feature Importance Levels for Three Non-Addicted Individuals

Individual-level inferences are not limited to SHAP; Bayesian approaches also offer powerful tools for this purpose. In this study, instance-based classifications were performed using the Bayesian Additive Regression Trees (BART) model, and the outputs for three different individuals are presented in Figure 8. In the figure, classical classification probabilities are shown as bar plots, while the posterior distributions of the predictions are visualized using density plots. For example, the first individual is classified as addicted with high probability, the second as non-addicted with high probability, and the third is classified as addicted with a probability close to the decision threshold. The uncertainty in the third example is evident not only from the proximity of the probability to 0.5 but also from the shape of the prediction distribution.

Posterior distributions provided by Bayesian models offer the opportunity to analyze the level of uncertainty through the distributional structure, unlike point probability estimates. In uncertainty assessment, classical metrics such as entropy can be used, or the behavior of the distribution around the 0.5 decision boundary can be examined. For instance, it can be tested whether the mean probability is significantly greater than 0.5, or the area under the density function above 0.5 can be evaluated. Alternatively, the prediction interval (e.g., the 95% quantile interval) can be calculated, and if this interval contains the value 0.5, the prediction can be considered “uncertain.”

In such uncertain cases, the model may abstain from prediction and refer the individual to expert evaluation. This enables selective classification, allowing machine learning systems to make decisions more safely and in a more controlled manner.

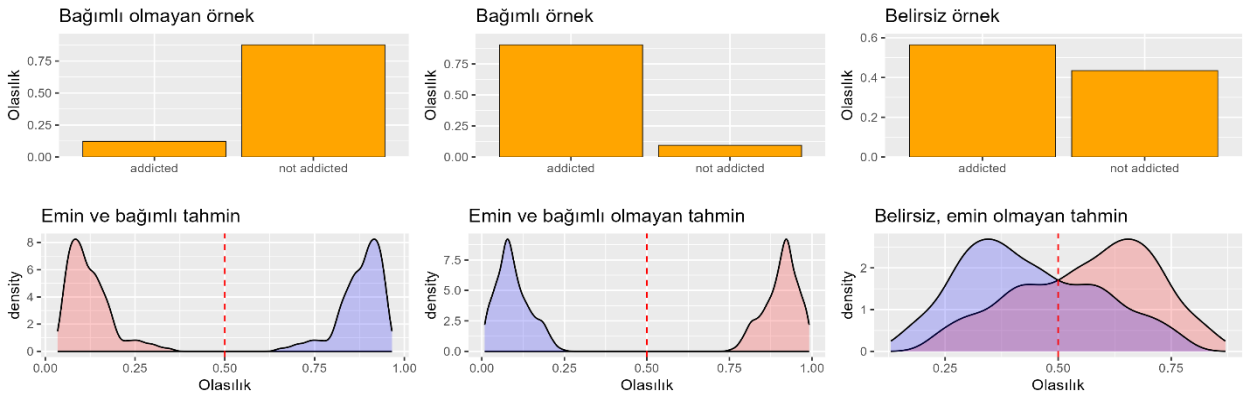

Figure 8. Probability Predictions and Posterior Distributions for Three Examples Using the BART Method

CONCLUSİONS AND DİSCUSSİON

This study aimed to evaluate variable effects, explain individual-level predictions, and model uncertainty-based decision processes by integrating statistical analysis and machine learning methods on mobile addiction data. The findings provide valuable insights at both the group and individual levels.

First, the distributions of variables between addicted and non-addicted groups were examined, and all variables were found to differ significantly between groups. This indicates that the variables serve as distinguishing factors in explaining addiction status. Subsequently, the classification performances of various machine learning models were tested, and all classifiers achieved high accuracy. However, high predictive performance does not necessarily imply interpretability or direct understanding of variable effects.

Therefore, both classical and modern methods were employed to more clearly evaluate the effects of explanatory variables. Linear effects were tested using logistic regression, while relative importance levels of variables were assessed using approaches such as Random Forest, the permutation method, MARS, and BORUTA. The rankings obtained from different methods were largely consistent, with certain variables—such as the number of notifications—emerging as the most distinguishing factors for addiction. Furthermore, the use of average ranks demonstrated that combining multiple methods yields a more balanced and reliable variable selection process.

In the individual-level analyses, SHAP was used to identify prominent features for each person, highlighting the potential of these insights for developing personalized intervention strategies. In addition, Bayesian approaches were used not only for prediction but also to assess the reliability of these predictions. Posterior distributions generated by the BART model enabled uncertainty analysis, and the implementation of selective classification systems was demonstrated. In this context, various decision criteria such as entropy, posterior mean deviation, area under the curve, and quantile intervals were proposed.

In conclusion, this study presents a robust and flexible analytical framework that combines classical statistical inference with modern machine learning techniques to improve understanding of mobile addiction at both group and individual levels. Making high-performing models more transparent and trustworthy contributes significantly to the development of personalized interventions and expert-assisted decision systems. Future work may extend this framework through interactive learning structures such as time-series monitoring, post-intervention change analysis, and active learning.

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SoftReMish: A Novel Activation Function for Enhanced Convolutional Neural Networks for Visual Recognition Performance (1219)

Mustafa Bayram Gücen[1*]

[1]Yildiz Technical University, Faculty of Arts and Sciences, Mathematics, Turkey

*Corresponding author e-mail: mgucen@yildiz.edu.tr

Abstract

In this study, we propose SoftReMish, a new activation function designed to improve the performance of convolutional neural networks (CNNs) in image classification tasks. Using the MNIST dataset, we implemented a standard CNN architecture consisting of two convolutional layers, max pooling, and fully connected layers. We evaluated SoftReMish against popular activation functions including ReLU, Tanh, and Mish by replacing the activation function in all trainable layers. The model performance was assessed in terms of minimum training loss and maximum validation accuracy. Results showed that SoftReMish achieved a minimum loss(3.14×10^{-8}) and a validation accuracy(%99.41) outperforming all other functions tested. These findings demonstrate that SoftReMish offers better convergence behavior and generalization capability, making it a promising candidate for visual recognition tasks.

Keywords: *Convolutional Neural Networks, Visual Recognition, Activation Functions*

INTRODUCTION

Activation functions play a vital role in the performance of deep neural networks, especially by introducing nonlinearity and enabling complex feature learning(Dubey et al., 2022, Wang et al., 2020). Traditional activation functions such as ReLU and Tanh have gained popularity due to their simplicity and computational efficiency. Despite their widespread use, these functions exhibit certain limitations ReLU is prone to the "dying neuron" problem, while Tanh can suffer from saturation, leading to vanishing gradients and slower convergence. Therefore, researchers have proposed alternative activation functions to mitigate these issues and enhance model performance (Zhou et al., 2020). While traditional functions such as ReLU, tanh and more recently Mish activation function have shown remarkable success, each comes with its own drawbacks in terms of smoothness, saturation and gradient behavior(Misra, 2019).

In this context, the present study introduces SoftReMish, a novel activation function that integrates the advantages of both smooth and non-linear transformations. The experimental results demonstrate that SoftReMish can effectively address the shortcomings of traditional functions and offers a more robust solution for complex learning tasks. Inspired by these observations, we present a new activation function called SoftReMish. This function combines the smoothness of Mish activation function with advanced nonlinear scaling that allows for better gradient propagation in deep networks (Misra, 2019).

We conducted extensive experiments on the MNIST dataset to evaluate the effectiveness of SoftReMish (LeCun et al., 1998). Our results show that networks using SoftRemish outperform those using ReLU and even Mish in terms of classification accuracy and training stability. The ability of the proposed function to maintain the gradient flow while preserving nonlinearity makes it a promising candidate for further investigation in more complex vision and language tasks.

The initial findings presented in this study suggest that a new perspective in activation function design is feasible. With further evaluation on different datasets and network architectures, the potential impact of SoftReMish in deep learning can be more thoroughly assessed. In this context, the proposed function is expected to be beneficial across a wide range of applications.

MATERIAL AND METHODS

Material

The material used in this study is the MNIST dataset, which consists of 60,000 training and 10,000 test images of handwritten digits ranging from 0 to 9. Each image is 28×28 pixels in size and in grayscale format. The dataset was selected due to its standardized structure, wide usage in benchmarking neural network performance, and suitability for evaluating the effectiveness of activation functions in classification tasks (LeCun et al., 1998).

Methods

Activation Functions and SoftReMish

Activation functions are essential components of deep neural networks, as they introduce non-linear transformations that allow the model to capture and represent complex patterns within the data. Among these, the Rectified Linear Unit (ReLU) is widely adopted in deep learning architectures because of its computational efficiency and straightforward implementation, making it effective for training deep models (Nair and Hinton, 2010). It is mathematically defined as:

$$f(x) = max(0, x)$$

This function returns the input value directly if it is positive; otherwise, it returns zero. ReLU helps mitigate the vanishing gradient problem encountered in deep networks and accelerates convergence during training(Krizhevsky et al., 2012). However, one of its known drawbacks is the "dying ReLU" problem, where neurons can become inactive if the input remains negative over time.

The Tanh activation function is a conventional, smooth and differentiable function that maps input values to the range (-1, 1). It is defined as:

$$f(x) = tanh(x) = (e^{x} - e^{-x}) / (e^{x} + e^{-x})$$

Compared to the sigmoid function, Tanh is zero-centered, which often leads to faster convergence. It is particularly useful in problems requiring normalized outputs. However, like the sigmoid, Tanh suffers from the vanishing gradient problem for large input values, as the gradients approach zero at both ends of the function.

The Mish activation function is a non-monotonic, smooth, and self-regularizing function that has demonstrated improved performance in various deep learning tasks. It is defined as:

$$f(x) = x \cdot tanh(ln(1 + e^{x})) = x \cdot tanh(softplus(x))$$

where the function $tanh(x) = (e^{x} - e^{-x}) / (e^{x} + e^{-x})$ (Misra, 2019). The function softplus is convex defined $softplus(x) = ln(1 + e^{x})$ (Dugas et al., 2009). Mish retains small negative values and avoids hard zero cut offs, unlike ReLU. Its smooth nature allows better gradient flow during backpropagation, potentially leading to improved model generalization and training dynamics.

Common activation functions such as Tanh, ReLU, and more recently Mish have demonstrated different strengths and limitations depending on the task and architecture. ReLU is computationally efficient and effective in many cases but suffers from the “dying ReLU” problem, where neurons can become inactive. Tanh offers smoother gradients but can lead to vanishing gradient issues in deep networks.

Mish, a self-regularized non-monotonic activation function, has shown promising results in preserving gradient flow and improving generalization.

SoftReMish is a novel activation function proposed as a smooth and bounded alternative that combines the benefits of Mish and exponential-based transformations. Mathematically defined as

$$f(x) = x.tanh(ln(1 + exp(\alpha x)))$$

SoftReMish introduces a steeper activation response in high-value domains while maintaining stability around the origin. In this study, alpha is set to 2 in order to perform the numerical calculations. This property helps in enhancing the learning capacity of the network, especially in early training phases. In our experiments, we compare the performance of SoftReMish against traditional and modern activation functions using the MNIST dataset, evaluating its effect on training convergence and accuracy.

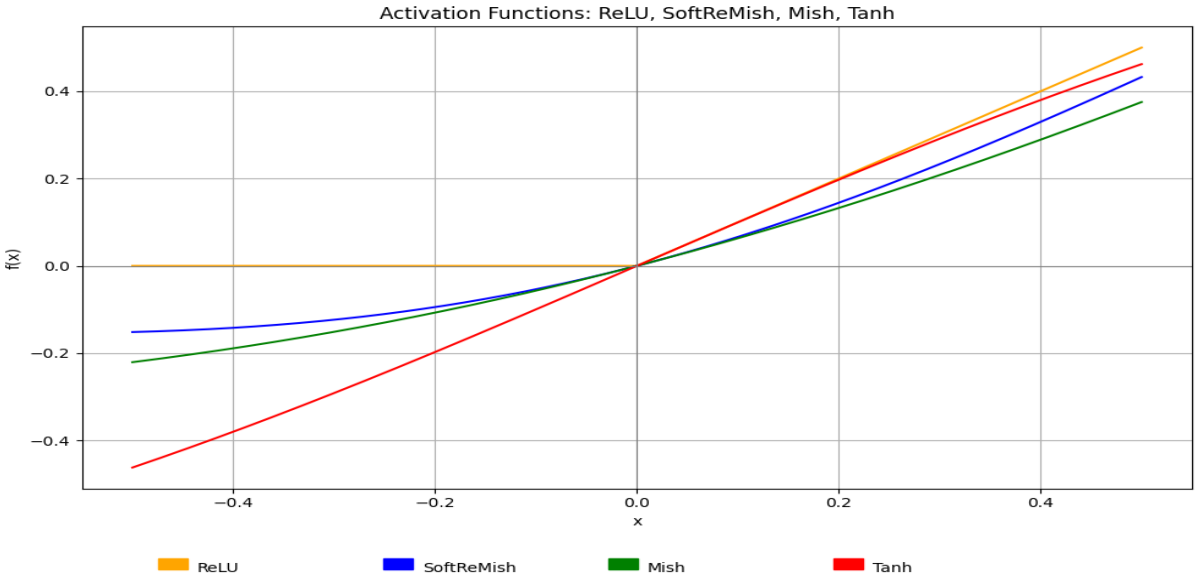

Figure 1. Graphs of Activation Functions ReLU, Tanh, Mish and SoftReMish

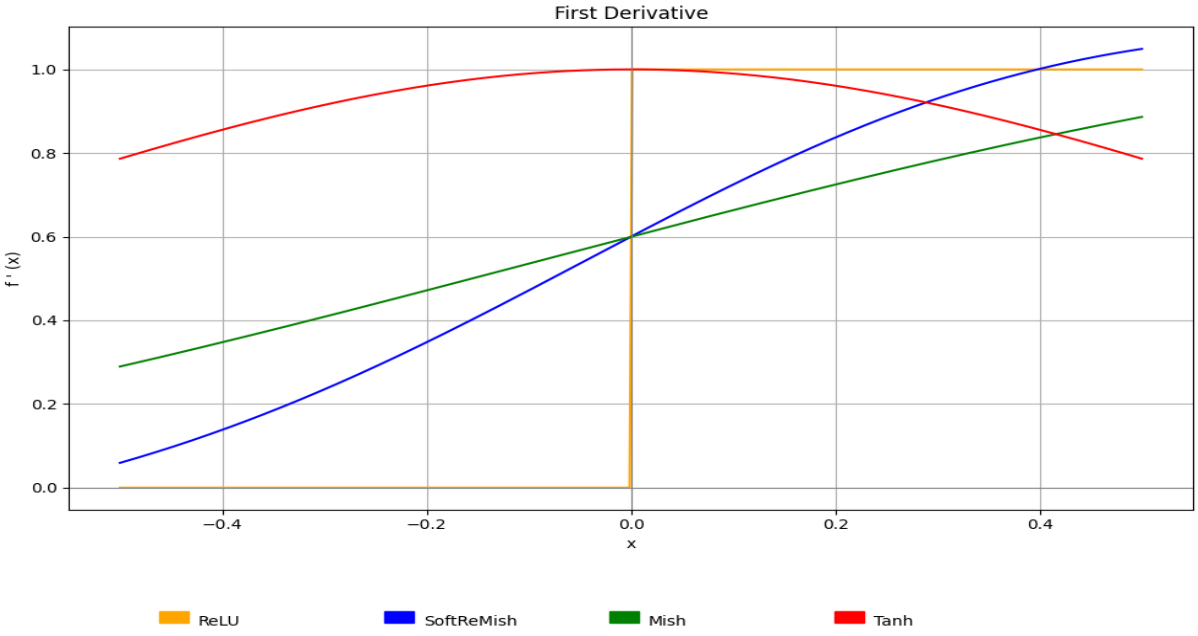

Figure 2. First Derivatives of Activation Functions ReLU, Tanh, Mish and SoftReMish

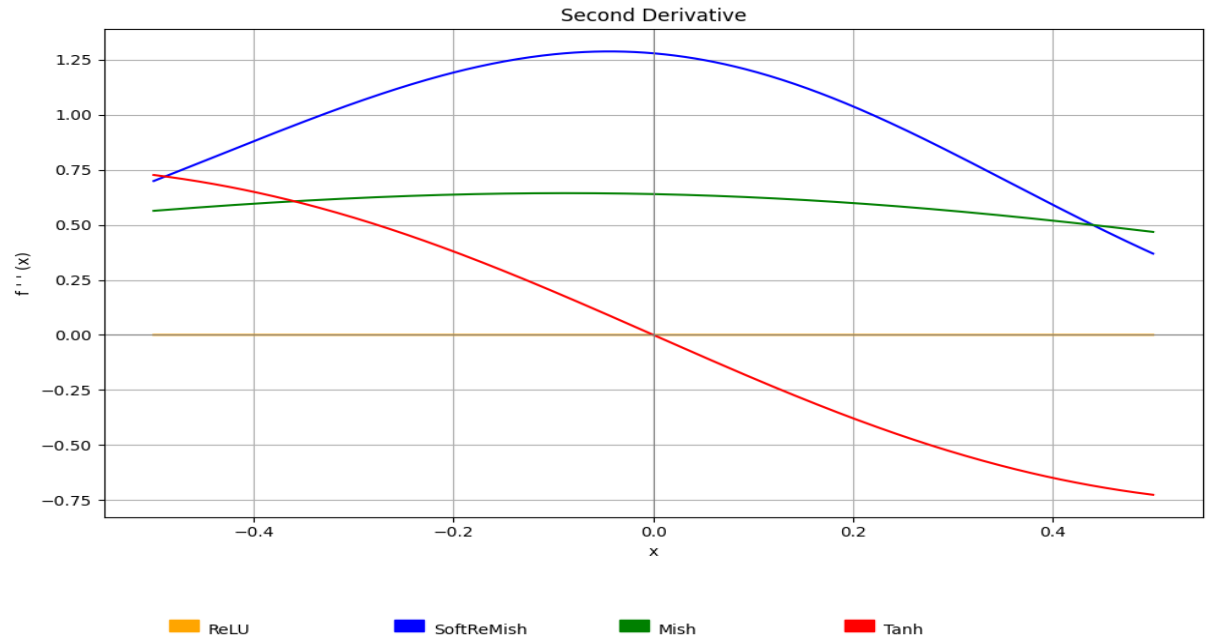

Figure 3. Second Derivatives of Activation Functions ReLU, Tanh, Mish and SoftReMish

Convolutional Neural Networks

Convolutional Neural Networks are the class of deep neural network models especially well-adapted for image classification tasks(Wang et al., 2020). They leverage convolutional layers to automatically learn spatial hierarchies of features, reducing the need for manual feature engineering. A typical CNN architecture consists of convolutional layers followed by pooling layers which downsample the spatial dimensions and dense layers for classification. By sharing weights across space and using local receptive fields, CNNs achieve parameter efficiency and translation invariance. Model explanation and layers defined detailed.

Conv2D (Convolutional Layer)

Convolutional layers are fundamental components of CNNs, especially effective in image processing tasks. This layer extracts local features from the input image. In the first convolutional layer, 32 filters of size 3×3 are applied to perform the convolution operation. The filters slide across the image in small regions, learning features like edges, textures, or patterns. This process extracts low-level features from the image.

The activation function is used to introduce non-linearity, which allows the model to learn complex relationships. Specifically, it outputs zero for any negative input, enabling the network to learn non-linear features.

This layer takes an input image of size 28×28×1 and begins learning higher-level features by performing convolutions with each filter.

MaxPooling2D (Max Pooling Layer)

The MaxPooling2D layer is commonly utilized following convolutional layers to decrease computational load while maintaining critical features of the input. By applying a 2×2 pooling window, this layer extracts the highest value from each subregion, effectively downsampling the spatial dimensions of the feature maps. This dimensionality reduction not only lessens the number of

parameters in the network thereby improving computational efficiency but also enhances the model's ability to handle minor spatial shifts in the input data, contributing to greater robustness.

Conv2D (Second Convolutional Layer)

The second convolutional layer processes the more abstract features extracted by the first convolutional layer. In this layer, 64 filters of size 3×3 are used. The goal of this layer is to learn more complex and higher-level features from the output of the previous layer.

This layer enables the neural network to learn deeper, more abstract representations of the inputs with the increased number of filters allowing for more nuanced feature extraction.

MaxPooling2D (Second Max Pooling Layer)

The second max pooling layer performs a similar operation to the first, reducing the spatial dimensions of the output further and helping to minimize overfitting. This additional pooling step ensures that the learned features become increasingly abstract, with less emphasis on fine-grained spatial details and more focus on the structure of the input data.

Flatten Layer

The flatten layer is used to convert the 2D feature maps obtained from the convolutional and pooling layers into a 1D vector. After the convolutional and pooling layers, the output is still in a multi-dimensional form. The flatten layer takes this output and transforms it into a 1D vector so that it can be fed into fully connected layers for further processing.

This step is crucial because fully connected layers require a 1D vector as input, and this transformation allows the network to transition from learning spatial features to making classification decisions.

Dense (Fully Connected Layer)

The fully connected layer (dense layer) connects each neuron to every neuron in the previous layer. This layer allows the network to learn more abstract relationships from the features learned in earlier layers. In this model, the first dense layer contains 128 neurons and uses the ReLU activation function.

The dense layer enables the network to combine and manipulate learned features to make higher-level decisions. It plays an essential role in abstracting the learned information into more complex decision-making patterns.

Dense (Output Layer)

The final dense layer represents the output layer of the network. It contains as many neurons as the number of classes in the classification problem, which is 10 in this case (for a 10-class classification task, such as digit recognition in the MNIST dataset). The softmax activation function have been used in this layer.

The Softmax activation function transforms the model's raw output scores into a normalized probability distribution, where each output neuron corresponds to the likelihood of the input belonging to a specific class. This function guarantees that all output values fall within the range [0, 1], and that their sum equals 1, enabling probabilistic interpretation for classification tasks.

The described model follows a conventional Convolutional Neural Network (CNN) framework, which is widely employed in image classification applications. Convolutional layers serve to extract fundamental features from the input data, while pooling layers reduce spatial resolution and emphasize the most salient patterns. Subsequently, fully connected layers integrate these extracted features to

perform the final classification. This architectural design has demonstrated strong performance in tasks such as handwritten digit recognition, facial recognition, and other visual classification challenges.

In this study, we implemented a CNN architecture with two convolutional and max pooling layers, followed by a fully connected layer and a final softmax output layer. We kept the overall architecture constant while varying the activation functions in each experiment to isolate their impact on performance. The models were trained and evaluated on the MNIST dataset, a standard benchmark for handwritten digit recognition. The results demonstrate how the choice of activation function can significantly affect validation loss, validation accuracy.

RESULTS

The comparative analysis of various activation functions demonstrated that the SoftReMish function outperformed the others in terms of both validation accuracy and validation loss. As shown in Table 1, the highest validation accuracy was achieved using the SoftReMish activation function with a value of 0.9941, followed by ReLU (0.9923), Tanh (0.9918), and Mish (0.9907). These results indicate that SoftReMish provides a slight yet significant improvement in classification performance compared to more conventional activation functions.

Table 1. Validation Accuracy Values Table

Activation Function	**Validation Accuracy**
SoftReMish	0.9941
ReLU	0.9923
Tanh	0.9918
Mish	0.9907

In addition to accuracy, validation loss values presented in Table 2 further support the superiority of the SoftReMish function. It yielded the lowest validation loss of 3.137582×10^{-8}, which is substantially lower than those observed for ReLU (3.215818×10^{-4}), Tanh (1.566297×10^{-4}), and Mish (7.632310×10^{-4}). This remarkable reduction in validation loss signifies better generalization and model stability during training.

Table 2. Validation Loss Values Table

Activation Function	**Validation Loss**
SoftReMish	3.137582×10^{-8}
ReLU	3.215818×10^{-4}
Tanh	1.566297×10^{-4}
Mish	7.632310×10^{-4}

Overall, the findings of this study suggest that the proposed SoftReMish activation function can be a promising alternative to traditional functions such as ReLU, Tanh, and Mish, particularly in applications where high accuracy and low loss are critical.

Figure 4. illustrates the validation loss values corresponding to each activation function evaluated in the study. The SoftReMish activation function achieved the lowest validation loss with a value of 3.137582×10^{-8}, indicating a highly stable and well-generalized model. In comparison, traditional activation functions such as ReLU, Tanh, and Mish resulted in significantly higher loss values, with Mish yielding

the highest at 7.632310×10^{-4}. The clear difference in loss values emphasizes the effectiveness of the SoftReMish function in reducing overfitting and enhancing model performance.

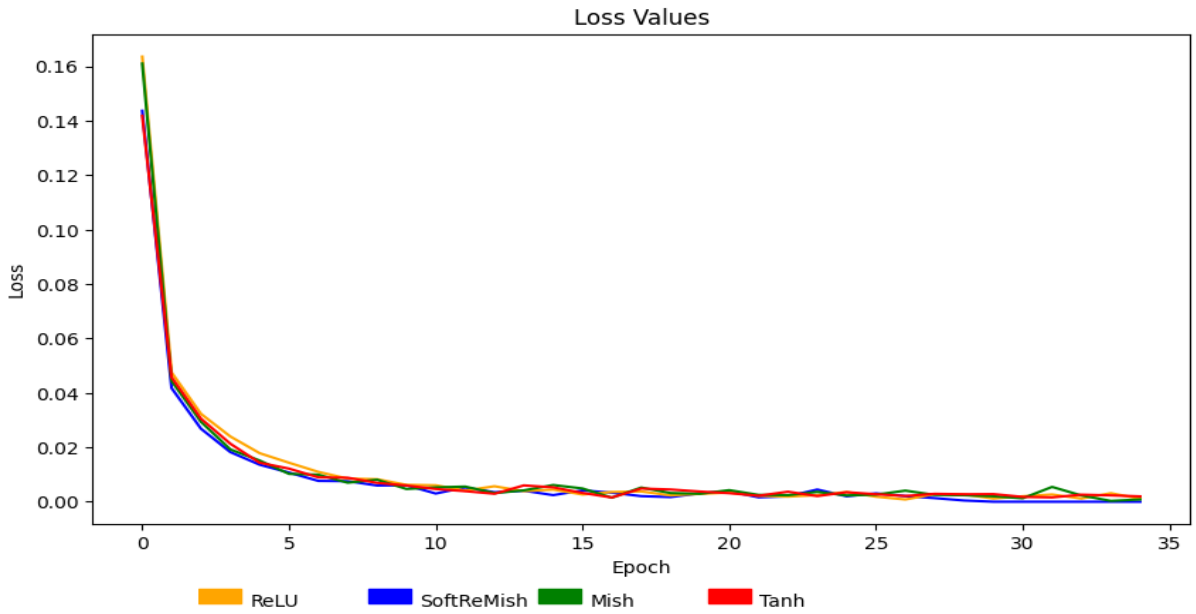

Figure 4. Validation loss values

Figure 5. presents the validation accuracy results for the tested activation functions. Among them, the SoftReMish activation function achieved the highest accuracy of 0.9941, surpassing ReLU (0.9923), Tanh (0.9918), and Mish (0.9907). These results confirm that SoftReMish not only minimizes loss but also contributes to a more accurate model. The improvements, although numerically small, are statistically and practically meaningful in high-precision tasks, reinforcing the advantage of using SoftReMish in deep learning models.

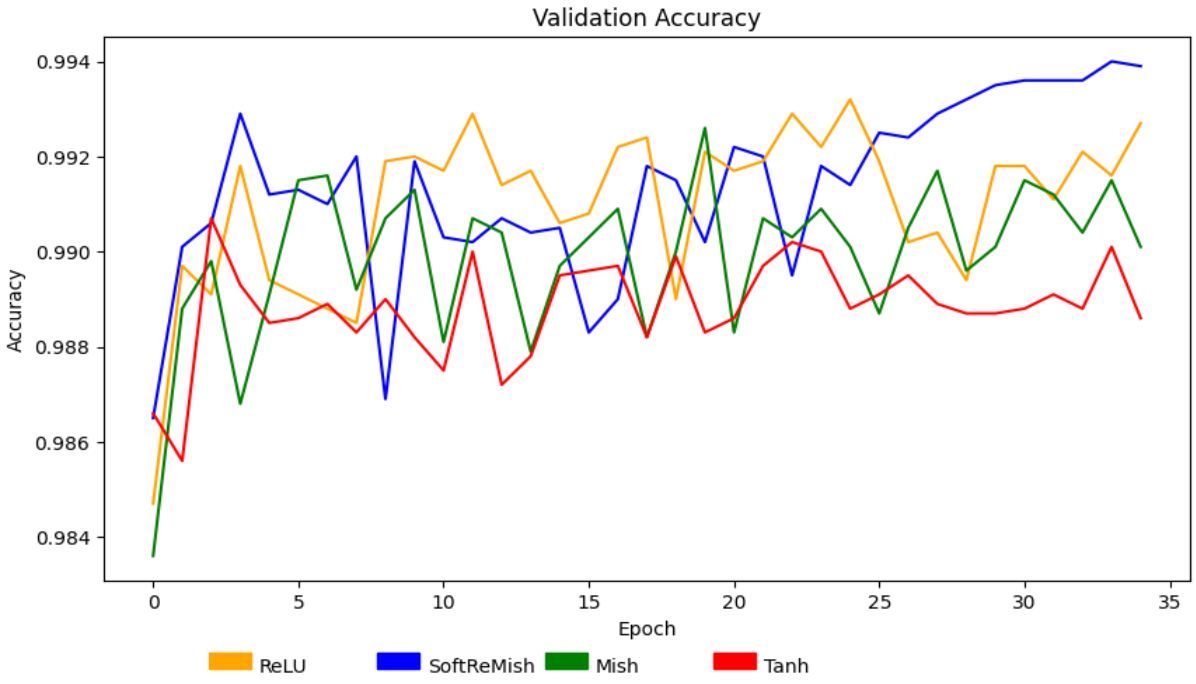

Figure 5. Validation accuracy values

All experiments were conducted using Python 3.11.12 on a Linux (kernel 6.1.123+) operating system. Model development and training were performed using an NVIDIA A100-SXM4-40GB GPU with CUDA 12.4 and NVIDIA driver version 550.54.15. The system was equipped with 83.48 GB of RAM and an x86_64 architecture CPU. GPU acceleration was utilized throughout training to optimize computation time. The libraries such as TensorFlow or PyTorch (depending on implementation) were used to construct and evaluate the CNN architecture for efficient experimentation. These results demonstrate that the SoftReMish activation function outperforms traditional activation functions in terms of both accuracy and loss. Therefore, SoftReMish can be considered a strong alternative for achieving higher accuracy and different generalization in deep neural networks.

CONCLUSION

In this study, the performance of four different activation functions—SoftReMish, ReLU, Tanh, and Mish—was evaluated based on their validation accuracy and validation loss in a deep learning model. The experimental results clearly indicate that the proposed SoftReMish activation function achieves superior results compared to the other functions tested. It provided the highest validation accuracy (0.9941) and the lowest validation loss (3.137582×10^{-8}) suggesting that it leads to better model generalization and training stability.

Traditional activation functions like ReLU and Tanh have been widely used due to their simplicity and computational efficiency. However, their limitations, such as dying neurons in ReLU or saturation issues in Tanh, can hinder model performance. Therefore, alternative activation functions have been proposed to address these shortcomings (Zhou et al., 2020). The results of this study show that SoftReMish, by combining the strengths of both smooth and non-linear transformations, can overcome some of these limitations and offer a more robust alternative for complex learning tasks.

Moreover, the remarkably low validation loss achieved by SoftReMish indicates that it can reduce overfitting, an important concern in deep neural networks. This makes it particularly suitable for applications where both accuracy and generalization are critical, such as medical diagnostics, autonomous systems, and financial forecasting.

In conclusion, the SoftReMish activation function holds significant promise for improving deep learning model performance. Future studies could explore its applicability across different architectures and datasets to further validate its effectiveness and to optimize its design for various real-world tasks.

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Simulink-Based Study of Fan-Driven Cooling: Understanding Airflow for Thermal Applications (1233)

Rahim Mammadzada[1*]

[1]Azerbaijan State Oil & Industry University, Faculty of Information Technologies and Control, Department of Instrumentation Engineering. Azerbaijan

*Correponding author e-mail: rahim.mammadzada02@gmail.com

Abstract

Air cooling with fans is widely used in electronics, vehicles, and industrial systems, but the dynamic response of fans is often simplified in simulation models. Many studies assume steady airflow and do not show how control inputs affect airflow and pressure over time. This limits the accuracy of thermal predictions and makes control system design more challenging. While tools like Simscape can model fan behavior in detail, they often rely on complete technical data that is not always available or practical. In this paper, a theoretical fan model was made using Simulink to show input and output behavior based on basic airflow principles. The model explains how changes in input signals like voltage or PWM control produce dynamic changes in airflow and pressure. By focusing only on the fan's dynamic response, this study offers a useful insight rather than a new tool, providing a practical base for improving fan control methods and making thermal models more accurate. The work highlights the advantage of dynamic modeling for systems that need to respond to changing thermal conditions. It also provides a starting point for future research aimed at developing better fan control strategies, enhancing cooling performance, and supporting more adaptive thermal management designs.

Keywords: *Fan Dynamics, Air Cooling, Simulink Modeling, Input-Output Behavior, Thermal Management*

INTRODUCTION

Fans play a key role in increasing convective heat transfer in systems ranging from electronic cooling to HVAC. In simulation environments like Simulink and Simscape (MathWorks, 2025. Simulink Documentation; MathWorks, 2025. Simscape Documentation), accurate airflow modeling is essential for predicting thermal behavior. However, commercial fan datasheets typically offer only basic values such as maximum airflow, static pressure, and rotational speed, while detailed aerodynamic parameters are often missing (Noctua, 2025).

In this paper, a practical approach is proposed to address the gap between limited datasheet information and the detailed inputs required by simulation tools. Using simplified equations based on fan affinity laws, the method captures how airflow and pressure change with speed. This allows for realistic simulation of fan behavior using minimal input, supporting effective design and analysis of thermal systems.

MATERIAL AND METHODS

Material

In MATLAB, fans are typically modeled using blocks from the Simscape Fluids library, such as the "Ideal Fan" or "Axial Fan" block (MathWorks, 2025. Simscape Documentation). These blocks require

parameters like the fan pressure rise curve, flow resistance coefficients, and efficiency values to define the fan's performance over various operating conditions. While these detailed inputs allow for precise airflow and pressure simulations, they assume that such data is available to the user. In practice, especially when using commercial fans, obtaining all the necessary parameters for these blocks is challenging, as most are not directly listed in standard datasheets (Holzinger and Kommenda, 2021; Turkeri and Kiselychnyk, 2023).

A typical commercial axial fan, commonly used in PC cooling systems is shown in Figure 1. Models like the Noctua NF-A12x25 illustrate the type of product that engineers and hobbyists often rely on in thermal systems (Noctua, 2025). These fans are widely available and come with datasheets that list basic performance metrics, but rarely provide detailed aerodynamic characteristics or full pressure–flow curves. This makes direct use in simulation environments difficult without additional assumptions or simplifications.

Figure 1. Fan Model Noctua NF-A12x25 (Noctua, 2025)

Methods

The Collection of the Data

To build the simplified fan model, key parameters such as minimum and maximum speed, airflow capacity, and static pressure are required. These values are available from the product datasheet and are listed in Table 1 (Noctua, 2025). They form the basis of the model's airflow and pressure equations, allowing the estimation of performance based solely on RPM input.

Table 1. Fan parameters provided by the manufacturer

Noctua NF-A12x25 Parameters	RPM		Air Flow Q_{Max}	Static Pressure P_{Max}	Max Input Power	Size in mm
	Min	N_{Max}				
Values	450 ±20%	2000 ±20%	60.1 CFM	2.34 mmH2O	1.68W	120x120 x25

Statistical Analysis

Fan modeling often starts with general performance estimation by using the basic data provided by the manufacturer. Here, a common assumption is that airflow Q increases proportionally with fan speed N, especially when pressure effects are minimal (Wikipedia, 2025; Intro to Pumps, 2025). As RPM rises, the amount of air moved grows nearly linearly, which matches typical fan behavior in low-resistance conditions. This relation is given by:

$$Q = k \cdot N$$

This basic equation offers a practical starting point for estimating airflow across different speeds when detailed pressure data is missing. Here k is calculated from known maximum values:

$$k = \frac{Q_{max}}{N_{max}}$$

However, when manufacture provide more detailed information like maximum pressure and fan dimensions, it is possible to write more accurate equation accounting for pressure changes. Apart from airflow Q, the pressure P is also affected by fan speed N. To capture this behavior, airflow Q is modeled as a function of both speed and pressure.

$$Q = k \cdot N \cdot \frac{1}{1 + \alpha \cdot P}$$

In this equation, P represents the pressure generated by the fan at a given speed, which increases with the square of RPM. The term α adjusts how much this built-up pressure reduces the airflow, helping the model reflect real fan behavior when system resistance is present. The pressure P is calculated by:

$$P = P_{max} \cdot \left(\frac{N}{N_{max}}\right)^2$$

Here we can simplify further by assigning a constant called k_p.

$$k_p = \frac{P_{max}}{N_{max}^2}$$

$$P = k_p \cdot N^2$$

With this simplification, the airflow equation becomes a simpler formula where airflow depends on N and pressure, dropping off smoothly as pressure rises.

$$Q = \frac{k \cdot N}{1 + \alpha \cdot k_p \cdot N^2}$$

Most manufacturers provide airflow Q in CFM, while the cubic meters per second is preferable in modeling. Thus, conversion between CFM to m³/s is necessary.

$$Q_{\frac{m^3}{s}} = Q_{CFM} \cdot 0.0004719$$

Air velocity v is calculated by dividing airflow Q by the area A through which air exits. This tells us how fast the air is blowing out, which is key for predicting cooling effect.

$$v = \frac{Q}{A}$$

Finally, when calculating the effective area $A_{effective}$, it's important to account for the part of the fan that moves air, usually defined as the area between the frame and hub diameters.

$$A_{effective} = \frac{\pi}{4} \cdot \left(D_{frame}^2 - D_{hub}^2\right)$$

RESULTS

The fan model uses basic specifications such as rated speed, airflow, and pressure to express performance directly as a function of RPM. This simplifies the estimation process and allows quick evaluation of fan suitability for a given system. The model structure supports implementation in

Simulink and direct integration with Simscape components. It provides a clear and efficient method for checking airflow and pressure behavior without needing full manufacturer data. The results confirm that simplified equations based on speed can be a reliable tool for early design and analysis tasks.

DISCUSSION AND CONCLUSION

This study presents a simplified fan modeling approach using affinity laws, where airflow and pressure are expressed as functions of rotational speed. The resulting equations isolate the speed as the key variable, allowing straightforward implementation in simulation environments like Simulink and seamless integration with Simscape components. This makes the method useful for estimating fan behavior under different operating conditions and evaluating system compatibility. Despite relying on minimal input data, the model offers sufficient flexibility and clarity for both early design phases and general-purpose thermal management studies. Future improvements may include parameter refinement based on real-world measurements or variable ambient conditions.

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Acknowledgment

Due to a service disruption on March 18, 2025, which affected multiple MathWorks applications, access to MATLAB and Simulink was temporarily unavailable. Consequently, it was not possible to perform simulations or generate plots for the fan model during the preparation of this paper. However, the proposed model can be implemented and validated in Simulink once the services are restored in the near future.

Conflict of Interest

The authors have declared that there is no conflict of interest.

Analysis of Key Health Indicators in Low-Income Countries Using Multi-Criteria Decision-Making Methods (1113)

Mulla Veli Ablay[1]

[1]Osmaniye Korkut Ata University, Rectorate, UZEM, Türkiye

*Corresponding author e-mail: veliablay@gmail.com

Abstract

Key health indicators are among the most critical metrics that reflect the effectiveness of a country's healthcare system, the quality of life of its population, and the level of access individuals have to healthcare services. In the study, the Entropy-WASPAS method was used to analyze the key health indicators of countries classified as low-income during the period from 2017 to 2021. The criteria for key health indicators include five fundamental measures: the share of current health expenditures in GDP, access to basic drinking water services, the general mortality rate, life expectancy at birth, and the neonatal mortality rate. The Entropy technique was used to determine the importance weights of the criteria, while the combination of the Weighted Sum Method (WSM) and the Weighted Product Method (WPM), known as the WASPAS technique, was employed to select the most suitable alternative. Additionally, in the WASPAS technique, the resistance of the alternatives was measured by using different λ values to rank them. According to the analysis based on 2021 data, the highest criterion was Current Health Expenditure, while the lowest criterion was Life Expectancy at Birth. In the performance ranking, Rwanda ranked first, while South Sudan ranked last. Over the past five years, minor changes in the performance rankings of the countries have been observed. It is believed that the findings will guide policymakers in their decision-making processes.

Keywords: *Health Indicators, Entropy, WASPAS*

INTRODUCTION

Health is not only a fundamental human right but also directly related to a country's level of development. In particular, it plays a significant role both socially and economically among the development indicators of countriesFundamental rights such as access to healthcare services, hygiene conditions, and nutritional opportunities, as well as the development of medical infrastructure—particularly in less developed countries—vary significantly, deepening inequalities in healthcare services (WHO, 2023). Health indicators are also included among the United Nations Sustainable Development Goals and play a significant role in determining countries' capacities to achieve these targets (United Nations, 2022).

Low-income countries face difficulties in accessing clean drinking water and basic healthcare services compared to developed countries. In addition, they face various challenges such as limited financial resources, political instability, and reduced quality of life. This situation negatively affects health indicators due to the high prevalence of infectious diseases, insufficient efforts to combat them, elevated mortality rates, and limited access to basic needs such as clean water and sanitation (UNDP, 2022).

When examining health issues in less developed countries, multiple health indicator criteria should be taken into account, as the impact of each criterion and potential solutions need to be thoroughly investigated. The method to be employed in this research is also of great importance. Multi-Criteria Decision-Making (MCDM) methods are powerful tools that allow for a holistic analysis regardless of the differing characteristics of performance indicators (Zavadskas, Turskis & Kildienė, 2014). In decision-making processes, the integration of MCDM techniques into the analysis ensures that the decisions made are data-driven, rational, and fair. The Entropy technique used in the study enables the objective determination of criterion weights independently of human influence (Çakır and Perçin, 2013). In addition, the WASPAS technique combines the weighted sum and weighted product model approaches to rank countries' performance according to different values of the λ parameter, providing reliable results (Zavadskas, Baušys, Lazauskas, 2015).

In this study, key health indicators in low-income countries were evaluated using the integrated Entropy-WASPAS method. The indicators used in the study include infant mortality rate, life expectancy at birth, maternal mortality rate, access to healthcare services, and access to clean water, with the objective weight of each criterion calculated using the Entropy method. Subsequently, countries were ranked according to their health performances using the WASPAS method.

The primary aim of this research is to analytically reveal the current state of health and to identify priority policy areas for low-income countriesThe findings obtained aim to contribute to the academic literature as well as provide practical, data-driven analyses for policymakersFurthermore, the adaptability of this methodological approach to other socioeconomic fields enhances the practical applicability of the study.

Literature Review

Key health indicators in low-income countries present significant challenges due to lack of access to healthcare services, inadequate infrastructure, and funding issues. The multidimensional nature of these indicators highlights the importance of Multi-Criteria Decision Making (MCDM) methods in the analysis process. In the literature, MCDM is used as an effective tool for the systematic and comparative evaluation of health data. In his study, Girginer (2013) analyzed the comparison of health indicators between EU countries and Turkey using the NHCA and MDSA techniques. The study found that the countries clustered based on health indicators exhibited similar characteristics. In their study, Chakraborty and Zavadskas (2014) examined the decision-making process in eight production problems using the WASPAS method. The study found that the WASPAS technique accurately ranked all alternatives. In their study, Alptekin and Yeşilaydın (2015) used the fuzzy clustering analysis technique to evaluate OECD countries in terms of health indicators. The study determined that the optimal number of clusters was five. The common characteristics of the states within each cluster were identified. In their study, Şener and Yiğit (2017) used the data envelopment analysis technique to investigate the health systems of OECD countries. The study found Austria to have the lowest score. Additionally, Turkey was identified among the countries with efficient scores. In his study, Altın (2020) used the WASPAS and VIKOR methods to investigate urban quality of life. A positive relationship was identified between the two methods. Furthermore, it was concluded that these two methods can be used interchangeably. In their study, Saygın and Kundakçı (2020) used the SWARA, WASPAS, and CODAS techniques to evaluate OECD countries in terms of health indicatorsIn the study, infant mortality rates were found to have the highest criterion weight, while the number of medical graduates had the lowest criterion weight. In the performance ranking of countries, Iceland was found to be in first place, while Luxembourg ranked last.

MATERIAL AND METHODS

Countries with available data were selected to examine key health indicators of low-income countries between 2017 and 2021. The data were compiled from the official website of the World Bank (The World Bank, 2025). This section presents the application steps, mathematical formulations, and explanations of the Entropy and WASPAS methods used in the study.

Entropy Method

The Entropy technique, one of the multi-criteria decision-making (MCDM) approaches, is an analytical method used to objectively determine the weights of each criterion and to rank these weights according to their level of importance. The entropy technique was first introduced by Rudolph Clausius in 1865 and later developed by Claude E. Shannon in 1948 (Zhang, Gu, Gu & Zhang, 2011). The application steps of the entropy method are as follows (Ömürbek, Eren, and Dağ, 2017):

Step 1: Decision Matrix

$$A_{ij} = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \dots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix} \quad (1)$$

Step 2: Normalized Decision Matrix

$$P_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}}} \quad ; A_j \quad (2)$$

Step 3: Entropy Value and E_{ij} Matrix

$$E_j = \left(\frac{-1}{\ln(m)}\right) \sum_{i=1}^{m} [P_{ij} \ln (P_{ij})]; \; A_j \quad (3)$$

Step 4: Determination of the Degree of Diversification, Dij Values

$$D_j = 1 - E_j; \; A_j \quad (4)$$

Step 5: Calculation of Entropy-Based Criterion Weights

$$w_j = \frac{d_j}{\sqrt{\sum_{j=1}^{n} d_j}}; \; A_j \quad (5)$$

WASPAS Method

The WASPAS method, one of the multi-criteria decision-making (MCDM) approaches, is a comprehensive method obtained by combining the Weighted Product Model (WPM) and the Weighted Sum Model (WSM). The application steps of the WASPAS method are as follows (Chakraborty & Zavadskas, 2014):

Step 1: Construction of the Decision Matrix

The first stage of the method involves the decision matrix composed of elements x_{ij}, denoted by X, which is obtained as shown in Equation 6.

$$X = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \dots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix} \quad (6)$$

Step 2: Normalization of the Decision Matrix

The values involved in the decision problem are standardized through the normalization procedures presented in Equations 7 and 8.

$$x_{ij}^{*}=\frac{x_{ij}}{\max\limits_{i}(x_{ij})},\quad i=1,2,\ldots,m\ ;\ j=1,2,\ldots,n \tag{7}$$

$$x_{ij}^{*}=\frac{\min\limits_{i}(x_{ij})}{x_{ij}},\quad i=1,2,\ldots,m\ ;\ j=1,2,\ldots,n \tag{8}$$

Step 3: Calculation of the Total Relative Importance of the iiith Alternative Based on the Weighted Sum Method

The total relative importance of the alternatives is calculated using the weighted sum method with the aid of Equation 9.

$$Q_i^{(1)}=\sum_{j=1}^{n} x_{ij}^{*}\ .\ w_j,\ \ i=1,2,\ldots,m;\ j=1,2,\ldots,n \tag{9}$$

Step 4: Calculation of the Total Relative Importance of the iiith Alternative Based on the Weighted Product Method

The total relative importance of the alternatives is calculated using the weighted product method with the aid of Equation 10.

$$Q_i^{(2)}=\prod_{j=1}^{n}(x_{ij}^{*})^{wj},\ \ i=1,2,\ldots,m;\ j=1,2,\ldots,n \tag{10}$$

Step 5: Calculation of the Weighted Aggregated Overall Criterion Value for the Weighted Sum and Weighted Product Models

The weighted aggregated criterion values are obtained by substituting the values derived from Equations 9 and 10 into Equation 11.

$$Q_i = 0{,}5\ .\ Q_i^{(1)} + 0{,}5.\ Q_i^{(2)} = 0{,}5.\sum_{j=1}^{n} x_{ij}^{*}\ .\ w_j + 0{,}5\ .\prod_{j=1}^{n}(x_{ij}^{*})^{wj} \tag{11}$$

Step 6: Calculation of the Relative Importance Values of the Decision Alternatives

The effective and accurate ranking of decision alternatives is performed with the aid of Equation 12.

$$Q_i = \lambda.\ Q_i^{(1)} + (1-\lambda).\ Q_i^{(2)} = \lambda.\sum_{j=1}^{n} x_{ij}^{*}\ .\ w_j + (1-\lambda)\ .\prod_{j=1}^{n}(x_{ij}^{*})^{wj} \tag{12}$$

The parameter λ\lambdaλ in Equation 12 takes a value between 0 and 1 and is calculated using Equation 13.

$$\lambda = \frac{\sigma^2(Q_i^{(2)})}{\sigma^2\left(Q_i^{(1)}\right)+\sigma^2(Q_i^{(2)})} \tag{13}$$

RESULTS

In this section of the study, the Entropy method was first employed to determine the criterion weights. Secondly, the WASPAS method was used to rank the performance of the alternatives. In the WASPAS method, the rankings of the countries for the year 2021 were determined for different values of λ\lambdaλ. To analyze the changes over the years, the rankings of the countries between 2017 and 2021 were determined for different values of λ\lambdaλ.

Calculation of Criterion Weights Using the Entropy Technique

The codes and optimization directions of the criteria used in the study are presented in Table 1.

Table 1: Criteria

Indicator	Code	Optimization
Current health expenditure	K1	Max

Death rate, crude	K2	Min
Life expectancy at birth	K3	Max
Mortality rate, neonatal	K4	Min
People using at least basic drinking water services	K5	Max

The steps of the Entropy technique to obtain the importance weights of the components used in examining the basic health indicators of low-income countries are as follows. The decision matrix is presented in Table 2.

Table 2: Decision Matrix

Country Names	K1	K2	K3	K4	K5
Afghanistan	12,6208	7,0270	63,0160	40,3000	68,0089
Chad	4,6474	13,0430	52,3080	34,5000	49,1250
Congo	4,0006	9,8650	59,4110	28,3000	36,0074
Ethiopia	3,4534	6,7670	64,8380	31,4000	44,4085
Gambia	3,3545	7,3960	62,9950	27,8000	82,7169
Guinea-Bissau	7,0005	8,6420	60,0660	37,6000	60,9967
Liberia	10,4727	8,9540	60,5530	32,3000	73,1546
Madagascar	4,8023	6,7900	65,0940	24,0000	49,6293
Malawi	7,5844	7,1490	62,9770	21,2000	67,8458
Mozambique	7,8376	8,6900	59,7710	28,4000	53,2772
Niger	5,5761	8,2390	62,1600	35,0000	46,6185
Rwanda	6,3900	6,3670	65,9410	19,0000	60,4193
Sierra Leone	9,3998	9,8800	58,6520	33,9000	59,6527
South Sudan	15,9569	11,0100	55,3090	39,8000	41,0973
Sudan	5,9177	6,7280	65,4450	28,6000	59,3287
Togo	6,4939	8,8030	60,1540	25,9000	65,6211
Uganda	3,9979	6,1690	62,1150	21,0000	51,0921

Source: The World Bank (2025)

Table 3 presents the normalized decision matrix values. These values were obtained from the decision matrix using Equation 2.

Table 3: Normalized Decision Matrix

Country Names	K1	K2	K3	K4	K5
Afghanistan	0,1780	0,0526	0,0596	0,0768	0,0771
Chad	0,0423	0,0895	0,0505	0,0679	0,0502
Congo	0,0309	0,0694	0,0569	0,0553	0,0342
Ethiopia	0,0262	0,0485	0,0625	0,0591	0,0484
Gambia	0,0260	0,0540	0,0597	0,0527	0,0823
Guinea-Bissau	0,0671	0,0624	0,0574	0,0724	0,0597
Liberia	0,1355	0,0633	0,0584	0,0643	0,0727
Madagascar	0,0286	0,0508	0,0620	0,0508	0,0510
Malawi	0,0604	0,0500	0,0605	0,0403	0,0688
Mozambique	0,0738	0,0629	0,0570	0,0553	0,0595
Niger	0,0474	0,0583	0,0592	0,0732	0,0465
Rwanda	0,0597	0,0449	0,0635	0,0367	0,0624
Sierra Leone	0,0698	0,0652	0,0577	0,0662	0,0621
South Sudan	0,0478	0,0791	0,0529	0,0833	0,0397
Sudan	0,0231	0,0487	0,0627	0,0553	0,0618
Togo	0,0453	0,0585	0,0592	0,0506	0,0677
Uganda	0,0381	0,0421	0,0603	0,0397	0,0557

The entropy values, which play a fundamental role in the calculation of criterion weights, are presented in Table 4. These values were calculated using Equation 3.

Table 4: Ej Values

K1	K2	K3	K4	K5
0,9354	0,9930	0,9994	0,9912	0,9918

The degree of diversification, a measure indicating the influence of each criterion in the decision-making process, is presented in Table 5. These degrees were calculated according to Equation 4.

Table 5: Dij Values

K1	K2	K3	K4	K5
0,0646	0,0070	0,0006	0,0088	0,0082

Table 6 presents the calculated weight values for each criterion. These weights were obtained using Equation 5. Criteria with a high degree of diversification carry greater weights, whereas criteria with a low degree of diversification have lower weights. This method enables the objective determination of the relative importance levels of the criteria.

Table 6: Criterion Weights

K1	K2	K3	K4	K5
0,7247	0,0785	0,0063	0,0986	0,0920

The weights of the criteria were calculated using the Entropy method. The weight rankings were found to be in the order of K1> K4> K5> K2> K3. As observed from Table 6, the criterion with the highest weight is health expenditures, while the criterion with the lowest weight is life expectancy at birth.

Evaluation of Country Performances Using the WASPAS Method

The normalized decision matrix, which is the first step of the WASPAS method used to analyze the performances of low-income countries and is based on the decision matrix in Table 2, is presented in Table 7.

Table 7: Normalized Decision Matrix

Country Names	K1	K2	K3	K4	K5
Afghanistan	1,0000	0,8002	0,9381	0,4780	0,9368
Chad	0,2378	0,4700	0,7950	0,5404	0,6101
Congo	0,1734	0,6059	0,8959	0,6641	0,4151
Ethiopia	0,1470	0,8673	0,9834	0,6214	0,5883
Gambia	0,1461	0,7791	0,9396	0,6960	1,0000
Guinea-Bissau	0,3767	0,6743	0,9028	0,5073	0,7252
Liberia	0,7612	0,6650	0,9194	0,5705	0,8832
Madagascar	0,1606	0,8273	0,9760	0,7220	0,6196
Malawi	0,3396	0,8416	0,9521	0,9110	0,8355
Mozambique	0,4147	0,6684	0,8979	0,6641	0,7227
Niger	0,2664	0,7219	0,9320	0,5014	0,5650
Rwanda	0,3355	0,9358	1,0000	1,0000	0,7577
Sierra Leone	0,3919	0,6449	0,9090	0,5541	0,7547
South Sudan	0,2685	0,5319	0,8320	0,4405	0,4827
Sudan	0,1299	0,8644	0,9878	0,6641	0,7510
Togo	0,2545	0,7195	0,9326	0,7250	0,8223
Uganda	0,2140	1,0000	0,9490	0,9255	0,6772

Using the normalized decision matrix data and entropy-based criterion weights, the results calculated based on the WSM method in accordance with Equation 9 are presented in Table 8. The values of $Q_i^{(1)}$ are also provided in the same table.

Table 8: Relative Importance Values According to the Weighted Sum Method

Country Names	K1	K2	K3	K4	K5	$\mathbf{Q_i^{(1)}}$
Afghanistan	0,7247	0,9752	1,0849	0,5391	0,7731	4,0970
Chad	0,1723	0,5728	0,9194	0,6095	0,5035	2,7774
Congo	0,1257	0,7383	1,0361	0,7490	0,3426	2,9917
Ethiopia	0,1065	1,0569	1,1373	0,7009	0,4854	3,4871
Gambia	0,1059	0,9495	1,0867	0,7850	0,8252	3,7522
Guinea-Bissau	0,2730	0,8217	1,0441	0,5721	0,5984	3,3094
Liberia	0,5517	0,8103	1,0633	0,6434	0,72889	3,7976
Madagascar	0,1164	1,0081	1,1287	0,8143	0,5113	3,5788
Malawi	0,2461	1,0256	1,1011	1,0275	0,6895	4,0897
Mozambique	0,3005	0,8145	1,0384	0,7490	0,5964	3,4989
Niger	0,1931	0,8797	1,0778	0,5656	0,4662	3,1823
Rwanda	0,2431	1,1404	1,1565	1,1279	0,6252	4,2931
Sierra Leone	0,2840	0,7859	1,0513	0,6250	0,6228	3,3690
South Sudan	0,1946	0,6481	0,9623	0,4968	0,3984	2,7002
Sudan	0,0942	1,0534	1,1424	0,7490	0,6197	3,6587
Togo	0,1844	0,8768	1,0786	0,8177	0,6785	3,6360
Uganda	0,1551	1,2186	1,0976	1,0439	0,5588	4,0740

Based on the normalized decision matrix and the criterion weights determined by the Entropy method, the results calculated using the WPM method with the aid of Equation 10 are presented in Table 9. The values of $Q_i^{(2)}$ are also provided in the same table.

Table 9: Relative Importance Values According to the Weighted Product Method

Country Names	K1	K2	K3	K4	K5	$\mathbf{Q_i^{(2)}}$
Afghanistan	1,0000	0,7622	0,9288	0,4350	0,9476	0,2918
Chad	0,3531	0,3985	0,7669	0,4995	0,6651	0,0359
Congo	0,2809	0,5430	0,8806	0,6303	0,4841	0,0410
Ethiopia	0,2492	0,8408	0,9808	0,5848	0,6454	0,0776
Gambia	0,2481	0,7378	0,9305	0,6645	1,0000	0,1132
Guinea-Bissau	0,4929	0,6186	0,8885	0,4651	0,7671	0,0967
Liberia	0,8206	0,6082	0,9074	0,5310	0,9026	0,2171
Madagascar	0,2656	0,7937	0,9723	0,6925	0,6736	0,0956
Malawi	0,4571	0,8105	0,9448	0,9002	0,8622	0,2717
Mozambique	0,5284	0,6120	0,8829	0,6303	0,7649	0,1377
Niger	0,3834	0,6723	0,9217	0,4591	0,6243	0,0681
Rwanda	0,4532	0,9224	1,0000	1,0000	0,7953	0,3324
Sierra Leone	0,5071	0,5859	0,8956	0,5139	0,7928	0,1084
South Sudan	0,3856	0,4633	0,8084	0,3967	0,5483	0,0314

Sudan	0,2279	0,8373	0,9859	0,63023	0,7895	0,0936
Togo	0,3709	0,6696	0,9225	0,6958	0,8509	0,1356
Uganda	0,3272	1,0000	0,9413	0,9164	0,7249	0,2046

Weighted aggregated criterion values were obtained using Equation 11. The performance rankings of the countries based on the values obtained from these calculations are presented in Table 10. According to the ranking, Rwanda holds the first position, while South Sudan is ranked last.

Table 10: Q_i Values

Country Names	Q_i	Rank
Afghanistan	2,1944	2
Chad	1,4066	16
Congo	1,5163	15
Ethiopia	1,7823	11
Gambia	1,9327	6
Guinea-Bissau	1,7030	13
Liberia	2,0073	5
Madagascar	1,8372	9
Malawi	2,1807	3
Mozambique	1,8183	10
Niger	1,6252	14
Rwanda	2,3128	1
Sierra Leone	1,7387	12
South Sudan	1,3658	17
Sudan	1,8761	8
Togo	1,8858	7
Uganda	2,1393	4

The more effective and accurate ranking of decision alternatives was calculated by considering the λ\lambdaλ parameter through Equation 12, and the results are presented in Table 11. An examination of Table 11 reveals that the performance rankings of the countries largely coincide with the rankings presented in Table 10. It is observed that small changes have occurred in the rankings of some countries.

Table 11: 2021 Rankings for Different λ\lambdaλ Values

Country Names	$\lambda = 0$	Rank	$\lambda = 0{,}25$	Rank	$\lambda = 0{,}75$	Rank	$\lambda = 1$	Rank
Afghanistan	0,2918	2	1,2431	2	3,1457	2	4,0970	2
Chad	0,0359	16	0,7212	16	2,0920	16	2,7774	16
Congo	0,0410	15	0,7787	15	2,2540	15	2,9917	15
Ethiopia	0,0776	13	0,9299	11	2,6347	11	3,4871	11
Gambia	0,1132	8	1,0229	6	2,8425	6	3,7522	6
Guinea-Bissau	0,0967	10	0,8998	13	2,5062	13	3,3094	13
Liberia	0,2171	4	1,1122	5	2,9025	5	3,7976	5
Madagascar	0,0956	11	0,9664	10	2,7080	9	3,5788	9

Malawi	0,2717	3	1,2262	3	3,1352	3	4,0897	3
Mozambique	0,1377	6	0,9780	9	2,6586	10	3,4989	10
Niger	0,0681	14	0,8466	14	2,4038	14	3,1823	14
Rwanda	0,3324	1	1,3226	1	3,3030	1	4,2931	1
Sierra Leone	0,1084	9	0,9236	12	2,5538	12	3,3690	12
South Sudan	0,0314	17	0,6986	17	2,0330	17	2,7002	17
Sudan	0,0936	12	0,9849	8	2,7674	7	3,6587	7
Togo	0,1356	7	1,0107	7	2,7609	8	3,6360	8
Uganda	0,2046	5	1,1719	4	3,1066	4	4,0740	4

The changes in the performance rankings of low-income countries over the years, according to different levels of the λ\lambdaλ parameter, are presented in Tables 12 and 13. Upon examining the data presented in these tables, some minor changes in the rankings of the countries have been observed.

Table 12: Changes in Country Rankings Over the Years for λ=0 and λ=0.25

	$\lambda = 0$					$\lambda = 0{,}25$				
Country Names	2017	2018	2019	2020	2021	2017	2018	2019	2020	2021
Afghanistan	2	1	1	2	2	2	1	1	2	2
Chad	14	14	13	11	16	15	15	14	12	16
Congo	16	17	17	15	15	16	17	17	15	15
Ethiopia	17	16	16	16	13	17	16	16	16	11
Gambia	15	15	14	13	8	14	14	13	13	6
Guinea-Bissau	9	8	8	5	10	9	8	8	5	13
Liberia	3	2	3	1	4	3	2	4	1	5
Madagascar	12	13	15	14	11	11	13	15	14	10
Malawi	5	4	4	6	3	4	4	3	6	3
Mozambique	6	6	5	8	6	7	6	5	8	9
Niger	11	10	10	10	14	12	12	10	10	14
Rwanda	7	5	6	4	1	6	3	6	4	1
Sierra Leone	4	7	2	3	9	5	7	2	3	12
South Sudan	1	3	7	7	17	1	5	7	7	17
Sudan	10	11	11	17	12	10	10	11	17	8
Togo	8	9	9	9	7	8	9	9	9	7
Uganda	13	12	12	12	5	13	11	12	11	4

Table 13: Changes in Country Rankings Over the Years for λ=0.75 and λ=1

	$\lambda = 0{,}75$					$\lambda = 1$				
Country Names	2017	2018	2019	2020	2021	2017	2018	2019	2020	2021
Afghanistan	2	1	1	2	2	2	1	1	2	2
Chad	16	15	15	13	16	16	16	16	13	16
Congo	17	17	17	17	15	17	17	17	17	15

Ethiopia	15	16	16	16	11	15	15	15	16	11
Gambia	14	14	13	12	6	13	13	13	12	6
Guinea-Bissau	9	8	8	5	13	10	8	8	6	13
Liberia	3	2	4	1	5	3	2	4	1	5
Madagascar	11	12	14	14	9	11	12	14	14	9
Malawi	4	4	2	6	3	4	4	2	5	3
Mozambique	7	6	6	8	10	7	6	6	8	10
Niger	13	13	12	11	14	14	14	12	11	14
Rwanda	6	3	5	3	1	6	3	5	3	1
Sierra Leone	5	7	3	4	12	5	7	3	4	12
South Sudan	1	5	7	7	17	1	5	7	7	17
Sudan	10	11	10	15	7	9	11	11	15	7
Togo	8	9	9	9	8	8	9	9	9	8
Uganda	12	10	11	10	4	12	10	10	10	4

DISCUSSION AND CONCLUSION

In this study, the basic health indicators of low-income countries between 2017 and 2021 were analyzed using the integrated Entropy-WASPAS multi-criteria decision-making method. The primary objective of the study is to objectively evaluate the performance of the health systems of low-income countries and, in this context, provide decision support to policymakers by conducting cross-country comparisons.

Within the scope of the analysis, five key health indicators were evaluated. These variables include the share of current health expenditures in GDP, the rate of access to basic drinking water services, the overall mortality rate, life expectancy at birth, and the neonatal mortality rate. The importance of these indicators was determined using the Entropy method; subsequently, countries were ranked and their performance evaluated using the WASPAS method. In the WASPAS method, the combined strength of both the WSM and WPM models, along with different values of λ, was used to test the sensitivity of countries' health indicator performance rankings.

When examining the weights of 2021 data for less developed countries according to the Entropy technique, it was found that the most significant weight was assigned to 'current health expenditure,' while the lowest weight was attributed to 'life expectancy at birth.' The obtained weights were used in the WASPAS technique to rank the health indicator performances of the countriesIn this ranking, Rwanda was found to have the highest score, while South Sudan had the lowest score. Additionally, although small variations were observed in the countries' performance rankings across the years 2017-2021 for different values of the λ parameter, no significant changes were detected.

In future studies, criteria such as the number of healthcare workers and doctors, which affect health indicators, can be included, and research can be conducted using different methods. Additionally, results obtained from multiple methods can be compared. Furthermore, not only annual but also periodic or regional analyses can be conducted to examine time-dependent trends in greater depth.

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The Suja q-Distribution (1114)

Hasan Hüseyin Gül[1*], Nurgül Okur[1]

[1]Giresun University, Faculty of Arts and Sciences, Statistics, Turkey

*Corresponding author e-mail: hasan.huseyin@giresun.edu.tr

Abstract

Quantum calculus, also known as calculus without the concept of limits, has gained significant attention in recent years. In particular, q-calculus has attracted interest due to its role in bridging mathematics with physical theories. The evolution of q-calculus from Euler's initial investigations to its contemporary formulation has been shaped by several significant milestones. Among these, the early 20th-century contributions of F. H. Jackson played a pivotal role in formalizing the foundations of q-calculus. Within probability theory, the q-distribution emerges as a q-analogue of a classical distribution, implying that the standard form of the distribution is obtained in the limit as q approaches 1. q-analogues of probability distributions establish a generalized framework that extends the conventional definitions while encompassing the classical cases as specific instances.

In this study, a new probability distribution, called the Suja q-distribution, is introduced. Its properties, including q-moments, q-variance, and q-cumulative functions, are discussed. In addition, q-versions of moments and associated statistical measures are derived for the proposed q-distribution. The study also presents the q-analogues of the reliability functions, the moment-generating function, the stress-strength reliability, and the cumulative distribution of the ordered statistics for the proposed q-distribution within this framework.

Keywords: *Suja distribution, q-calculus, q-distribution, q-probability theory.*

INTRODUCTION

Probability q-distributions generalize classical probability distributions, offering a wider range of possibilities. Key contributions to the theory of basic discrete q-distributions include the works of Dunkl, 1981, Crippa et al., 1997, Kupershmidt, 2000, Kemp, 2002, Charalambides, 2016, Kyriakoussis and Vamvakari, 2017, among others. Significant research on q-continuous distributions, including the Gaussian and generalized gamma q-distributions by Diaz et al., 2009, 2010, the Erlang q-distributions by Charalambides, 2016, the gamma and beta q-distributions by Boutouria et al., 2018, the Lindley q-distribution in two forms was introduced by Imed, 2023.

Recently, there has been a surge in the proposal of lifetime distributions for modeling lifetime data by various statisticians. In this context, the Suja distribution was introduced by Shanker, 2017 with its PDF and CDF for $\tau \geq 0$ and $\theta > 0$:

$$f(\tau;\theta) = \frac{\theta^5}{\theta^4 + 24}(1 + \tau^4)e^{-\theta\tau},$$

$$F(\tau;\theta) = 1 - \left(1 + \frac{\theta^4\tau^4 + 4\theta^3\tau^3 + 12\theta^2\tau^2 + 24\theta\tau}{\theta^4 + 24}\right)e^{-\theta\tau}.$$

The primary motivation of this study is to construct the q-analogue of Suja distribution, and its distributional and statistical properties are investigated.

MATERIAL AND METHODS

This section outlines the principles of q-calculus, and q-probability theory. In this entire study, unless otherwise stated, it is assumed that $0 < q < 1$. Readers are referred to the relevant literature.

Definition (Kac and Cheung, 2002). Let x, q be real numbers. The q-number $[x]_q$ is defined as

$$[x]_q = \frac{1 - q^x}{1 - q}, (q \neq 1).$$

For $n \in \mathbb{N}$, a natural q-number, it reduces to $[n]_q = \sum_{k=0}^{n-1} q^k$.

Definition (Kac and Cheung, 2002). The q-Gauss binomial formula is given by

$$(x + y)_q^n = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q q^{\binom{k}{2}} y^k x^{n-k}\ , -\infty < x, y < \infty.$$

The q-binomial coefficients are provided for $k = 0,1, \dots, n$ by

$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{[n]_q!}{[n-k]_q![k]_q!}, [n]_{k,q} = \frac{[n]_q!}{[n-k]_q!}\ , [n]_q! = [n]_q[n-1]_q \dots [2]_q[1]_q.$$

Definition (Vamvakari, 2023). The q-analogues of the exponential function are presented

$$E_q^\tau = \sum_{k=0}^{\infty} q^{\binom{k}{2}} \frac{\tau^k}{[k]_q!} = \prod_{k=0}^{\infty} \left(1 + (1-q)q^k\tau\right), (\tau \in \mathbb{R}),$$

$$e_q^\tau = \sum_{k=0}^{\infty} \frac{\tau^k}{[k]_q!} = \prod_{k=0}^{\infty} \frac{1}{(1-(1-q)q^k\tau)}, \left(|\tau| < \frac{1}{1-q}\right).$$

Definition (Kac and Cheung, 2002). The q-derivative of f is defined as

$$D_q f(\tau) = \frac{d_q\big(f(\tau)\big)}{d_q(\tau)} = \frac{f(q\tau) - f(\tau)}{q\tau - \tau}, \ (q \neq 1\ ),$$

where $d_q(.)$ is the q-differential operator.

Definition (Kac and Cheung, 2002). The well-known Jackson q-integral of f is given by

$$\int_0^b f(\tau) d_q\tau = (1-q) \sum_{n=0}^{\infty} q^n b f(q^n b)\,, b > 0,$$

Definition (Kac and Cheung, 2002). The generalized q-integral is given by

$$\int_{-\infty}^{\infty} f(\tau) d_q\tau = (1-q) \sum_{n \in \mathbb{Z}} q^n f(q^n).$$

Definition (Kac and Cheung, 2002, De Sole and Kac, 2003). The q-analogues of the gamma functions are given for $\alpha > 0$, $n \in \mathbb{N}$

$$\Gamma_q(\alpha) = \int_0^{[\infty]_q} \tau^{\alpha-1} E_q^{-q\tau} d_q\tau\,, \qquad \gamma_q(\alpha) = \int_0^{\infty} \tau^{\alpha-1} e_q^{-\tau} d_q\tau.$$

Identities derived from the q-gamma functions can be obtained

$$\Gamma_q(\alpha+n)=[\alpha]_{n,q}\Gamma_q(\alpha), \qquad \gamma_q(\alpha+n)=q^{-\binom{n}{2}-\alpha n}[\alpha]_{n,q}\gamma_q(\alpha),$$
$$\Gamma_q(n+1)=[n]_q!, \qquad \gamma_q(n+1)=q^{-\binom{n+1}{2}}[n]_q!$$

Definition (Vamvakari, 2023). A random variable ξ is considered q-continuous if there exists a non-negative function $f_q^{\xi}(\tau)$ for $\tau \geq 0$ such that

$$P\{a<\xi\leq b\}=\int_a^b f_q^{\xi}(\tau)d_q\tau.$$

The q-cumulative distribution function (q-CDF) of the non-negative q-continuous random variable ξ is defined for $\tau>0$

$$F_q^{\xi}(\tau)=P(\xi\leq\tau)=\int_0^{\tau} f_q^{\xi}(u)d_qu,$$

satisfying the relation $P(\alpha<\xi\leq\beta)=F_q^{\xi}(\beta)-F_q^{\xi}(\alpha)$. Then,

$$f_q(\tau)=D_qF_q(\tau)=\frac{F_q^{\xi}(\tau)-F_q^{\xi}(q\tau)}{(1-q)\tau}=\frac{P(q\tau\leq\xi\leq\tau)}{(1-q)\tau},\ (q\neq 1\ ).$$

RESULTS

In this section, the modeling of the Suja q-distribution, its moments, related measures, moment generating function, reliability functions, the CDF of order statistics, and stress-strength reliability are presented under separate subheadings.

Modeling of the Suja q-distribution

In this subsection, the Suja q-distribution is constructed.

Definition 1. Let ξ be a random variable characterized by

$$f_q^{i_m}\left(\tau;[\theta]_q\right)=\sum_{k\in\{1,5\}}\pi_q^{i(k)}\mathrm{g}_q^{\sigma_m(k)}\left(\tau;k,[\theta]_q\right)$$
$$F_q^{i_m}\left(\tau;[\theta]_q\right)=\sum_{k\in\{1,5\}}\pi_q^{i(k)}\mathrm{G}_q^{\sigma_m(k)}\left(\tau;k,[\theta]_q\right)$$

for $\tau\geq 0$, $i=I,II, m=1,2,3,4$ with $\sum_{k\in\{1,5\}}\pi_q^{i(k)}=1$, where

$$\pi_q^{i(k)}=\begin{cases}\dfrac{[k-1]_q!\,[\theta]_q^{1-k}}{\sum_{k\in\{1,5\}}[k-1]_q!\,[\theta]_q^{1-k}}, & i=I\\ \dfrac{q^{-\binom{k}{2}}[k-1]_q!\,[\theta]_q^{1-k}}{\sum_{k\in\{1,5\}}q^{-\binom{k}{2}}[k-1]_q!\,[\theta]_q^{1-k}}, & i=II\end{cases}$$

$$\mathrm{g}_q^{i}\left(\tau,k,[\theta]_q\right)=\begin{cases}\dfrac{[\theta]_q^k}{[k-1]_q!}\tau^{k-1}E_q^{-q[\theta]_q\tau}\mathbf{1}_{[0,[\infty]_q]}(\tau), & i=I\\ \dfrac{q^{\binom{k}{2}}[\theta]_q^k}{[k-1]_q!}\tau^{k-1}e_q^{-[\theta]_q\tau}\mathbf{1}_{[0,\infty)}(\tau), & i=II\end{cases}$$

$$G_q^i(\tau,k,[\theta]_q)=\begin{cases}1-\sum_{z=0}^{k-1}\frac{\left([\theta]_q\tau\right)^z}{[z]_q!}E_q^{-[\theta]_q\tau}, & i=I\\ 1-\sum_{z=0}^{k-1}\frac{q^{\binom{z}{2}}\left([\theta]_q\tau\right)^z}{[z]_q!}e_q^{-[\theta]_q\tau}, & i=II\end{cases}$$

and $\sigma_m(k)$ is a function that determines which q-PDF of the gamma q-distribution (for $i=I,II$) is used for distinct combinations of m and k.The distribution obeying the aforementioned q-PDF is called "Suja q-Distribution".

Remark 1. From Definition 1, the Suja q-distribution appears in eight convex combinations:

$$f_q^{I_1}(\tau;[\theta]_q)=\pi_q^{I(1)}\mathrm{g}_q^{I}(\tau;1,[\theta]_q)+\pi_q^{I(5)}\mathrm{g}_q^{I}(\tau;5,[\theta]_q)$$
$$f_q^{I_2}(\tau;[\theta]_q)=\pi_q^{I(1)}\mathrm{g}_q^{I}(\tau;1,[\theta]_q)+\pi_q^{I(5)}\mathrm{g}_q^{II}(\tau;5,[\theta]_q)$$
$$f_q^{I_3}(\tau;[\theta]_q)=\pi_q^{I(1)}\mathrm{g}_q^{II}(\tau;1,[\theta]_q)+\pi_q^{I(5)}\mathrm{g}_q^{I}(\tau;5,[\theta]_q)$$
$$f_q^{I_4}(\tau;[\theta]_q)=\pi_q^{I(1)}\mathrm{g}_q^{II}(\tau;1,[\theta]_q)+\pi_q^{I(5)}\mathrm{g}_q^{II}(\tau;5,[\theta]_q)$$
$$f_q^{II_1}(\tau;[\theta]_q)=\pi_q^{II(1)}\mathrm{g}_q^{I}(\tau;1,[\theta]_q)+\pi_q^{II(5)}\mathrm{g}_q^{I}(\tau;5,[\theta]_q)$$
$$f_q^{II_2}(\tau;[\theta]_q)=\pi_q^{II(1)}\mathrm{g}_q^{I}(\tau;1,[\theta]_q)+\pi_q^{II(5)}\mathrm{g}_q^{II}(\tau;5,[\theta]_q)$$
$$f_q^{II_3}(\tau;[\theta]_q)=\pi_q^{II(1)}\mathrm{g}_q^{II}(\tau;1,[\theta]_q)+\pi_q^{II(5)}\mathrm{g}_q^{I}(\tau;5,[\theta]_q)$$
$$f_q^{II_4}(\tau;[\theta]_q)=\pi_q^{II(1)}\mathrm{g}_q^{II}(\tau;1,[\theta]_q)+\pi_q^{II(5)}\mathrm{g}_q^{II}(\tau;5,[\theta]_q)$$

Hence, the Suja q-PDFs are obtained as follows:

$$f_q^{I_1}(\tau;[\theta]_q)=\frac{[\theta]_q^5}{[\theta]_q^4+[4]_q!}(1+\tau^4)E_q^{-q[\theta]_q\tau},\qquad 0<\tau<[\infty]_q,$$

This study focuses solely on q-distributions characterized by $f_q^{I_1}$.

The graphs of the Suja q-PDF for $f_q^{I_1}$ are depicted in the figures below:

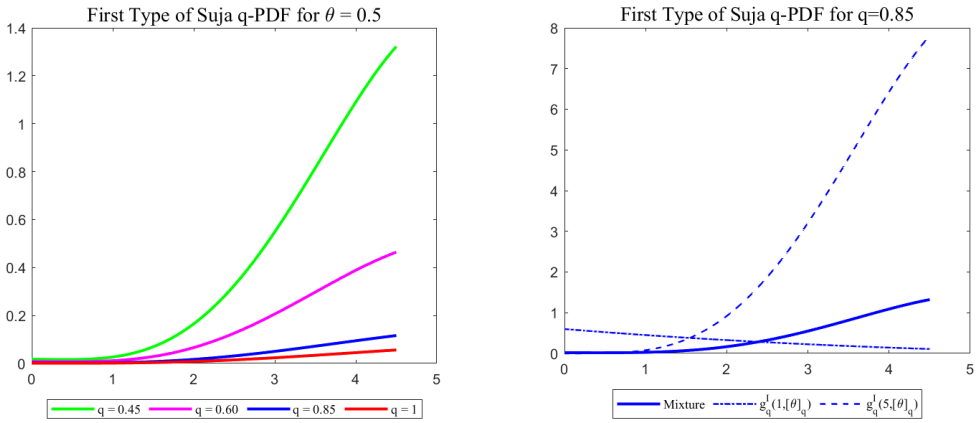

Figure 1. Graphs of the first type of Suja q-PDF and its components.

Theorem 1. Let the random variable ξ be characterized by $f_q^{I_1}(\tau;[\theta]_q)$. Its q-CDF is given by:

$$F_q(\tau;[\theta]_q) = 1 - \left(1 + \frac{\sum_{z=0}^{3}[4]_{z,q}\tau^{4-z}[\theta]_q^{4-z}}{[\theta]_q^4 + [4]_q!}\right)E_q^{-[\theta]_q\tau}$$

Moments, Related Measures, and Moment Generating Function of the Suja q-distribution

In this subsection, we investigate the rth q-moments, related measures, and the q-moment generating function for the Suja q-distribution.

Definition 2. (Vamvakari, 2023). If $\mathbb{E}_q|\xi^r| < \infty$ for all positive integers r, the r-th q-moment of ξ is:

$$\mu_q^{(r)} = \mathbb{E}_q(\xi^r) = \int_0^\infty \tau^r \, f_q^{\xi}(\tau) d_q\tau.$$

Particularly, $\mathbb{E}_q(\xi) = \mu_q,\ \mathbb{V}_q(\xi) = \mu_q^{(2)} - (\mu_q)^2$.

Definition 3. (Okur and Djongmon, 2025). Let ξ be a q-continuous non-negative random variable. If $\mathbb{E}_q\left|(\xi - \mu_q)_q^r\right| < \infty$ for all positive integers r, it can be defined as

$$m_q^{(r)} = \mathbb{E}_q(\xi - \mu_q)_q^r = \sum_{s=0}^{r}\begin{bmatrix} r \\ s \end{bmatrix}_q q^{\binom{s}{2}}(-1)^s \mu_q^s \mu_q^{(r-s)},\ s \le r$$

Theorem 2. Let ξ follow the Suja q-distribution. Its rth q-moment and q-central moment are given

$$\mu_q^{(r)} = \frac{[\theta]_q^5}{[\theta]_q^4 + [4]_q!}\left(\frac{[r]_q!}{[\theta]_q^{r+1}} + \frac{[r+4]_q!}{[\theta]_q^{r+5}}\right)$$

$$m_q^{(r)} = \sum_{s=0}^{r}\begin{bmatrix} r \\ s \end{bmatrix}_q q^{\binom{s}{2}}(-1)^s \left(\frac{[\theta]_q^4 + [5]_q!}{[\theta]_q([\theta]_q^4 + [4]_q!)}\right)^s \frac{[\theta]_q^4[r-s]_q! + [r-s+4]_q!}{[\theta]_q^{r-s}([\theta]_q^4 + [4]_q!)}$$

Definition 4. (Okur and Djongmon, 2025). Let ξ be a q-continuous non-negative random variable. Its q-MGF is expressed in two distinct forms as follows:

$$\mathbb{M}_q^I(t) = \mathbb{E}_q\left(E_q^{qt\xi}\right) = \int_0^\infty E_q^{qt\tau} f_q(\tau) d_q\tau, \qquad \mathbb{M}_q^{II}(t) = \mathbb{E}_q\left(e_q^{t\xi}\right) = \int_0^\infty e_q^{t\tau} f_q(\tau) d_q\tau$$

for those values of t for which $\mathbb{E}_q\left(E_q^{qt\xi}\right)$ and $\mathbb{E}_q\left(e_q^{t\xi}\right)$ are well defined. If these q-expectations do not converge for values of t close to 0, the q-MGF is considered undefined.

Theorem 3. Let ξ follow the Suja q-distribution. The q-MGFs of ξ can be given by:

$$\mathbb{M}_q^I(t) = \frac{[\theta]_q^5}{[\theta]_q^4 + [4]_q!}\sum_{r=0}^{\infty}\frac{q^{\binom{r+1}{2}}t^r}{[r]_q!}\left(\frac{[r]_q!}{[\theta]_q^{r+1}} + \frac{[r+4]_q!}{[\theta]_q^{r+5}}\right)$$

$$\mathbb{M}_q^{II}(t) = \frac{[\theta]_q^5}{[\theta]_q^4 + [4]_q!}\sum_{r=0}^{\infty}\frac{t^r}{[r]_q!}\left(\frac{[r]_q!}{[\theta]_q^{r+1}} + \frac{[r+4]_q!}{[\theta]_q^{r+5}}\right)$$

Reliability Functions of the Suja q-distribution

In this subsection, we give the q-reliability functions (q-RFs) for the Suja q-distribution.

Definition 5. (Okur and Djongmon, 2025). Let ξ be a q-continuous non-negative random variable. Its q-RFs can be defined as

1) the q-survival function (q-CCDF) $\mathbb{S}_q(\tau) = P(\xi > \tau) = 1 - F_q(\tau)$,

2) the q-hazard rate function (q-HRF)

$$h_q(\tau) = \frac{P(q\tau \le \xi \le \tau | \xi \ge q\tau)}{(1-q)\tau} = \frac{f_q(\tau)}{\mathbb{S}_q(q\tau)},$$

3) the q-mean residual life function (q-MRLF)

$$mrl_q(\tau) = \mathbb{E}_q(\xi - \tau | \xi > \tau) = \frac{1}{\mathbb{S}_q(\tau)} \int_{\tau}^{\infty} \mathbb{S}_q(qu) d_q u.$$

Theorem 4. Let ξ follow the Suja q-distribution. Its q-reliability functions are given by:

$$\mathbb{S}_q(\tau; [\theta]_q) = \left(1 + \frac{\sum_{z=0}^{3} [4]_{z,q} \tau^{4-z} [\theta]_q^{4-z}}{[\theta]_q^4 + [4]_q!}\right) E_q^{-[\theta]_q \tau}$$

$$h_q(\tau; [\theta]_q) = \frac{[\theta]_q^5 (1+\tau^4)}{[\theta]_q^4 + \sum_{z=0}^{4} [4]_{z,q} (q\tau)^{4-z} [\theta]_q^{4-z}}$$

$$mrl_q(\tau; [\theta]_q) = \frac{[\theta]_q^4 + \sum_{z=0}^{3} \sum_{y=0}^{4-z} q^{4-z} [4]_{z,q} [4-z]_{y,q} \tau^{4-z-y} [\theta]_q^{-(z+y-3)}}{[\theta]_q^4 + \sum_{z=0}^{4} [4]_{z,q} \tau^{4-z} [\theta]_q^{4-z}}$$

Order Statistics of the Suja q-distribution

In this subsection, we introduce the q-ordered statistics CDFs of the Suja q-distribution.

Definition 6. (Vamvakari, 2023). Suppose $\Xi = (\xi_1, \xi_2, \dots, \xi_n)$ represents an n-dimensional q-continuous random variable, with corresponding k-th ordered random variables denoted as $\xi_{(k)}$, $1 \le k \le n$. Under the following condition, $\xi_{(k)}$ are defined as k-th q-ordered random variables:

$$\xi_{(1)} < q\xi_{2)} < \xi_{(2)} < \cdots < \xi_{(n-1)} < q\xi_{(n)}.$$

Note that, given that the non-ordered q-continuous random variables: $\xi_1, \xi_2, \dots, \xi_n$ are dependent and not identically distributed but share the same functional form, and satisfy the specified q-ordering.

Lemma 1. (Vamvakari, 2023). Consider the random variables $\xi_1, \xi_2, \dots, \xi_n$, where each ξ_i follows $F_q^{\xi_i}$. *The q-CDFs of $\xi_{(n)}$, $\xi_{(1)}$, $\xi_{(k)}$ $(1 \le k \le n)$* are expressed, respectively

$$F_q^{\xi_{(n)}}(\tau) = \prod_{i=1}^{n} F_q^{\xi_i}\left(q^{i-1}\tau\right)$$

$$F_q^{\xi_{(1)}}(\tau) = 1 - \prod_{i=1}^{n} \left(1 - F_q^{\xi_i}(\tau)\right)$$

$$F_q^{\xi_{(k)}}(\tau) = \sum_{r=k}^{n} \sum_{1<i_1<\cdots<i_r<n} \prod_{j=1}^{r} F_q^{\xi_j}\left(q^{j-1}\tau\right) \prod_{m=r+1}^{n} \left(1 - F_q^{\xi_m}\left(q^{i_m-(m-r)}\tau\right)\right)$$

Theorem 5. Let $\xi_1, \xi_2, \dots, \xi_n$ be a random sample of size n from the Suja q-distribution, and $\xi_{(1)} < \xi_{(2)} < \cdots < \xi_{(n)}$ denote the corresponding order statistics. The q-CDFs of the random variables $\xi_{(1)}$, $\xi_{(n)}$ and $\xi_{(k)}$, $1 \le k \le n$, are given by :

$$F_q^{\xi_{(n)}}(\tau;[\theta]_q) = \prod_{i=1}^{n}\left(1-\left(1+\frac{\sum_{z=0}^{3}[4]_{z,q}\left(q^{i-1}\tau\right)^{4-z}[\theta]_q^{4-z}}{[\theta]_q^4+[4]_q!}\right)E_q^{-[\theta]_q q^{i-1}\tau}\right)$$

$$F_q^{\xi_{(1)}}(\tau;[\theta]_q) = 1-\prod_{i=1}^{n}\left(1+\frac{\sum_{z=0}^{3}[4]_{z,q}\tau^{4-z}[\theta]_q^{4-z}}{[\theta]_q^4+[4]_q!}\right)E_q^{-[\theta]_q\tau}$$

$$F_q^{\xi_{(k)}}(\tau;[\theta]_q) = \sum_{r=k}^{n}\sum_{1<i_1<\cdots<i_r<n}\prod_{j=1}^{r}\left(1-\left(1+\frac{\sum_{z=0}^{3}[4]_{z,q}\left(q^{j-1}\tau\right)^{4-z}[\theta]_q^{4-z}}{[\theta]_q^4+[4]_q!}\right)E_q^{-[\theta]_q q^{j-1}\tau}\right)$$
$$\times\prod_{m=r+1}^{n}\left(1+\frac{\sum_{z=0}^{3}[4]_{z,q}\left(q^{i_m-(m-r)}\tau\right)^{4-z}[\theta]_q^{4-z}}{[\theta]_q^4+[4]_q!}\right)E_q^{-[\theta]_q\tau}E_q^{-[\theta]_q q^{i_m-(m-r)}\tau}$$

Stress Strength Reliability of the Suja q-distribution

In this subsection, we present the q-stress-strength reliability the Suja q-distribution.

Definition 7. (Okur and Djongmon, 2025). Let ξ and η are two independent random variable, then the stress-strength reliability can be defined by

$$\mathbb{R}_q = P(\xi>\eta) = \int_0^{\infty} P(\xi>\tau|\,\eta=\tau)\,f_q^{\xi}(\tau)d_q\tau = \int_0^{\infty} f_q^{\xi}(\tau)\mathrm{F}_q^{\eta}(\tau)d_q\tau.$$

Theorem 6. Let ξ and η are two independent random variables, with $[\theta_1]_q$ and $[\theta_2]_q$ parameters, respectively, follow the Suja q-distribution. Its stress-strength reliability can be given by

$$\mathbb{R}_q = \frac{[\theta_1]_q^5}{[\theta_1]_q^4+[4]_q!}\sum_{r=0}^{\infty}\frac{q^{\binom{r}{2}}\left(-[\theta_2]_q\tau\right)^r}{[r]_q!}\left(\begin{array}{c}\left(\dfrac{[r]_q!}{\theta_1]_q^r}+\dfrac{[r+4]_q!}{\theta_1]_q^{r+4}}\right)\\ +\dfrac{\sum_{z=0}^{3}[4]_{z,q}[\theta_2]_q^{4-z}}{[\theta_2]_q^4+[4]_q!}\left(\dfrac{[4-z+r]_q!}{[\theta_1]_q^{4-z+r+1}}+\dfrac{[8-z+r]_q!}{[\theta_1]_q^{8-z+r+1}}\right)\end{array}\right)$$

DISCUSSION AND CONCLUSION

In conclusion, the evolution of q-distributions naturally aligns with the development of q-calculus, which generalizes classical calculus and extends beyond traditional analysis. In this paper, we introduce the Suja q-distribution, and provide a detailed analysis of its distributional properties, including its modeling, the moments, moment generating functions, reliability functions, ordere statistics and stress-strenght reliability. In conclusion, the evolution of q-distributions represents a natural progression in the development of q-calculus. q-calculus serves as a parametric generalization of classical calculus, with the classical framework being recovered in the limit as $q \to 1$. This property makes q-calculus a more expansive and inclusive framework, extending beyond the confines of traditional analysis. Our findings suggest that the proposed q-distribution holds significant promise and may have widespread applications across various fields. In future research, we aim to explore the finite muxture and compound q-distribution.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

Both authors contributed equally to the finalization of this paper.

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Parameter Estimation for the Unit Generalized Rayleigh Distribution: Theory and Simulation Study (1115)

Hasan Hüseyin Gül[1*]

[1]Giresun University, Faculty of Arts and Sciences, Department of Statistics, Turkey

*Corresponding author e-mail: hasan.huseyin@giresun.edu.tr

Abstract

In the context of data modeling, many traditional statistical distributions often fail to provide an adequate fit when applied to real-world datasets. To address this issue, researchers have developed new distributions by employing various techniques aimed at enhancing the adaptability of existing models. These newly proposed statistical frameworks offer improved flexibility and are applicable across diverse fields, including communication systems, reliability studies, biomedical and medical lifetime analysis, finance, and insurance.

In recent years, analyzing bounded random variables within the interval (0, 1), such as proportions and extreme values, has gained significant attention. Unit models have become widely adopted across various disciplines due to their efficiency, often outperforming baseline distributions without requiring additional parameters. Consequently, an increasing number of studies in the literature have focused on unit distributions.

In this paper, the Unit Generalized Rayleigh (UGR) distribution, one of the unit distributions, is examined. fundamental characteristics of the UGR distribution, including its probability density function, cumulative distribution function, and hazard function, are investigated in detail. Additionally, four parameter estimation methods—Maximum Likelihood Estimation (MLE), Least Squares (LS), Weighted Least Squares (WLS), and the Anderson-Darling (AD) method—are employed to estimate the unknown parameters of the distribution. A Monte Carlo simulation study is conducted to evaluate and compare the performance of these estimators.

Keywords: *Unit generalized Rayleigh distribution, Maximum likelihood estimation, Unit models, Parameter estimation.*

INTRODUCTION

Numerous probability distributions have been introduced in the statistical literature to represent real-world phenomena across various disciplines. While the majority of these distributions are defined either over the positive real line or the entire real axis, many practical applications—particularly in fields such as economics, medicine, and biology—involve data restricted to a bounded interval. This is often the case with variables representing rates, indices, proportions, or scores. In such scenarios, it is more appropriate to employ distributions with bounded support. Despite the frequent occurrence of bounded data in practice, there remains a noticeable scarcity of bounded distributions compared to the vast array of unbounded ones. To address this gap, particularly in the last decade, there has been growing interest among researchers in developing new probability distributions defined on bounded intervals.

Lately, there has been a noticeable rise in efforts to develop flexible probability distributions tailored for data bounded between 0 and 1. This growing interest stems from the need to accurately model quantities such as proportions, percentages, and probabilities. In various applied fields, datasets confined

to the (0, 1) interval are frequently encountered, creating substantial demand for suitable parametric, semi-parametric, and regression-based modeling techniques. Importantly, unit distributions offer additional modeling flexibility across the unit interval, often without increasing the number of parameters in the baseline distribution.

The following are some of the most important unit distributions with diverse numbers of parameters: unit inverse Gaussian (Ghitany et al., 2018), unit Weibull (Mazucheli et al., 2018b), unit Birnbaum-Saunders (Mazucheli et al., 2018), unit Lindley (Mazucheli et al., 2019), unit Burr XII (Korkmaz and Chesneau, 2021), unit exponentiated half logistic (Hassan et al., 2022), unit power Burr X (Fayomi et al., 2023), unit inverse exponentiated Weibull (Hassan and Alharbi, 2023) and unit generalized Rayleigh (El-Raouf and AbaOud, 2023).

In this paper, the unit generalized Rayleigh (UGR) distribution, one of the unit distributions, is examined. fundamental characteristics of the UGR distribution, including its probability density function, cumulative distribution function, and hazard function, are investigated in detail. Additionally, four parameter estimation methods—Maximum Likelihood Estimation (MLE), Least Squares (LS), Weighted Least Squares (WLS), and the Anderson-Darling (AD) method—are employed to estimate the unknown parameters of the distribution. A Monte Carlo simulation study is conducted to evaluate and compare the performance of these estimators.

MATERIAL AND METHODS

Unit Generalized Rayleigh Distribution

The probability distribution function (pdf) and cumulative distribution function (cdf) of the UGR distribution can be written, respectively, as

$$F(x) = \left\{1 - e^{-\theta\left(\frac{x}{1-x}\right)^2}\right\}^{\alpha}, \qquad 0 < x < 1, \tag{1}$$

$$f(x) = \frac{2\alpha\theta x}{(1-x)^3} e^{-\theta\left(\frac{x}{1-x}\right)^2} \left\{1 - e^{-\theta\left(\frac{x}{1-x}\right)^2}\right\}^{\alpha-1}. \tag{2}$$

The survival function (SF) and hazard rate function (HRF) of UGR distribution are expressed, respectively, as

$$S(x;\alpha,\theta) = 1 - F(x) = 1 - \left\{1 - e^{-\theta\left(\frac{x}{1-x}\right)^2}\right\}^{\alpha}, \tag{3}$$

$$h(x;\alpha,\theta) = \frac{f(x)}{S(x)} = \frac{\frac{2\alpha\theta x}{(1-x)^3} e^{-\theta\left(\frac{x}{1-x}\right)^2} \left\{1 - e^{-\theta\left(\frac{x}{1-x}\right)^2}\right\}^{\alpha-1}}{1 - \left\{1 - e^{-\theta\left(\frac{x}{1-x}\right)^2}\right\}^{\alpha}}. \tag{4}$$

The probability density function plot of the UGR distribution for some parameter values is given in Figure 1.

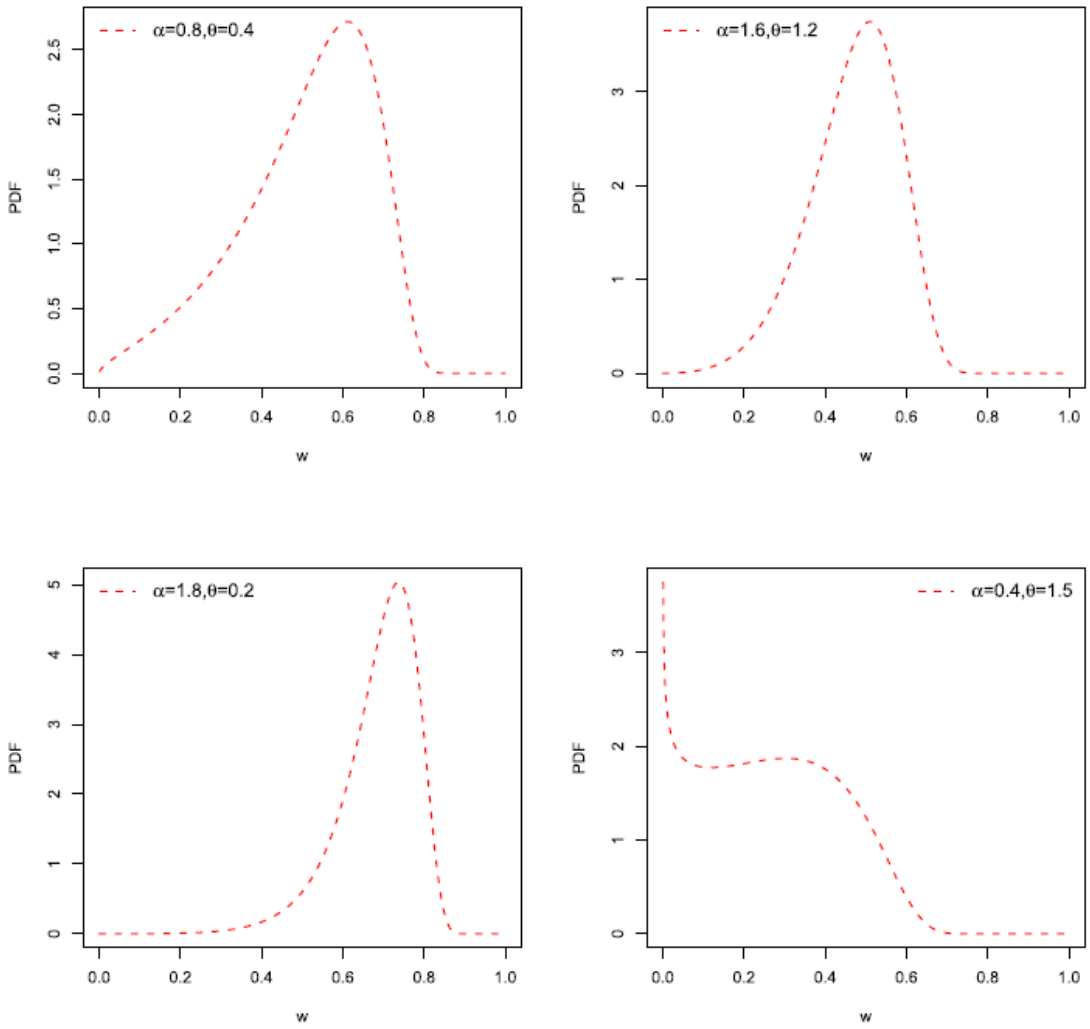

Figure 1. The plot for pdf of UGR distribution for different values of parameters.

Methods

Maximum Likelihood Estimation

Let $x_1, x_2, \dots, x_n$ be a random sample of size *n* coming from the UGR. The likelihood function is

$$L(x;\alpha,\theta) = \prod_{i=1}^{n} f(x)$$

$$= (2\alpha\theta)^n \prod_{i=1}^{n} \frac{x_i}{(1-x_i)^3} e^{-\theta\left(\frac{x_i}{1-x_i}\right)^2} \left\{1 - e^{-\theta\left(\frac{x_i}{1-x_i}\right)^2}\right\}^{\alpha-1}. \quad (5)$$

The associated log-likelihood function is

$$LL(x;\alpha,\theta) = -nlog(2\alpha\theta) + \sum_{i=1}^{n} log\left(\frac{x_i}{(1-x_i)^3}\right) - \theta \sum_{i=1}^{n} \left(\frac{x_i}{1-x_i}\right)^2$$

$$-(\alpha-1) \sum_{i=1}^{n} log\left(1 - e^{-\theta\left(\frac{x_i}{1-x_i}\right)^2}\right), \quad (6)$$

$\hat{\alpha}$ and $\hat{\theta}$ are obtained, respectively, by solving two non-linear equations,

$$\frac{\partial LL(x;\alpha,\theta)}{\partial \alpha} = -\frac{n}{\alpha} - \sum_{i=1}^{n} log\left(1 - e^{-\theta\left(\frac{x_i}{1-x_i}\right)^2}\right), \quad (7)$$

$$\frac{\partial LL(x;\alpha,\theta)}{\partial \theta} = -\frac{n}{\theta} - \sum_{i=1}^{n}\left(\frac{x_i}{(1-x_i)^3}\right) + -(\alpha-1)\sum_{i=1}^{n} log\left(\frac{\left(\frac{x_i}{1-x_i}\right)^2 e^{-\theta\left(\frac{x_i}{1-x_i}\right)^2}}{1-e^{-\theta\left(\frac{x_i}{1-x_i}\right)^2}}\right). \quad (8)$$

It is clear that the MLE of α and θ can't be solved explicitly, and approximate numerical methods such as Newton–Raphson method have to produce the MLEs of α and θ.

Least Squares and Weighted Least Squares Estimations

LS and WLS estimators were proposed by Swain et al. (1988) to estimate the parameters of Beta distributions. Suppose that $F(X_{(j)})$ denotes the distribution function of the ordered random variables $X_1, X_2, \ldots, X_n$ where $\{X_1, X_2, \ldots, X_n\}$ is a random sample of size n from the distribution function $F(\cdot)$. The EKK estimators can be obtained by minimizing Eq. (9):

$$S(x;\alpha,\theta) = \sum_{i=1}^{n}\left(F\left(X_{(i)}\right) - \frac{i}{n+1}\right)^2. \quad (9)$$

The EKK estimators of the unknown parameters of the UGR distribution can be obtained by minimizing Eq. (10):

$$S(x;\alpha,\theta) = \sum_{i=1}^{n}\left(\left\{1 - e^{-\theta\left(\frac{x_i}{1-x_i}\right)^2}\right\}^{\alpha} - \frac{i}{n+1}\right)^2. \quad (10)$$

WLS estimators can also be obtained by minimizing Eq. (11):

$$W(x;\alpha,\theta) = \sum_{i=1}^{n} w_i\left(F\left(X_{(i)}\right) - \frac{i}{n+1}\right)^2. \quad (11)$$

burada $w_i = \frac{(n+1)^2(n+2)}{i(n-i+1)}$. The WLS estimators of the unknown parameters of the UGR distribution can be obtained by minimizing Eq. (12):

$$W(x;\alpha,\theta) = \sum_{i=1}^{n} w_i\left(\left\{1 - e^{-\theta\left(\frac{x_i}{1-x_i}\right)^2}\right\}^{\alpha} - \frac{i}{n+1}\right)^2. \quad (12)$$

Anderson Darling Estimation

AD estimator was proposed by D'Agostino and Stephens (1986) and Luceno (2006). The AD estimator can be obtained by minimizing Eq. (13):

$$AD = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\left\{logF\left(X_{(i)}\right) + log\left(1 - F\left(X_{(n+1-i)}\right)\right)\right\}. \quad (13)$$

AD estimators of the unknown parameters of the UGR distribution can be obtained by minimizing Eq. (14):

$$AD = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\left\{log\left\{1 - e^{-\theta\left(\frac{x_i}{1-x_i}\right)^2}\right\}^{\alpha} + log\left(1 - \left\{1 - e^{-\theta\left(\frac{x_{(n+1-i)}}{1-x_{(n+1-i)}}\right)^2}\right\}^{\alpha}\right)\right\}. \quad (14)$$

RESULTS

In this section, a Monte Carlo simulation study is performed to compare the performance of ML, LS, WLS and AD estimators of the unknown parameters of the UGR distribution. All simulations were

performed using the Matlab programming language for 10,000 Monte Carlo runs. Sample sizes n=20, 50, 100 and parameter values (α,θ)=(0.8,0.4), (0.4,1.5). The performance of the estimators was compared according to the bias and mean square error (MSE) criteria. The results are given in Table 1 and Table 2.

Table 1. Bias and MSE values of estimators.

	(α,θ)=(0.8,0.4)			
	$\hat{\alpha}$		$\hat{\theta}$	
Estimators	Bias	MSE	Bias	MSE
n=20				
ML	0.1206	0.0946	0.0578	0.0251
LS	0.0346	0.1540	0.0064	0.0308
WLS	0.0465	0.1184	0.0152	0.0277
AD	0.0580	0.1073	0.0256	0.0272
n=50				
ML	0.0444	0.0272	0.0217	0.0078
LS	0.0144	0.0349	0.0042	0.0103
WLS	0.0198	0.0314	0.0076	0.0086
AD	0.0200	0.0282	0.0093	0.0081
n=100				
ML	0.0211	0.0119	0.0110	0.0034
LS	0.0055	0.0162	0.0010	0.0048
WLS	0.0109	0.0134	0.0055	0.0039
AD	0.0091	0.0124	0.0046	0.0037

Table 2. Bias and MSE values of estimators.

	(α,θ)=(0.4,1.5)			
	$\hat{\alpha}$		$\hat{\theta}$	
Estimators	Bias	MSE	Bias	MSE
n=20				
ML	0.0476	0.0165	0.3312	0.7825
LS	0.0121	0.0231	0.1163	1.0599
WLS	0.0161	0.0197	0.1113	0.8010
AD	0.0207	0.0177	0.1503	0.7931
n=50				
ML	0.0179	0.0053	0.1222	0.1911
LS	0.0039	0.0067	0.0346	0.2720
WLS	0.0071	0.0058	0.0481	0.2186
AD	0.0086	0.0055	0.0609	0.1983
n=100				
ML	0.0079	0.0023	0.0568	0.0803
LS	0.0017	0.0031	0.0142	0.1206
WLS	0.0045	0.0026	0.0303	0.0977
AD	0.0041	0.0025	0.0281	0.0910

- For all estimators, both the bias and MSE values decrease as the sample size increases from 20 to 100, indicating consistency and improved reliability of parameter estimation with larger data.
- Across both parameter settings, the AD estimator provides a good balance between low bias and low MSE, particularly in larger samples, making it a strong candidate for both α and θ parameter estimation.

- LS often yields the smallest bias values, especially for α; however, its MSE values are not consistently the lowest, suggesting that LS prioritizes bias reduction at the expense of increased variability in the estimates.
- For the estimation of θ, especially when the sample size is 50 or 100, ML and WLS methods tend to perform well in terms of MSE, highlighting their strength in predicting scale-related parameters under moderate-to-large sample sizes.

DISCUSSION AND CONCLUSION

In this study, the UGR distribution was examined in depth, with a focus on its key statistical properties and parameter estimation techniques. Four different estimation methods- MLE, LS, WLS, and AD-were applied to estimate the unknown parameters of the UGR distribution. A comprehensive Monte Carlo simulation was carried out to assess and compare the performance of these estimators under different parameter settings and sample sizes.

Simulation results revealed that all estimators improve in terms of bias and MSE as the sample size increases, demonstrating consistency. The LS estimator frequently produced the smallest bias values, especially for the shape parameter α, although its MSE was not always the lowest, indicating a trade-off between bias and variance. The AD method exhibited a balanced performance with relatively low bias and MSE values, particularly in moderate and large samples. Meanwhile, MLE and WLS were found to be effective for estimating the scale parameter θ, especially in larger sample sizes.

Overall, the findings suggest that no single estimation method dominates across all scenarios; however, AD and WLS offer reliable and stable performance, making them favorable choices for practical applications involving the UGR distribution.

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Conflict of Interest

If there is no conflict of interest of the authors, it should be written as "The authors have declared that there is no conflict of interest".

Author Contributions

Contributions of the authors to the study can be explained.

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A Computational Study on Sobol' Sequences (1116)

Bahri TOKMAK[1*], Ömür UGUR[2]

[1]Middle East Technical University, Institute of Applied Mathematics, Scientific Computing, Turkey
[2] Middle East Technical University, Institute of Applied Mathematics, Scientific Computing, Turkey

*Corresponding author e-mail: tokmakbahri@gmail.com

Abstract

This study presents a computational comparison between Quasi-Monte Carlo (QMC) methods based on Sobol' sequences and traditional Monte Carlo (MC) methods using the Mersenne Twister (MT) generator. While Sobol' sequences are widely recognized for outperforming MT in terms of convergence, our results reveal notable deficiencies when applied to high-dimensional Geometric Asian option pricing. To investigate this behavior, we conduct moment and correlation analyses, identifying a bias in the incremental construction of Sobol' paths—a bias that is absent in MT and can be alleviated through skipping initial points, scrambling, or Brownian Bridge (BB) techniques. All simulations are implemented in Python, with additional acceleration achieved through Graphics Processing Unit (GPU)-based parallel computing environments.

Keywords: *Quasi-Monte Carlo, Sobol' sequence, GPU acceleration, Brownian Bridge, Asian option*

INTRODUCTION

The motivation for this work stems from the complexities of pricing exotic derivatives under models demanding numerous time steps, thereby creating highly dimensional Brownian motion trajectories. In financial engineering, it is often impossible to derive closed-form solutions for the valuation of financial products, especially those categorized as exotic options. As a result, numerical techniques—and Monte Carlo (MC) simulation methods in particular—play a critical role. As discussed extensively by Glasserman (Glasserman, 2014), the appeal of Monte Carlo methods lies in their general applicability, especially in cases where analytical solutions are infeasible or unavailable. In MC simulations, the proper use of random number generators is vital, as incorrectly generated random sequences can lead to severe mispricing of financial products, potentially resulting in significant financial losses for institutions and individuals involved in their trading.

Traditional Monte Carlo method relies on pseudo-random number generators (PRNGs), such as the widely used Mersenne Twister, to simulate stochastic processes. However, the convergence rate of classical MC is typically $\mathcal{O}\left(\mathrm{N}^{-1/2}\right)$, where N is the number of samples. This achieving high accuracy requires generating millions of paths, which can be computationally intensive and slow to converge (Glasserman, 2014 and Jäckel, 2002).

Quasi-Monte Carlo (QMC) methods aim to improve upon this by replacing pseudo-random numbers with deterministic low-discrepancy sequences, such as Sobol' or Halton sequences. These sequences are designed to fill the simulation space more evenly, reducing clustering and improving convergence to approximately $\mathcal{O}\left(\frac{(\log N)^d}{N}\right)$ as stated in (Jäckel, 2002). In particular, Sobol' sequences have gained significant attention due to their simplicity, extensibility in high dimensions, and well-understood mathematical properties.

Despite their theoretical advantages, QMC methods are sensitive to implementation details. As noted by Glasserman (Glasserman, 2004), implementation choices such as scrambling—to improve uniformity and mitigate artifacts—and the ordering of dimensions can significantly influence the accuracy and stability of QMC results. Additionally, techniques like Brownian bridge (BB) construction are often employed in QMC to reorder variance allocation in time-dependent simulations, thereby improving efficiency for path-dependent derivatives.

This study provides a practical and empirical evaluation of Sobol' sequences in high-dimensional Monte Carlo simulations. We benchmark the performance of Sobol implementation against the Mersenne Twister, across a set of controlled experiments that include:

- One- and multi-dimensional integration tests
- Geometric Asian option pricing under a geometric Brownian motion model
- Moment and correlation analysis among Sobol' dimensions
- Effects of scrambling, skipping initial points, and Brownian bridge
- Graphics Processing Unit (GPU)-accelerated simulations

Our main objective is to determine under what conditions QMC methods, particularly Sobol' sequences, truly outperform traditional pseudo-random simulations in practice, and to investigate the performance gains offered by GPU-based parallel computation in high-dimensional Monte Carlo simulations.

BACKGROUND AND RELATED WORK

Monte Carlo Methods

Monte Carlo methods are widely used to approximate expectations by drawing random samples from a given probability distribution (Glasserman, 2004). These methods operate by repeatedly sampling and evaluating a function of interest, thereby estimating integrals or probabilities numerically. As the number of samples increases, the estimate converges to the true value by virtue of the law of large numbers. Furthermore, the central limit theorem enables quantification of the uncertainty in the estimate by providing asymptotic confidence intervals and standard errors.

To illustrate the method, let us use the example from Section 1.1.2 of (Glasserman, 2004). Consider the pricing of a European call option using the Black Scholes framework. The payoff of the option depends on the terminal value of the underlying stock, which follows a lognormal distribution as a result of modeling the asset price dynamics with a Geometric Brownian Motion (GBM). By generating standard normal samples, we can simulate the terminal stock price and compute the discounted payoff. The process is summarized in Algorithm 1 (see Figure 1), which shows the basic steps for estimating the expected present value of the option using Monte Carlo simulation. In this algorithm, r denotes the risk-free interest rate, σ is the volatility of the underlying asset, S_0 is the initial asset price, K is the strike price, T is the time to maturity, and $Z_i \sim \mathcal{N}(0,1)$ represents independent standard normal variables.

Algorithm 1 Monte Carlo estimation of expected present value of a European call option

1: **for** $i \leftarrow 1$ to n **do**
2: Generate $Z_i \sim \mathcal{N}(0,1)$
3: $S_i(T) \leftarrow S_0 \cdot \exp\left(\left(r - \frac{1}{2}\sigma^2\right)T + \sigma\sqrt{T} \cdot Z_i\right)$
4: $C_i \leftarrow e^{-rT} \cdot \max(S_i(T) - K, 0)$
5: **end for**
6: Estimate: $\hat{C}_n = \frac{1}{n}\sum_{i=1}^{n} C_i$

Figure 1. Monte Carlo estimation of a European call option

In classical MC simulation, PRNGs such as the widely used Mersenne Twister are employed to sample independent and identically distributed random variables from target distributions. Although these methods exhibit a convergence rate of $\mathcal{O}(N^{-1/2})$, their efficiency diminishes in problems that require high precision or suffer from the curse of dimensionality. See, for instance (Glasserman, 2004 and Jäckel, 2002).

To address these limitations, QMC methods utilize *low-discrepancy sequences (LDS)* to fill the integration domain more uniformly than pseudo-random samples. This deterministic structure can improve the convergence rate to $\mathcal{O}\left(\frac{(\log N)^d}{N}\right)$ for d-dimensional integrals under sufficient smoothness conditions (Glasserman, 2004, Jäckel, 2002 and Sobol' and Kucherenko, 2005). The discrepancy of a point set intuitively measures its deviation from uniformity, where lower discrepancy yields better space-filling.

Pseudo-Random Generators: Mersenne Twister

The Mersenne Twister is one of the most commonly used PRNGs due to its long period $2^{19937} - 1$, fast generation speed, and guaranteed equidistribution in at least 623 dimensions (Jäckel, 2002). While its output is not truly random, it produces sequences that behave uncorrelated and independent for practical purposes, making it suitable for many high-dimensional Monte Carlo simulations.

In comparative studies, MT serves as a baseline for measuring QMC effectiveness, particularly in terms of convergence rate, variance, and dimensional correlation.

Low-Discrepancy Sequences: Sobol'

One of the most widely adopted low-discrepancy sequences in QMC methods is the Sobol' sequence, originally introduced in (Sobol', 1967) by I. M. Sobol' in 1967 and further developed in 1976. See (Sobol', 1976). These sequences are constructed using *direction numbers* and *Gray code* ordering to ensure uniform coverage over the unit hypercube $[0,1)^d$. Due to their simplicity of construction, scalability to high dimensions, and successful applications in finance, Sobol' sequences have become a standard choice in QMC simulations (Jäckel, 2002).

Despite their advantages, the effectiveness of Sobol' sequences in high-dimensional settings is not always guaranteed. High-dimensional implementations, such as BRODA's Sobol65536 generator as documented in (Broda, 2025), aim to scale QMC techniques to tens of thousands of dimensions. However, empirical studies (Sobol', 1967 and Silva and Barbe, 2005) have observed issues such as dimensional correlation and loss of uniformity, which can negatively impact simulation accuracy. This is especially problematic in financial applications, where small deviations in distribution quality may lead to significant pricing and risk estimation errors.

Several practical enhancements have been proposed to improve the robustness of Sobol' sequences. Among these, three key factors are frequently highlighted:

- The use of *scrambling*, which introduces controlled randomness to eliminate structural artifacts and enable error estimation (Owen, 1998)
- Proper *dimensional ordering*, which assigns the most influential variables (if known) to the earliest dimensions, where Sobol' sequences typically exhibit better uniformity properties (Silva and Barbe, 2005)
- Careful treatment of the *initial points*, especially the very first point (typically $(0,0,\ldots,0)$ (Owen, 2021)

Scrambling was first introduced by Owen (Owen, 1998) to address the deterministic structure of low-discrepancy sequences, which limits error estimation through confidence intervals. In scrambling, the digits of the b-ary expansion of each point are randomly permuted in a recursive and structured manner. At the j-th digit, there are b^{j-1} partitions, each independently shuffled. This technique improves space-filling properties while retaining the low-discrepancy nature of the original sequence.

Another discussed topic in literature is the practice of skipping initial Sobol' points (also known as fast-forwarding), which involves discarding the first few values of the sequence—typically starting from the all-zero point $(0,0,\dots,0)$. This is often motivated by the observation that early Sobol' points can lead to undesirable properties, particularly when passed through nonlinear transformations such as the inverse normal CDF Φ^{-1}, to produce normally distributed variates. Under certain conditions, skipping has been shown to reduce numerical integration error, albeit without changing its asymptotic order (Radovic et al., 1996). However, the optimal number of points to skip is problem-dependent and cannot easily be determined. Moreover, as Owen (Owen, 2021) notes, indiscriminate skipping—especially when combined with scrambling—may disrupt the digital net structure of the sequence and lead to worse convergence behavior. As such, the handling of initial points demands careful consideration and should ideally be validated within the specific simulation context.

A more algorithmic enhancement that complements Sobol' sequences is the Brownian bridge construction, a technique designed to improve the dimensional efficiency of path simulations. Brownian Bridge reorders the discretization of stochastic processes to concentrate variance in the early time steps. This reordering aligns well with Sobol' sequences, whose early dimensions are more uniformly distributed, allowing the most critical components of the process to benefit from the best uniformity. As a result, Brownian bridge improves both dimensional ordering and convergence properties (Glasserman, 2004, Kucherenko and Shah, 2007 and Bianchetti et al., 2015).

Given the high computational demands of large-scale QMC simulations, leveraging parallel computing architectures has become essential. In particular, GPUs offer a powerful platform for accelerating simulations due to their high degree of parallelism. Recent work by Bernemann et al (Bernemann et al., 2011) demonstrated the feasibility and effectiveness of GPU-accelerated Sobol-based simulations for pricing exotic derivatives and performing calibration tasks.

In this study, we build upon these methodological advances by evaluating GPU-based implementations of Sobol' and Mersenne Twister generators. Our focus is on understanding both the theoretical conditions and practical configurations under which Sobol' sequences, possibly enhanced with Brownian Bridge and scrambling, can outperform traditional pseudo-random approaches. The ultimate goal is to achieve high numerical accuracy while benefiting from the significant speedups offered by parallel hardware.

METHODOLOGY

This section outlines the computational setup, simulation framework, and evaluation metrics used in this study to compare Sobol-based QMC methods with classical MC methods using PRNGs, specifically the Mersenne Twister.

Tools and Computational Environment

All experiments are implemented in Python 3.12 and executed on a machine equipped with an NVIDIA GPU (CUDA enabled). The following libraries are used for simulation and numerical computation:

- *scipy*: To generate low-discrepancy Sobol' sequences, with support for scrambling and skipping.

- *numpy*: For numerical operations and random number generation using Mersenne Twister.
- *cupy*: To enable GPU-accelerated vectorized simulation paths.
- *torch*: Used specifically to generate Sobol' sequences directly on the GPU, as CuPy does not provide native support for GPU-based quasi-random sequence generation

The *Sobol* class in Scipy incorporates several important features for high-dimensional QMC simulations. It supports up to 21201 dimensions by utilizing precomputed direction numbers generated by Kuo et al. (Joe and Kuo, 2008). This capability is particularly useful in financial applications involving long time horizons, such as Asian options with daily monitoring over extended periods.

The implementation also includes a two-stage scrambling technique (Matousek, 1998) to enhance uniformity and reduce structural artifacts in the sequence. Such scrambling consists of:

- *Left Linear Matrix Scrambling (LMS)*, where direction numbers are transformed via a non-singular lower-triangular matrix to maintain low discrepancy while improving uniformity.
- *Digital Random Shift*, which applies a uniform digital shift across all dimensions to introduce randomness.

By combining these features—extended dimensional support and scrambling techniques—Scipy Sobol implementation is well-suited for high-dimensional simulations in financial engineering applications involving complex payoff structures.

Experimental Design

To provide a fair comparison between PRNG-based Monte Carlo and Sobol-based QMC, each experiment follows the same simulation logic, differing only in the source of randomness and variance reduction techniques. The following factors are controlled across experiments:

- *Sample size:* Fixed to powers of two (e.g., 4096, $2^{19}, 2^{20}$) to suit Sobol's structure.
- *Dimensionality:* Ranges from 1D to 21201D depending on the test, constrained by the maximum dimensionality supported by Python. In practical scenarios such as Asian option pricing, the number of dimensions corresponds to the number of discretization steps over time (e.g., 3650 for 10 years of daily steps).
- *Skip values:* Various skip levels are tested (e.g., 1024, 2^{19}) to investigate convergence sensitivity.
- *Scrambling:* Tests are conducted to assess the impact of randomized Sobol' sequences.
- *Brownian Bridge:* Time steps are simulated using Brownian Bridge construction to reallocate variance into lower dimensions.
- *GPU usage:* High-dimensional path simulations and payoff evaluations are accelerated using GPU parallelization to reduce computation time.

Evaluation Metrics

To assess simulation accuracy and efficiency, the following quantitative metrics are recorded, where $\widehat{V}_i$ is the estimated value from trial i, and V_{true} is the known theoretical value (e.g., for Asian options), and N is the sample size (or, number of replications):

$$\text{Mean Absolute Error (MAE)}: = \frac{1}{N}\sum_{i=1}^{N}\left|\widehat{V}_i - V_{\text{true}}\right| \tag{1}$$

$$\text{Maximum Absolute Error (MaxAE)} = \max_{1\le i\le N}\left|\widehat{V}_i - V_{\text{true}}\right| \tag{2}$$

$$\text{Mean Error (Bias)} = \frac{1}{N}\sum_{i=1}^{N}\left(\widehat{V}_i - V_{\text{true}}\right) \tag{3}$$

The elapsed time is computed using Python's time.time() function before and after simulation blocks. We remark that in high-dimensional tests, correlation matrices and variance of intermediate quantities are also recorded to study internal simulation stability. By standardizing the above setup, we ensure reproducibility and comparability across different random number sources, variance reduction techniques, and hardware acceleration strategies.

INTEGRAL TEST

To systematically evaluate the numerical accuracy of QMC and traditional MC methods, we conduct a series of one-dimensional and multi-dimensional integration experiments.

One-Dimensional Integral Test

To assess the basic numerical behavior of QMC methods in low-dimensional settings, we begin by evaluating a classical family of one-dimensional integrals of the form

$$(n+1)\int_0^1 x^n\, dx = 1, \quad n = 1,2,\dots,20 \tag{4}$$

for which the analytical value is known to be exactly one for all integer values of n. This benchmark serves as a reliable testbed for quantifying integration error and convergence characteristics of different sampling methods, including pseudo-random sequences and low-discrepancy sequences such as Sobol'.

Each integral is approximated numerically using 4096 sample points. The performance of both methods is compared by computing the absolute error between the estimated and exact values across the tested range of n. For each n, the maximum and mean absolute errors are recorded in Table 1.

Table 1. Absolute errors for one-dimensional monomial integrals using Sobol and MT

n	Sobol Max	Sobol Mean	MT Max	MT Mean
1	0.00018	0.00012	0.03549	0.00716
2	0.00028	0.00018	0.05918	0.01114
3	0.00037	0.00024	0.07578	0.01416
4	0.00046	0.00031	0.08617	0.01666
5	0.00055	0.00037	0.09518	0.01884
6	0.00064	0.00043	0.10459	0.02079
7	0.00073	0.00049	0.11263	0.02256
8	0.00083	0.00055	0.11959	0.02421
9	0.00092	0.00061	0.12588	0.02574
10	0.00101	0.00067	0.13414	0.02720
11	0.00110	0.00073	0.14182	0.02858
12	0.00119	0.00079	0.14896	0.02991
13	0.00128	0.00085	0.15560	0.03117
14	0.00138	0.00092	0.16178	0.03239
15	0.00147	0.00098	0.16757	0.03357
16	0.00156	0.00104	0.17320	0.03470
17	0.00165	0.00110	0.17849	0.03580
18	0.00174	0.00116	0.18347	0.03687
19	0.00183	0.00122	0.18818	0.03791
20	0.00193	0.00128	0.19262	0.03892

The results demonstrate that Sobol' sequences yield significantly lower integration error compared to Mersenne Twister across all values of n. In particular, the maximum and mean absolute errors associated with Sobol' are consistently an order of magnitude smaller than those of MT, especially for higher-degree monomials as clearly seen in Table 1. These findings reinforce the established theoretical understanding that low-discrepancy sequences exhibit superior performance in deterministic numerical integration, especially in low-dimensional settings.

Multi-Dimensional Integral Test

To assess the accuracy and robustness of QMC methods in higher dimensions, we conducted a series of experiments based on the family of multi-dimensional integrals given in Equation 5:

$$\int_0^1 \cdots \int_0^1 \prod_{i=j}^{j+n-1} \left(1 + c_i \cdot (x_i - 0.5)\right) dx_j \cdots dx_{j+n-1} = 1 \tag{5}$$

evaluated for various values of j, n, and constant coefficients c_i The integral is designed to emphasize the role of boundary sampling: the larger the values of c_i, the more weight is placed on the integrand near the edges of the $[0,1]^n$ hypercube. This makes it an ideal stress test for comparing Sobol' sequences to MT, especially in terms of their ability to adequately capture extreme regions of the domain.

We tested several values of n ranging from 3 to 21,201 dimensions. In the small and moderate dimensional cases ($n < 21201$), we employed a sliding window approach over a large Sobol' matrix of shape (4096×21201): for each offset j, we extracted n consecutive dimensions, computed the numerical integral, and recorded the corresponding absolute error. In the full-dimensional case ($n = 21201$), a batch-based method is adopted. We performed 100 independent simulations of 4096 Sobol' samples, each preceded by a skip of $j \cdot 4096$, and reported the maximum and average absolute error across batches.

All Sobol' sequences are generated with scrambling disabled and appropriate skip values. Pseudo-random samples are initialized with batch-specific seeds to ensure independence. Each integrand is evaluated, having fixed $c_i = c$, as:

$$f(x) = \prod_{i=1}^{n} \left(1 + c \cdot (x_i - 0.5)\right) \tag{6}$$

The results are summarized in Table 2, which reports the observed maximum and mean absolute errors for both Sobol and Mersenne Twister methods across different configurations.

Table 2. Multi-dimensional integral errors for various (n, c) combinations

n and c	Sobol Max Error	Sobol Mean Error	MT Max Error	MT Mean Error
3 and 0.5	0.0157	0.00023	0.0153	0.0031
5 and 0.5	0.0192	0.00047	0.0218	0.0041
5 and 0.5 (skip 2^{19})	0.0192	0.00027	0.0218	0.0041
5 and 0.3	0.0068	0.00023	0.0132	0.0024
30 and 0.3	0.0258	0.00263	0.0334	0.0063
1000 and 0.01	0.0021	0.00073	0.0042	0.0014
21201 and 0.0002	0.000068	0.000004	0.0004	0.0001

These results clearly demonstrate the advantage of Sobol' sequences, particularly as dimensionality increases and the coefficient c decreases. In low-dimensional settings, both methods perform reasonably

well, although Sobol still shows a lower mean absolute error. However, in high dimensions—especially with $n = 21201$ and $c = 0.0002$—Sobol significantly outperforms Mersenne Twister, with an error reduction factor exceeding 25 times.

An additional experiment applying a large skip of 2^{19} in the $n = 5,\ c = 0.5$ case also confirmed that skipping early Sobol' points leads to meaningful error reduction (from 0.00047 to 0.00027 in average error), without affecting the maximum deviation. See Table 2, thereby, we may claim that skipping can be beneficial for high-accuracy results.

Overall, these findings validate the superiority of Sobol' sequences equily well also in high-dimensional numerical integration.

GEOMETRIC ASIAN OPTION PRICING

To investigate the comparative performance of QMC methods and classical MC methods in a realistic financial context, we conducted a series of experiments pricing a geometric Asian call option under a Geometric Brownian Motion (GBM) framework: Black-Scholes setting.

The underlying asset price S_t is modeled according to the classical GBM process, defined by the stochastic differential equation:

$$dS_t = rS_t\,dt + \sigma S_t\,dW_t \tag{7}$$

where r denotes the constant risk-free rate, σ is the constant volatility of the asset, and W_t represents a standard Brownian motion. Discretization is performed using a log-Euler scheme:

$$\ln S_{t_i} = \ln S_{t_{i-1}} + \left(r - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}\cdot z_i \tag{8}$$

where $z_i \sim \mathcal{N}(0,1)$ are independent standard normal variates generated via either pseudo-random or quasi-random sequences.

The geometric Asian call option payoff is based on the geometric mean of asset prices over discrete monitoring dates:

$$\text{Payoff} = \max(\text{GA} - K, 0)\,, \quad \text{where} \quad \text{GA} = \left(\prod_{i=1}^{n} S_{t_i}\right)^{1/n} \tag{9}$$

with K denoting the strike price. In our experimental setup, we set $S_0 = 1,\ r = 0.05,\ \sigma = 0.4$ and $K = 1$. The maturity T corresponds to 21201 timesteps, equivalent to 10 monitoring points per day over approximately 5.8085 years:

$$t_i = \frac{i}{365 \times 10}, \quad i = 1, \ldots, 21201 \tag{10}$$

The theoretical value of the geometric Asian option is available in closed form. It is computed using an adjusted volatility and maturity:

$$\sigma_{\text{adj}} = \frac{\sigma}{\sqrt{3}}, \quad T_{\text{adj}} = \frac{T(r + \sigma^2/6)}{2}, \quad K_{\text{adj}} = Ke^{T_{\text{adj}}} \tag{11}$$

yielding the option price:

$$\text{Price} = e^{-T_{\text{adj}}} \cdot \text{BlackScholes}\left(S_0, K_{\text{adj}}, r, \sigma_{\text{adj}}, T_{\text{adj}}\right) \tag{12}$$

where the Black-Scholes pricing formula for European call options (Black and Scholes, 1973) is employed:

$$\text{BlackScholes}(S_0, K, r, \sigma, T) = S_0\Phi(d_1) - Ke^{-rT}\Phi(d_2), \quad (13)$$

$$d_1 = \frac{\ln(S_0/K) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Here, $\Phi(\cdot)$ denotes the cumulative distribution function (CDF) of the standard normal distribution.

For the option under consideration the theoretical formula returns 0.19715. Simulations are conducted using two approaches: Sobol' sequences, and Mersenne Twister based pseudo-random numbers. Each simulation consists of 16 independent trials, each using 4096 paths, leading to a total of 65536 simulated paths per method.

The Mersenne Twister results yields a mean estimated option price of 0.19575, compared to the theoretical price of 0.19715. The average absolute error across trials is 0.00597, with a maximum absolute error of 0.01702. The variance of the estimated prices is 0.00005398, corresponding to a standard deviation of 0.00735. These results indicate a relatively tight distribution of price estimates around the theoretical value.

In contrast, simulations using the Sobol' sequence produced a mean estimated option price of 0.23363, which deviates from the theoretical price of 0.19715 by an absolute difference of 0.03648. Despite using a large number of simulated paths (65536), this level of discrepancy is considered suboptimal, as such sample sizes are typically expected to yield tighter convergence. The average absolute error across trials is 0.04617, and the maximum absolute error reaches to 0.33548. The price estimates shows a variance of 0.00755007 and a standard deviation of 0.08689. Furthermore, the observed minimum and maximum prices (0.18357 and 0.53263, respectively) indicate a much wider dispersion compared to the Mersenne Twister results, reflecting greater instability and less reliable convergence behavior.

This discrepancy in performance prompted a deeper investigation into the underlying causes of the observed deficiencies in the Sobol' sequence simulations. To this end, the next section conducts a series of statistical analyses aimed at diagnosing potential issues inherent to the structure of Sobol-generated paths. In particular, we focus on two key diagnostic tools: moment analysis, to evaluate the marginal distributions of individual Sobol' dimensions, and correlation analysis, to detect any unexpected dependencies between dimensions that could impair the sequence's uniformity and effectiveness.

ROOT CAUSE ANALYSIS OF THE OBSERVED DISCREPANCIES

Moment Analysis of Sobol' Sequences

In order to better understand the statistical properties of Sobol' sequences in high-dimensional simulations, we perform a detailed investigation of the first four moments --- mean, variance, skewness, and kurtosis --- of the sequences. Two cases are analyzed: the raw Sobol' sequences on $[0,1)$, and their transformation into standard normal space via the inverse cumulative distribution function (ICDF), Φ^{-1}.

We generated samples at sizes 2^{12}, 2^{16}, $2^{20}, 2^{24}$ and 2^{28}, corresponding to 4096, 65536, 1048576, 16777216, and 268435456 samples, respectively.

For the raw Sobol' samples (i.e., before applying ICDF), the results are summarized below and depicted in Table 3.

- The sample mean converges to 0.5 as the number of samples increases, aligning with the expected value for a uniform distribution.
- The sample variance also converges to $\frac{1}{12} \approx 0.08333333$, the theoretical variance of the

uniform distribution.

- The skewness remaines effectively zero across all sample sizes, indicating symmetric distributions.
- The kurtosis stabilizeds at 1.8, as theoretically expected for uniform random variables.

Table 3. Moments of Raw Sobol' Sequence $([0,1))$

n_{samples}	Mean	Variance	Skewness	Kurtosis
2^{12} (4096)	0.499969	0.08333329	1.06×10^{-10}	1.79999865
2^{16} (65536)	0.499992	0.08333333	6.29×10^{-12}	1.79999999
2^{20} (1048576)	0.499999	0.08333333	2.74×10^{-14}	1.8
2^{24} (16777216)	0.499999	0.08333333	1.08×10^{-16}	1.8
2^{28} (268435456)	0.5	0.08333333	-2.77×10^{-19}	1.8

After applying the inverse cumulative normal function Φ^{-1} to transform the Sobol' points to standard normal deviates, the moments shown in Table 4 are observed to behave as follows:

- The mean converges rapidly towards 0, as expected for a standard normal distribution.
- The variance converges to 1, as theoretically expected.
- The skewness remaines near 0, indicating symmetry of the resulting normal distributions.
- The kurtosis approaches 3, matching the kurtosis of a standard normal distribution.

Table 4. Moments of Transformed Sobol' Sequence $(\mathcal{N}(0,1))$

n_{samples}	Mean	Variance	Skewness	Kurtosis
2^{12} (4096)	-2.23×10^{-4}	0.99908158	−0.002273	2.97942869
2^{16} (65536)	-5.99×10^{-5}	0.99993473	−0.000829	2.99790646
2^{20} (1048576)	-4.45×10^{-6}	0.99999588	-8.43×10^{-5}	2.9998241
2^{24} (16777216)	-3.11×10^{-7}	0.99999974	-7.54×10^{-6}	2.99998616
2^{28} (268435456)	-2.13×10^{-8}	0.99999998	-6.30×10^{-7}	2.99999896

These empirical findings align with the theoretical properties established in the literature.

Correlation Analysis in Sobol' Sequences

It is well-known that Sobol' sequences can exhibit substantial correlations between different dimensions, meaning that the generated quasi-random numbers in different dimensions are correlated. In the context of this study, references to inter-dimensional correlation specifically refer to the correlation between the quasi-random numbers assigned to different dimensions. This phenomenon is already mentioned in the early works on low-discrepancy sequences (Sobol', 1967). In (Sobol' et al., 2012), it is noted that, for a particular implementation of Sobol' numbers, "a test done with 2500 dimensions showed that 2449 pairs of consecutive dimensions have correlation greater than 70% (in absolute value)."

In our study, with 21201 dimensions and 4096 samples, we perform a similar analysis. Among the $\frac{21201 \times 21200}{2} = 224{,}730{,}600$ possible distinct dimension pairs, we observe that 415,391 pairs exhibit a correlation greater than 0.1, corresponding to approximately 0.18% of all pairs. Furthermore, 91,668 pairs show correlations exceeding 0.5, and 91,631 pairs exceed 0.6, both representing approximately 0.04% of all possible pairs. When considering even stronger correlations, 91,493 pairs show correlations greater than 0.7, but this number dropped sharply to only 110 pairs when the threshold is raised to 0.8, accounting for a negligible fraction of 0.00005% and highlighting that while moderate correlations are

relatively common, extremely strong correlations remain very rare. Specifically among consecutive dimensions, we identified 10 pairs with correlation above 0.7. This is in stark contrast to the findings of (Sobol' et al., 2011), where nearly all consecutive pairs in their test showed high correlation—highlighting the improved behavior of the *Scipy Sobol* implementation in high-dimensional settings.

This strong correlation behavior in Sobol' sequences contrasts sharply with the performance of pseudo-random generators such as Mersenne Twister. In a comparable experiment using Mersenne Twister with 4096 samples across 21201 dimensions, the maximum absolute correlation observed is only 0.091 (between dimensions 15689 and 15966).

Additionally, it is important to highlight that the magnitude of these correlations in Sobol' sequences diminishes significantly as the number of samples increases. For instance, considering the dimension pair (1229, 6014), the correlation is $\rho = 0.9375$ with 4096 samples but drops dramatically to $\rho = 0.0249$ when the number of samples are increased to 20000. This behavior is consistent with the theoretical expectations regarding the asymptotic behavior of Sobol' sequences.

Some representative scatter plots of correlated dimension pairs are shown in Figure 2. These visualizations illustrate how severe clustering can occur when the random variables associated with different dimensions are strongly correlated.

Following the application of the inverse standard normal transformation to the Sobol' samples, we performed a similar correlation analysis. Among the 224,730,600 possible distinct dimension pairs, 847,904 pairs exhibit correlations greater than 0.1, corresponding to approximately 0.38% of all pairs. Furthermore, 91,668 pairs show correlations exceeding 0.5, while 91,394 pairs exceed 0.6, both representing approximately 0.04% of all possible pairs and indicating that moderate levels of dependency remain relatively prevalent. However, as the correlation threshold is increased beyond 0.6, a sharp decline is observed: only 189 pairs exhibited correlations greater than 0.7, accounting for a minuscule proportion of approximately 0.00008% of all pairs. This sharp drop between 0.6 and 0.7 thresholds differs slightly from the pattern observed before the transformation, where a comparable decline is only observed between 0.7 and 0.8 thresholds. These findings suggest that the inverse transformation modifies the underlying dependency structure of Sobol' sequences, effectively reducing the prevalence of very high correlations (greater than 0.7) and shifting dependencies into the correlation range (above 0.6).

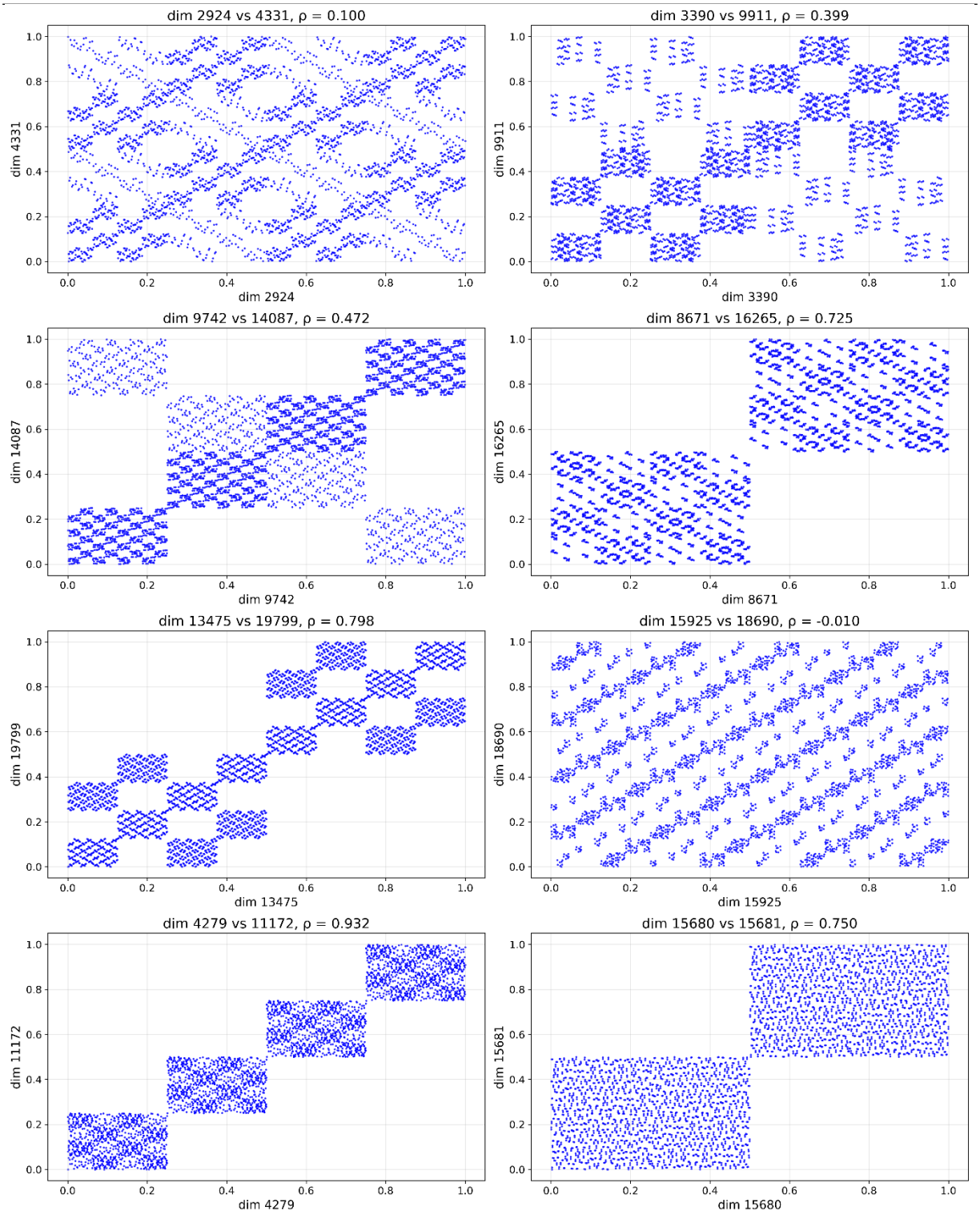

Figure 2. Scatter plots of selected dimension pairs from Sobol' sequence.

Variance Bias in Incremental Construction

In classical Monte Carlo simulation of Brownian motion, the standard discretization method constructs the terminal value W_T by sequentially summing small increments generated from independent Gaussian variables. This method, commonly referred to as the incremental construction, approximates W_T as:

$$W_T^I = \sum_{i=1}^{N} \sqrt{dt}\ z_i \tag{14}$$

where $z_i \sim \mathcal{N}(0,1)$ are independent standard Gaussian random variables, N is the number of time steps, and $dt = T/N$.

In an ideal Monte Carlo settings, if the z_i are truly independent, the following properties would hold:

$$E[z_i] = 0, \quad \forall i \tag{15}$$

$$\mathrm{V}[z_i] = 1, \quad \forall i \tag{16}$$

$$E[z_i z_j] = 0, \quad \text{for } i \neq j \tag{17}$$

where $(E[\cdot])$ and $(V[\cdot])$ denote the expectation and variance operators, respectively. Consequently, the mean and variance of (W_T^I) satisfy:

$$\mathrm{E}[W_T^I] = 0 \tag{18}$$

$$V[W_T^I] = T \tag{19}$$

However, when quasi-random sequences like Sobol' numbers are used to generate z_i via the inverse normal CDF, strict independence between coordinates is no longer guaranteed. Particularly at small sample sizes or high dimensions, residual correlations between z_i and z_j $(i \neq j)$ can introduce a bias into the variance of W_T^I.

To quantify this effect, assume that the empirical variance of each z_i is $v \lesssim 1$, and that the empirical correlation between different dimensions is $\rho_{ij} = E[z_i z_j] - E[z_i]E[z_j]$. Then, the variance of W_T^I becomes:

$$V[W_T^I] = \sum_{i=1}^{N} dt\ V[z_i] + 2\sum_{i<j} dt\ \rho_{ij} \tag{20}$$

$$= T\left(v + \frac{2}{N}\sum_{i<j}\rho_{ij}\right) \tag{21}$$

We define the variance distortion term C induced by the correlations as:

$$C = \frac{2}{N}\sum_{i<j}\rho_{ij} \tag{22}$$

such that the total variance is expressed as $T(v + C)$.

To complement the theoretical analysis, we computed the variance v, the term C caused by inter-dimensional correlations, and the total variance $v + C$ for both Sobol' and Mersenne Twister sequences across different dimensions. The sample size is fixed at 4096 simulations. The tested dimensionalities includes n_{dims} = 100, 500, 1000, 2000, and 21201, covering a wide range from low to extremely high dimensions. In Table 5, the empirical results are summarized. As observed, the Sobol' sequences exhibit significant deviations in the $v + C$ values, particularly for high dimensions such as $n_{\text{dims}} = 21201$, where $v + C$ exceeds 4.0. This reflects the impact of dimension-dependent correlations inherent to Sobol' sequences. In contrast, Mersenne Twister sequences show relatively stable behavior, with $v + C$ values remaining close to 1 even at high dimensions, confirming their low-correlation pseudo-random nature.

Table 5. Variance and variance distortion for Sobol' and Mersenne Twister sequences at $n_{\text{samples}} = 4096$

Method	n_{dims}	v	C	$v + C$
Sobol	100	0.999275	-0.044254	0.955021
Sobol	500	0.999275	-0.141961	0.857314
Sobol	1000	0.999278	-0.308172	0.691105
Sobol	2000	0.999277	-0.386930	0.612347
Sobol	21201	0.999282	3.061985	4.061268
MT	100	0.998997	-0.013125	0.985872
MT	500	0.998894	0.014969	1.013864
MT	1000	0.998440	-0.003114	0.995326
MT	2000	0.998587	-0.018400	0.980186
MT	21201	0.999428	0.001260	1.000688

Effect of Scrambling, Skipping, and Brownian Bridge on Variance Stability

In the previous sections, we observed that in the absence of any enhancements, Sobol' sequences exhibited significant deterioration in the total variance $v + C$ as the dimensionality increased. In this section, we explore how three different techniques — scrambling, skipping initial points, and Brownian Bridge construction — affect the variance stability.

Variance Stability under Scrambling

When scrambling is enabled, Sobol' sequences show a remarkable improvement in variance properties. As shown in Table 6, even for $n_{\text{dims}} = 21201$, the total variance $v + C$ is corrected to approximately 1.0025. Across all tested dimensions (n_{dims} = 100, 500, 1000, 2000, 21201), the deviations of $v + C$ from 1 remain within ± 0.006 margins, indicating stability.

Thus, the results observed in our tests validate the theoretical expectations: scrambling substantially improves the variance stability of Sobol' sequences, making them highly reliable even when applied to very high-dimensional integration problems.

Variance Stability under Initial Points Skipping

Applying a large skip of $2^{19} = 524{,}288$ points (without scrambling) results in a notable improvement in the stability of total variance estimates. For lower dimensions (n_{dims} = 100, 500), the total variance $v + C$ remains very close to the theoretical value of 1, with deviations on the order of 10^{-3} or less. Even in extremely high-dimensional settings, such as n_{dims} = 21201, the total variance remains stable around 0.9934 (see Table 6), representing a substantial improvement over the non-skipped case.

These results suggest that skipping a substantial number of initial Sobol' points can also substantially improve numerical behavior across a wide range of dimensions. In our tests, skipping is consistently observed to have a positive impact on stability, supporting the understanding that Sobol' sequences tend to perform better when an initial portion of the sequence is discarded.

Several studies, including Owen (Owen, 2021), have pointed out that skipping initial points in *scrambled* Sobol' sequences may disrupt the underlying randomized net structure, potentially degrading convergence. While our findings demonstrate the practical benefits of skipping in *non-scrambled* settings, further research is needed to better understand how skipping interacts with scrambling in different simulation contexts, particularly with respect to determining how many points should be skipped, which may vary significantly depending on the integrand, dimensionality, and target accuracy.

Variance Stability under Brownian Bridge Construction

Brownian bridge (BB) construction is a widely used technique to improve the efficiency of path generation in Monte Carlo and Quasi-Monte Carlo simulations. By carefully redistributing the variance contributions across dimensions, BB can significantly stabilize the numerical behavior, particularly in high-dimensional settings.

Consider a standard Brownian motion process $W(t)$ over the interval $[0, T]$, with fixed endpoints:

- $W(0) = 0$
- $W_T^{\mathrm{BB}} = \sqrt{T} z_1$, where $z_1 \sim \mathcal{N}(0,1)$

It follows that $E\left(W_T^{\mathrm{BB}}\right) = 0$ and $V\left(W_T^{\mathrm{BB}}\right) = T$ since $E(z_1) = 0$ and $V(z_1) = 1$.

Intermediate values of the Brownian motion are not generated sequentially in time. Instead, the process recursively fills in midpoints of the largest remaining intervals. Suppose we already have values at two time points T_i and T_{i+1}, and we want to generate the value at a time $t \in (T_i, T_{i+1})$. This is done using:

$$W(t) = \frac{T_{i+1} - t}{T_{i+1} - T_i} W(T_i) + \frac{t - T_i}{T_{i+1} - T_i} W(T_{i+1}) + \sqrt{\frac{(T_{i+1} - t)(t - T_i)}{T_{i+1} - T_i}} \cdot z_k \qquad (23)$$

where each $z_k \sim \mathcal{N}(0,1)$ is an independent standard normal random variable.

The first two terms represent a linear interpolation, while the third term introduces stochasticity consistent with Brownian motion's properties. In theory, when ideal random numbers are used, the Brownian Bridge construction should maintain stability across dimensions without degradation as dimension increases. Minor deviations from the ideal variance are primarily attributed to residual correlations among quasi-random samples. That is, while Brownian bridge organizes the variance efficiently, quasi-random sequences like Sobol' are not perfectly independent across dimensions, and small correlations can slightly affect variance. See (Sobol' and Kucherenko, 2005, Kucherenko and Shah, 2007 and Bianchetti et al., 2015) for more information.

In our numerical experiments, we observe that when Brownian Bridge construction is applied to Sobol' sequences, the total variance $v + C$ remains very close to 1 across all dimensions (see Table 6). Even at $n_{\mathrm{dims}} = 21201$, the total variance is approximately 0.9989, reflecting agreement with theoretical expectations. These results validate that Brownian Bridge dramatically improves variance stability in high-dimensional settings, and small observed deviations are consistent with the minor correlation effects inherent to quasi-random sequences rather than flaws in the Brownian Bridge algorithm itself.

Table 6. Summary of Total Variance $v + C$ Under Different Enhancements

Method	**Scrambling Enabled**			
	Dimensions	**Variance**	**Distortion**	**Total**
Sobol	100	1.000169	0.002468	1.002636
Sobol	500	0.999994	-0.015104	0.984890
Sobol	1000	0.999974	0.006229	1.006202
Sobol	2000	1.000021	0.003872	1.003893
Sobol	21201	0.999992	0.002492	1.002484

Table 6. Continued

Method	**Skip = 2^{19} (524288 points)**			
	Dimensions	**Variance**	**Dimensions**	**Total**
Sobol	100	1.000097	0.001626	1.001723

Sobol	500	1.000026	-0.000100	0.999926
Sobol	1000	1.000002	-0.004902	0.995100
Sobol	2000	1.000004	-0.014498	0.985507
Sobol	21201	0.999997	-0.006642	0.993355
Method	**Brownian Bridge Enabled**			
	Dimensions	**Variance**	**Distortion**	**Total**
Sobol	100	0.999275	-0.000388	0.998886
Sobol	500	0.999275	-0.000234	0.999040
Sobol	1000	0.999278	0.000012	0.999290
Sobol	2000	0.999277	-0.000324	0.998953
Sobol	21201	0.999282	-0.000391	0.998892

EFFECT OF SCRAMBLING, SKIPPING, AND BROWNIAN BRIDGE ON ASIAN OPTION PRICING

To evaluate the practical impact of various Sobol' sequence enhancement techniques on financial simulations, we conducted a series of controlled experiments focusing on the pricing of a Geometric Asian option. The theoretical price, previously established in Section "geometric asian option pricing", serves as a benchmark for assessing pricing accuracy.

In this study, we specifically tested the effects of scrambling, initial skipping, and Brownian bridge construction. Four different Sobol configurations are examined:

- *Baseline*: No scrambling, no Brownian Bridge.
- *Scrambled Sobol*: Scrambling enabled, no Brownian Bridge.
- *Brownian Bridge*: No scrambling, Brownian Bridge enabled.
- *Large Skip Only*: Skip = 2^{19} (524,288 points), No scrambling, no Brownian Bridge.

Each configuration, except the initial skip case, is evaluated over 16 independent trials with 4096 paths per trial to ensure statistical consistency. The initial skip case configuration used a single batch of 4096 paths without repetition across trials.

The results, summarized in Table 7, reveal clear and consistent trends regarding the effectiveness of these enhancement techniques. Additionally, results for the standard Mersenne Twister simulation have been included in the table to provide reference for comparison with theoretical MC.

The baseline configuration, without scrambling or Brownian bridge, performs poorly: the mean estimated price is 0.23363, deviating significantly from the theoretical value. The maximum absolute error reaches to 0.33548, and the variance across trials is as high as 0.00755. These large errors are primarily attributed to structural artifacts and correlations inherent in the raw Sobol' sequence.

Enabling scrambling leads to a substantial improvement. The mean price converges to 0.19775, with a maximum error of only 0.00908, and The variance is reduced significantly, reaching approximately to 1.542×10^{-5}. This supports theoretical findings that scrambling improves uniformity, reduces bias, and allows for effective variance estimation.

Applying Brownian bridge construction without scrambling further enhances performance. The mean price achieved is 0.19711, with a maximum absolute error of just 0.00121, and the variance is reduced to 2.4×10^{-7}. This dramatic variance stabilization is consistent with the theoretical expectation that Brownian bridge reallocates variance contributions, improving the convergence behavior, particularly in high-dimensional settings.

Finally, applying a large skip of 2^{19} points—without scrambling or Brownian bridge—also produces notably accurate results. The mean price is 0.19608, and the maximum absolute error is just 0.00107, indicating a meaningful reduction in simulation bias. While skipping alone may not match the variance stabilization achieved by Brownian bridge, it remains an effective and simple strategy for improving coverage and reducing structural artifacts in Sobol'-based simulations.

Table 7. Comparison of Different Configurations in Geometric Asian Option Pricing

Configuration	Mean Price	Avg Error	Max Error	Variance
MT	0.19575	0.00597	0.01702	0.00005398
No Scramble, No BB	0.23363	0.04617	0.33548	0.00755007
Scramble = True, No BB	0.19775	0.00319	0.00908	0.00001542
BB = True, No Scramble	0.19711	0.00041	0.00121	0.00000024
Skip = 2^{19} (1 batch)	0.19608	0.00107	0.00107	N/A

In summary, our experiments demonstrate that proper configuration of Sobol' sequences is crucial for achieving high precision in QMC simulations in finance. Scrambling significantly reduces bias and variance, while Brownian Bridge construction further stabilizes variance by optimizing the allocation of variability across dimensions. Initial skipping alone can offer measurable improvements, but the best results are achieved when Brownian Bridge technique are employed. These findings reinforce the necessity of combining enhancement strategies to fully exploit the potential of quasi-Monte Carlo methods in high-dimensional option pricing problems.

GPU-ACCELERATED SIMULATIONS

The use of GPUs in computational finance has been extensively explored in the literatüre (Dempster et al., 2018). The inherently parallel structure of Monte Carlo simulations for path-dependent option pricing makes them well-suited for GPU-enabled parallel computing frameworks like CUDA. In our study, we harness the parallel processing power of GPUs to enhance the speed and efficiency of option pricing computations.

In our implementation, we primarily utilize the CuPy library to perform all array operations, random number transformations, and Monte Carlo path simulations on the GPU. CuPy provides a highly efficient, NumPy-compatible interface that allows straightforward migration of CPU-based codes to CUDA-enabled devices with minimal adjustments. PyTorch is employed exclusively for generating Sobol' sequences directly on the GPU, as CuPy currently lacks a native GPU-based Sobol generator. By using PyTorch's *Sobol Engine* for quasi-random number generation and relying on CuPy for the remaining computational tasks, we combine the strengths of both libraries to maximize performance and maintain numerical accuracy in high-dimensional Monte Carlo simulations.

This section presents a comparative evaluation of GPU-accelerated Monte Carlo and Quasi-Monte Carlo simulations in the context of pricing Geometric Asian options (BS framework). To assess the impact of GPU acceleration, we conducted a series of controlled experiments focusing on runtime performance. Specifically, we benchmarked CPU-based versus GPU-based implementations across three scenarios: Mersenne Twister random number generation, Sobol' sequence generation, and Sobol' sequence generation combined with Brownian Bridge construction. These experiments are designed to isolate and quantify the computational advantages offered by GPU parallelization while keeping the pricing methodology and simulation parameters consistent. In all cases, CPU implementations are executed serially and serve as a baseline for evaluating the speedup and efficiency improvements achieved through GPU parallelization.

Algorithm 2 (see Figure 3) models the core simulation loop for Monte Carlo and Quasi-Monte Carlo simulations, where random numbers are either sampled from a standard normal distribution (for Mersenne Twister) or generated via Sobol' sequences followed by an inverse transformation. Each path is constructed through cumulative summation of the simulated increments and subsequently used to compute the option payoff. Algorithm 3 (see Figure 4) specifically describes the construction of Brownian Bridge increments using Sobol' sequences, to allocate early Sobol' dimensions to the most critical parts of the simulated Brownian motion path.

Algorithm 2 Per-thread vectorized path simulation using either Mersenne Twister or Sobol' sequences

$dt \leftarrow T/N$
$drift \leftarrow (r - 0.5\sigma^2) \cdot dt$
$vol \leftarrow \sigma \cdot \sqrt{dt}$
if Generator = Mersenne Twister **then**
    $Z \leftarrow$ StandardNormal[PATHS][N] ▷ Generated using `cupy.random`
else if Generator = Sobol **then**
    $U \leftarrow$ Sobol[PATHS][N] ▷ From `torch.quasirandom.SobolEngine`
    $Z \leftarrow \sqrt{2} \cdot \texttt{erfinv}(2U - 1)$ ▷ Inverse CDF transform
end if
$increments \leftarrow drift + vol \cdot Z$ ▷ Element-wise vectorized operation
$log\_paths \leftarrow \texttt{cumsum}(increments, \text{axis} = 1)$ ▷ Sequential in time, parallel across paths
$log\_paths \leftarrow \texttt{concat}([0], log\_paths)$ ▷ Insert $W_0 = 0$
$S \leftarrow S_0 \cdot \exp(log\_paths)$ ▷ Path prices
$GA \leftarrow \exp\left(\frac{1}{N}\sum_{i=1}^{N} \log S_i\right)$ ▷ Geometric mean per path
$payoff \leftarrow \max(GA - K, 0)$ ▷ Vectorized payoff calculation
$price \leftarrow \exp(-rT) \cdot \texttt{mean}(payoff)$

Figure 3. Illustrative figure of per-thread vectorized path simulation workflow using Mersenne Twister or Sobol' sequences

Algorithm 3 Brownian Bridge construction using Sobol-generated normal deviates

$U \leftarrow$ Sobol[PATHS][N] ▷ Sobol samples from `torch.quasirandom.SobolEngine`
$Z \leftarrow \sqrt{2} \cdot \texttt{erfinv}(2U - 1)$ ▷ Inverse CDF transform
Initialize $W_t[n] \leftarrow \sqrt{T} \cdot Z[t][0]$ ▷ Set terminal value
Build midpoint schedule using recursive bisection
$cur\_z \leftarrow 1$
for each midpoint (i, l, r) **do**
    $\Delta \leftarrow t_r - t_l$
    $a \leftarrow \frac{(t_r - t_i)\cdot W_t[l] + (t_i - t_l)\cdot W_t[r]}{\Delta}$
    $b \leftarrow \sqrt{\frac{(t_i - t_l)(t_r - t_i)}{\Delta}}$
    $W_t[i] \leftarrow a + b \cdot Z[t][cur\_z]$
    $cur\_z \leftarrow cur\_z + 1$
end for
$dw[t][i] \leftarrow (W_t[i+1] - W_t[i]) \cdot \sqrt{N/T}$ ▷ Construct Brownian increments

Figure 4. Illustration of Brownian Bridge path generation based on Sobol-normal samples

We also remark that Table 8 summarizes the execution times recorded for each simulation setup under a single batch execution.

Table 8. Execution time comparison for MC and QMC simulations (in seconds)

Method	Implementation	Time (s)

MT	CPU	3.148
MT	GPU	*0.095*
Sobol	CPU	4.889
Sobol	GPU	*0.570*
Sobol + BB	CPU	71.266
Sobol + BB	GPU	*5.788*
Sobol + Scramble	CPU	5.582
Sobol + Scramble	GPU	*1.085*
Sobol + (Skip = 2^{19})	CPU	18.108
Sobol + (Skip = 2^{19})	GPU	*13.552*

The results in Table 8 highlight the considerable computational advantages offered by GPU acceleration across various simulation methods. Among these, the most significant improvement is observed in standard Monte Carlo simulations using Mersenne Twister, where the GPU implementation completes the task in just 0.095 seconds, compared to 3.148 seconds on the CPU—a 33-fold speedup. This dramatic reduction demonstrates the efficiency of GPU-based parallel random number generation for large-scale simulations.

We next examine the performance of quasi-Monte Carlo simulations using Sobol' sequences. The GPU implementation achieves a runtime of 0.57 seconds, offering an 8.5-fold speedup relative to its CPU counterpart. This improvement underscores the effectiveness of GPU acceleration even when using low-discrepancy sequences, which are traditionally more structured and less amenable to parallelism than pseudo-random number generators. In the scrambled Sobol case, GPU runtime increases modestly to 1.085 seconds due to the additional computational cost of digital scrambling, yet it remains significantly faster than CPU execution.

Brownian Bridge construction, when applied alongside Sobol' sequences, introduces additional computational overhead due to its recursive midpoint structure. Nonetheless, the GPU implementation reduces runtime from 71.266 seconds on the CPU to 5.788 seconds, achieving a 12-fold acceleration. Despite this gain, BB simulations remain more time-consuming overall, as the recursive dependencies inherently limit parallelism on GPU architectures.

A more detailed breakdown of execution times reveals the primary computational bottlenecks in each configuration. For Sobol' simulations on GPU, approximately 86% of the total time is spent on generating quasi-random numbers, while the remaining time is used for path construction. In the scrambled Sobol case, the extra 0.515 seconds of overhead stems from scrambling operations. In contrast, applying a large skip (e.g., 2^{19}) leads to a total runtime of 13.552 seconds on GPU, indicating that while skipping improves sequence quality, it is computationally inefficient in practice.

In Brownian Bridge simulations, the performance bottleneck shifts away from number generation. Approximately 90% of the GPU runtime is spent on recursive midpoint interpolation, while only about 8% is used for generating the Sobol' sequence. This shift clearly shows that the recursive structure of BB—not the sampling method—is the dominant contributor to total execution time in this configuration.

These observations suggest that the combination of CuPy and PyTorch---both high-level GPU libraries---offers an effective and practical solution for accelerating Monte Carlo and Quasi-Monte Carlo simulations without requiring low-level custom CUDA kernel programming. High-level libraries like CuPy and PyTorch handle kernel generation and GPU memory management automatically, whereas low-level CUDA programming requires manually writing and optimizing custom kernels. Using such high-level libraries allows for quick implementation while maintaining sufficient computational

performance for pricing single exotic options, such as the Geometric Asian option considered in this study.

However, it is important to note that this analysis focuses on a single product type. In scenarios where portfolios of multiple exotic options are to be priced, or risk metrics such as Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) are to be computed, the overall dimensionality (i.e., the total number of time steps × assets) —and thus the computational complexity— can become extremely large. In such cases, relying solely on high-level libraries may no longer be sufficient, and more optimized implementations involving explicit kernel configurations could be necessary to fully exploit the available GPU resources.

Lastly, the remarkable speedup observed with GPU-accelerated Mersenne Twister simulations suggests that further studies combining Mersenne Twister random number generation with advanced variance reduction techniques under full GPU parallelization could make valuable contributions to the computational finance literature. Moreover, exploring algorithmic modifications to the Brownian Bridge construction that improve its compatibility with parallel architectures presents another promising direction for enhancing the efficiency of quasi-Monte Carlo methods.

CONCLUSION

This study provides a comprehensive computational analysis of Quasi-Monte Carlo methods using Sobol' sequences in comparison to traditional Monte Carlo simulations based on the Mersenne Twister generator. While Sobol' sequences are theoretically known to offer superior convergence rates, our investigation reveals notable challenges when applying them to high-dimensional problems—arising, for instance, in financial simulations such as Geometric Asian option pricing.

The theoretical background and related literature highlight the known strengths of Sobol' sequences, but also hint at their sensitivity to dimension ordering and structural artifacts. Through one- and multi-dimensional integral tests, we validate the accuracy and convergence behavior in integral calculations using Sobol' sequences. However, when transitioning to the high-dimensional setting of Asian option pricing, significant deviations are observed, especially in baseline Sobol configurations.

To investigate the source of the observed discrepancies, we conduct detailed moment and correlation analyses; they reveal a persistent bias inherent in the incremental construction of Sobol' sequences. This bias is found to be effectively mitigated through various enhancement techniques, including scrambling, initial skipping, and Brownian Bridge construction. Among these, Brownian bridge yields the most accurate option price estimates, while scrambling offers both improved accuracy and straightforward parallel implementation. Although initial skipping alone provides noticeable improvements, its overall effectiveness is more limited relative to Brownian Bridge technique.

Finally, we implement GPU-accelerated versions of all simulation methods, achieving substantial speedups across all configurations. The most striking gains are observed in Mersenne Twister-based simulations, where GPU parallelism yields a 33-fold runtime reduction of the base case. Sobol with Brownian Bridge simulations also benefits significantly from GPU acceleration, though to a lesser extent due to algorithmic limitations in parallelizing recursive path construction.

Overall, our findings reinforce the importance of proper configuration when applying QMC methods to high-dimensional problems. Moreover, they highlight the combined value of theoretical insight, algorithmic enhancement, and hardware-level acceleration in achieving both numerical precision and computational efficiency in modern financial simulations.

FUTURE WORK

The findings of this study suggest several directions for future research that could further enhance the efficiency, accuracy, and applicability of GPU-accelerated Monte Carlo and Quasi-Monte Carlo methods in computational finance.

First, the remarkable performance gains achieved through GPU-accelerated Mersenne Twister (MT) simulations indicate a valuable opportunity for further enhancement. In particular, the use of high-level GPU libraries such as CuPy enables the realization of substantial speedups without requiring low-level CUDA programming, making efficient pseudo-random number generation easily accessible within Python environments. While MT-based simulations offer excellent computational speed due to their lightweight nature, they are not inherently variance-reducing. Therefore, integrating MT with advanced variance reduction techniques—such as control variates, stratification, or antithetic sampling—under full GPU parallelization could substantially improve simulation accuracy without sacrificing computational efficiency. This combination may serve as a practical and scalable alternative to Quasi-Monte Carlo methods in applications.

Second, although the Brownian Bridge construction is a well-established variance reduction technique, its recursive nature limits full parallelization, especially on GPU architectures. One fruitful direction for future work would be to investigate alternative algorithmic structures or approximations that retain the variance allocation benefits of Brownian Bridge while reducing memory bottlenecks or enabling greater parallel throughput. Additionally, it would be worth exploring whether custom low-level CUDA kernel implementations—specifically designed to optimize memory access and thread scheduling—could further enhance the performance of Brownian Bridge-based simulations beyond what is achievable with high-level GPU libraries alone.

Third, some studies, particularly in the context of *scrambled* Sobol' sequences, caution that skipping initial points may interfere with the randomized net structure and degrade convergence (Owen, 2021). Nonetheless, other works such as (Radović et al., 1996) report reduced integration error when early low-quality points are avoided. Our results support this view in the *non-scrambled* case: skipping a large number of initial points (e.g., 2^{19}) improved accuracy in high-dimensional simulations without Brownian Bridge. In addition to the interaction between skipping and scrambling, the question of how many points should be skipped remains problem-dependent, underscoring the need for more systematic analysis across different integrands and dimensionalities to better understand when skipping improves or degrades performance.

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Appendix

GPU and CUDA specifications

CUDA toolkit version 12.6 was used for all of the simulations. The online documentation for this version is available at (https://docs.nvidia.com/cuda/archive/12.6.0/index.html). All simulations were conducted on a single NVIDIA GeForce RTX 4070 Laptop GPU. The specifications of the device are summarized in Table 9.

Table 9. Hardware specifications of the GPU used in simulations (NVIDIA GeForce RTX 4070)

Title	Title
Device Name	NVIDIA GeForce RTX 4070 Laptop GPU
CUDA Driver / Runtime Version	12.6 / 12.6
Compute Capability	8.9
CUDA Cores	4608
Global Memory	8 GB (8585 MB)
Shared Memory per Block	49 KB
Constant Memory	64 KB
L2 Cache Size	32 MB
Memory Clock Rate	8001 MHz
Memory Bus Width	128-bit
GPU Max Clock Rate	2175 MHz
Maximum Texture Dimension (1D)	131072
Maximum Texture Dimension (2D)	(131072, 65536)
Maximum Texture Dimension (3D)	(16384, 16384, 16384)
Maximum Threads per Multiprocessor	1536
Maximum Threads per Block	1024
Max Block Dimensions (x, y, z)	(1024, 1024, 64)
Max Grid Dimensions (x, y, z)	(2147483647, 65535, 65535)
Warp Size	32
Support for Concurrent Copy and Execution	Yes
Unified Memory (UVA) Support	Yes
ECC Support	No

Python Environment and Library Versions

All simulations were implemented using Python~3.12.4, supported by a set of high-performance numerical and GPU-accelerated libraries. The core packages and their versions are summarized in Table 10.

Table 10. Python Environment and Library Versions

Library	Version
Python	3.12.4
NumPy	1.26.4
SciPy	1.13.1
CuPy	13.3.0
PyTorch	2.5.1

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

Both authors contributed equally to this work.

An Overview of Optimization Approaches for Waste Management Problems (1124)

Murat Yesilkaya[1*]

[1]Tokat Gaziosmanpaşa University, Faculty of Engineering and Architecture, Department of Industrial Engineering, Türkiye

*Corresponding author e-mail: murat.yesilkaya@gop.edu.tr

Abstract

Waste management has become an important research area, especially in recent years, in line with sustainability goals. Optimization models developed in this field are increasingly gaining interest and stand out as both a current and strategic field of study in the operations research discipline. The waste management process includes a complex network structure starting from waste generation centers and extending to treatment, recycling, separation, and final disposal facilities. In this process, operational decisions such as transportation planning, appropriate technology selection and the establishment of new facilities are evaluated under constraints such as capacity, time, budget, flow balance and demand. This paper studies the development and trends of mathematical optimization approaches in the field of waste management through bibliometric analysis and a brief literature review. 596 articles obtained from the Web of Science (WOS) database were analyzed using VOSviewer and Biblioshiny with R software. The analyses cover multidimensional data such as the most frequently used keywords, published journals, influential authors, collaborating countries and thematic development areas. The findings show that there has been a significant increase in the number of publications, especially after the COVID-19 pandemic, since 2019, and the themes of optimization, uncertainty, multi-objective programming, healthcare waste, and sustainability have come to the fore. It has been observed that operations research methods such as stochastic programming, chance-constrained programming and mixed integer models are at the forefront along with the most frequently used uncertainty-themed keywords. In the thematic map analysis, it has been determined that topics such as the facility location problem and multi-objective modeling are developing as niche areas. The results obtained can guide operations research researchers working in the field of waste management in terms of monitoring current trends and identifying new research opportunities.

Keywords: *waste management, mathematical programming models, optimization*

INTRODUCTION

Waste management (WM) is a multidimensional environmental management process that includes the collection, transportation, recycling, reuse, and final disposal of waste generated by people's domestic, industrial, and agricultural activities. Today, there is a significant increase in the amount of waste, especially due to population growth and changes in consumption habits. According to the World Bank's 2020 data, approximately 2.1 billion tons of urban solid waste are produced annually, and this amount is expected to reach 3.8 billion tons in 2050 (Statista, 2020). Only 19% of this waste is recycled, and a significant portion is disposed of with methods that are dangerous for the environment and public health. This situation is a global problem that threatens not only environmental but also economic and social sustainability. Effectively managing WM processes is important for countries, especially in order to implement sustainable and circular economy concepts that have been politically discussed in many countries.

One of the important approaches used in planning WM processes is the development of optimization models. These models, which have an important place in the discipline of operations research, provide systematic solutions to decision makers in many critical decisions such as the collection and transportation of waste, the determination of waste collection routes, and the location selection of recycling centers (Ghiani, et al 2014). Stochastic and robust optimization models, especially developed in recent years, offer suggestions for policymakers in environments dominated by uncertainty (Akbarpour, et al 2021). This study aims to examine mathematical programming-based optimization models used in the field of WM using the bibliometric analysis method and to discuss the fundamental contributions of these models.

MATERIAL AND METHODS

The proposed bibliometric analysis review relevant publications on mathematical programming models for WM. Web of Science (WOS) is used as the data source. Some filtering is done on WOS (Table 1). A search is conducted with keywords reflecting WM and mathematical programming models and types, and as a result, 596 articles published in the last 20 years in this field are found, and these articles are analyzed. This number of articles also reveals that optimization models in WM are a popular research topic and are still open to research.

Table 1. Literature search strategy

Sources	**Web of Science (WOS)**
Subject area	All
Document type	Article
Period Time	2004-2024
Query String	(ALL=(”waste management" ”linear programming model”) OR ALL=(”waste management""Non linear programming model”) OR ALL=(”waste management""Multi-objective programming”) OR ALL=(”waste management""Multi objective programming”) OR ALL=(”waste management" ”Fuzzy programming") OR ALL=(”waste management" ”Stochastic programming”) OR ALL=(”waste management""bilevel programming”) OR ALL=(”waste management""Chance Constrained”) OR ALL=(”waste management" ”Robust Optimization”) OR ALL=(”waste management""mathematical programming model”) OR ALL=(”waste management""mathematical programming”) OR ALL=(”waste management""Mixed integer programming”) OR ALL=(”waste management" ”operation research”) OR ALL=(”waste management""mathematical model” ”optimization model”) OR ALL=(”waste management" ”deterministic programming”) OR ALL=(”waste management""multi-objective optimization model”) OR ALL=(”waste management" ”goal programming”) OR ALL=(”waste management""mathematical optimization”))
Total	596 papers

The obtained dataset includes a large number of bibliographic features related to the articles. This data is analyzed using VOSviewer and Biblioshiny with R, which are bibliometric analysis software widely used in the literature (Salas-Navarro, et al 2022). The software in question produces visuals based on color, network structure, and dimensions using various data belonging to the articles; thus, it provides a comprehensive perspective to researchers by revealing the relationships between studies and prominent research trends. These visuals are discussed in various aspects.

RESULTS

Optimization in WM aims to determine the most appropriate strategies in terms of economic, environmental, social and risk factors by using existing resources in the most efficient way. In this process, a network extending from waste generated centers to treatment, transformation, separation and disposal centers is taken into account. Operational decisions such as transportation, technology selection, new facility installation is optimized under constraints such as capacity, duration, budget, flow balance and demand (Abou Najm and El-Fadel, 2004). The aim is not only to minimize costs, but also to reduce environmental impacts, minimize risks and increase social benefits. In recent years, various parameters such as waste amount and costs have been included in models under uncertainty. The structure created in this context offers decision makers applicable and sustainable solution alternatives. In this context, optimization models are a strategic tool for the design of WM systems. In the field of WM, mathematical programming models have been introduced for various waste types such as

municipal waste, solid waste, hazardous waste, medical waste, disaster-related waste, and debris waste (Tirkolaee, 2024).

We now present the results from the bibliometric analysis. Figure 1 shows the distribution of a total of 596 publications (article) that include optimization models in the field of WM by year. A significant increase in the number of publications has been observed since 2018, and the highest number of publications in the last 20 years was reached in 2021. This increase can be attributed to the increase in model studies on medical WM due to the impact of the COVID-19 pandemic.

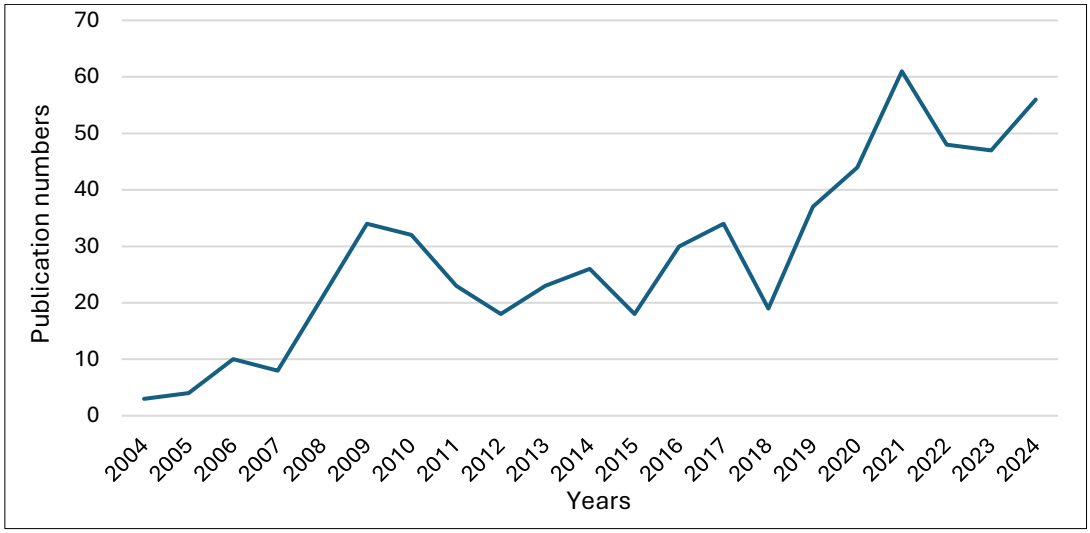


Figure 1. Publication years

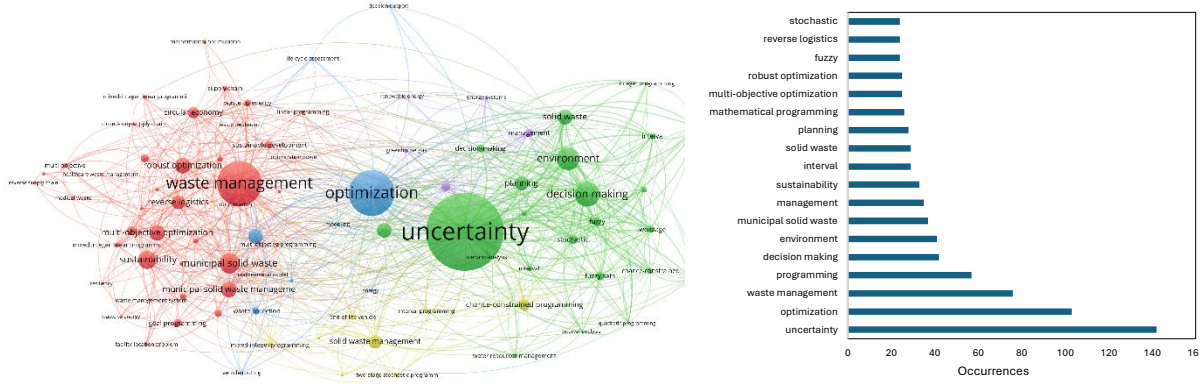
Table 2 shows the journals in which the articles were published. The articles examined were published in a total of 190 different journals. The three journals with the highest number of publications are "Journal of Cleaner Production", "Waste Management", and "Waste Management and Research", which account for approximately 20% of the total publications. These journals are multidisciplinary journals that publish in both environmental sciences and operations research. The other journals are seen to focus mainly on operations research.

Table 2: Most productive journals

Sources	Articles	%
Journal of Cleaner Production	53	8.9
Waste Management	41	6.9
Waste Management and Research	21	3.5
Engineering Optimization	16	2.7
Journal of Environmental Management	16	2.7
Journal of the Air Waste Management Ass.	16	2.7
Resources Conservation and Recycling	16	2.7
Environmental Engineering Science	14	2.3
Journal of Environmental Informatics	14	2.3
Sustainability	14	2.3
Computers and Industrial Engineering	12	2.0
Science of the Total Environment	12	2.0
Stochastic Environmental Research	12	2.0
Energy	11	1.8
Annals of Operations Research	10	1.7
European Journal of Operational Research	9	1.5
Sustainable Cities and Society	9	1.5
All Other Journals	300	50.3

Figure 2(a) visualizes the frequency of keywords with node sizes, while Figure 2(b) numerically shows the frequencies of the most frequently used words. The most frequently used keyword was "uncertainty"

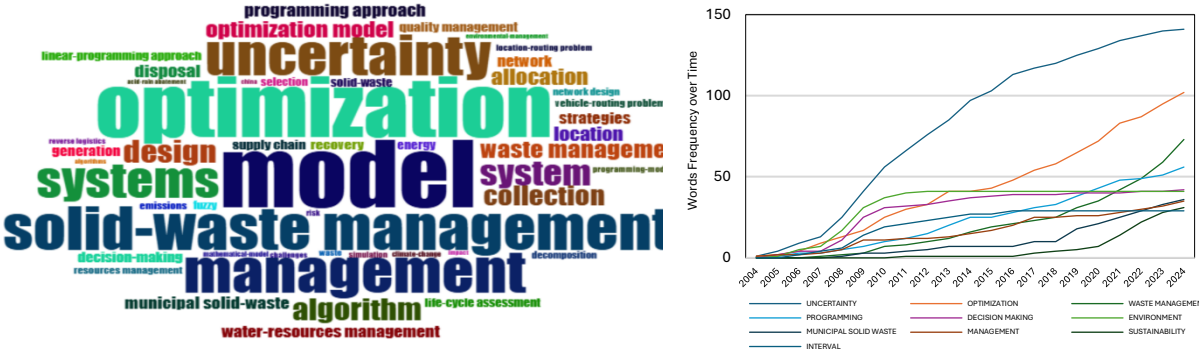
with 142 times. This was followed by "optimization" with 103 times and "waste management" with 76 times. The keywords were divided into clusters, each representing a different color. Terms belonging to the same cluster exhibit strong relationships and are grouped in the same color. The fact that the term "uncertainty" is the most frequently used keyword reveals that there are significant uncertainties at the parameter level in WM models. This word is in the green cluster and has a strong relationship with terms such as stochastic programming, fuzzy approaches, chance-constrained programming, environment, planning, decision making and two-stage programming. This situation shows that uncertainty-based modeling approaches are widely preferred in the waste management literature. In addition, the term "waste management" is included in the red cluster and is closely related to concepts such as robust optimization, sustainability, circular economy, multi-objective optimization, solid waste management, goal programming, reverse logistics and supply chain.



(a) (b)

Figure 2. Co-occurence of all keywords

Figure 3(a) visualizes the word cloud obtained from the keywords; Figure 3(b) presents the change of the keywords over the years. One of the most frequently used terms, "uncertainty", has been showing a continuous increase, especially since 2007. The term "optimization" has entered a significant upward trend after 2014. The term "sustainability", which remained at a low level until 2020, has started to show a significant increase since this year.

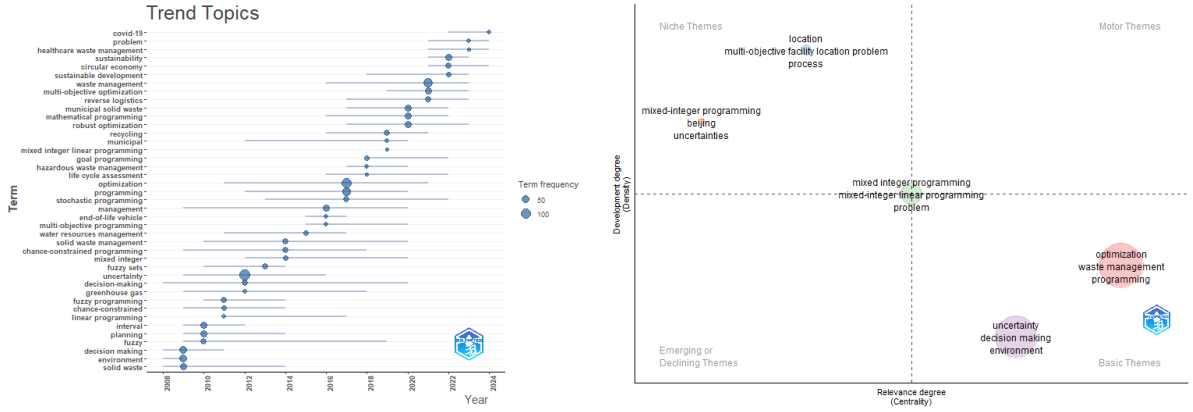


(a) (b)

Figure 3. Most used keywords

Figure 4(a) shows the trend changes of keywords over the years. It is observed that terms such as "COVID-19", "healthcare waste management", and "sustainability" have gained popularity, especially with the COVID-19 pandemic experienced in 2020. Figure 4(b) presents the thematic map of keywords, which is divided into four quadrants: motor themes, niche themes, emerging or declining themes and basic themes. While there is no prominent keyword in the motor themes and emerging or declining themes quadrants, the basic themes section includes standard and widely used terms such as

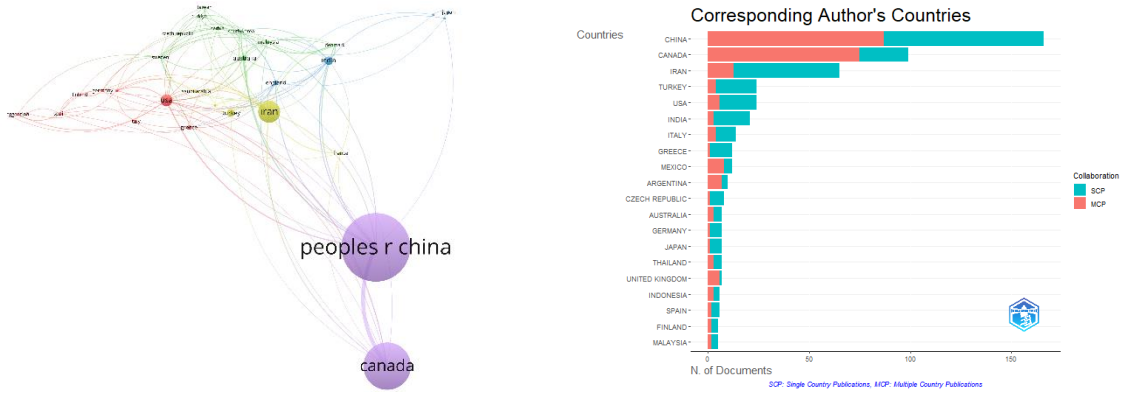
optimization, waste management, uncertainty and decision making. The niche themes quadrant includes more specific and emerging topics such as multi-objective modeling, facility location problem, uncertainty and mixed integer programming.



(a) (b)

Figure 4. Trend keywords and thematic map

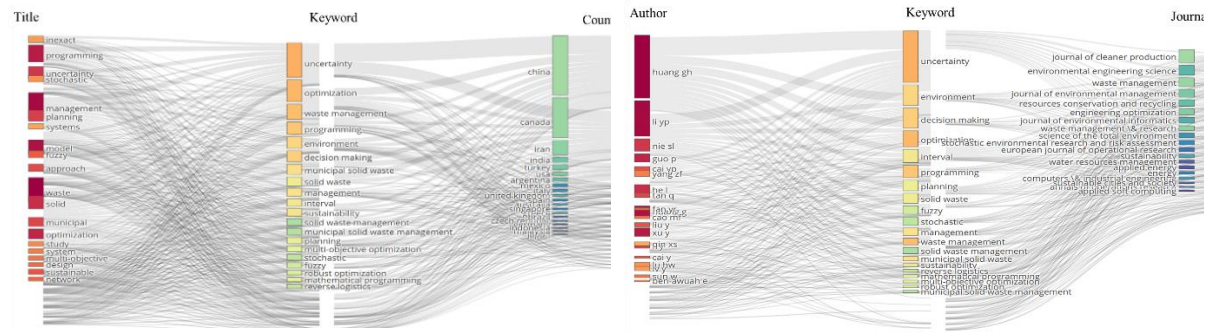
Figure 5(a) shows the countries with the most publications. China is by far the country with the most publications, followed by Canada, Iran, USA, India and Turkey. Figure 5(b) shows the collaborations between countries by country of the corresponding authors. While nearly half of China's publications are co-authored, Canada and Turkey stands out with more individual, i.e. single-country publications.



(a) (b)

Figure 5. Countries with the most publications

The three-field plot is an analysis method used to visualize the relationships between three different parameters. In Figure 6(a), the article titles are in the left area, the keywords in the middle area, and the countries in the right area. In this visualization, China has a particularly strong relationship with the keywords "uncertainty" and "stochastics". In Figure 6(b), the authors are in the left area, the keywords in the middle area, and the journals are in the right area. In this graph, the journal "Journal of Cleaner Production" stands out with its theme of "uncertainty" and its clear relationship with the author "Huang G.H."



(a) (b)

Figure 6. Three-field plot among keywords

CONCLUSION

WM is a critical area in terms of environmental sustainability and resource efficiency, and optimization-based approaches provide effective decision support mechanisms in this area. This study presents a bibliometric analysis of optimization-based operations research approaches in the field of WM. The findings reveal that the publications are largely in operations research-focused journals and have shown a remarkable increase, especially after 2019. Keyword analyses show that the concepts of uncertainty, optimization, multi-objective modeling and sustainability are prominent, and techniques such as stochastic programming, chance-constrained programming and mixed integer models are frequently used.

According to thematic map analysis, multi-objective modeling, facility location problem and mixed models with uncertainty stand out as niche themes in the literature. These areas can pave the way for future studies on more precise modeling of environmental uncertainties, sustainable facility location and integrated waste logistics. In this context, the study can be beneficial for researchers both to follow current trends and to identify new research.

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Threshold Selection Method of Machine Learning and Deep Neural Networks for Plant Growth Data Classification (1162)

Mustafa Bayram Gücen[1*]

[1]Yildiz Technical University, Faculty of Arts and Sciences, Mathematics, Turkey

*Corresponding author e-mail: mgucen@yildiz.edu.tr

Abstract

This research focuses on predicting plant growth milestones using a dataset containing key environmental and agricultural parameters such as soil type, sunlight duration, irrigation frequency, fertilizer type, temperature, and humidity. Several supervised machine learning models were applied to classify plant growth stages: Gaussian Naive Bayes, Bernoulli Naive Bayes, Logistic Regression, k-Nearest Neighbors, Decision Tree, Random Forest, Support Vector Classifier, and Deep Neural Network. A threshold optimization technique was used to fine-tune the classification results. Among all the models, Random Forest Classifier provided the best overall performance with an accuracy of 91.0% and a ROC AUC score of 0.9624. Despite being more computationally intensive, Deep Neural Network also produced promising results in terms of precision and F1 score. ROC AUC curves were constructed to demonstrate the comparative effectiveness of the models. The findings highlight the potential of both classical and deep learning approaches in modeling plant growth, providing meaningful support for agricultural planning and greenhouse management.

Keywords: *Deep Learning, Threshold Selection Method, Machine Learning methods, Deep Neural Networks, Plant Growth Datasets*

INTRODUCTION

Agriculture is a cornerstone in the development of human civilization, and crop productivity and predictability of production are critical to food security and economic stability. Plant growth is governed by a complex interaction of environmental and agricultural variables, including soil properties, temperature, humidity, sunlight exposure, irrigation schedules, and fertilizer types. Accurate prediction of plant growth stages based on these parameters enables timely interventions in greenhouse management and field agriculture. In recent years, the advancement of supervised machine learning (ML) and deep learning (DL) algorithms has provided powerful tools for modeling biological processes. Known for their ability to learn complex nonlinear patterns, these methods are increasingly used in precision agriculture to predict growth outcomes, diagnose plant diseases, and optimize resource allocation.

While ML and DL models can classify and predict with high accuracy, their real-world performance often depends on the decision threshold used to convert probabilistic outputs into discrete class predictions (Kiritchenko, 2005, Gharroudi et al., 2015). This threshold is usually set to 0.5 by default; however, this default may not be optimal, especially in imbalanced datasets or when the cost of misclassification is asymmetric (Read J et al., 2011). The threshold selection process (systematically adjusting this value) can significantly improve model metrics such as precision, recall, and F1 score. These techniques allow models to be better calibrated for specific applications and their predictions to be aligned with the specific needs of the selected domains. Especially in agriculture, where decisions

have direct economic and ecological consequences, optimizing classification thresholds can increase the reliability of prediction systems.

AI-based models, particularly machine learning and deep learning algorithms, are increasingly used in agricultural yield prediction, offering high accuracy despite environmental complexities (Jabed and Murad, 2024). Deep learning architectures like LGCNN have demonstrated high precision in soil classification and corresponding crop and fertilizer recommendations, supporting sustainable agriculture (Swathi et al., 2024). Comprehensive reviews on rice yield prediction further emphasize the importance of machine learning and deep learning approaches, coupled with data sources for model accuracy and generalizability (De Clercq and Mahdi, 2024), which underlines the relevance of our study on threshold selection methods for plant growth data classification.

In this study, various supervised machine learning algorithms, including Naive Bayes variants, Logistic Regression, Decision Trees, Random Forest, k-Nearest Neighbors, Support Vector Classifiers, and Deep Neural Networks, were used to predict plant growth using a multi-feature dataset (Sharma R et al., 2023). A custom threshold optimization loop was developed to iteratively fine-tune the classification performance, resulting in significant improvements in F1 score across models. Among the tested approaches, the Random Forest Classifier emerged as the most effective approach, achieving 91.0% accuracy and a ROC AUC score of 0.9624. Deep Neural Network also gave competitive results, especially in terms of precision and F1 score, but required more computational resources and more time.

The findings of this research show that both machine learning models and deep learning methods can be used in agricultural analysis. More importantly, the introduction of a threshold selection strategy can be used to improve model calibration and performance. These improvements are particularly valuable for precision agriculture, where even small improvements in prediction accuracy can lead to better crop outcomes, resource efficiency, and reduced costs. This study highlights that methodological innovations in machine learning can be used to support sustainable agriculture and improve decision-making processes in both open-field and controlled-environment farming systems.

MATERIAL AND METHODS

Dataset

The dataset utilized for this study is the Plant Growth Data Classification dataset, which is available from the Kaggle platform(Kaggle, 2024). This dataset is particularly useful for tasks that involve predicting or classifying plant growth milestones based on various environmental and management factors. The goal is to predict the growth stage or milestone that a plant reaches, with respect to several factors that influence plant growth. These factors include soil type, sunlight exposure, watering frequency, fertilizer type, temperature, and humidity levels.

The dataset contains several key features that are relevant for understanding how each factor impacts plant growth:

Soil Type: Describes the type or composition of soil in which the plants are grown, which is an essential variable influencing root development and nutrient absorption.

Sunlight Hours: Specifies the number of hours of sunlight the plants receive. Sunlight plays a crucial role in photosynthesis and overall plant health.

Water Frequency: This feature indicates the frequency of watering, which impacts the moisture levels in the soil and directly affects the plant's ability to thrive.

Fertilizer Type: Details the type of fertilizer used to nourish the plants, which contributes to plant nutrition and influences growth rate and quality.

Temperature: The ambient temperature under which the plants are grown. Temperature is a critical environmental factor that affects plant metabolism, germination rates, and overall growth patterns.

Humidity: The moisture level in the environment surrounding the plants. Humidity influences transpiration rates and water uptake in plants.

The Growth_Milestone variable is the target column, which consists of descriptions or markers that indicate the various stages or significant milestones in the plant growth process.

This dataset provides valuable insights into how different environmental and management factors can affect plant development, making it an essential resource for agricultural optimization and greenhouse management studies.

Methods

Threshold Selection Method

This study introduces a comprehensive analysis function developed to evaluate the classification performance of supervised learning algorithms. The proposed method analyzes the performance of a model on training and test datasets using a range of metrics, while also performing a multi-stage threshold optimization procedure to determine the optimal classification threshold. The model is evaluated in detail based on performance measures such as accuracy, precision, recall, F1-score, ROC-AUC and log-loss.

In classification problems, model outputs are typically converted into binary decisions based on a predefined threshold. However, using a fixed threshold (e.g., 0.50) may not yield optimal results for every dataset. Therefore, dynamically adjusting the threshold is a critical step in enhancing model performance. This study proposes a comprehensive classification model evaluation procedure that includes threshold optimization.

Initially, the model is trained on the training dataset, and predictions are generated on the test set using various threshold values. This process is conducted through a three-stage optimization procedure:

Initial Scan: The threshold is coarsely scanned over the range [0.0001, 0.9999].

Intermediate Scan: A more refined search is performed around the best threshold identified in the initial scan.

Fine-Tuning: A high-precision optimization is conducted within ±0.01 of the previously selected threshold.

At each step, the F1-score is calculated for different threshold values, and the threshold yielding the highest F1-score is passed to the next phase.

Performance Evaluation

The model's accuracy on both the training and test datasets is computed. Subsequently, performance metrics such as recall, precision, and F1-score are evaluated for fixed thresholds (0.25, 0.50, and 0.75). These metrics are then recalculated based on the optimal threshold to compare performance. Additionally, the model’s overall classification capability is assessed using the ROC-AUC score and log-loss metric.

By determining the optimal threshold, the function improves the model’s performance on the test dataset. Particularly for imbalanced datasets, this method often results in higher F1-scores compared to fixed-threshold classifications. Furthermore, the detailed outputs provided allow for a clear understanding of the model’s strengths and weaknesses.

This methodology enables the evaluation of classification algorithms not only in terms of accuracy but also through more comprehensive performance measures. The threshold optimization process contributes to more accurate classification decisions. The performance metrics obtained after implementation also facilitate comparative analysis across different models.

RESULTS

The performance of various classification models was evaluated using multiple metrics, including Accuracy, Precision, Recall, F1-score, ROC AUC, Logloss, and computation time. Table 1 presents the obtained values for each model.

Among the models tested, Random Forest achieved the highest overall performance, with an accuracy of 0.910, a F1-score of 0.913, and the highest ROC AUC value of 0.962, indicating strong classification capability. Furthermore, it maintained a relatively low log loss (0.270), suggesting that its probability estimates were well-calibrated. However, it had the highest computation time (69.22 seconds), which might be a limiting factor in real-time or resource-constrained applications.

Support Vector Classifier (SVC) also showed strong performance, with an accuracy of 0.890 and an F1-score of 0.892. It maintained a high ROC AUC (0.939) and demonstrated a good balance between recall and precision, although it required more computation time compared to simpler models.

Logistic Regression provided a competitive result with an accuracy of 0.885 and the highest recall (0.93) among all models, making it a suitable choice in cases where identifying positive cases is particularly critical. Its F1-score (0.894) and ROC AUC (0.934) further support its robustness.

The Decision Tree model performed reasonably well, with an accuracy of 0.865 and F1-score of 0.870, but it showed signs of overfitting, typically associated with single-tree models. It had a low computation time (2.42 seconds), which may be advantageous in applications requiring fast decisions.

K-Nearest Neighbors (KNN) and Naive Bayes models (GaussianNB and BernoulliNB) showed moderate performance, with accuracies in the range of 0.835–0.845. While these models are computationally efficient, their lower AUC and F1 scores make them less favorable for tasks requiring high predictive accuracy.

The Deep Neural Network (DeepNN) achieved a decent accuracy (0.850) and recall (0.89) but had a relatively lower ROC AUC (0.850) and significantly higher log loss (0.843). Most notably, its computation time (743.96 seconds) was substantially higher than all other models, which limits its practical applicability without GPU acceleration or parallel processing.

Table 1. Performance Metrics Table

Model	**Obtained Values**						
	Accuracy	**Precision**	**Recall**	**F1-score**	**RocAuc**	**Logloss**	**Time**
GaussianNB	0.845	0.854369	0.88	0.866995	0.91330	0.473460	0.317094
BernoulliNB	0.835	0.892473	0.83	0.860104	0.86210	0.559107	0.311612
LogisticRegression	0.885	0.861111	0.93	0.894231	0.93390	0.324209	16.91
KNN	0.845	0.833333	0.85	0.841584	0.91360	0.386108	2.31
DecisionTree	0.865	0.841121	0.90	0.869565	0.89190	0.374964	2.42
RandomForest	0.910	0.936842	0.89	0.912821	0.96245	0.270159	69.22
SVC	0.890	0.915789	0.87	0.892308	0.93930	0.313458	0.58
DeepNN	0.850	0.824074	0.89	0.855769	0.85000	0.842643	743.96

Threshold adjustment played a significant role in optimizing model performance (Table 2). For instance, the optimal threshold for the GaussianNB model was 0.190, which is significantly lower than the default

0.5, indicating a skewed class probability distribution. In contrast, the Random Forest and SVC models maintained thresholds closer to the default (0.534 and 0.524, respectively), reinforcing their stable probability calibration.

Table 2. Threshold Selections

Model	Selected Threshold
GaussianNB	0.190
BernoulliNB	0.176
LogisticRegression	0.333
KNN	0.385
DecisionTree	0.084
RandomForest	0.534
SVC	0.524
DeepNN	0.470

Figure 1 shows the ROC curves for all models, highlighting the superior area under the curve for Random Forest and SVC, followed by Logistic Regression. These curves visually confirm the quantitative superiority of ensemble and margin-based methods over others in this classification task.

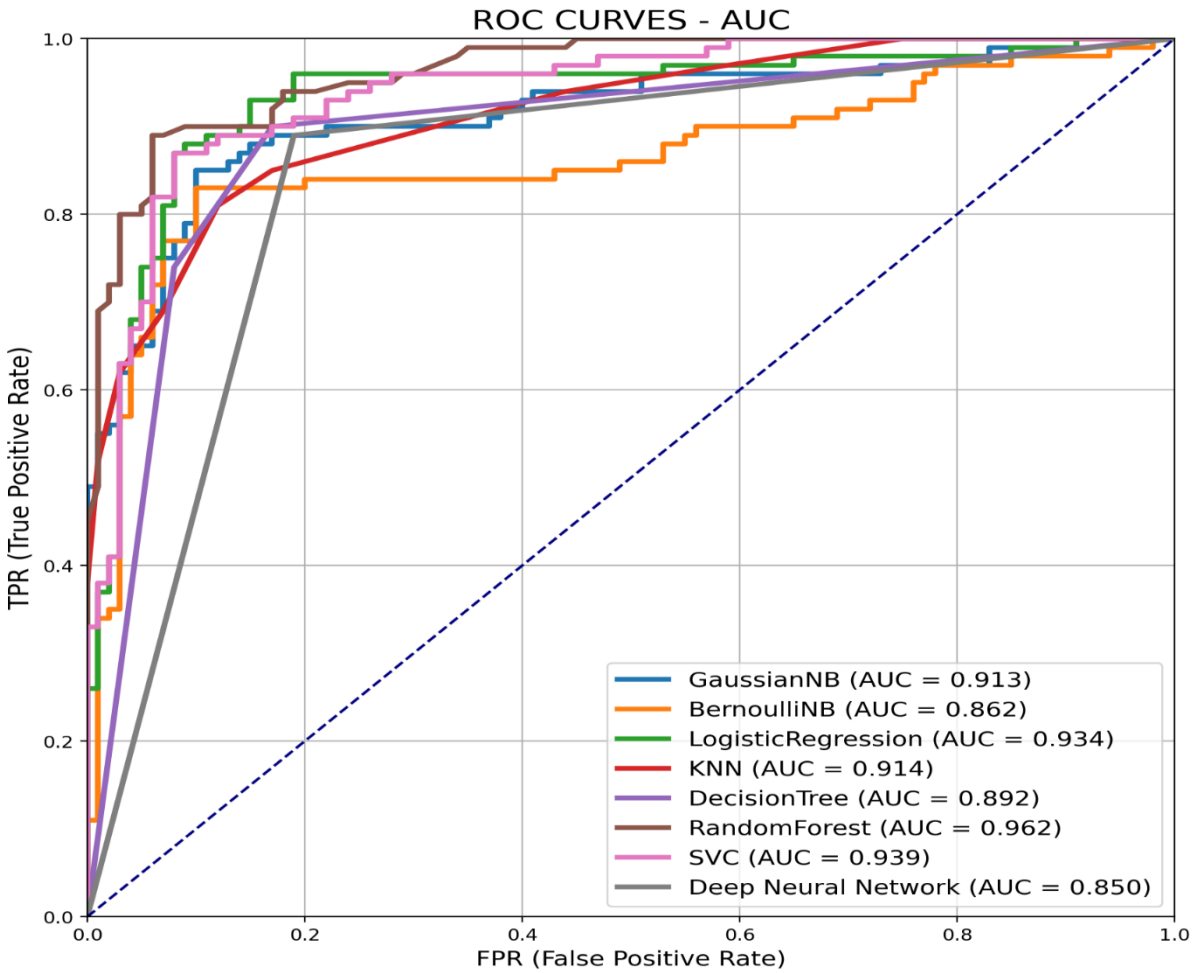

Figure 1. Roc Curves and AUC values

The Python environment was based on Python 3.11.12 and ran on Linux kernel 6.1.123+. The system was equipped with an NVIDIA A100-SXM4-40GB GPU, 83.48 GB of RAM, and an x86_64 CPU architecture.

The GPU had a CUDA version of 12.4 and was detected using TensorFlow's device listing. No other GPU processes were running during the experiments, ensuring full availability of the computing resources. The NVIDIA driver version was 550.54.15.

During the experiments, both the CPU and GPU were utilized simultaneously. While the GPU handled the majority of the computationally intensive tasks such as matrix operations, tensor manipulations, and deep learning model training, the CPU was responsible for auxiliary operations. These included data preprocessing, control flow management, and facilitating communication between the CPU and GPU. This hybrid usage ensured efficient performance and optimal utilization of hardware resources.

This setup provided a high-performance environment for deep learning model training and inference, particularly useful for large-scale data processing and accelerated neural network computations.

In conclusion, Random Forest and SVC emerged as the most reliable models in terms of both predictive performance and robustness. However, the choice of model should be guided by the specific requirements of the application domain, particularly when balancing performance with interpretability and computational efficiency.

DISCUSSION AND CONCLUSION

In this study, various classification algorithms were systematically evaluated to determine their effectiveness in a binary classification task. The analysis focused on multiple performance metrics such as accuracy, precision, recall, F1-score, ROC AUC, log loss, and computational efficiency.

The results clearly show that ensemble-based methods, particularly Random Forest, consistently outperform other models across most metrics. With the highest ROC AUC (0.962) and F1-score (0.913), Random Forest not only provides accurate classifications but also ensures balanced performance across precision and recall. However, its computational cost is notably higher, which might limit its use in real-time applications or environments with limited processing power.

Support Vector Classifier (SVC) and Logistic Regression also demonstrated strong and balanced performance. SVC provided high precision (0.916) and a competitive ROC AUC (0.939), while Logistic Regression stood out with the highest recall (0.93), making it highly suitable for applications where minimizing false negatives is critical. These models also had relatively lower log loss values, indicating that their probabilistic predictions were reliable.

Simpler models such as GaussianNB and BernoulliNB performed reasonably well with very low computational cost, which may make them preferable in lightweight or embedded systems where efficiency is prioritized over accuracy.

The Deep Neural Network (DeepNN) showed promising accuracy and recall; however, its extremely high computation time (743.96 seconds) and elevated log loss suggest that further optimization is required before it can be used efficiently in production environments. Its performance could potentially be improved through better hyperparameter tuning and use of other GPU resources.

A key insight from the threshold selection analysis is the impact of customized probability thresholds on classification performance. Adjusting thresholds away from the default (0.5) significantly improved performance in models like GaussianNB and DecisionTree, underscoring the importance of calibration and post-processing in machine learning pipelines.

In conclusion, this comparative study emphasizes that there is no one-size-fits-all model, and the choice of algorithm should be driven by the specific context and constraints of the application. While Random Forest and SVC offer high performance for accuracy-critical applications, models like Naive Bayes and KNN may still serve well in scenarios demanding simplicity and speed. Future work could involve further optimizing the deep learning model, applying feature engineering and exploring ensemble combinations.

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The Effect of Disaster Awareness on AFAD Volunteer Intention: A Study on University Students (1166)

Ahmet Mumcu[1]

[1]Tokat Gaziosmanpaşa University, Faculty of Health Sciences, Emergency and Disaster Management, Turkey

[*]Corresponding author e-mail: ahmet.mumcu@gop.edu.tr

Abstract

Disasters affect almost every segment of society in social, environmental and economic dimensions, and also cause loss of life and property. AFAD volunteers who act together with the public organizations in the disaster management process are of great importance for the successful conclusion of the disaster management process. Although disasters occur so frequently in Turkey, which is located in an earthquake zone, the fact that people are not sufficiently conscious, have insufficient disaster awareness and do not attach much importance to the consequences of disasters also affects the participation of individuals in disaster volunteer activities. In this context, the aim of the research is to determine the effect of the disaster awareness level of university students on the intention to become an AFAD volunteer. The universe of the research consists of students studying at Tokat Gaziosmanpaşa University. The research sample consists of 384 students who were determined by the convenience sampling method and who were not AFAD volunteers. Voluntary consent forms were obtained from the students who accepted to participate in the research. First, the Disaster Awareness Scale developed by Dikmenli et al. (2013) was applied to the participants in the sample group. After this stage, the participants were given the definition of AFAD Volunteering. Afterwards, questions were asked about the three-item volunteer intention developed by Blau and Holladay (2006) and translated into Turkish by Doğrul (2019). In this way, the participants' intentions to become AFAD volunteers will be determined. As a result of the statistical analyzes made with the structural equation model in the research, it was determined that pre-disaster awareness and educational disaster awareness and post-disaster awareness have a positive and significant effect on the intention to become AFAD volunteers. As a result of the research, suggestions were developed for ensuring disaster awareness in university students.

Keywords: *Disaster Awareness, AFAD Volunteer, University Student*

INTRODUCTION

Disasters are the results of natural, technological and human-induced events that unexpectedly interrupt normal life, stop human activities and have destructive effects, and where local resources are not sufficient (https://www.afad.gov.tr/aciklamali-afet-yonetimi-terimleri-sozlugu). It has a structure that includes various disasters due to the geographical and geological structure of our country. Therefore, many losses of life and property have occurred. Despite the fact that disasters occur so frequently in Turkey. Disaster awareness refers to the knowledge, awareness and understanding that individuals and societies have about disasters. Due to disaster awareness, individuals know what to do before, during and after a disaster. But people do not have sufficient awareness, disaster education levels are not sufficient and the consequences of disasters are not given much importance, which constitutes a great loss for us. Therefore, all people should constantly and regularly learn what to do before, during and

after a disaster, apply this learned information to every moment of their lives and be ready for disasters at all times. Disaster training is provided to the public by AFAD in each province. The trainings cover basic information and concepts, preparations to be made before a disaster, correct behavior during a disaster, the first moments after a disaster, fire and evacuation. Almost everyone should take these trainings. Because we should know that a disaster should be managed correctly.

Another important issue for the success of the disaster management process is the disaster volunteers working in AFAD or various civil society organizations. Especially in earthquakes where 3rd and 4th level, in other words national and international support is needed, volunteers play an active role in the disaster management process. In cases where public and civil society organizations do not have sufficient intervention capacity, AFAD volunteers who have received professional training undertake important tasks especially in the intervention and recovery stages of the disaster management process.

AFAD Volunteers are people who contribute to community service activities before, during and after disasters and emergencies, completely in line with their own will, for the purpose of solidarity and assistance, without considering their individual interests, without any financial expectations, with the sole desire to be beneficial to society, using their physical strength, time, knowledge, skills and experience (https://www.afad.gov.tr/aciklamali-afet-yonetimi-terimleri-sozlugu). Can an individual who does not have disaster awareness, in other words, an individual who does not have basic disaster education, who does not know what to do before, during and after a disaster, become an AFAD volunteer? Isn't disaster awareness the first thing that must be possessed before becoming an AFAD volunteer? This study was conducted based on this research question. By choosing university students as the research universe, the disaster awareness levels of the educated young population who can be an important part of the disaster management process will be determined and the effect of disaster awareness levels on their intention to become an AFAD volunteer will be determined.

The research hypotheses are based on Ajzen Planned Behavior Theory (PBT). According to this, while the main reason why individuals perform a behavior is intention, it is stated that attitudes towards this behavior are explained by attitudes towards the behavior, personal norms and perceived behavioral control (Ajzen, 2006). According to the theory, behavior is not expected from people who do not have a intention. If we reconcile this theory with our research hypotheses, we can say that the essence of AFAD volunteers' behavior is volunteering intention. In other words, volunteering behavior is not expected from people who do not have a tendency to volunteer. If we explain the subject based on the three important assumptions of the planned behavior theory, it is expected that people who support volunteering and find it positive will volunteer. According to the second assumption, whether the people in the social environment approve of the volunteering behavior is the determinant of the intention to do that behavior. According to the third assumption, it states that volunteering behavior is in one's own will and intention (Ajzen, 1991).

According to literature research conducted in recent years on the subject, Avcı et al. (2020) determined that nursing students are knowledgeable about disasters in their study conducted to determine the knowledge and awareness levels of students studying in the nursing department. However, they determined that the students' awareness levels were insufficient due to the fact that they and their surroundings were not prepared for disasters at a high rate, there was no disaster and emergency kit in the place where they lived, and there were no alternative shelter opportunities.

Şahin et al. (2018) aimed to determine the level of disaster preparedness and disaster awareness of students studying in different departments of Burdur Mehmet Akif Ersoy University Faculty of Economics and Administrative Sciences. As a result of the surveys they applied, they determined that although the students' disaster awareness levels were high, their preparedness levels were low.

İnal et al. (2012) conducted a study to determine the behaviors of individuals between the ages of 18-23 in coping with disasters, their preparedness for disasters, and their knowledge and awareness levels on this subject. In the study, they determined that the students' average score was quite low and concluded that there was a statistically significant difference between disaster education and knowledge score.

Aydemir (2021) aimed to raise awareness of the society about disasters or emergencies with Civil Society Organizations, to prevent or reduce disaster damages, to make preparations, to learn about intervention studies and to ensure that voluntary services participate in the process. It aimed to reach professional manpower in disaster operations by providing training programs to volunteers. As a result of the study, considering the contributions of volunteer resources in disaster management, it was revealed that these services should be expanded and volunteering should be emphasized. It was concluded that it was necessary to increase the activities for the training of volunteers in order to create qualified manpower.

METHOD

Purpose and Hypothesis

The aim of the study is to determine the effect of disaster awareness on the intention to become an AFAD volunteer. The research model is shown in Figure 1. Three separate hypotheses were created in the study for this purpose.

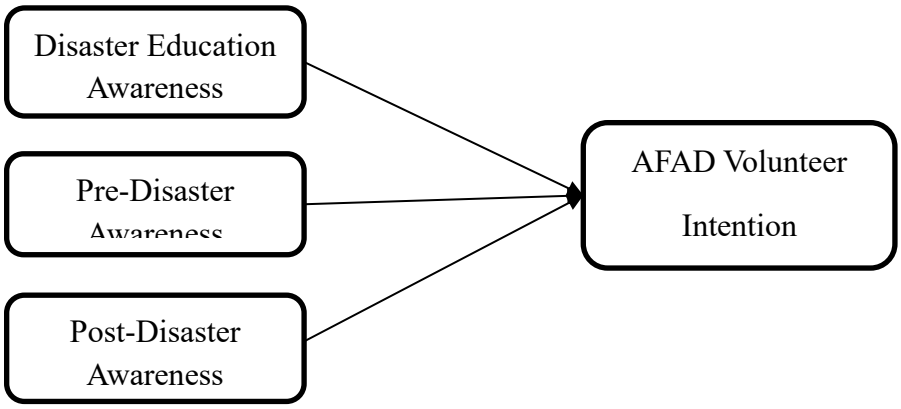

Figure 1. Research Model

H1: Disaster education awareness has an effect on the intention to become an AFAD volunteer.

H2: Pre-disaster awareness has an effect on the intention to become an AFAD volunteer.

H3: Post-disaster awareness has an effect on the intention to become an AFAD volunteer.

Research Sample

The universe of the research consists of students at Tokat Gaziosmanpaşa University. The research sample consists of 384 students who were determined by the convenience sampling method and who were not AFAD volunteers. Participants were 58% female and 42% male. The average age of the participants was 20.9.

The Collection of the Data

Voluntary consent forms were obtained from the students who accepted to participate in the research. First, the Disaster Awareness Scale developed by Dikmenli et al. (2013) was applied to the participants in the sample group. After this stage, the participants were given the definition of AFAD Volunteering.

Later, three-item questions regarding volunteer intention, developed by Blau and Holladay (2006) and translated into Turkish by Doğrul (2019), were asked. These questions were converted into the AFAD volunteer format by the researcher.

Disaster Awareness Scale: The scale was developed by Dikmenli, Yakar, and Konca (2018). The Disaster Awareness Perception scale is a Likert-type scale. The items in the Disaster Awareness Perception Scale were scored as follows: (1) “strongly disagree”, (2) “disagree”, (3) “undecided”, (4) “agree” and (5) “strongly agree”. The scale consists of 36 items in total and there are 27 positive and 9 negative items. Items numbered 12, 22, 23, 24, 25, 26, 27, 28, and 29 were determined as reverse coded and these items were reverse coded before data analysis. The disaster awareness perception scale consists of 4 factors. These are disaster education awareness consisting of 13 items, pre-disaster awareness consisting of 8 items, false disaster awareness consisting of 8 items, and post-disaster awareness consisting of 7 items. The false disaster awareness dimension, which was not related to the purpose of the study, was removed from the scale. As a result of the primary level confirmatory factor analysis conducted on the scale, the goodness of fit index values of the scale were determined as x^2/df: 3.845; GFI: 0.912; CFI: 0.954 IFI: 0.956 and RMSEA: 0,77. As a result of the reliability analysis, it was determined that disaster education awareness had 0.854; pre-disaster awareness 0.932; post-disaster awareness 0.882 Croanbach alpha internal consistency coefficient.

AFAD Volunteer Intention Scale: The three-item volunteer intention scale developed by Blau and Holladay (2006) and translated into Turkish by Doğrul (2019) will be used in the study. Scale expressions will be included in the survey form as "I intend to be an AFAD volunteer in the future. I hope to be an AFAD volunteer in the future. I would like to be an AFAD volunteer in the future." As a result of the primary level confirmatory factor analysis applied to the scale consisting of a single factor and three statements, the goodness of fit index values was reached as x^2/df: 2.845; GFI: 0.962; CFI: 0.984, IFI: 0.987 and RMSEA:0,57. The internal consistency coefficient of the scale was determined as 0.930.

Findings

As a result of the normality distribution test conducted on the statements included in the research, it was determined that the data were between the reference value range of-1.5 and +1.5 skewness and kurtosis coefficient values. After this stage, the structural equation model analysis of the research hypotheses was carried out. The structural equation model including the research model is shown in Figure 2.

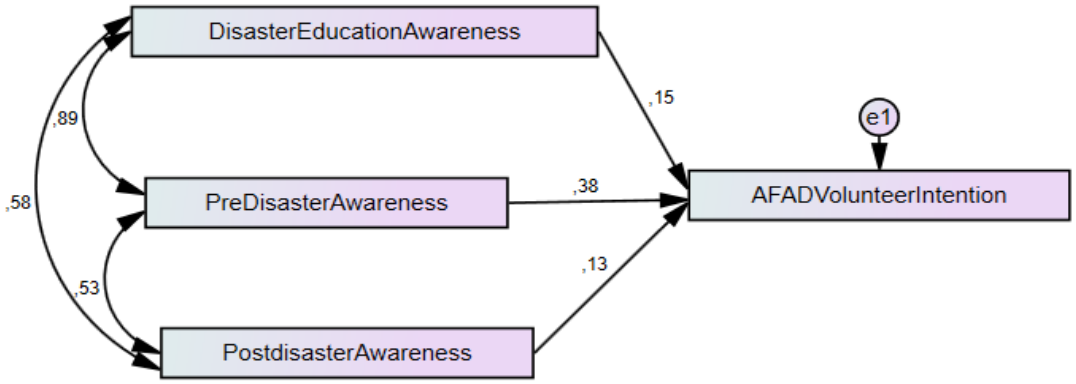

Figure 2. Structural Equation Model

Goodness of fit values of the tested structural equation model are x^2/df: 1.725; GFI: 0.952; CFI: 0.964 IFI: 0.967 RMSEA: 0.62. Data regarding the paths of the structural equation model are presented in Table 1.

Table 1. Regression Weights of Paths of the Research Model

Tested Hypothesis			Standardized Regression Weights(β)	*p*
Educational Disaster Awareness	→	AFAD Volunter Intention	,154	***
Pre-Disaster Awareness	→	AFAD Volunter Intention	,382	***
Post-Disaster Awareness	→	AFAD Volunter Intention	,135	***

***p<0,001

RESULTS

In this study, the relationship between AFAD volunteering, which is a critical factor for the successful completion of the integrated disaster management process, and disaster awareness was tested with the structural equation model. Ajzen (1991) theory of planned behavior states that the primary explanatory factor for a behavior or action to occur is the intention of that person regarding that behavior. In this respect, the important antecedent for AFAD volunteer is the formation of that disaster volunteer intention in the person. The research hypotheses were created in the study assuming that disaster awareness may have an effect on the formation of this intention. The research results revealed that disaster education awareness, pre-disaster awareness and post-disaster awareness have positive and significant effects on AFAD volunteering intention. The volunteering behavior in which an individual can contribute to the disaster management process by taking responsibility is possible with the person having a basic level of disaster awareness. An individual with a basic level of education regarding disaster and awareness of what should be done before and after the disaster will be able to demonstrate the behavior of being a disaster management volunteer. In the study, university students were especially preferred as the research universe. The disaster awareness and volunteer activities of the dynamic young population of the society have an important place in responding to natural and technological disasters. In this respect, basic disaster awareness should be created in young people who will be public administrators, business people and parents of the future. This will reveal their conscious and voluntary behaviors regarding disaster management. The study has time and space limitations as in many studies. The research was conducted with data obtained from only one public university within a limited time frame.

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Comparative Analysis of Growth Curve Models in Livestock Experiments (1171)

Aslı Ateş[1*], G. Tamer Kayaalp[1]

[1]Çukurova University, Faculty of Agriculture, Department of Animal Sciencs, Adana, Türkiye

*Corresponding author e-mail: asliates91@gmail.com

Abstract

This study was conducted to define the live weight growth in quails. A section form a trial unit of a study conducted in the Animal Sciences Department of the Faculty of Agriculture, Çukurova University was used in the study. Hatchling live weights and 5-week live weights of 6 quails were recorded in the study. Coefficients of determination (R^2), mean square errors and parameter values of growth curves were determined in Linear, Gompertz and Brody growth models. The model with the highest coefficient of determination (R^2)and the lowest mean square error (MSE) was preferred to determine the best fitting model in the growth period. Coefficients of determination of live weight were determined as 0.936, 0.989 and 0.973 in Brody, Gompertz and Linear models, respectively. Mean square errors were found as 736.263, 122.612 and 296.205, respectively. Considering these values, it was seen that the best fitting model was obtained from the Gompertz growth model. It can be said that the other two models can also describe growth sufficiently.

Keywords: *Growth curve, quail, gompertz model, brody model, linear model*

INTRODUCTION

Growth is at the top of the list of biological characteristics in living things. The time-dependent change of living things in a certain period of time is called the growth curve (Efe, 1990). Growth can be affected by genetic factors and is also related to environmental conditions. Therefore, the growth curve is the changes in the efficiency formed by these relationships over time (Göven, 2019).

Growth is affected by many factors. For this reason, the shape of the growth curve is determined by factors such as the type of organism, environmental conditions, and the trait to be examined. In order to best evaluate the data on the growth obtained, the appropriate model must be selected. It cannot be said that a single model for growth curves is the most appropriate model for a single breed (Efe, 1990), (Göven, 2019).

Growth curves are used to examine the growth characteristics of living things at different times. Growth curves include parameters such as the adult weight of the living thing, the speed of maturation, and the ratio of the weight reached after birth to the adult weight. With the help of these parameters, the biological growth process of a living thing can be defined (Oda, 2017).

Growth curve models are frequently used in many fields such as economics, biology, zootechnics, and medicine. A long time has been spent on defining growth and the growth curve. In this process, it has been observed that growth is non-linear. Therefore, non-linear growth curve models such as Brody, Bertalanffy, Gompertz, and Richards have emerged. In non-linear models, iterative methods are generally preferred for parameter estimates (Ekiz and Ekni, 2007), (Ersöz and Alpan, 1994).

MATERIAL AND METHODS

Material

In this study, a section of data from a study at Çukurova University Animal Science Department of the Faculty of Agriculture was used. The hatching and 5-week live weights of 6 quails were recorded.

Methods

The growth curve for a trait represents all the factors that affect that trait. In other words, there are many factors that affect growth. For example, when we consider a person's weight, this weight is affected by many factors such as genetic structure, age, gender, metabolic rate, eating habits, exercise, daily calorie intake and burnt. Therefore, growth models cannot be expressed with simple equations.

A common feature of many growth models is that they use two biologically relevant parameters: one determines the location in the general growth space at a given reference age, and the other is the growth rate, which depends on body size (Efe, 1990).

In selecting the model, the data structure, the ease of estimation of the model and the biological interpretation of the estimated parameters should be taken into account. In the model under examination, the difference between the real value and the observed estimate value should be as small as possible (Akbaş, 1995).

In transforming the data, a mathematical function suitable for the purpose is determined and parameter estimates are made and comments are made according to the estimates. One method used in parameter estimation is to take the derivative of the function separately according to each parameter and equate it to zero. Another method is to take the initial estimated value of the a parameter by taking the value closest to the largest one from the data belonging to the observation value. As the initial value of the b parameter, the first value belonging to the observed data and the a parameter are written in place in the function. For the c parameter, the a and b parameters are written in place in the function and the parameters are estimated using an age or first time value (Bilgin and Esenbuğa, 2003), (Gbangboche et al., 2008).

There are some cases where growth is not symmetrical with respect to the inflection point. Animal growth, population growth, etc. The Gompertz model is often preferred in such studies. It has been observed that this model fits well with this type of data structure (Çolak, 2005), (Linton et al., 1995), (Bem et al., 2018).

There are data types where growth is at a decreasing rate. The model that is suitable for this data structure is generally the Brody growth model (Bilgin and Esenbuğa, 2003).

The linear growth model is used in cases where the growth course is constant. In other words, it is a mathematical concept that describes situations where a quantity increases or decreases at a constant rate over time (İzgi et al., 2020).

Table 1: Brody, Gompertz and Linear growth models

Brody	$Y = a.(1 - be^{-ct})$
Gompertz	$Y = ae^{-e^{b-ct}}$
Linear	Y = a+bt

y: size at time t

t: time

a: asymptotic size

b: initial size of the organism

c: growth constant

e=2.71828≈2.72: Euler number

The results were obtained using the SPSS package program. While the linear model was included in the package program, the Brody and Gompertz models were not included in the program, and the analysis results were obtained by writing the formulas in the appropriate places.

RESULTS

In the Brody model, the a parameter was selected as the smallest value greater than the observation values and was found to be 308. Then, the a parameter was written in its place in the function, t=0 was taken and equalized to the first observation value, and the b parameter was found to be -36.69. Then, the a and b parameters were written in their place in the function, t=1 was taken and equalized to the first week value, and the c parameter was found to be 0.03.

Table 2: Regression Variance Analysis of Brody Model

ANOVA[a]

Source	Sum of Squares	df	Mean Squares
Regression	1022857,201	3	340952,400
Residual	24296,695	33	736,263
Uncorrected Total	1047153,896	36	
Corrected Total	379588,640	35	

Dependent variable: y

a. R squared = 1 - (Residual Sum of Squares) / (Corrected Sum of Squares) = ,936.

According to Table 2, the regression sum of squares was found to be 1022857.201, the regression degree of freedom was 3 and the regression mean squares was 340952.4. The error sum of squares was found to be 24296.695, the error degree of freedom was 33 and the error mean squares was 736.263. The coefficient of determination (R^2) was found to be 0.936.

In the Gompertz model, the a parameter was selected as the smallest value greater than the observation values and was found to be 308. Then, the a parameter was written in its place in the function, t=0 was taken and equalized to the first observation value, and the b parameter was found to be -1.297. Then, the a and b parameters were written in their place in the function, t=1 was taken and equalized to the first week value, and the c parameter was found to be -0.017.

Table 3: Regression Variance Analysis of Gompertz Model

ANOVA[a]

Source	Sum of Squares	df	Mean Squares
Regression	1042927,468	3	347642,489
Residual	4046,187	33	122,612
Uncorrected Total	1046973,656	36	
Corrected Total	379517,335	35	

Dependent variable: y

a. R squared = 1 - (Residual Sum of Squares) / (Corrected Sum of Squares) = ,989.

According to Table 3, the regression sum of squares was found to be 1042927.468, the regression degree of freedom was 3 and the regression mean squares was 347642.489. The error sum of squares was found

to be 4046.187, the error degree of freedom was 33 and the error mean squares was 122.612. The coefficient of determination (R^2) was found to be 0.989.

Table 4: Regression Variance Analysis of Linear Model

ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	369516,145	1	369516,145	1247,312	<,001[b]
	Residual	10072,495	34	296,250		
	Total	379588,640	35			

a. Dependent Variable: y

b. Predictors: (Constant), t

Table 5: Model Summary of Linear Model

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	,987[a]	,973	,973	17,21191	,973	1247,312	1	34	<,001

a. Predictors: (Constant), t

After the analysis of the linear model, the regression sum of squares was found to be 369516.145, the regression degree of freedom was 1 and the regression mean squares was found to be 369516.145. The error sum of squares was found to be 10072.495, the error degree of freedom was found to be 34 and the error mean squares was found to be 296.250. The coefficient of determination (R2) was found to be 0.973.

DISCUSSION AND CONCLUSION

The coefficient of determination (R^2) provides information about how effective the independent variable is on the dependent variable. A high R^2 value indicates that the independent variable has a high effect on the dependent variable. A low value indicates that the independent variable has a low effect on the dependent variable.

The coefficient of determination (R^2) of the Brody model is 0.936 and has a high effect. The coefficient of determination (R^2) of the Gompertz model is 0.989 and has a high effect. The coefficient of determination (R^2) of the Linear model is 0.973 and has a high effect. In all three models, the effect of the independent variable on the dependent variable is high. The highest effect is seen in the Gompertz model.

The mean square error gives information about how much error was made in the predictions. A small value of the mean square error indicates that the predictions were made with less error.

The mean square error of the Brody model was found to be 736.263. The mean square error of the Gompertz model was found to be 122.612. The mean square error of the Linear model was found to be 296.250. In this case, it can be said that the Gompertz model makes predictions with less error.

According to these results, when the coefficients of determination (R^2) were examined, the coefficient of determination in the Gompertz model was found to be higher. When the mean square error was examined, the value in the Gompertz model was found to be smaller. Accordingly, it was seen that the

Gompertz model was more suitable for the data set examined. When writing a mathematical model suitable for this data set and analyzing for growth models, it was seen that the Gompertz model described this data set well.

In this study, a small number of data sets were examined in Gompertz, Brody, Linear growth models. Analyses can be made by increasing the number of data. A large number of data can increase the power of predictions. In addition, analyses can be made in different growth models. Linear and non-linear models can be investigated. In addition to the models examined here, when growth models such as Bertalanffy, Logistic, Richards, Gaussian, Logarithmic, Cubic, Exponential, Logistic are examined, the subject can be better understood and stronger interpretations can be made.

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Examination of Reliability Analysis Methods (1172)

Aslı Ateş[1*], G. Tamer Kayaalp[1]

[1]Çukurova University, Faculty of Agriculture, Department of Animal Sciences, Adana, Türkiye

*Corresponding author e-mail: asliates91@gmail.com

Abstract

The reliability feature that shows the stability of the measurement items is expected to be present in measuremet tools. In this study, the analysis and interpretations of some of the reliability analysis methods were examined. For the Cronbach alpha coefficient and split-half coefficient, a 10 item mathematics course attitude scala was applied to 5th grade students of Kahramanmaraş 5 Nisan Secondary School. For the Kurder-Richardson (KR20) coefficient, a 10 question test was applied to 25 8th grade students at Kahramanmaraş 5 Nisan Secondary School. For the Kurder-Richardson (KR21) coefficient application, the results of the Elective Mathematics and Science Applications course written exam of the 6/E grade of Kahramanmaraş 5 Nisan Secondary School were used. The Cronbach alpha coefficient was analyzed in the SPSS package program and was found to be 0.427. It was seen that the reliability of the test was low. The split-half coefficient was analyzed in the SPSS package program and was found to be 0.647. It can be said that the reliability of the test is at a moderate level. Kurder-Richardson (KR20) coefficient was analyzed in the SPSS package program and found to be 0.688. It can be said that the reliability of the test is at a medium level. Kurder-Richardson (KR21) coefficient was analyzed in Minitab and found to be 0.459. It can be said that the test has a low level of reliability.

Keywords: *Reliability, cronbach alpha, split-half, kr20, kr21*

INTRODUCTION

The desired feature of measurement statistical units is represented by numbers within the framework of certain rules. In many areas, physical methods are used to reflect a formation in its real form. Such as length, volume, weight. These types of features can be measured directly with tools such as meters and scales. While concrete features can be represented with clear numbers, it is more difficult to represent abstract features. Abstract features (behavioral, judgmental, attitude, knowledge, taste, skill, performance, success, etc.) cannot be measured directly with physical tools. Different measurement tools (scales) are developed and indirect measurement is made in order to measure features that cannot be measured directly (Ercan and Kan, 2004).

Since it is not possible to measure the feature with a single item in scales, the scale contains k items/questions. An indirect measurement method is used according to the answers given to these items. Scales are tools that try to quantify the characteristics of an individual such as attitude, personality traits, taste, behaviors, subjective judgments, etc. Since these tools address non-objective situations, it is necessary to be meticulous in the preparation and application of these tools. Tests that measure knowledge are also scales. Affective, psychological, attitude, etc. may contain more objective criteria than scales measuring features (Aba and Kulakaç, 2016), (Lia et al.,2020).

Some features are desired for the developed scales. These are validity and reliability. Usability is also expected. Validity is when a test measures the feature it wants to measure correctly and without mixing it with other features. In other words, it is when a measuring tool measures the feature it wants to measure

correctly. For example, using a meter to measure length. When we use a meter to measure weight, the scale is not valid (Altınova and Duyan, 2013).

Reliability: It means that a measuring tool gives the same or similar results (at least 70%) when we apply it repeatedly under the same conditions. In addition, reliability is the level of purity of the measuring tool from errors (especially random errors). It is the measure of the consistency in the scale and the closeness of the scale to reality. It can also be defined as the repeatability of the test and its consistency in repetitions. A valid test must be reliable. However, a reliable test may not be valid. (Ercan and Kan, 2004), (Yaşar, 2014), (Koçak et al., 2014), (Arastaman et al., 2018).

Reliability analysis is the process of interpreting the results of studies and studies conducted to evaluate the reliability of measurement tools (Çalışkan and Çınar, 2012).

In order to increase reliability, it is necessary to minimize the error. Error sources are determined and controlled. Errors may be factors other than the measurement tool or they may originate from the measurement tool. Reliability must be examined in those originating from the measurement tool (Ercan and Kan, 2004).

In this study, some of the reliability analysis methods were introduced and their analysis and interpretation were included.

MATERIAL AND METHODS

Material

In this study, a 10-item attitude scale towards mathematics was applied to students in 5/A/B/D classes at Kahramanmaraş 5 Nisan Secondary School for Cronbach Alpha analysis. A different 10-item mathematics attitude scale was applied to students in 5/A/B/D classes at 5 Nisan Secondary School for Split-Half analysis. A 10-item test was applied to 25 8th grade students at Kahramanmaraş 5 Nisan Secondary School for KR-20 analysis. The results of the second written exam of the Mathematics and Science Applications course of 6/E classes at Kahramanmaraş 5 Nisan Secondary School were used for KR-21 analysis.

Methods

There are different methods for reliability analysis. The appropriate analysis method can be selected according to features such as scale features and data structure.

Cronbach Alpha

It is the general reliability coefficient that measures the power of a scale containing k items with continuous, intermittent, consecutive 4 or usually 5-choice answers. It is a value found by dividing the total variance of the items in the scale by the general variance. Likert-type scales are frequently used. Cronbach alpha coefficient evaluates the general reliability of the scale. It measures the internal consistency of the items in the scale. It is also an indicator of the homogeneity of the items in the scale. It evaluates whether the k items in the scale form a whole to explain a homogeneous structure. Cronbach alpha coefficient takes a value between 0 and 1. As the coefficient increases, the reliability of the test increases. A high coefficient shows that the items in the scale are consistent with each other and measure the same feature (Çalışkan and Çınar, 2012), (Tavakol and Dennick, 2011).

Cronbach alpha reliability coefficient is calculated with the formula below.

$$\alpha = \frac{k}{k-1}\left(1 - \frac{\sum_{i=1}^{k} Si.Si}{Sp.Sp}\right)$$

$S_i.S_i = S_i^2$: Item Variance

$S_p.S_p = S_p^2$: General Variance

If the Cronbach alpha coefficient is between 1 and 0.80, it can be said that it has high reliability, if it is between 0.79 and 0.60, it has medium reliability, and if it is between 0.59 and 0.40, it can be said that it is not reliable (Yaşar, 2014). In addition, when the coefficient is above 0.70, it can be said that the reliability is at an acceptable level (Karakaya Özer, 2021).

Split-Half

A method based on examining the applied measurement tool by dividing it into two equal parts. The halves can be applied to the subjects at the same time or the items in the applied scale can be divided into two equal parts. The halves are obtained by dividing the items in the scale into two equal parts. If the number of items in the scale is even, the halves are in the form of k1=k2=k/2. If the number of items in the scale is odd, the halves are in the form of k1=(k+1)/2), i.e. k1= 1,2,…,k1 and k2=k-k1, i.e. k2=k1+1, k1+2,…,k. The correlation coefficient r between the two variables consisting of the sum of the items in both halves is found. Reliability is estimated with the correlation between the scores obtained from the halves (Ercan and Kan, 2004), (Cronbach, 1946).

Kurder-Richardson KR20

It is a coefficient that measures internal consistency in scales with items that include dichotomous responses. It examines whether the items are homogeneous. It is a method based on the estimation of internal consistency of the items in the scale with each other and with the entire scale. It is assumed that all items in the scale measure the same feature. Item responses are coded as true/false, yes/no, positive/negative 1/0. It is used in cases where the test items are not of similar difficulty. If the number of samples is 30 or more, it is considered a large sample (Ercan and Kan, 2004), (Soto and Charter, 2009).

For large samples; $S^2_x = \frac{\sum_{i=1}^{N}(Xi - Xort)2}{N}$

For small samples; $S^2_x = \frac{\sum_{i=1}^{n}(Xi - Xort)2}{n-1}$

$$KR20 = \frac{k}{k-1}\left(1 - \frac{\sum_{i=1}^{k} pi.qi}{S2x}\right)$$

k: number of items in the scale

pi: number of positive/correct answers

qi: number of negative/incorrect answers

Sx^2: variance of the scale

The KR20 coefficient takes a value between 0 and 1. As the coefficient approaches 1, internal consistency increases. As it approaches 0, it decreases. The internal consistency/homogeneity of the scale can also be expressed as a percentage between 0 and 100 by expressing it as KR20*100.

Kurder-Richardson KR21

It is a coefficient that measures internal consistency in scales with items that include dichotomous answers. The answers given to the items are equally weighted and are answered as yes/no, positive/negative, yes/no, 0/1. It is a method based on the estimation of internal consistency of the items in the scale with each other and with the entire scale. It is assumed that all items in the scale measure the same feature. It is used in cases where the difficulty level of the test items is the same (Ercan and Kan, 2004), (Foster, 2022).

For large samples; $S^2_x = \frac{\sum_{i=1}^{N}(Xi - Xort)2}{N}$

For small samples $S^2_x = \frac{\sum_{i=1}^{n}(Xi - Xort)2}{n-1}$

$$KR21 = \frac{k}{k-1}\left(1 - \frac{nXort - X2ort}{S2x}\right)$$

k: number of items in the scale

Xmean: mean of positive answers

S^2_x: variance of the scale

n: number of units applied

The KR21 coefficient takes a value between 0 and 1. As the coefficient approaches 1, internal consistency increases. As it approaches 0, it decreases. The internal consistency/homogeneity of the scale can also be expressed as a percentage between 0 and 100 by expressing it as KR21*100.

The results were obtained using the SPSS package program and the Minitab package program.

RESULTS

For Cronbach Alpha coefficient analysis, a 10-item mathematics attitude scale was applied to 59 students in 5/A/B/D classes at Kahramanmaraş 5 Nisan Secondary School. The answers were 1-strongly disagree, 2-disagree, 3-undecided, 4-agree, 5-strongly agree. The analysis was done in the SPSS package program.

Table 1: Cronbach Alpha Coefficient

Reliability Statistics

Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
,427	,441	10

As a result of the analysis, Cronbach alpha coefficient was found to be 0.427.

For the Split-Half coefficient analysis, a 10-item mathematics attitude scale was applied to 59 students in 5/A/B/D classes at Kahramanmaraş 5 Nisan Secondary School. A different scale was applied than the scale applied in the Cronbach alpha coefficient analysis. The answers were 1-strongly disagree, 2-disagree, 3-undecided, 4-agree, 5-strongly agree. The analysis was done in the SPSS package program.

Table 2: Split-Half Coefficient

Reliability Statistics

Cronbach's Alpha	Part 1	Value	,145
		N of Items	5[a]
	Part 2	Value	,080
		N of Items	5[b]
	Total N of Items		10
Correlation Between Forms			,478
Spearman-Brown Coefficient	Equal Length		,647
	Unequal Length		,647
Guttman Split-Half Coefficient			,647

a. The items are: M1, M2, M3, M4, M5.
b. The items are: M6, M7, M8, M9, M10.

As a result of the analysis, the Split-Half coefficient was found to be 0.647. Here, items 1,2,3,4,5 constituted the first half and items 6,7,8,9,10 constituted the second half.

A 10-question test was applied to 25 8th grade students at Kahramanmaraş 5 Nisan Middle School for the KR20 coefficient analysis. Correct answers were coded as 1 and incorrect answers as 0. The analysis was performed in the SPSS package program. Here, the KR20 coefficient was accepted as the Cronbach alpha coefficient and the results were obtained.

Table 3: KR20 Coefficient

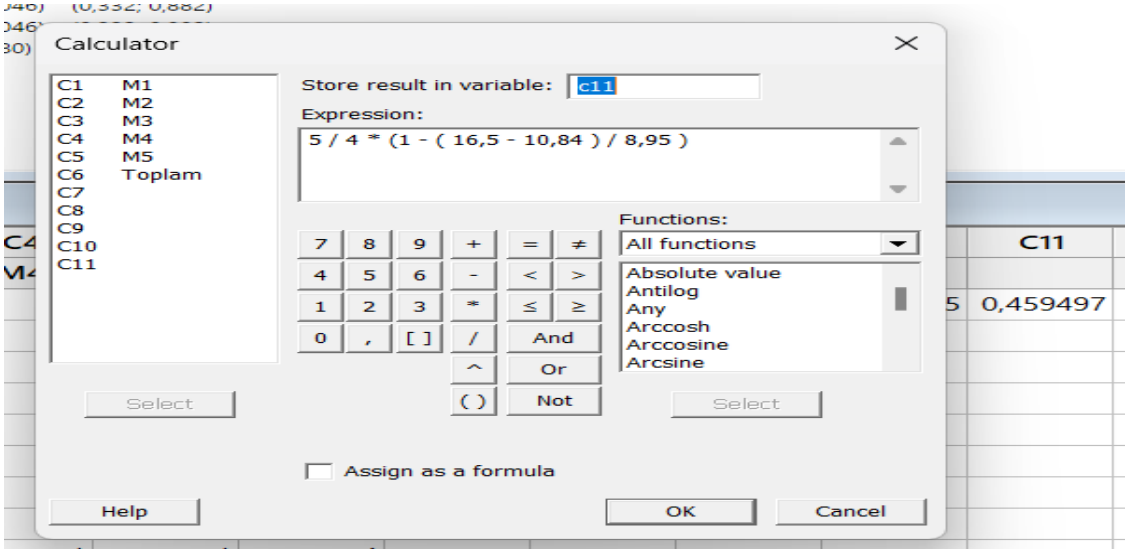
Reliability Statistics

Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
,688	,717	10

As a result of the analysis, the KR20 coefficient was found to be 0.688.

For the KR21 coefficient analysis, the second written results of the Mathematics and Science Applications course of 23 students in the 6/E class of Kahramanmaraş 5 Nisan Secondary School were used. Correct answers were coded as 1, wrong answers as 0. Minitab package program was used for coefficient analysis.

Table 4: KR21 Coefficient



As a result of the analysis, the KR21 coefficient was found to be 0.459.

DISCUSSION AND CONCLUSION

Cronbach's alpha coefficient was analyzed in the SPSS package program and found to be 0.427. It was found that the reliability of the test was low. The split-half coefficient was analyzed in the SPSS package program and found to be 0.647. It can be said that the reliability of the test was at a moderate level. The

Kurder-Richardson (KR20) coefficient was analyzed in the SPSS package program and found to be 0.688. It can be said that the reliability of the test was at a moderate level. The Kurder-Richardson (KR21) coefficient was analyzed in Minitab and found to be 0.459. It can be said that the test has a low level of reliability.

Here, the reliability of the Cronbach's alpha coefficient was found to be low. Some items can be removed from the test to increase reliability. Here, the application was made to 59 students. The reliability coefficient can increase when the number of students to whom the application was made is increased.

The reliability of the split-half coefficient was found to be at a moderate level. Reliability can be increased by removing some items or increasing the number of students to whom it was applied.

The KR20 coefficient was found to be at a medium level but very close to an acceptable level. The application was made to 25 students. When the number of students applied is increased and reanalyzed, the reliability coefficient may be high.

The KR21 coefficient was found to be at a low level. The results of 23 students were used. The reliability coefficient may increase when the results of more students are used.

Although the scales and tests applied were prepared in accordance with the purpose, the reliability coefficient may not have been at the desired level because the application was made to a small number of students. It is expected that the reliability coefficients will be higher if the data is taken from more students.

The analyses can be repeated by increasing the number of students. Different methods than the methods examined in this study can be examined for reliability analysis.

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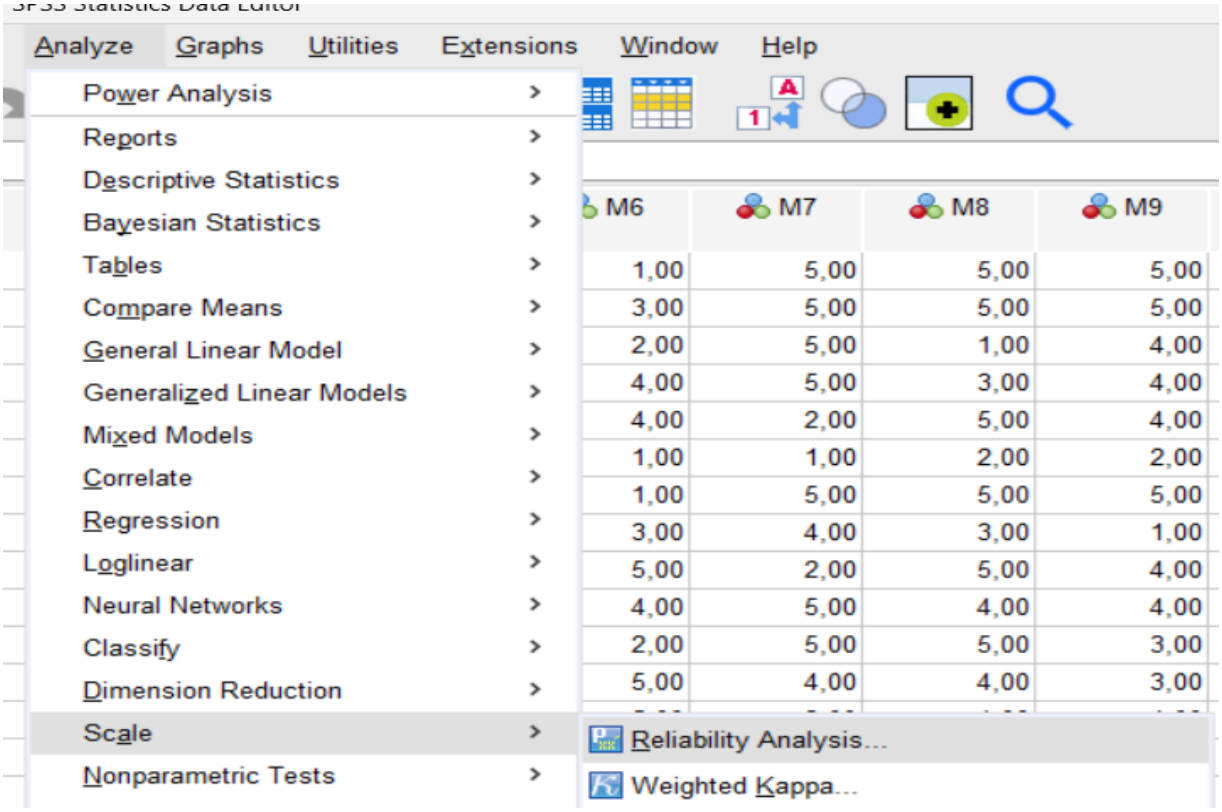
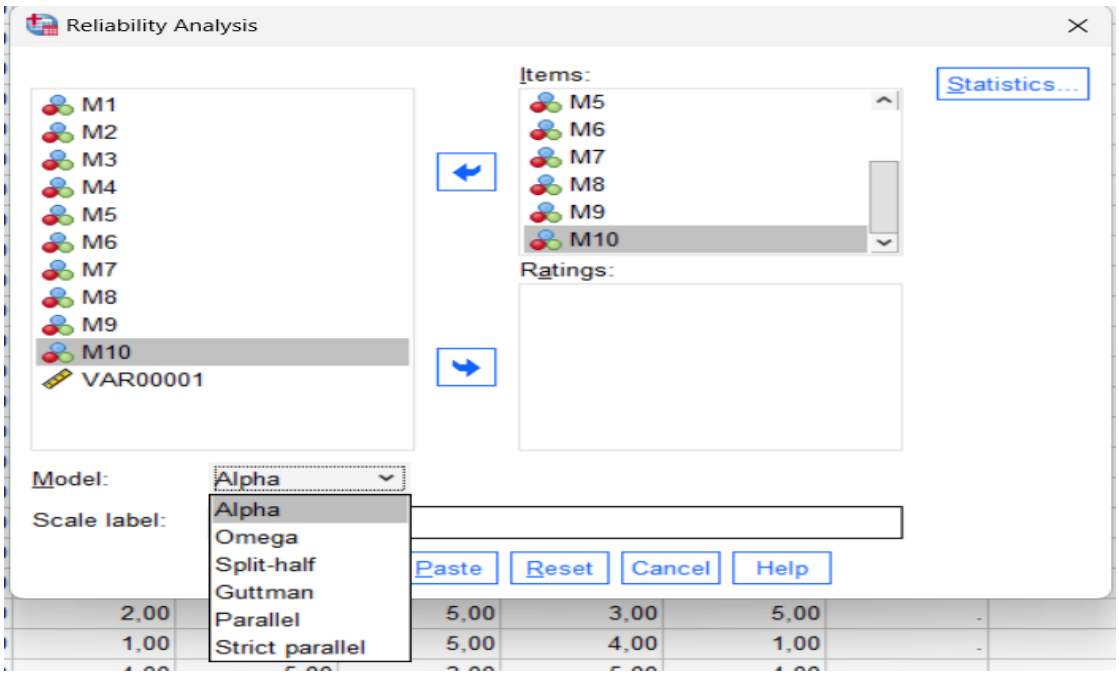
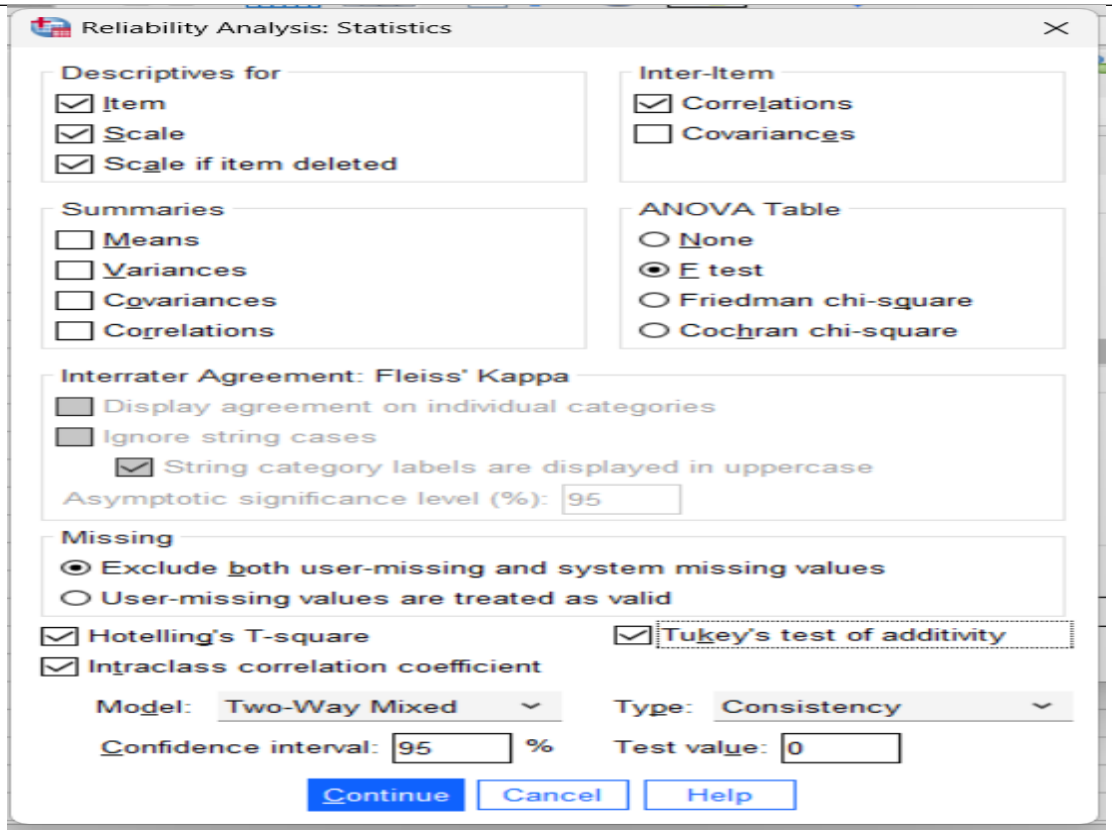
ATTACHMENTS



After data entry for Cronbach alpha analysis, the analyze/scale/reliability analysis command sequence is followed.



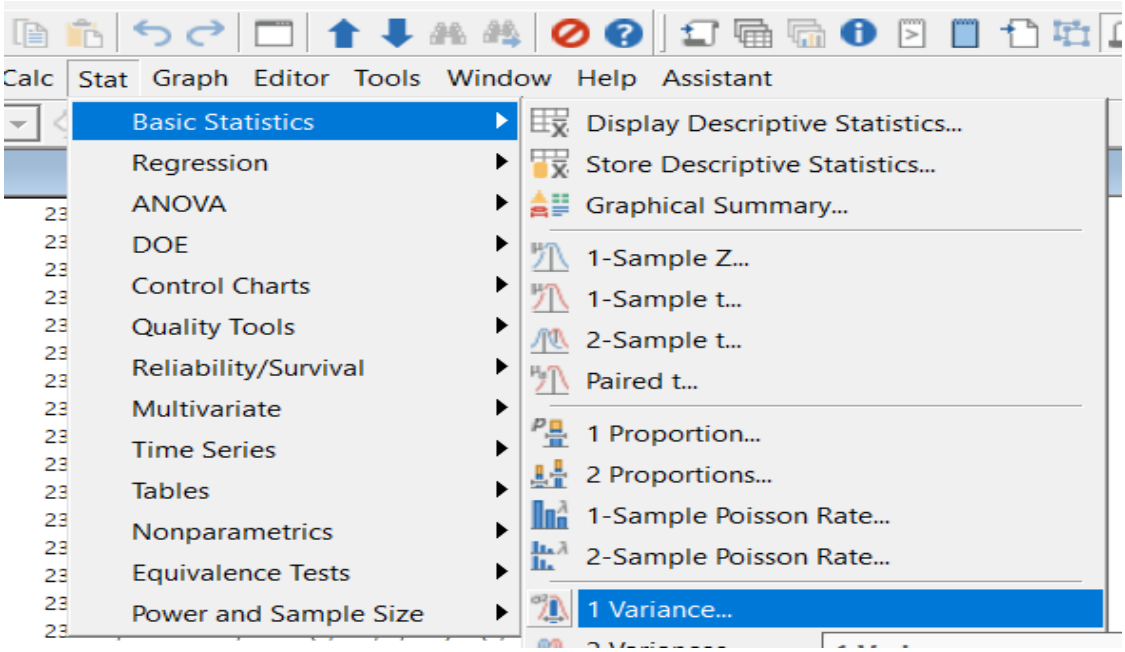
After following the command line, in the window that opens, the items on the left are moved to the right with the arrow. Then alpha is selected in the model box. Then statistic is clicked on the top right.



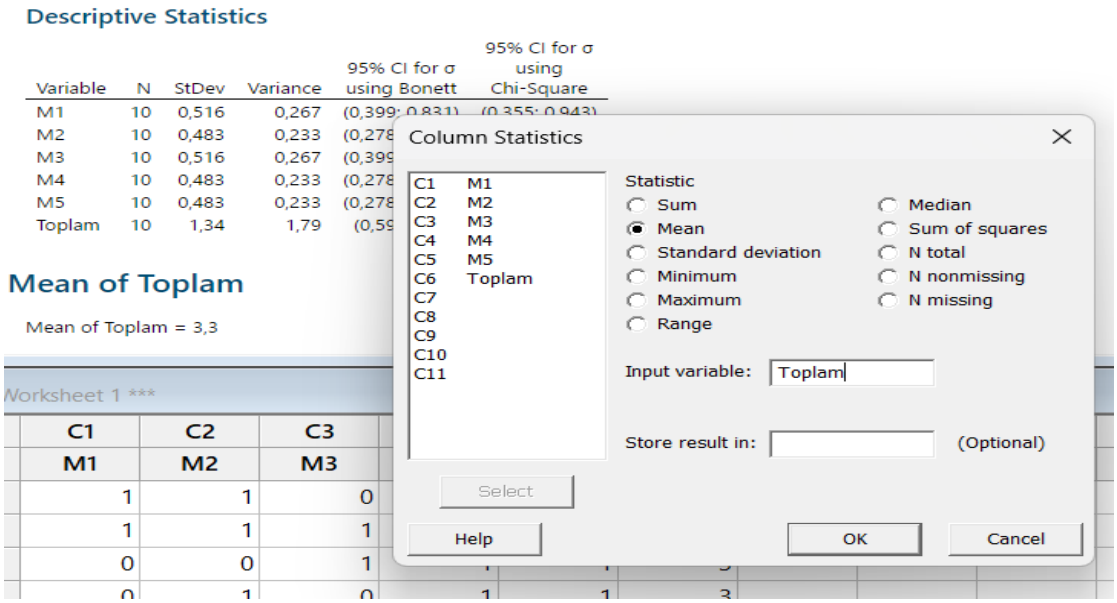
In the window that opens, the information that you want to include in the output is marked. Then, continue/ok is clicked and the analysis results are seen.

For split half analysis, Cronbach alpha analysis steps are followed. In the model section of the reliability analysis window, split half is selected.

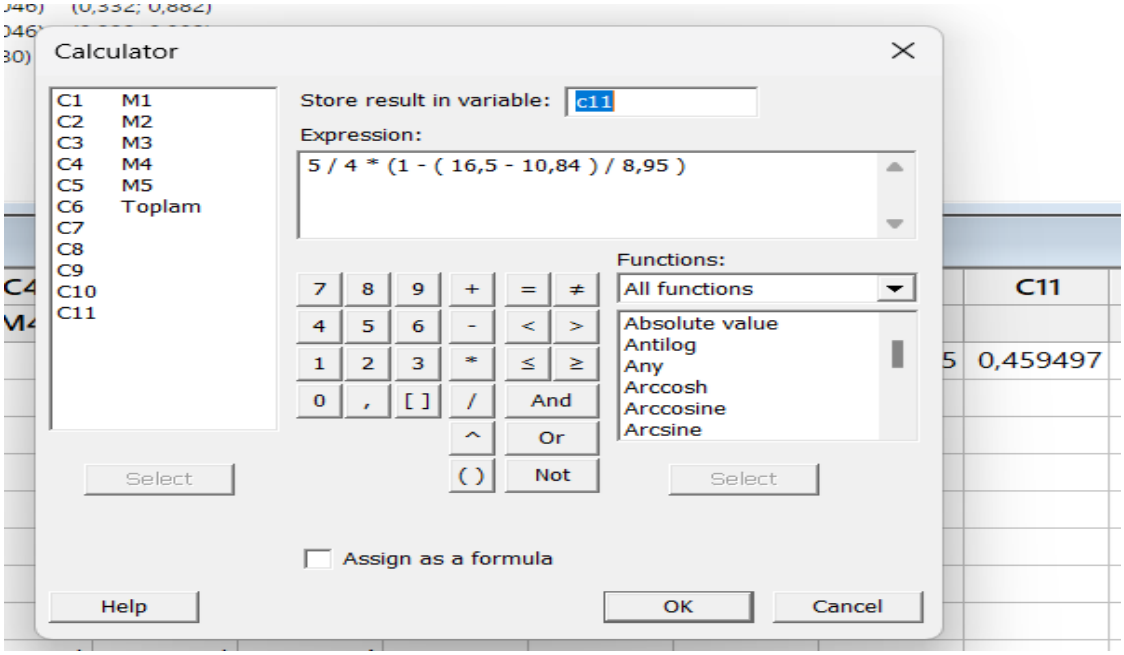
The KR20 coefficient can be calculated in SPSS like the Cronbach Alpha coefficient and the alpha value can be taken as the kr20 coefficient and commented on. After data entry, the analyze/scala/reliability analysis command line is followed.



After data entry into Minitab, the stat/basic/statistic/variance command line is followed to calculate item variances and test score variance. Then, ok is clicked.



After the data is entered into minitab, the calc/column statistic command sequence is followed to find the average of the test scores. Other values required for the formula are calculated in the same way.



After finding the values required for the KR21 coefficient, the calc/calculator command line is followed. In the opened window, the cell in which we want to see the result is written in the store result in variable section. The formula values for the kr21 coefficient are entered in the Expression section as shown below and the arrow is clicked.

Multi-Frequency and Multi-Protocol RFID Card Reader Device: Hardware and Application Design (1174)

Mustafa Talip Koyuncu[1*], Burak Demir[1], Muhammet Fatih Aslan[2], Akif Durdu[3]

[1] Butkon Asansör San. Tic. A.Ş., Research and Development, Konya, Turkiye
[2] Karamanoglu Mehmetbey University, Faculty of Engineering, Electrical and Electronics Engineering, Karaman, Turkiye
[3] Konya Technical University, Faculty of Engineering, Electrical and Electronics Engineering, Konya, Turkiye

* Corresponding author e-mail: mustafa.talip.koyuncu@gmail.com

Abstract

In this study, a dual frequency supported Radio Frequency Identification (RFID) card reader device that can operate at both 125 kHz and 13.56 MHz is developed for use in access control systems. Most of the existing systems operate at a single frequency band and therefore offer limited card compatibility. But, the proposed design supports multiple RFID standards and offers a more flexible and user-friendly solution. The reader functioning at 125 kHz can successfully detect cards based on the EM4100 protocol, whereas at 13.56 MHz, it effectively communicates with MIFARE Classic, MIFARE Ultralight, and Near Field Communication (NFC)-compatible cards. In the hardware design, two RFID modules have been optimized and integrated under the control of a single microcontroller. The system is capable of automatically detecting the card's frequency, switching to the appropriate protocol, and performing secure identity verification processes. Additionally, the device has been designed to be resistant to electromagnetic interference and capable of operating inside a metal enclosure, taking into account harsh environmental conditions. Thus, it can also exhibit reliable performance in areas such as industrial facilities, parking systems and dense metal environments. Thanks to this innovative approach, not only has compatibility with existing card infrastructures been achieved, but also an important step has been taken for long-lasting and secure access control systems.

Keywords: *Access Control Systems, Industrial Applications, MIFARE, NFC, RFID,*

INTRODUCTION

Radio Frequency Identification (RFID) is a technology and system that uses radio waves (wireless) to remotely identify and track objects. RFID systems consist of tag, reader, and back-end database components. Data transmission is carried out via electromagnetic fields (Piramuthu, 2007). The tag contains object identification (ID) data, the reader reads this data, and the back-end database is used to store product information (Khan et al., 2024). In the early days, RFID was only used to basically identify an object (for example, to understand who a product is) or to track its location. However, later, chip RFID went far beyond optical barcode technology. Thanks to radio waves, out-of-sight communication was provided. Therefore, the object could be read even when it was in the box. In addition, unlike barcodes, RFID chips could store more information, such as production date, manufacturer, temperature history, and transportation information. In addition, RFID systems can automatically identify an object when it enters the reading area without the need for human intervention (Lasantha et al., 2023).

Today, RFID technology is widely and effectively used in different sectors. RFID has gained application in many sectors such as manufacturing (e.g. inventory tracking), logistics (Wanhua, 2020), healthcare

(Cheng et al., 2021), retail (e.g. stock tracking), construction, transportation systems (Casella et al., 2022). It offers a contactless, fast, and reliable solution, especially in security-oriented applications such as access control. Its main advantages include automation capability, multiple tag recognition, and reliable performance even in harsh environmental conditions. For example, it can identify many RFID tags simultaneously in fast-flowing production lines or at points with busy passages. In addition, RFID tags that are resistant to harsh environmental conditions such as high temperature, humidity, dust, and impact offer suitable solutions for industrial environments (Zhao et al., 2022).

RFID systems are divided into three basic groups according to their operating frequencies: Low Frequency (LF, 125–134 kHz), High Frequency (HF, 13.56 MHz), and Ultra High Frequency (UHF, 860–930 MHz). Generally, a certain type of passive RFID tag can only be read by a reader of the same type. LF RFID systems have a short reading range but work more stably in challenging environments such as metal and liquid. HF systems have a short reading range of about 12 cm and are widely used in contactless smart cards and access systems. UHF RFID systems provide a reading range of up to 3 meters (Ashour et al., 2023).

In RFID systems, readers are generally designed to operate only at a certain frequency, which leads to significant limitations in system integration. Most RFID readers operate either with only LF, only HF or only UHF protocols. This requires users to install multiple systems and increases hardware costs and system complexity. Because multiple readers or complex access systems must be installed to read RFID tags operating at different frequencies. This poses a significant problem in terms of traceability and flexibility, especially in industrial applications. In addition, due to frequency dependency, systems may need to be adapted separately to legal frequency restrictions in different geographical regions. In this context, the demand for hybrid systems is increasing today, because installation, maintenance and user management become more efficient thanks to devices that can support both HF cards such as MIFARE and LF cards such as EM4100. In addition, the importance of such systems for Internet of Things (IoT) integrations, multi-device compatibility and comprehensive security systems is rapidly increasing (Hirvonen et al., 2008).

In the past literature, it has been frequently emphasized that RFID systems mostly operate at only a single frequency, which creates serious limitations on system flexibility and hardware costs. In this context, Cui et al. (2019) discussed the place of RFID technology in IoT systems in terms of energy efficiency, data transmission and sensor integration in a review study. In the study, they detailed the working principles of both HF and UHF RFID systems. They analyzed the impact of different antenna designs and RF energy harvesting methods on RFID sensor performance. The authors emphasized that RFID-based sensors are generally optimized only for HF or UHF systems, and multi-frequency systems are not yet widespread, which limits interoperability in applications. Similarly, Bajaj et al. (2024) detailed the limitations of modern RFID reader antenna technologies in a review article. They also presented research trends in the development of multi-frequency systems. The authors noted that LF, HF, and UHF bands each serve different application requirements, but unfortunately most current readers are optimized for only a single frequency band. Based on this, the authors clearly emphasized the need for multiple readers, especially in environments where a wide variety of cards may be used, such as logistics, supply chain, and healthcare systems. In a separate study, Fischer et al. (2019) developed a multi-frequency (868 MHz and 915 MHz) RFID reader system in the UHF band. The system was configured on a software-defined radio (SDR) platform in accordance with international standards. With this system, the authors aimed to increase the reading success in multipath environments. Experimental results showed that successful reading was achieved from the same tag at both frequencies. Thanks to frequency diversity, blind spots known as “dead zones” were significantly reduced. This study technically demonstrated the advantages of multi-frequency systems in terms of

reliability, especially in harsh industrial environments. Similarly, Bournine et al. (2015) developed a multi-frequency RFID reader prototype operating at HF and UHF frequencies. They also integrated this device into a cloud-based traceability system. In the designed system, they combined Adafruit PN532 HF module and HYM730 UHF module using Arduino Due platform. They managed the frequencies sequentially with microcontroller software. For a secure access model, after user authentication with HF tag, transactions were allowed via UHF. This study presented a multi-frequency RFID that supports different RFID protocols on the same device and can be integrated into traceability processes.

This study aims to develop a multi-frequency and multi-protocol RFID reader system that can operate at both LF (125 kHz) and HF (13.56 MHz) frequencies on a single device. The developed system can successfully identify EM4100 based low-frequency cards and MIFARE Classic, MIFARE Ultralight and NFC compatible high-frequency cards. Thus, this application aims to provide a flexible and multi-purpose solution in response to the limited frequency compatibility of traditional RFID readers, similar to previous studies. The reader side of the RFID allows two different RFID modules to be automatically controlled by a single microcontroller. In this way, users can perform access transactions without thinking about which card they use in the system. In addition, the device's electromagnetic interference-resistant structure and its ability to operate even in metal enclosures make it suitable for use in harsh industrial conditions.

MATERIAL AND METHODS

Hardware Components

In the proposed system, a dual-frequency RFID card reader architecture is developed to enable the detection of both low-frequency (125 kHz) and high-frequency (13.56 MHz) RFID cards. At the core of the system is the NUC1311L series microcontroller based on the Advanced RISC Machine (ARM) Cortex-M0 core. It also manages the communication with the peripheral components via Serial Peripheral Interface (SPI), Inter-Integrated Circuit (I^2C) and Universal Asynchronous Receiver/Transmitter (UART) protocols.

For the LF section, an analog front end (AFE) circuit was designed according to the EM4100 protocol. The circuit consists of a resonant inductance-capacitance (LC) network operating at 125 kHz driven by a 50% duty cycle pulse-width modulation (PWM) signal generated by the microcontroller. A specially designed air core coil antenna optimized for high quality (Q) factor and mechanical stability was used. The multi-turn antenna design is shown in Figure 1. The inductance and resonant frequency were verified using an inductance-capacitance-resistance (LCR) meter and the antenna was tuned with a series capacitor. The magnetic signal induced in the LC circuit is passed through a low noise differential preamplifier, followed by an envelope detector and a comparator circuit to convert the signal to a digital format suitable for further processing. The digitized data is transferred to the digital input pin of the microcontroller and made available for timing analysis in the software layer.

For the HF section, a Texas Instruments TRF7970A RFID transceiver was used. This module supports International Organization for Standardization / International Electrotechnical Commission (ISO/IEC) 14443 and ISO/IEC 15693 standards, enabling communication with MIFARE Classic, MIFARE Ultralight, and near field communication (NFC) compatible cards. A planar spiral printed-circuit-board (PCB) antenna, designed according to calculation tools and radio frequency (RF) design guidelines, was directly interfaced with the TRF7970A module. The PCB antenna design is shown in Figure 2. Critical parameters such as trace width, spacing, and impedance matching were carefully considered to minimize signal loss and maximize read range. SPI communication was established between the TRF7970A and the microcontroller, and an interrupt request (IRQ) line was configured to trigger card detection events.

Figure 1. Multi-turn coil antenna (125kHz)

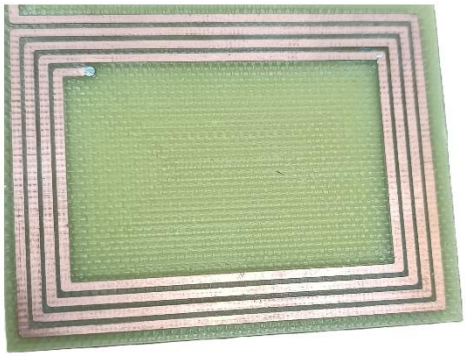


Figure 2. PCB antenna design (13.56Mhz)

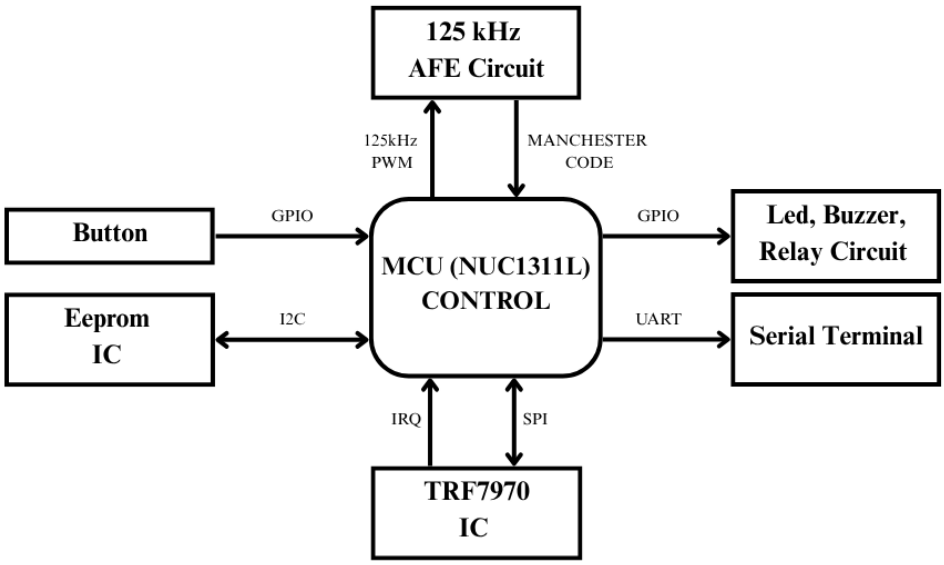


Figure 3. General Block Diagram of the Hardware

In addition to the main RFID modules, the system is equipped with light-emitting diode (LED) indicators, a buzzer circuit, and a relay module for access control feedback. Card registration and deletion are performed via a push-button interface, and card identification numbers (ID) are stored in an external electrically erasable programmable read-only memory (EEPROM) chip. The system compares each scanned card with the EEPROM database to verify authorization status. All relevant information, including the scanned card ID and authorization status, is transmitted via UART to a connected terminal for real-time monitoring. Figure 3 shows the overall block diagram of the design.

Software Components

RFID cards operating at 125 kHz typically use data encoding methods such as Manchester or Non-Return-to-Zero (NRZ). Manchester decoding is used in this study due to its reliability in detecting bit transitions. The microcontroller monitors the rising and falling edges of the input signal to determine each bit: a rising edge is interpreted as a logical '1', a falling edge as a logical '0'. This method allows the transmitted data stream to be accurately reconstructed. After the 64-bit Manchester encoded data stream is decoded, cyclic redundancy check (CRC) verification is performed to ensure data integrity. After successful verification, the 64-bit stream is processed to extract a 32-bit unique identifier (UID) used for access control logic.

The TRF7970A module for high-frequency cards manages communication using ISO/IEC compliant protocols. When a card enters the detection range, the module generates an IRQ, which triggers the

microcontroller to initiate SPI-based data reception. The incoming data stream is parsed according to the card's protocol structure and the UID is extracted. After verification and format conversion, the UID is reduced to a 32-bit card ID for uniformity in access processing.

In both frequency domains, the system ensures that only valid and correctly formatted card IDs advance to the access control evaluation phase. To support dual-frequency operation, the system alternates between LF and HF scan cycles. The microcontroller initiates a 200 ms LF scan window and actively checks 125 kHz cards. If no cards are detected within this window, the LF scanner is disabled and the HF scanner is enabled for the next 200 ms. This alternating cycle continues indefinitely. If a card is detected during any of the scan stages, the system temporarily stops scanning to process the card data. Once processing is complete, the alternating scan routine continues from the next cycle.

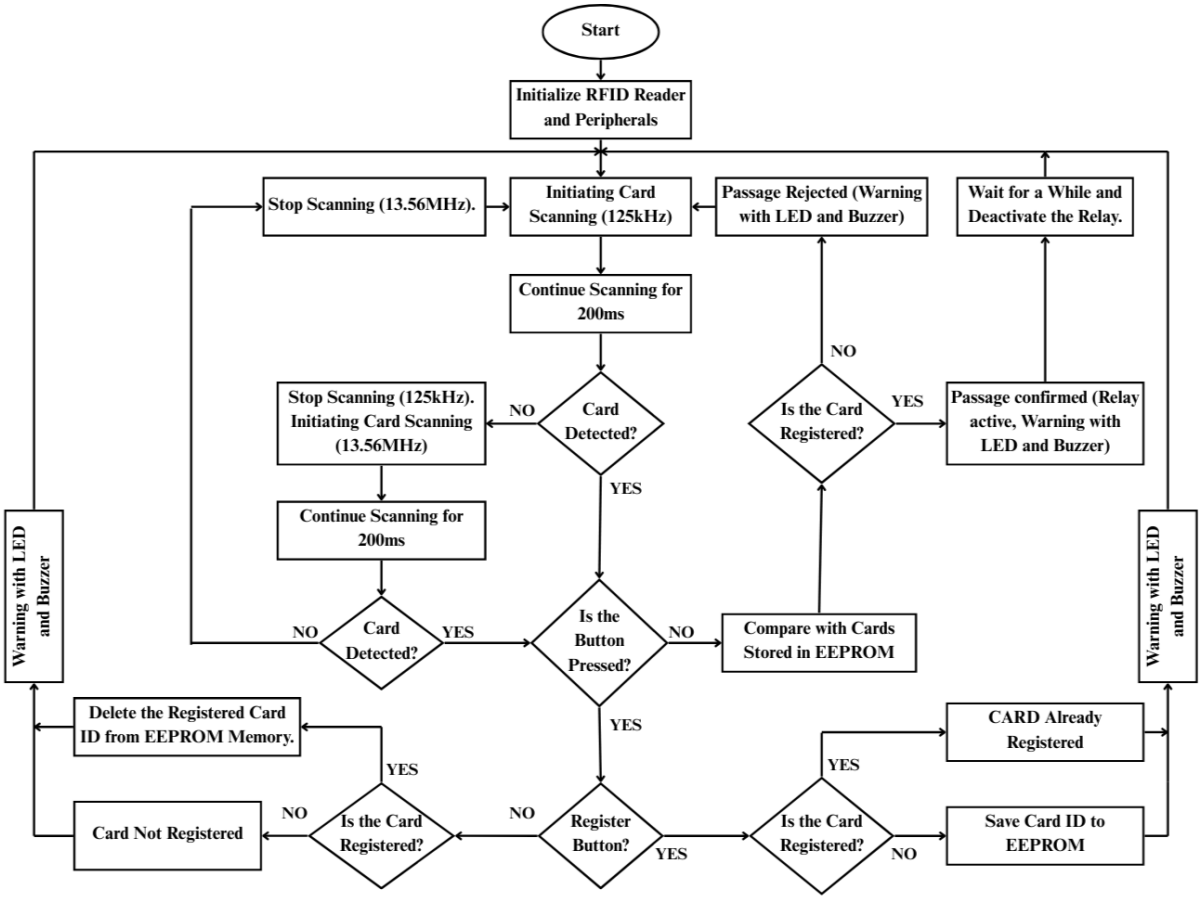


Figure 4. Algorithm Flowchart

The access logic of the system includes card registration, deletion and authorization control. A user can press a physical button to register a new card or delete an existing card. When a card is presented in this mode, the system checks its existence in the EEPROM database. If the card is not registered, it is added to the database. If it is already registered, a "duplicate" warning is given. If deletion is requested, the card is removed from the EEPROM or a "not found" message is returned if it is not stored at all. In normal operation, each card scanned is compared with the list stored in the EEPROM. If the card is recognized, the system authorizes access by triggering the relay, turning on the green LED and briefly sounding the beeper. For unrecognized cards, the system activates the red LED and emits a long beeper tone. All card events and system responses are recorded and transmitted to a serial terminal via UART for monitoring and debugging purposes. Figure 4 shows the general flowchart for the software algorithm.

RESULTS

In this study, a dual-frequency RFID card reader system capable of operating at both 125 kHz and 13.56 MHz was successfully designed, implemented, and tested. All tests and experimental analyzes were carried out at BUTKON R&D center (BUTKON, 2025). The system demonstrated reliable performance in reading EM4100-compliant LF cards, including MIFARE Classic, MIFARE Ultralight, and NFC-enabled cards, as well as ISO/IEC 14443-compliant HF cards. Combining both frequency bands under the control of a single microcontroller enabled seamless protocol transition and real-time access control functionality. Figure 5 shows the designed circuit board, and Figure 6 shows the card ID information read as a result of the tests.

Figure 5. The prototype of the designed circuit

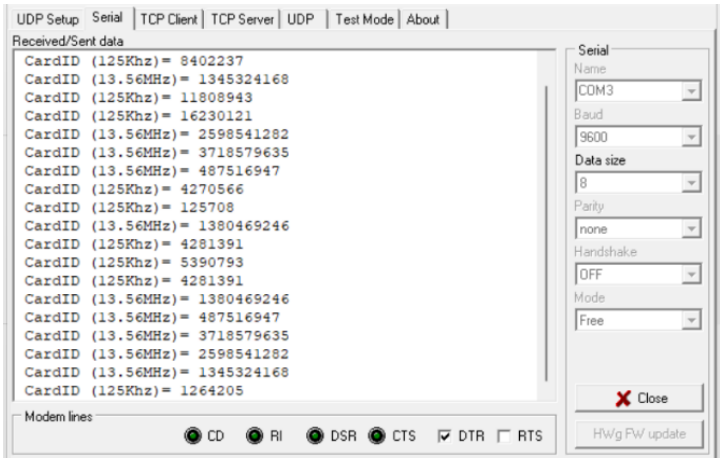


Figure 6. Serial Terminal Output

Considerable emphasis was placed on robust hardware design practices, including careful PCB layout with ground plane separation and controlled return paths to reduce electromagnetic interference. LC resonator and planar PCB antenna structures were optimized to target resonant frequencies and validated through experimental testing using LCR meters and functional range measurements. Terminated antenna designs provided stable performance and adequate read distances in both frequency domains.

Overall, the developed system provides a compact, scalable and cost-effective solution for modern access control applications. Its ability to support multiple card types, conduct secure authentication and withstand harsh environmental conditions makes it suitable for use in factories, smart building entry points and high-traffic areas. The dual-frequency design ensures long-term compatibility with a wide range of existing RFID infrastructures and simplifies system management by combining different card technologies on a unified reader platform.

CONCLUSION AND DISCUSSION

In this study, a multi-frequency and multi-protocol card reader prototype that can read both low-frequency (LF, 125 kHz) and high-frequency (HF, 13.56 MHz) RFID cards was developed. The developed system successfully identified EM4100-based LF cards and HF/NFC compatible cards such as MIFARE Classic and Ultralight; and automatically switched between frequencies. The tests showed that the cards could be read correctly and that the system could operate stably even in metal enclosures against electromagnetic interference. In this way, it offers a practical, flexible and reliable solution for security-oriented applications such as device access control.

While the current system successfully achieves multi-frequency RFID card detection and access control functionality, future studies could focus on extending its capabilities through integration with IoT platforms. Adding IoT support would enable real-time remote monitoring, centralized data logging, and

improved traceability of access events. Secure communication protocols can be implemented to transmit card activity to cloud-based databases, allowing for advanced data analytics and audit capabilities. In addition, the development of a user-friendly web or mobile application interface could simplify user registration, card management, and remote authorization processes. Such improvements would enhance the system's scalability and usability in complex or distributed environments, such as smart buildings, campuses, or industrial facilities. Incorporating features like biometric verification or multi-factor authentication in future iterations may also further strengthen system security and versatility.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

Mustafa Talip Koyuncu: Methodology, investigation, visualization, writing.

Burak Demir: Methodology, investigation

Muhammet Fatih Aslan: Writing, supervision.

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Structural and Statistical Analysis of Finite Mixture Models Based on q-Calculus (1179)

Nurgül Okur[1*]

[1]Giresun University, Faculty of Science and Arts, Department of Statistics, Türkiye

*Corresponding author e-mail: nurgul.okur@giresun.edu.tr

Abstract

The foundations of q-analysis date back to the 1740s, when Euler introduced the theory of partitions, also referred to as additive analytic number theory. Over the years, the discovery of q-calculus applications in fields such as operator theory, combinatory, probability theory, and many others has sparked tremendous interest in this mathematical framework.

Mixture distributions are probabilistic models in which a data set is assumed to originate from multiple underlying distributions, each contributing with a certain probability. These distributions are commonly used to model complex data structures more accurately.

This paper introduces q-finite mixture models as a novel extension of the classical finite mixture family, motivated by recent progress in q-calculus and generalized probability distributions. By incorporating a deformation parameter q, the proposed mixture models offer enhanced modeling flexibility for a variety of stochastic phenomena. The fundamental distributional and statistical properties of the suggested q- mixture models are systematically are explored.

Keywords: *Finite mixture model, q-probability theory, q-parameter.*

INTRODUCTION

Quantum calculus, also known as q-calculus or calculus without limit is a generalization of classical calculus that originated in the early 20th century, although its roots can be traced even further back. Euler studied the q-analog of Newton's infinite series and made foundational contributions. Jacobi formulated the Gauß-Jacobi triple product identity. Gauß introduced the q-binomial coefficients and established identities involving them. Jackson defined the concept of the q-integral. Ernst provided a comprehensive historical overview and proposed a new approach to q-calculus. Cheung and Kac authored the monograph Quantum Calculus, further developing the field.

Mixture distributions are probabilistic models in which observations are assumed to originate from multiple underlying distributions, each with a certain probability. The evolution of q-distributions represents a natural progression in the development of q-calculus. q-calculus serves as a parametric generalization of classical calculus, with the classical framework being recovered in the limit as $q \to 1$. Significant contributions to the theory of q-distributions including such as Dunkl, 1981, Crippa et al., 1997, Kupershmidt, 2000, Kemp, 2002, Charalambides, 2016, including the Gaussian and generalized gamma q-distributions by Diaz et al., 2009, 2010, the Erlang q-distributions by Charalambides, 2016, the gamma and beta q-distributions by Boutouria et al., 2018, the Lindley q-distribution in two forms was introduced by Bouzida, 2023.

In response to recent progress in the study of generalized probability q-distributions, this paper presents q-finite mixture distribution with their fundamental statistical and distributional characteristics.

MATERIAL AND METHODS

This section outlines the principles of q-calculus, and q-probability theory. In this entire study, unless otherwise stated, it is assumed that $0 < q < 1$. Readers are referred to the relevant literature.

Definition 1. (Kac and Cheung, 2002). Let x, q be real numbers. The q-number $[x]_q$ is defined as

$$[x]_q = \frac{1 - q^x}{1 - q}.$$

Definition 2. (Kac and Cheung, 2002). The q-Gauss binomial formula is given by

$$(x + y)_q^n = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q q^{\binom{k}{2}} y^k x^{n-k}, \qquad -\infty < x, y < \infty.$$

The q-binomial coefficients are provided for $k = 0,1, \dots, n$ by

$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{[n]_q!}{[n-k]_q![k]_q!}, \qquad [n]_{k,q} = \frac{[n]_q!}{[n-k]_q!}, \qquad [n]_q! = [n]_q[n-1]_q \dots [2]_q[1]_q.$$

Definition 3. (Kac and Cheung, 2002). The q-analogues of the exponential function are presented

$$E_q^\tau = \sum_{k=0}^{\infty} q^{\binom{k}{2}} \frac{\tau^k}{[k]_q!} = \prod_{k=0}^{\infty} \left(1 + (1-q)q^k\tau\right), \qquad \tau \in \mathbb{R},$$

$$e_q^\tau = \sum_{k=0}^{\infty} \tau = \prod_{k=0}^{\infty} \frac{1}{(1-(1-q)q^k\tau)}, \qquad |\tau| < \frac{1}{1-q}.$$

Definition 4. (Kac and Cheung, 2002). The q-derivative of f is defined as

$$D_q f(\tau) = \frac{f(q\tau) - f(\tau)}{q\tau - \tau}$$

Definition 5. (Kac and Cheung, 2002). The well-known Jackson q-integral of f is given by

$$\int_0^b f(\tau) d_q\tau = (1-q) \sum_{n=0}^{\infty} q^n b f(q^n b), \qquad b > 0.$$

Definition 6. (Vamvakari, 2023). X is considered q-continuous if there exists $f_q^X(x)$ such that

$$P\{a < X \leq b\} = \int_a^b f_q^X(x) d_q x, \qquad x \geq 0.$$

The q-cumulative distribution function (q-CDF) of X is defined for $x > 0$

$$F_q^X(x) = P(X \leq x) = \int_0^x f_q^X(u) d_q u,$$

satisfying the relation $P(\alpha < X \leq \beta) = F_q^X(\beta) - F_q^X(\alpha)$. Then, $f_q(x) = D_q F_q(x)$.

Definition 7. (Vamvakari, 2023). Under the condition $\xi_{(1)} < q\xi_{2)} < \xi_{(2)} < \dots < \xi_{(n-1)} < q\xi_{(n)}$ the random variable $\xi_{(\nu)}$ is defined as ν-th q-ordered random variable. The q-CDFs of the q-ordered statistics $\xi_{(n)}$, $\xi_{(1)}$, $\xi_{(\nu)}$ $(1 \leq \nu \leq n)$ are expressed, respectively

$$F_q^{\xi_{(n)}}(\tau) = \prod_{i=1}^{n} F_q\left(q^{i-1}\tau\right), \qquad F_q^{\xi_{(1)}}(\tau) = 1 - \prod_{i=1}^{n}\Big(1 - F_q(\tau)\Big),$$

$$F_q^{\xi_{(v)}}(\tau) = \sum_{w=v}^{n} \sum_{1<i_1<\cdots<i_w<n} \prod_{j=1}^{r} F_q\left(q^{j-1}\tau\right) \prod_{m=w+1}^{n} \Big(1 - F_q\left(q^{i_m-(m-w)}\tau\right)\Big)$$

Definition 8. (Vamvakari, 2023). Let $\mathbb{E}_q|\xi^r| < \infty$ for all positive integers r. Then,

$$\mu_q^{(r)} = \mathbb{E}_q(\xi^r) = \int_0^{\infty} \tau^r \, f_q^{\xi}(\tau) d_q\tau\,, \qquad \mathbb{E}_q(\xi) = \mu_q, \qquad \mathbb{V}_q(\xi) = \mu_q^{(2)} - \left(\mu_q\right)^2.$$

Definition 9. (Okur and Djongmon, 2025). Let ξ be a q-continuous non-negative RV, and $\mathbb{E}_q\left|\left(\xi - \mu_q\right)_q^r\right| < \infty$ for all positive integers r. Then,

$$m_q^{(r)} = \mathbb{E}_q\left(\xi - \mu_q\right)_q^r = \sum_{s=0}^{r} \begin{bmatrix} r \\ s \end{bmatrix}_q q^{\binom{s}{2}} (-1)^s \mu_q^s \mu_q^{(r-s)}\,, \qquad s \le r.$$

Definition 10. (Okur and Djongmon, 2025). Let ξ be a q-continuous non-negative RV. Then, its q-MGF is expressed in two distinct forms as follows:

$$\mathbb{M}_q^I(t) = \mathbb{E}_q\left(E_q^{qt\xi}\right) = \int_0^{\infty} E_q^{qt\tau} f_q(\tau) d_q\tau, \qquad \mathbb{M}_q^{II}(t) = \mathbb{E}_q\left(e_q^{t\xi}\right) = \int_0^{\infty} e_q^{t\tau} f_q(\tau) d_q\tau.$$

Definition 11. (Djongmon and Okur, 2025). Let ξ be a q-continuous non-negative RV. Then,

4) the q-survival function (q-CCDF) $\mathbb{S}_q(\tau) = P(\xi > \tau) = 1 - F_q(\tau)$,

5) the q-hazard rate function (q-HRF)

$$h_q(\tau) = \frac{P(q\tau \le \xi \le \tau | \xi \ge q\tau)}{(1-q)\tau} = \frac{f_q(\tau)}{\mathbb{S}_q(q\tau)},$$

6) the q-mean residual life function (q-MRLF)

$$mrl_q(\tau) = \mathbb{E}_q(\xi - \tau | \xi > \tau) = \frac{1}{\mathbb{S}_q(\tau)} \int_{\tau}^{\infty} \mathbb{S}_q(qu) d_q u.$$

Definition 12. (Djongmon and Okur, 2025). Let X and Y be two independent q-continuous non-negative random variables. The q-stress-strenght reliability (q-SSR) is given by:

$$\mathbb{R}_q = P(X > Y) = \int_0^{\infty} P(X > \tau |\, Y = \tau)\, f_q^X(\tau) d_q\tau = \int_0^{\infty} f_q^X(\tau) \mathrm{F}_q^Y(\tau) d_q\tau.$$

RESULTS

This section outlines a q-finite mixture model, including structral and statistical properties.

Modeling of a q-Finite Mixture Model

A q-finite mixture model (denoted q-FMM) is a q-probabilistic model composed of multiple component q-distributions combined with certain weights. The general form of mixture q-PDF is described as:

$$f_q^{mix(i_m)}(x) \quad = \sum_{k=1}^{K} \pi_{q_i}^{(k)} \cdot f_{q_i}^{C_m(k)}\left(x; \lambda^{(k)}\right), \qquad q_i \in \{q, 1/q\}, \qquad 0 < q < 1$$

where:

- $i \in \{I, II\}$: type of q-mixture model such that q, for $i = I$ and $1/q$, for $i = II$
- $m \in \{1,2,\cdots, 2^n\}$: number of q-mixture model
- K: number of components,
- $\pi_{q_i}^{(k)}$: mixing proportion of the k-th component, $\pi_{q_i}^{(k)} \geq 0$, $\sum_{k=1}^{n} \pi_{q_i}^{(k)} = 1$
- C_m: selecting function the appropriate q or $1/q$-PDF for the k-th density component
- $\lambda_{(k)}$: parameter vector of the k-th density component
- $f_q^{(k)}(x; \lambda^{(k)})$: the k-th q-density component
- $f_{1/q}^{(k)}(x; \lambda^{(k)})$: the k-th $1/q$-density component

As $q \to 1$, the q-finite mixture model converges to its ordinary form.

Structure Properties of the q-Finite Mixture Model

The q-finite mixture model represents a general modeling framework that subsumes both q-homogeneous and q-hybrid variants, offering enhanced flexibility for representing heterogeneous systems characterized by different q-parametrizations. Let $\mathbb{I}_A(k)$ be the indicator function equal to 1 if $k \in A$, and 0 otherwise, and $\mathbb{I}_B(k)$ is similarly defined for complementary set $B \subset \{1,2, \dots, K\}$, where $A \cup B = \{1,2, \dots, K\}$.

q-Homogeneous Finite Mixture Model. A q-homogeneous mixture model refers to a class of mixture models in which all component distributions are derived exclusively from a single formulation family—either the original q-formulation or its reciprocal counterpart based on $1/q$. The homogeneity of the component structure ensures a consistent parametric behavior across the mixture, which is particularly advantageous for analytical tractability and interpretability within the same functional family. Hence, it can be formulated:

$$f_q^{mix(i_m)}(x) = \sum_{k=1}^{K} \left(\pi_q^{(k)} \cdot \mathbb{I}_A(k) + \pi_{1/\mathrm{q}}^{(k)} \cdot \mathbb{I}_B(k) \right) \cdot f_{q_i}^{(k)}(x; \lambda^{(k)}).$$

q-Hybrid Finite Mixture Model. A q-hybrid mixture model is a type of mixture model that incorporates component distributions originating from both the original q-formulation and its reciprocal counterpart based on $1/q$. Unlike the q-homogeneous model, the q-hybrid structure allows for the coexistence of multiple generative mechanisms within a single framework. Thus, it can be defined as:

$$f_q^{mix(i_m)}(x) = \sum_{k=1}^{K} \pi_{q_i}^{(k)} \cdot \left(f_q^{(k)}(x; \lambda^{(k)}) \cdot \mathbb{I}_A(k) + f_{1/q}^{(k)}(x; \lambda^{(k)}) \cdot \mathbb{I}_B(k) \right).$$

Statistical Properties of the q-Finite Mixture Model

The corresponding q-mixture statistical characteristics are given by

- q-cumulative distributional function of model and the q-ordered statistics

$$F_q^{mix(i_m)}(x) = \sum_{k=1}^{K} \pi_{q_i}^{(k)} \cdot F_{q_i}^{C_m(k)}(x; \lambda^{(k)}), F_q^{\xi_{(1)}(mix(i_m))}(x) = \sum_{k=1}^{K} \pi_{q_i}^{(k)} \cdot F_{q_i}^{\xi_{(1)}(C_m(k))}(x; \lambda^{(k)})$$

$$F_q^{\xi_{(n)}(mix(i_m))}(x)=\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot F_{q_i}^{\xi_{(n)}(C_m(k))}\left(x;\lambda^{(k)}\right), F_q^{\xi_{(v)}(mix(i_m))}(x)=\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot F_{q_i}^{\xi_{(v)}(C_m(k))}\left(x;\lambda^{(k)}\right)$$

- q-moment, q-central moment, q-expectation and q-variance:

$$\mu_q^{(r)(mix(i_m))}=\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot\mu_{q_i}^{(r)(C_m(k))},\qquad m_q^{(r)(mix(i_m))}=\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot m_{q_i}^{(r)(C_m(k))}$$

$$\mathbb{E}_q^{mix(i_m)}(\xi)=\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot\mathbb{E}_{q_i}^{C_m(k)}\left(\xi^{(k)}\right),\qquad \mathbb{V}_q^{mix(i_m)}(\xi)=\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot\mathbb{V}_{q_i}^{C_m(k)}\left(\xi^{(k)}\right)$$

- q-moment generating function:

$$\mathbb{M}_q^{I(mix(i_m))}=\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot\mathbb{M}_{q_i}^{I(C_m(k))}(t),\qquad \mathbb{M}_q^{II(mix(i_m))}=\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot\mathbb{M}_{q_i}^{II(C_m(k))}(t)$$

- q-reliability functions and q-stress-strength reliability:

$$\mathbb{S}_q^{mix(i_m)}(x)=\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot\mathbb{S}_{q_i}^{C_m(k)}\left(x;\lambda^{(k)}\right),\qquad h_q^{mix(i_m)}(x)=\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot h_{q_i}^{C_m(k)}\left(x;\lambda^{(k)}\right)$$

$$mrl_q^{mix(i_m)}(x)=\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot mrl_{q_i}^{C_m(k)}\left(x;\lambda^{(k)}\right),\qquad \mathbb{R}_q^{mix(i_m)}=\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot\mathbb{R}_{q_i}^{C_m(k)}$$

An Illustirative Example (q-Exponential Finite Mixture Model)

A q-exponential finite mixture model (q-EFMM) is constructed by combining multiple q-exponential distributional components through a weighted linear combination. These weights reflect the relative contributions of each component and may be generalized using the q-algebra framework. The resulting model provides a flexible representation for heavy-tailed data. Its general form is:

$$\xi\sim\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot\mathrm{Exp}_q\left(x;\lambda^{(k)}\right),\qquad q_i\in\{q,1/q\},\qquad 0<q<1$$

where $\mathrm{Exp}_q\left(x;\lambda^{(k)}\right)$ represents th exponential q-distribution and the corresponding q and $1/q$-component densities are:

$$f_q^{(k)}\left(x;\lambda^{(k)}\right)=\lambda^{(k)}E_q^{-q\lambda^{(k)}x}\cdot\mathbb{I}_{[0,1/1-q]}(x),\qquad f_{1/q}^{(k)}\left(x;\lambda^{(k)}\right)=\lambda^{(k)}e_q^{-\lambda^{(k)}x}\cdot\ \mathbb{I}_{[0,\infty)}(x)$$

For $K=2$, let us describe two types of the q-mixture exponential model with a fixed mixing proportion. For the sake of simplification, let

$$\pi_q^{(1)}=\omega_q,\pi_q^{(2)}=1-\omega_q\ ,\qquad \pi_{1/q}^{(1)}=\omega_{1/q},\pi_{1/q}^{(2)}=1-\omega_{1/q}$$

Thus, the parameters of the q-density components may either be identical or non-identical. In such a case, $\lambda^{(1)}=\lambda^{(2)}=\lambda>0$ or alternatively, $\lambda^{(1)}=\alpha\neq\lambda^{(2)}=\beta$, where $\alpha,\beta>0$. Accordingly, the distributional characteristics of the q-exponential mixture model (q-EMM) are given by:

Table 1. The exponential mixture q-PDFs with identical component parameters

m	The Homogeneous Mixture q-PDF	The Hybrid Mixture q-PDF
1	$\omega_q \lambda E_q^{-q\lambda x} + (1-\omega_q)\lambda E_q^{-q\lambda x} = \lambda E_q^{-q\lambda x}$	$\omega_q \lambda E_q^{-q\lambda x} + (1-\omega_q)\lambda e_q^{-\lambda x}$
2	$\omega_q \lambda e_q^{-\lambda x} + (1-\omega_q)\lambda e_q^{-\lambda x} = \lambda e_q^{-\lambda x}$	$\omega_q \lambda e_q^{-\lambda x} + (1-\omega_q)\lambda E_q^{-q\lambda x}$
3	$\omega_{1/q} \lambda E_q^{-q\lambda x} + (1-\omega_{1/q})\lambda E_q^{-q\lambda x} = \lambda E_q^{-q\lambda x}$	$\omega_{1/q} \lambda E_q^{-q\lambda x} + (1-\omega_{1/q})\lambda e_q^{-\lambda x}$
4	$\omega_{1/q} \lambda e_q^{-\lambda x} + (1-\omega_{1/q})\lambda e_q^{-\lambda x} = \lambda e_q^{-\lambda x}$	$\omega_{1/q} \lambda e_q^{-\lambda x} + (1-\omega_{1/q})\lambda E_q^{-q\lambda x}$

Table 2. The exponential mixture q-PDFs with non-identical component parameters

m	The Homogeneous Mixture q-PDF	The Hybrid Mixture q-PDF
1	$\omega_q \alpha E_q^{-q\alpha x} + (1-\omega_q)\beta E_q^{-q\beta x}$	$\omega_q \alpha E_q^{-q\alpha x} + (1-\omega_q)\beta e_q^{-\beta x}$
2	$\omega_q \alpha e_q^{-\alpha x} + (1-\omega_q)\beta e_q^{-\beta x}$	$\omega_q \alpha e_q^{-\alpha x} + (1-\omega_q)\beta E_q^{-q\beta x}$
3	$\omega_{1/q} \alpha E_q^{-q\alpha x} + (1-\omega_{1/q})\beta E_q^{-q\beta x}$	$\omega_{1/q} \alpha E_q^{-q\alpha x} + (1-\omega_{1/q})\beta e_q^{-\beta x}$
4	$\omega_{1/q} \alpha e_q^{-\alpha x} + (1-\omega_{1/q})\beta e_q^{-\beta x}$	$\omega_{1/q} \alpha e_q^{-\alpha x} + (1-\omega_{1/q})\beta E_q^{-q\beta x}$

The graphs below illustrate the q-exponential mixture model for $\omega = 0.75$ and $\lambda = 2$, with varying values of the q-parameter as outlined in Table 1.

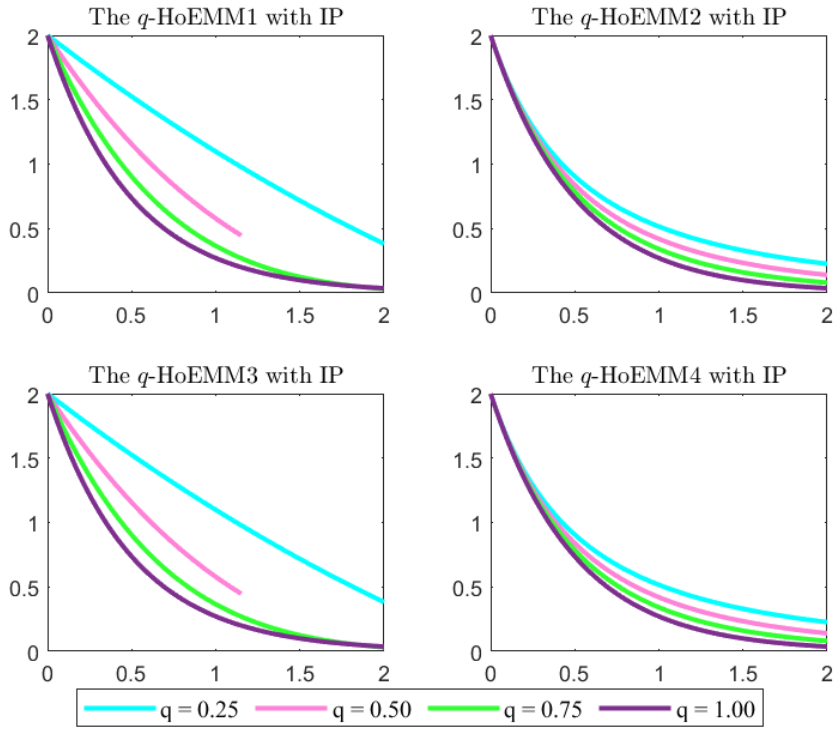

Figure 1. Graph of the homogeneous exponential mixture q-PDFs for $\omega = 0.75$ and $\lambda = 2$

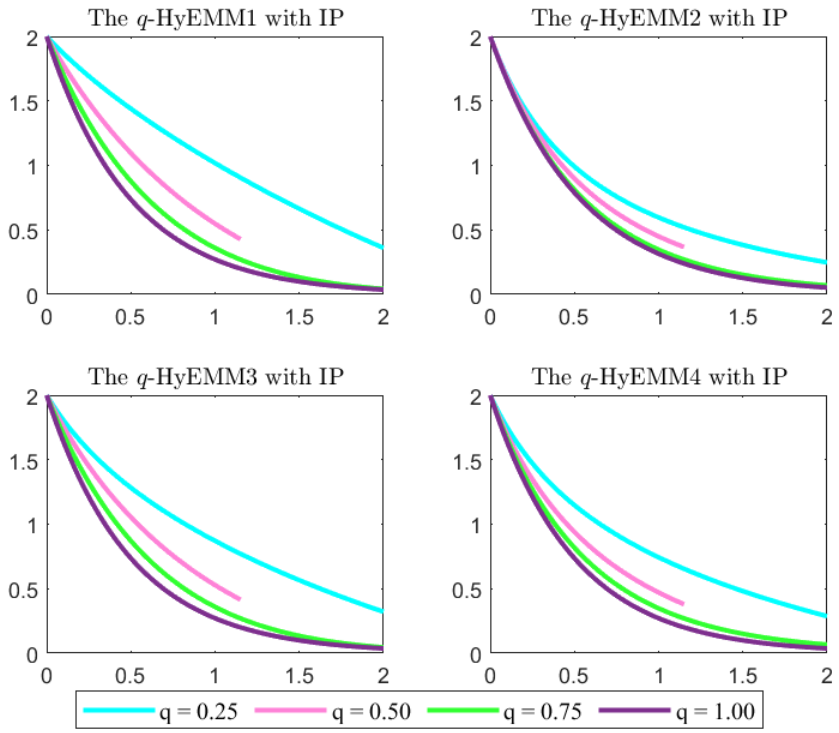

Figure 2. Graph of the hybrid exponential mixture q-PDFs for $\omega = 0.75$ and $\lambda = 2$

Presented below is the graph of the q-exponential mixture model corresponding to $\omega = 0.75$ and $\alpha = 1, \beta = 3,$ under varying q-parameter values as specified in Table 2. The homogeneous and hybrid configurations are depicted in Figures 3 and 4, respectively.

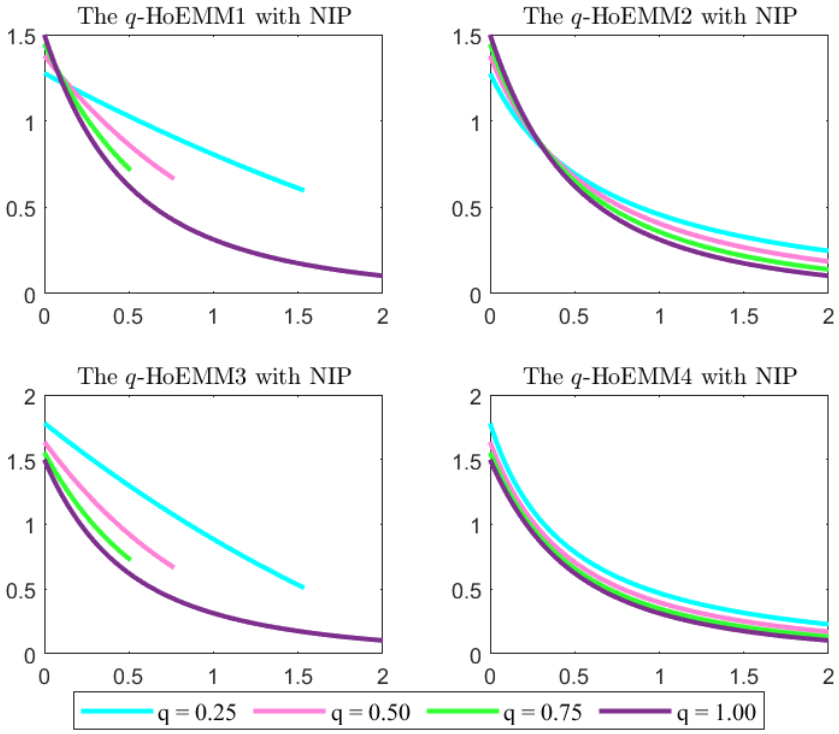

Figure 3. Graph of the homogeneous exponential mixture q-PDFs for $\omega = 0.75$ and $\alpha = 1, \beta = 3$

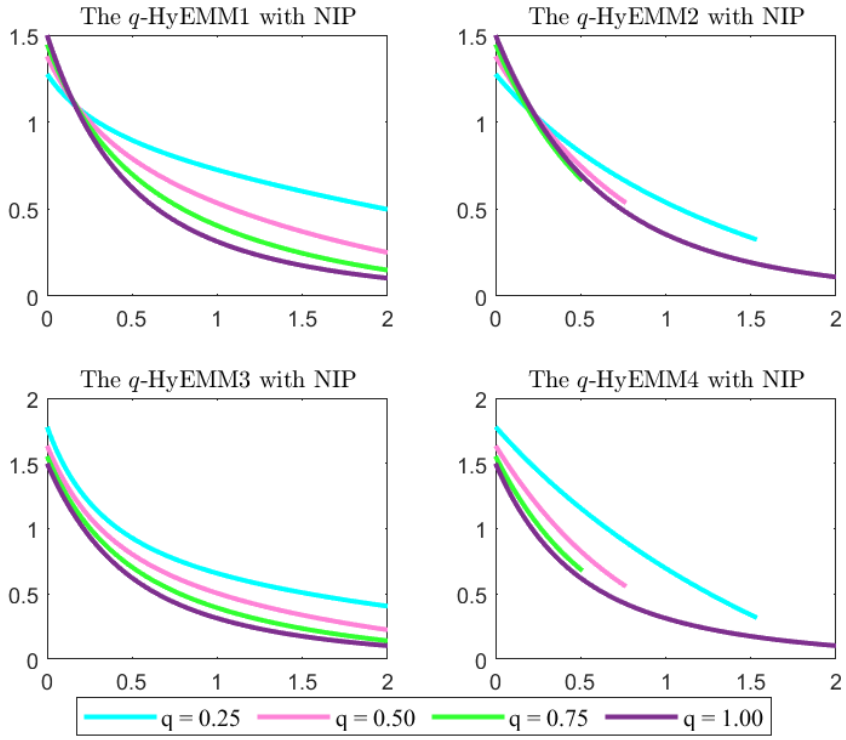

Figure 4. Graph of the hybrid exponential mixture q-PDFs for $\omega = 0.75$ and $\alpha = 1, \beta = 3$

Above, q-HoEMMm $(m = 1 - 4)$ and q-HyEMMm $(m = 1 - 4)$ denote homogeneous and hybrid q-exponential mixture models, respectively. NIP and IP stand for non-identical and identical parameters. The plots show that increasing q enhances model convexity, and all q-exponential mixtures reduce to the standard form as $q \to 1$.

DISCUSSION AND CONCLUSION

Probability q-distributions provide a flexible and dynamic framework that generalizes classical probability distributions by introducing the q-parameter. This parameter allows for a broader class of probabilistic models, enriching both theoretical understanding and practical applications.

In this paper, we introduce q-finite mixture model, and and provide a detailed analysis of the structural and statistical properties. As a representative example to elucidate the core concept, this study presents an exponential mixture model with a fixed mixing proportion, along with a discussion of its properties. Our findings suggest that the proposed q-distribution holds significant promise and may have widespread applications across various fields. In future research, we aim to explore the finite mixture and compound q-distribution.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

Both authors contributed equally to the finalization of this paper.

Homogeneous and Hybrid q-Mixture Forms of the Uma Distribution (1180)

Nurgül Okur[1*], Hasan Hüseyin Gül[2]

[1]Giresun University, Faculty of Science and Arts, Department of Statistics, Türkiye
[2]Giresun University, Faculty of Science and Arts, Department of Statistics, Türkiye

*Corresponding author e-mail: nurgul.okur@giresun.edu.tr

Abstract

Probability q-distributions provide a flexible and dynamic framework that generalizes classical probability distributions by introducing the q-parameter. This parameter allows for a broader class of probabilistic models, enriching both theoretical understanding and practical applications.

This study presents a comprehensive framework for the q-analogues of the Uma distribution and provides an in-depth analysis of the specific cases corresponding to the homogeneous and hybrid types. Although these distributions exhibit similar probability density functions, they exhibit significant differences in terms of parameter constraints, generative mechanisms, and underlying statistical properties.

The study encompasses the modeling of the q-analogues of both the probability density function and the cumulative distribution function, along with a detailed exploration of their shapes through rigorous mathematical analysis. Furthermore, the fundamental statistical and distributional characteristics of these distributions are thoroughly examined. Ultimately, these findings offer valuable insights for the continued exploration and application of q-Uma distributions across various fields.

Keywords: *q-analogue, q-parameter, q-distribution, homogeneous, hybrid.*

INTRODUCTION

Over the past few years, there has been a significant rise in the proposal of lifetime distributions for modeling lifetime data by various statisticians. In this context, the Uma distribution was introduced by Shanker in 2017, along with its probability density function (PDF) and cumulative distribution function (CDF), for the parameters $x \geq 0$ and $\lambda > 0$:

$$f(x;\lambda) = a(1 + x + x^3)e^{-\lambda x}, \qquad a = \frac{\lambda^4}{\lambda^3 + \lambda^2 + 3!},$$

$$F(x;\lambda) = 1 - \left\{1 + \frac{a}{\lambda^4}[(\lambda x)^3 + 3(\lambda x)^2 + (\lambda^2 + 3!)\lambda x]\right\} e^{-\lambda x}.$$

In probability theory, the concept of q-distributions expands upon classical distributions, offering a wider array of possibilities. Key contributions to the theory of basic discrete q-distributions include the works of Dunkl, 1981, Crippa et al., 1997, Kupershmidt, 2000, Kemp, 2002, Charalambides, 2016, Kyriakoussis and Vamvakari, 2017, among others. Significant research on q-continuous distributions, including the Gaussian and generalized gamma q-distributions by Diaz et al., 2009, 2010, the Erlang q-distributions by Charalambides, 2016, the gamma and beta q-distributions by Boutouria et al., 2018, the Lindley q-distribution in two forms was introduced by Bouzida, 2023.

This research develops a comprehensive framework for the q-analogues of the Uma distribution, along with an extensive analysis of the particular cases that correspond to the homogeneous and hybrid types.

MATERIAL AND METHODS

This section outlines the principles of q-calculus, and q-probability theory. In this entire study, unless otherwise stated, it is assumed that $0 < q < 1$. Readers are referred to the relevant literature.

Definition 1. (Kac and Cheung, 2002). Let x, q be real numbers. The q-number $[x]_q$ is defined as

$$[x]_q = \frac{1 - q^x}{1 - q}.$$

For $n \in \mathbb{N}$, a natural q-number, it reduces to $[n]_q = \sum_{k=0}^{n-1} q^k$.

Definition 2. (Kac and Cheung, 2002). The q-Gauss binomial formula is given by

$$(x + y)_q^n = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q q^{\binom{k}{2}} y^k x^{n-k}, \qquad -\infty < x, y < \infty.$$

The q-binomial coefficients are provided for $k = 0,1, \dots, n$ by

$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{[n]_q!}{[n-k]_q![k]_q!}, \qquad [n]_{k,q} = \frac{[n]_q!}{[n-k]_q!}, \qquad [n]_q! = [n]_q[n-1]_q \dots [2]_q[1]_q.$$

Definition 3. (Kac and Cheung, 2002). The q-analogues of the exponential function are presented

$$E_q^\tau = \sum_{k=0}^{\infty} q^{\binom{k}{2}} \frac{\tau^k}{[k]_q!} = \prod_{k=0}^{\infty} \left(1 + (1-q)q^k\tau\right), \qquad \tau \in \mathbb{R},$$

$$e_q^\tau = \sum_{k=0}^{\infty} \frac{\tau^k}{[k]_q!} = \prod_{k=0}^{\infty} \frac{1}{(1-(1-q)q^k\tau)}, \qquad |\tau| < \frac{1}{1-q}.$$

Definition 4. (Kac and Cheung, 2002). The q-derivative of f is defined as

$$D_q f(\tau) = \frac{f(q\tau) - f(\tau)}{q\tau - \tau}.$$

Definition 5. (Kac and Cheung, 2002). The well-known Jackson q-integral of f is given by

$$\int_0^b f(\tau) d_q\tau = (1-q) \sum_{n=0}^{\infty} q^n b f(q^n b), \qquad b > 0,$$

Definition 6. (Kac and Cheung, 2002). The generalized q-integral is given by

$$\int_{-\infty}^{\infty} f(\tau) d_q\tau = (1-q) \sum_{n\in\mathbb{Z}} q^n f(q^n).$$

Definition 7. (Kac and Cheung, 2002, De Sole and Kac, 2003). The q-analogues of the gamma functions are given for $\alpha > 0$, $n \in \mathbb{N}$

$$\Gamma_q(\alpha) = \int_0^{[\infty]_q} x^{\alpha-1} E_q^{-qx} d_q x, \qquad \gamma_q(\alpha) = \int_0^{\infty} x^{\alpha-1} e_q^{-x} d_q x.$$

Identities derived from the q-gamma functions can be obtained

$$\Gamma_q(\alpha + n) = [\alpha]_{n,q}\Gamma_q(\alpha) \qquad \gamma_q(\alpha + n) = q^{-\binom{\alpha+n}{2}}[\alpha]_{n,q}\gamma_q(\alpha)$$

$$\Gamma_q(n+1) = [n]_q! \qquad \gamma_q(n+1) = q^{-\binom{n+1}{2}}[n]_q!$$

Definition 8. (Vamvakari, 2023). A random variable X is considered q-continuous if there exists a non-negative function $f_q^X(x)$ for $x \geq 0$ such that

$$P\{a < X \leq b\} = \int_a^b f_q^X(x) d_q x.$$

The q-cumulative distribution function (q-CDF) of X is defined for $x > 0$

$$F_q^X(x) = P(X \leq x) = \int_0^x f_q^X(u) d_q u,$$

satisfying the relation $P(\alpha < X \leq \beta) = F_q^X(\beta) - F_q^X(\alpha)$. Then,

$$f_q(x) = D_q F_q(x) = \frac{F_q^X(x) - F_q^X(qx)}{(1-q)x} = \frac{P(qx \leq X \leq x)}{(1-q)x}, \ (q \neq 1\ ).$$

Definition 9. (Vamvakari, 2023). Let $\mathbb{E}_q|X^r| < \infty$ for all positive integers r. Then,

$$\mu_q^{(r)} = \mathbb{E}_q(X^r) = \int_0^\infty x^r \, f_q^X(x) d_q x\,, \qquad \mathbb{E}_q(X) = \mu_q, \qquad \mathbb{V}_q(X) = \mu_q^{(2)} - \mu_q^2.$$

RESULTS

This section presents the Uma q-distribution, including its homogeneous and hybird forms.

Modeling of the Uma q-distribution

A q-mixture model of the Uma distribution is a q-probabilistic parametric model composed of multiple component Uma q-distributions combined with certain parametric weights. The general form of the q-parametric mixture model of the Uma distribution is for $x \geq 0$ characterized by

$$f_q^{i_n}(x;\lambda) = \sum_{\alpha \in D} p_q^i(\alpha)\, g_q^{C_n}(x;\alpha,\lambda), \ (i = I, II;\ n = 1,2,\ldots,8)$$

where

- $\alpha \in D$: Parameter value in the set of components $D = \{1,2,4\}$
- $p_q^i(\alpha)$: Mixing parametric proportion of the α-th component, $p_q^i(\alpha) \geq 0$, $\sum_{\alpha \in D} p_q^i(\alpha) = 1$ and

$$p_q^I(\alpha) = \frac{\Gamma_q(\alpha)\lambda^{1-\alpha}}{\sum_{k \in D} \Gamma_q(\alpha)\lambda^{1-\alpha}}, \qquad p_q^{II}(\alpha) = \frac{\gamma_q(\alpha)\lambda^{1-\alpha}}{\sum_{k \in D} \gamma_q(\alpha)\lambda^{1-\alpha}}$$

- $g_q^{C_n}(x;\alpha,\lambda)$: Gamma q-PDF of the α-th component and

$$g_q^I(x;\alpha,\lambda) = \frac{\lambda^\alpha}{\Gamma_q(\alpha)} x^{\alpha-1} E_q^{-q\lambda x} \mathbf{1}_{[0,[\infty]_q]}(x), \qquad g_q^{II}(x;\alpha,\lambda) = \frac{\lambda^\alpha}{\gamma_q(\alpha)} x^{\alpha-1} e_q^{-\lambda x} \mathbf{1}_{[0,\infty)}(x)$$

- C_n: Selecting function the appropriate q or $1/q$-PDF.

As $q \to 1$, the q-PDF $f_q^{i_n}$ converges to the ordinary Uma PDF. The function $f_q^{i_n}$ satisfies the necessary conditions to qualify as a q-PDF. Furthermore, the corresponding q-CDF is:

$$F_q^{i_n}(x;\lambda) = \sum_{\alpha \in D} p_q^i(\alpha)\, G_q^{C_n}(x;\alpha,\lambda), \ (i = I, II;\ n = 1,2,\ldots,8)$$

where $G_q^i(x,\alpha,\lambda)$ is the gamma q-CDF of the α-th component, and

$$G_q^I(x,\alpha,\lambda)=\frac{\Gamma_q(\alpha,\lambda x)}{\Gamma_q(\alpha)},\qquad G_q^{II}(x,\alpha,\lambda)=\frac{\gamma_q(\alpha,\lambda x)}{\gamma_q(\alpha)}$$

The q-parametric mixture model of the Uma distribution is a broad family of distributions that includes the original q-formulation, the reciprocal formulation based on $1/q$, as well as their hybrid combinations. Let $p_q^i(\alpha)$ and $g_q^i(x;\alpha,\lambda)$ be $p_q^{i(\alpha)}$ and $g_q^{i(\alpha)}$, and let us define the notations:

$$a_q^I=\frac{\lambda^4}{\lambda^3+\lambda^2+[3]_q!},\qquad a_q^{II}=\frac{\lambda^4}{\lambda^3+q^{-1}\lambda^2+q^{-6}[3]_q!}$$

Homogeneous Type of the Uma q-distribution

This section introduces the homogeneous Uma q-distribution and presents its distributional charachteristics as outlined in Table 1.

Table 1. The homogeneous Uma q-PDF and q-CDF

n	The first type of HoUma q-PDF $f_q^{I_n}(x;\lambda)$
1	$p_q^{I(1)}g_q^{I(1)}+p_q^{I(2)}g_q^{I(2)}+p_q^{I(4)}g_q^{I(4)}=a_q^I(1+x+x^3)E_q^{-q\lambda x}$
2	$p_q^{I(1)}g_q^{II(1)}+p_q^{I(2)}g_q^{II(2)}+p_q^{I(4)}g_q^{II(4)}=a_q^I(1+qx+q^6x^3)e_q^{-\lambda x}$
n	The first type of HoUma q-CDF $F_q^{I_n}(x;\lambda)$
1	$1-\left\{1+\frac{a_q^I}{\lambda^4}\left[(\lambda\tau)^3+[3]_q(\lambda\tau)^2+[3]_q!\,\lambda\tau+\lambda^3\tau\right]\right\}E_q^{-\lambda\tau}$
2	$1-\left\{1+\frac{a_q^I}{\lambda^4}\left[q^3(\lambda\tau)^3+q[3]_q(\lambda\tau)^2+[3]_q!\,\lambda\tau+\lambda^3\tau\right]\right\}e_q^{-\lambda x}$
n	The second type of HoUma q-PDF $f_q^{II_n}(x;\lambda)$
1	$p_q^{II(1)}g_q^{I(1)}+p_q^{II(2)}g_q^{I(2)}+p_q^{II(4)}g_q^{I(4)}=a_q^{II}(1+q^{-1}x+q^{-6}x^3)E_q^{-q\lambda x}$
2	$p_q^{II(1)}g_q^{II(1)}+p_q^{II(2)}g_q^{II(2)}+p_q^{II(4)}g_q^{II(4)}=a_q^{II}(1+x+x^3)e_q^{-\lambda x}$
n	The second type of HoUma q-CDF $F_q^{II_n}(x;\lambda)$
1	$1-\left\{1+\frac{a_q^{II}}{\lambda^4}\left[q^{-6}\left((\lambda\tau)^3+[3]_q(\lambda\tau)^2+[3]_q!\,\lambda\tau\right)+q^{-1}\lambda^3\tau\right]\right\}E_q^{-\lambda\tau}$
2	$1-\left\{1+\frac{a_q^{II}}{\lambda^4}\left[q^{-6}\left(q^3(\lambda\tau)^3+q[3]_q(\lambda\tau)^2+[3]_q!\,\lambda\tau\right)+q^{-1}\lambda^3\tau\right]\right\}e_q^{-\lambda\tau}$

The graphs below illustrate the homogeneous Uma q-PDF for $\lambda=0.5$, with varying values of the q-parameter as outlined in Table 1.

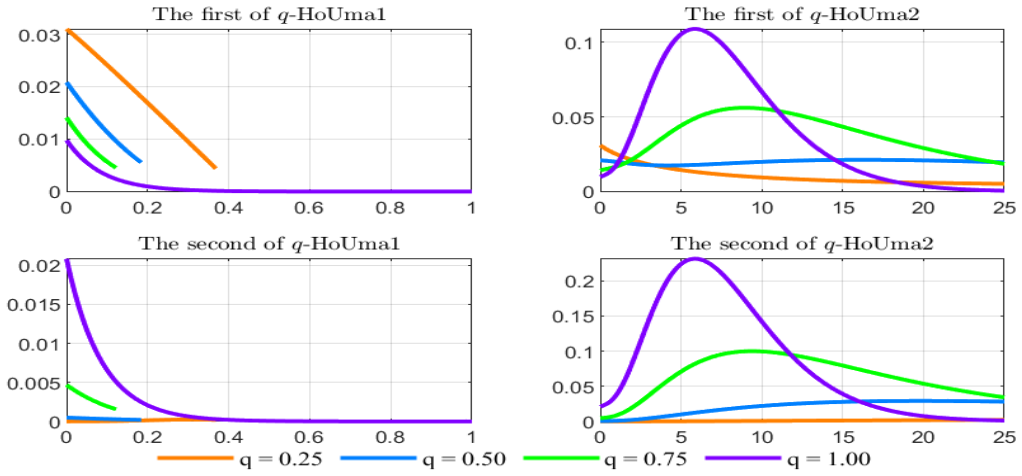


Figure 1. Graphs of the homogeneous Uma q-PDF for $\lambda=0.5$ and varying values of the q-parameter

The plots above illustrate two types of q-HoUma distributions (for $m = 1,2$), representing the q- PDFs of the homogeneous q-Uma distribution. It is observed that increasing the parameter q enhances the convexity of the model in the case of q-HoUma1, while it increases the concavity in the case of q-HoUma2. Moreover, all q-Uma distributions converge to their standard form as $q \to 1$.

Table 1. The homogeneous Uma q-moment, q-expected value and q-variance

n	The first type of HoUma q-Moment $\mu_q^{(r)(I_n)}$
1	$a_q^I\left([r]_q!\,\lambda^{-(r+1)} + [r+1]_q!\,\lambda^{-(r+2)} + [r+4]_q!\,\lambda^{-(r+5)}\right)$
2	$a_q^I\left(q^{-\binom{r+1}{2}}[r]_q!\,\lambda^{-(r+1)} + q^{1-\binom{r+2}{2}}[r+1]_q!\,\lambda^{-(r+2)} + q^{6-\binom{r+5}{2}}[r+4]_q!\,\lambda^{-(r+5)}\right)$
n	The first type of HoUma q-expected value
1	$a_q^I\left(\lambda^{-2} + [2]_q!\,\lambda^{-3} + [5]_q!\,\lambda^{-6}\right)$
2	$a_q^I\left(q^{-1}\lambda^{-2} + q^{-2}[2]_q!\,\lambda^{-3} + q^{-9}[5]_q!\,\lambda^{-6}\right)$
n	The first type of HoUma q-variance
1	$a_q^I\left([2]_q!\,\lambda^{-3} + [3]_q!\,\lambda^{-4} + [6]_q!\,\lambda^{-7}\right) - \left(a_q^I\right)^2\left(\lambda^{-2} + [2]_q!\,\lambda^{-3} + [5]_q!\,\lambda^{-6}\right)^2$
2	$a_q^I\left(q^{-1}[2]_q!\,\lambda^{-3} + q^{-2}[3]_q!\,\lambda^{-4} + q^{-9}[6]_q!\,\lambda^{-7}\right) - \left(a_q^I\right)^2\left(q^{-1}\lambda^{-2} + q^{-2}[2]_q!\,\lambda^{-3} + q^{-9}[5]_q!\,\lambda^{-6}\right)^2$
n	The second type of HoUma q-Moment $\mu_q^{(r)(II_n)}$
1	$a_q^{II}\left([r]_q!\,\lambda^{-(r+1)} + q^{-1}[r+1]_q!\,\lambda^{-(r+2)} + q^{-6}[r+4]_q!\,\lambda^{-(r+5)}\right)$
2	$a_q^{II}\left(q^{-\binom{r+1}{2}}[r]_q!\,\lambda^{-(r+1)} + q^{-\binom{r+2}{2}}[r+1]_q!\,\lambda^{-(r+2)} + q^{-\binom{r+5}{2}}[r+4]_q!\,\lambda^{-(r+5)}\right)$
n	The second type of HoUma q-Moment $\mu_q^{(r)(II_n)}$
1	$a_q^{II}\left(\lambda^{-2} + q^{-1}[2]_q!\,\lambda^{-3} + q^{-6}[5]_q!\,\lambda^{-6}\right)$
2	$a_q^{II}\left(q^{-1}\lambda^{-2} + +q^{-3}[2]_q!\,\lambda^{-3} + q^{-15}[5]_q!\,\lambda^{-6}\right)$
n	The second type of q-variance
1	$a_q^{II}\left([2]_q!\,\lambda^{-3} + q^{-1}[3]_q!\,\lambda^{-4} + q^{-6}[6]_q!\,\lambda^{-7}\right) - \left(a_q^{II}\right)^2\left(\lambda^{-2} + q^{-1}[2]_q!\,\lambda^{-3} + q^{-6}[5]_q!\,\lambda^{-6}\right)^2$
2	$a_q^{II}\left(q^{-1}[2]_q!\,\lambda^{-3} + +q^{-3}[3]_q!\,\lambda^{-4} + q^{-15}[6]_q!\,\lambda^{-7}\right) - \left(a_q^{II}\right)^2\left(q^{-1}\lambda^{-2} + +q^{-3}[2]_q!\,\lambda^{-3} + q^{-15}[5]_q!\,\lambda^{-6}\right)^2$

Hybrid Type of the Uma q-distribution

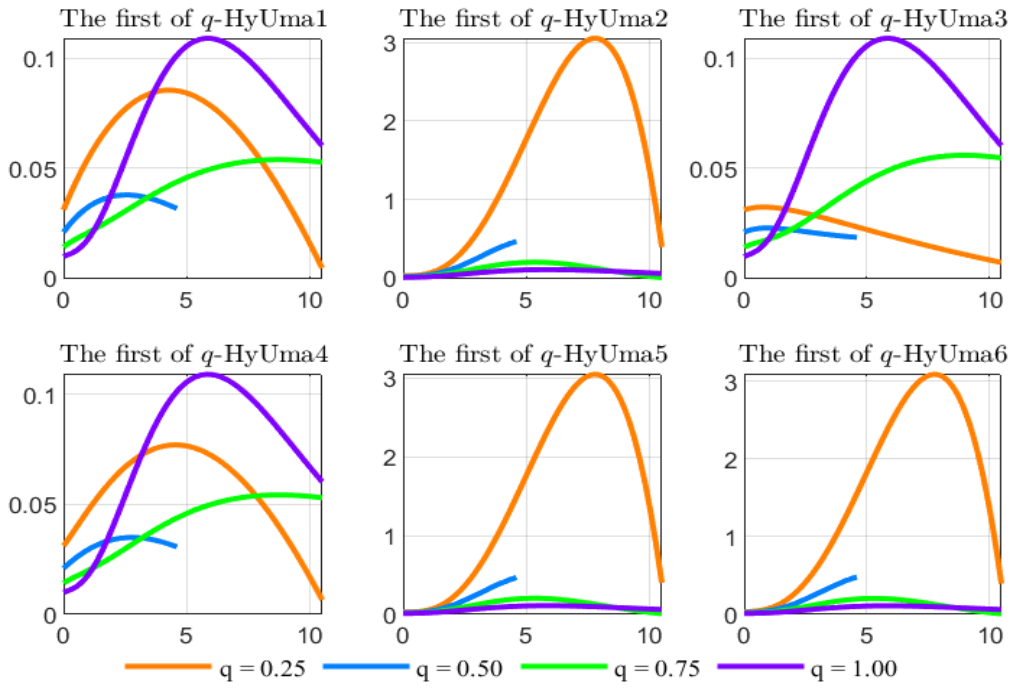
This section introduces the hybrid Uma q-distribution and presents its distributional charachteristics in Table 3.

Table 3. The hybrid Uma q-PDF and q-CDF

n	The first type of HyUma q-PDF $f_q^{I_n}(x;\lambda)$
1	$p_q^{I(1)}g_q^{I(1)} + p_q^{I(2)}g_q^{I(2)} + p_q^{I(4)}g_q^{II(4)} = a_q^I\left\{(1+x)E_q^{-q\lambda x} + q^6x^3e_q^{-\lambda x}\right\}$
2	$p_q^{I(1)}g_q^{I(1)} + p_q^{I(2)}g_q^{II(2)} + p_q^{I(4)}g_q^{I(4)} = a_q^I\left\{(1+x^3)E_q^{-q\lambda x} + qxe_q^{-\lambda x}\right\}$
3	$p_q^{I(1)}g_q^{I(1)} + p_q^{I(2)}g_q^{II(2)} + p_q^{I(4)}g_q^{II(4)} = a_q^I\left\{E_q^{-q\lambda x} + (qx+q^6x^3)e_q^{-\lambda x}\right\}$
4	$p_q^{I(1)}g_q^{II(1)} + p_q^{I(2)}g_q^{I(2)} + p_q^{I(4)}g_q^{II(4)} = a_q^I\left\{xE_q^{-q\lambda x} + (1+q^6x^3)e_q^{-\lambda x}\right\}$
5	$p_q^{I(1)}g_q^{II(1)} + p_q^{I(2)}g_q^{II(2)} + p_q^{I(4)}g_q^{I(4)} = a_q^I\left\{x^3E_q^{-q\lambda x} + (1+x)e_q^{-\lambda x}\right\}$
6	$p_q^{I(1)}g_q^{II(1)} + p_q^{I(2)}g_q^{I(2)} + p_q^{I(4)}g_q^{I(4)} = a_q^I\left\{(x+x^3)E_q^{-q\lambda x} + e_q^{-\lambda x}\right\}$
n	The first type of HoUma q-CDF $F_q^{I_n}(x;\lambda)$
1	$1 - \frac{a_q^I}{\lambda^4}\left\{[\lambda^3 + \lambda^2(1+\lambda\tau)]E_q^{-\lambda\tau} + \left[q^3(\lambda\tau)^3 + q[3]_q(\lambda\tau)^2 + [3]_q!\,\lambda\tau + [3]_q!\right]e_q^{-\lambda\tau}\right\}$
2	$1 - \frac{a_q^I}{\lambda^4}\left\{\lambda^2(1+q\lambda\tau)e_q^{-\lambda\tau} + \left[(\lambda\tau)^3 + [3]_q(\lambda\tau)^2 + [3]_q!\,\lambda\tau + [3]_q! + \lambda^3\right]E_q^{-\lambda\tau}\right\}$

n	The first type of HyUma q-PDF $f_q^{I_n}(x;\lambda)$
3	$1-\frac{a_q^I}{\lambda^4}\{\lambda^3E_q^{-\lambda\tau}+[q^3(\lambda\tau)^3+q[3]_q(\lambda\tau)^2+[3]_q!\,\lambda\tau+[3]_q!+\lambda^2(1+\lambda\tau)]e_q^{-\lambda\tau}\}$
4	$1-\frac{a_q^I}{\lambda^4}\{\lambda^2(1+\lambda\tau)E_q^{-\lambda\tau}+[q^3(\lambda\tau)^3+q[3]_q(\lambda\tau)^2+[3]_q!\,\lambda\tau+[3]_q!+\lambda^3]e_q^{-\lambda\tau}\}$
5	$1-\frac{a_q^I}{\lambda^4}\{[\lambda^3+\lambda^2(1+q\lambda\tau)]e_q^{-\lambda\tau}+[(\lambda\tau)^3+[3]_q(\lambda\tau)^2+[3]_q!\,\lambda\tau+[3]_q!]E_q^{-\lambda\tau}\}$
6	$1-\frac{a_q^I}{\lambda^4}\{\lambda^3e_q^{-\lambda\tau}+[(\lambda\tau)^3+[3]_q(\lambda\tau)^2+[3]_q!\,\lambda\tau+[3]_q!+\lambda^2(1+\lambda\tau)]E_q^{-\lambda\tau}\}$
n	The second type of HyUma q-PDF $f_q^{II_n}(x;\lambda)$
1	$p_q^{II(1)}g_q^{I(1)}+p_q^{II(2)}g_q^{I(2)}+p_q^{II(4)}g_q^{II(4)}=a_q^{II}\{(1+q^{-1}x)E_q^{-q\lambda x}+x^3e_q^{-\lambda x}\}$
2	$p_q^{II(1)}g_q^{I(1)}+p_q^{II(2)}g_q^{I(2)}+p_q^{II(4)}g_q^{I(4)}=a_q^{II}\{(1+q^{-6}x^3)E_q^{-q\lambda x}+xe_q^{-\lambda x}\}$
3	$p_q^{II(1)}g_q^{I(1)}+p_q^{II(2)}g_q^{I(2)}+p_q^{II(4)}g_q^{II(4)}=a_q^{II}\{E_q^{-q\lambda x}+(x+x^3)e_q^{-\lambda x}\}$
4	$p_q^{II(1)}g_q^{II(1)}+p_q^{II(2)}g_q^{I(2)}+p_q^{II(4)}g_q^{II(4)}=a_q^{II}\{q^{-1}xE_q^{-q\lambda x}+(1+x^3)e_q^{-\lambda x}\}$
5	$p_q^{II(1)}g_q^{II(1)}+p_q^{II(2)}g_q^{II(2)}+p_q^{II(4)}g_q^{I(4)}=a_q^{II}\{q^{-6}x^3E_q^{-q\lambda x}+(1+x)e_q^{-\lambda x}\}$
6	$p_q^{II(1)}g_q^{II(1)}+p_q^{II(2)}g_q^{I(2)}+p_q^{II(4)}g_q^{I(4)}=a_q^{II}\{(q^{-1}x+q^{-6}x^3)E_q^{-q\lambda x}+e_q^{-\lambda x}\}$
n	The second type of HoUma q-CDF $F_q^{II_n}(x;\lambda)$
1	$1-\frac{a_q^{II}}{\lambda^4}\{[\lambda^3+q^{-1}\lambda^2(1+\lambda\tau)]E_q^{-\lambda\tau}+q^{-6}[q^3(\lambda\tau)^3+q[3]_q(\lambda\tau)^2+[3]_q!\,\lambda\tau+[3]_q!]e_q^{-\lambda\tau}\}$
2	$1-\frac{a_q^{II}}{\lambda^4}\{q^{-1}\lambda^2(1+q\lambda\tau)e_q^{-\lambda\tau}+[q^{-6}((\lambda\tau)^3+[3]_q(\lambda\tau)^2+[3]_q!\,\lambda\tau+[3]_q!)+\lambda^3]E_q^{-\lambda\tau}\}$
3	$1-\frac{a_q^{II}}{\lambda^4}\{\lambda^3E_q^{-\lambda\tau}+[q^{-6}(q^3(\lambda\tau)^3+q[3]_q(\lambda\tau)^2+[3]_q!\,\lambda\tau+[3]_q!)+q^{-1}\lambda^2(1+\lambda\tau)]e_q^{-\lambda\tau}\}$
4	$1-\frac{a_q^{II}}{\lambda^4}\{q^{-1}\lambda^2(1+\lambda\tau)E_q^{-\lambda\tau}+[q^{-6}(q^3(\lambda\tau)^3+q[3]_q(\lambda\tau)^2+[3]_q!\,\lambda\tau+[3]_q!)+\lambda^3]e_q^{-\lambda\tau}\}$
5	$1-\frac{a_q^{II}}{\lambda^4}\{[\lambda^3+q^{-1}\lambda^2(1+q\lambda\tau)]e_q^{-\lambda\tau}+q^{-6}[(\lambda\tau)^3+[3]_q(\lambda\tau)^2+[3]_q!\,\lambda\tau+[3]_q!]E_q^{-\lambda\tau}\}$
6	$1-\frac{a_q^{II}}{\lambda^4}\{\lambda^3e_q^{-\lambda\tau}+[q^{-6}((\lambda\tau)^3+[3]_q(\lambda\tau)^2+[3]_q!\,\lambda\tau+[3]_q!)+q^{-1}\lambda^2(1+\lambda\tau)]E_q^{-\lambda\tau}\}$

The hybrid configurations of the Uma q-PDF for $\lambda = 0.5$, with varying values of the q-parameter as outlined in Table 3. are depicted in Figures 2.



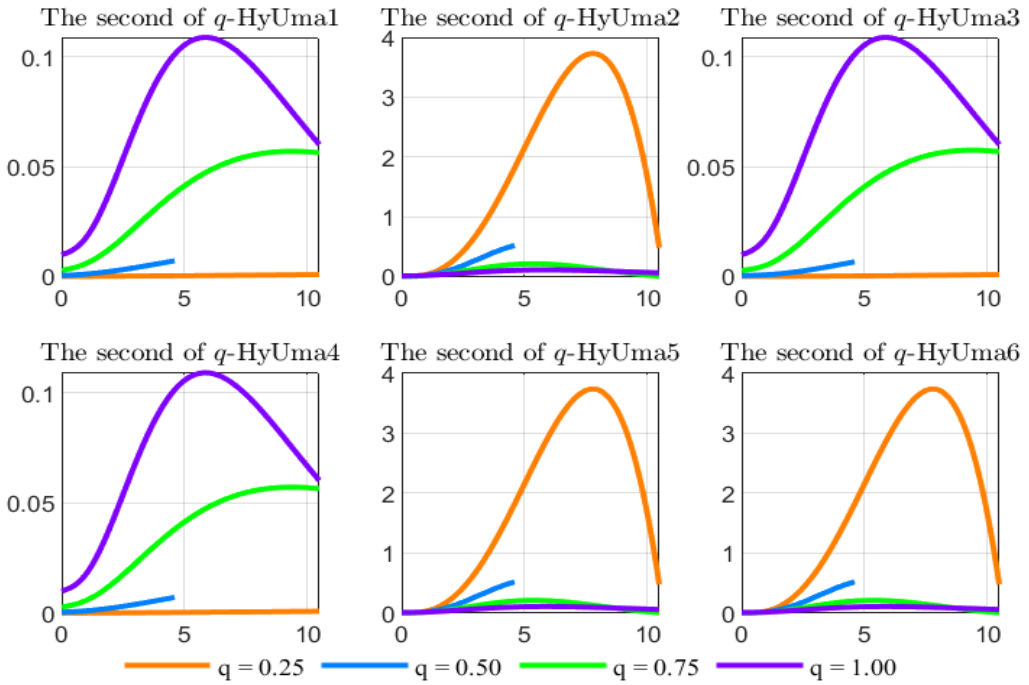


Figure 2. Graphs of the hybrid Uma q-PDF for $\lambda = 0.5$ and varying values of the q-parameter

Two hybrid q-Uma models, each consisting of six q-PDFs, are introduced. The 2nd, 5th, and 6th q-PDFs exhibit similar patterns, while the 1st, 3rd, and 4th form another group. This similarity arises from the multiplication of the q-exponential function $E_q^{-q\lambda x}$ by x^3.

Furthermore, the derivation of the hybrid q-moment and the corresponding statistical measures can be carried out in an analogous manner for the homogeneous type.

DISCUSSION AND CONCLUSION

The evolution of q-distributions represents a natural progression in the development of q-calculus. q-calculus serves as a parametric generalization of classical calculus, with the classical framework being recovered in the limit as q→1. In this paper, we introduce the Uma q-distribution, and and provide a detailed analysis of the structural and statistical properties, including its modeling, shape, moment of the specific cases corresponding to the homogeneous and hybrid types. Our findings suggest that the proposed q-distribution holds significant promise and may have widespread applications across various fields. In future research, we aim to explore the finite mixture and compound q-distribution.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

Both authors contributed equally to the finalization of this paper.

Multidimensional Analysis of Inflation Dynamics in Turkey: A Bayesian VAR and Markov Models Approach (1038)

Melih Aslan[1*], İrem Olgun[1], Beyza Şahin[1]

[1]Ondokuz Mayıs University, Faculty of Economics and Administrative Sciences, Economics, 55200, Samsun, Turkey

*Corresponding author e-mail: 21120593@stu.omu.edu.tr

Abstract

This study aims to examine in detail how the industrial production index, financial services confidence index, weighted average cost of funding, the average interest rates applied by banks on consumer loans, and household transfers affect inflation and the relationships among these series in Turkey. A comprehensive time series analysis was conducted using monthly data from the period 2012:07 to 2024:11. During the analysis process, the stationarity levels of the series were determined using ADF and PP unit root tests, and the results indicated that all series were stationary at the first difference, following an I(1) process. The Markov Regime Switching Model was employed to model different periods, and a Bayesian Vector Autoregression (BVAR) model was constructed to analyze the impulse response functions of the series. The findings reveal that increases in the interest rates applied to consumer loans and the industrial production index lead to higher inflation by creating demand and cost pressures. Additionally, the joint implementation of monetary and fiscal policies is found to be highly significant. This study provides a comprehensive time series analysis to deepen the understanding of the interaction between inflation and macroeconomic variables. It also concludes that avoiding high interest rates, which restrict household expenditures and increase production costs, can play a significant role in reducing price increases.

Keywords: *Inflation, Markov regime switching model, Vector autoregression, Impulse-Response analysis*

INTRODUCTION

Inflation is recognised as an important macroeconomic indicator for the economic stability of a country. High inflation adversely affects economic growth and leads to fluctuations in domestic demand and investments. Recent developments in the Turkish economy require a closer examination of the relationship between inflation and monetary policy.

Between 2012 and 2024, the Turkish economy experienced various crises due to global developments, domestic political fluctuations and economic policy changes. In 2013, the unrest and social instability caused by the Gezi Park protests had a negative impact on investment confidence and led to a decline in economic activity. In 2016, Turkey experienced a failed military coup attempt. This failed attempt raised concerns about the rule of law and led to an economic slowdown. In 2018, high inflation, a large deficit and concerns about the independence of the central bank led to a currency crisis, causing uncertainty in the economy and negatively affecting both businesses and consumers. In 2019, the Covid-19 pandemic, which affected the world, caused disruptions in various sectors and significantly affected the Turkish economy.

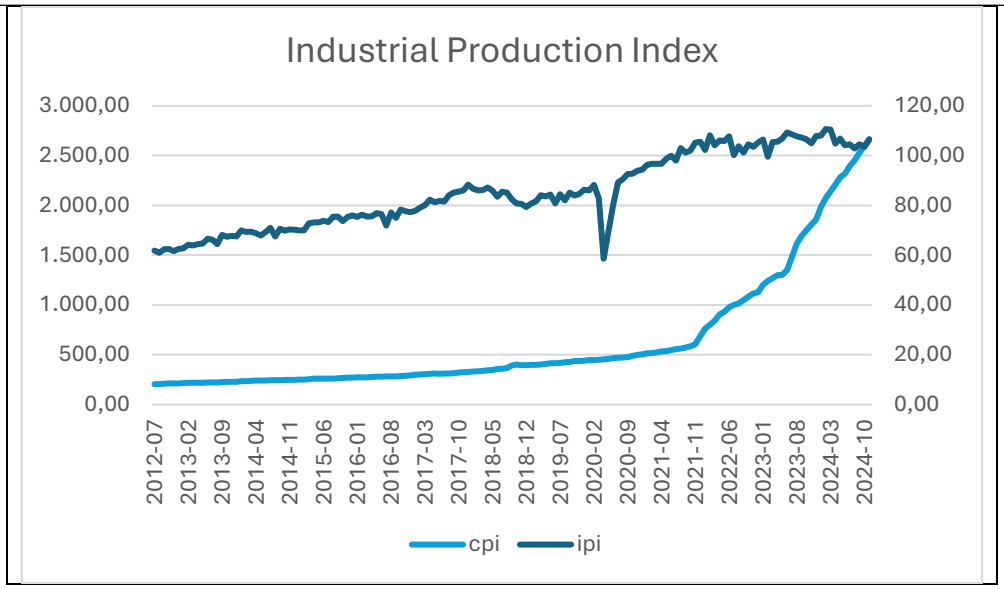

Figure 1: Industrial Production Index and Consumer Prices Index Graphs

As one of the main indicators of economic growth, the industrial production index reveals changes in production activities. The December 2020 industrial production index reveals that Turkey's transition to the Presidential Government System in 2018 and the diplomatic crisis with the US caused sudden fluctuations in the exchange rate. The rapid depreciation of the Turkish Lira caused inflation rates to rise, industrial production to decline and domestic demand and investments to weaken. This process continued in 2019. In 2020, the emergence of the COVID-19 pandemic led to a sharp decline in industrial production in Turkey, as in global economic activity. However, a rapid economic recovery was observed in 2021. Industrial production started to rise again with the increase in domestic and foreign demand. In 2022 and beyond, high inflation, increases in interest rates and geopolitical uncertainties caused fluctuations in industrial production and economic growth.

Consumer loans are a type of repayable loan obtained from financial institutions used for purchasing or spending on goods and services other than housing. Consumer loans allow individuals to use their future income. This facility has an impact on macroeconomic variables. By making spending easier, consumer loans support economic growth by increasing consumption. However, excessive borrowing can lead to financial instability and trigger crises.

The Central Bank of the Republic of Turkey (CBRT) uses various tools to ensure and maintain price stability. One of the most important indicators among these tools is the CBRT Weighted Average Funding Cost (WAFC). WAFC refers to the weighted average interest rate of the funds provided by the CBRT to banks. It is used to determine the impact of the interest rate policies implemented by the CBRT on market interest rates. During the period between 2012-2015, the CBRT tried to keep the cost of funding low in the market. The recovery from the 2008 financial crisis was continuing, and economic difficulties caused interest rates to remain at certain levels. The low funding cost continued. After 2015, due to the depreciation of the Turkish lira, the CBRT opted to increase interest rates. During this period, the WAFC began to rise. Due to the pandemic experienced in 2019, the CBRT went for an interest rate cut, and accordingly, the WAFC decreased. In 2023-2024, interest rates were increased, and it has been observed that the WAFC rose rapidly during this period.

Ensuring economic stability, fairness in income distribution and increasing social welfare are important for a country. Transfers to households are of great importance in achieving this objective. Transfers to households are payments made to individuals or families by the state or institutions. These payments include various benefits such as pensions, unemployment pensions, orphan and orphan pensions,

scholarships paid to students, agricultural supports and health supports. Depending on the economic developments in 2012-2024, transfers to households increase. The COVID-19 pandemic between these years and the high inflation after 2022 have led to significant increases in transfers to support households.

Confidence indices are used to measure the confidence in financial markets. One of these is the Financial Services Confidence Index. This index is developed by the answers given to the Financial Services Survey Questions applied to business people. Between 2012 and 2017, the index was above 100, indicating that the sector had a good period. In 2018, there was a sharp decline due to the currency crisis. The COVID-19 pandemic in 2020, the currency crisis in 2022 and the earthquake in 2023 had a negative impact on the Turkish economy. In 2024, although the recovery continued, volatility continued.

LITERATURE REVIEW

Durmuş and Şahin (2019) examined the relationship between consumer loans and selected macroeconomic variables in Turkey using quarterly data from 2006:Q1 to 2018:Q2. The causality relationship between variables was analyzed using the Toda-Yamamoto approach-based Granger causality test. The test results indicate a bidirectional causality between interest rates and loans, as well as a unidirectional causality from money supply to loans. It was observed that macroeconomic variables such as economic growth, employment and unemployment rates, interest rates, inflation rates, exchange rates, public and private sector expenditures, savings rates, and monetary aggregates have adverse effects on consumer loans, particularly during economic cycles or crisis periods.

Shinwari and Özdemir (2022) aimed to investigate the dynamics between selected macroeconomic variables, such as the Consumer Price Index (CPI) and Industrial Production Index (IPI), and housing prices in Turkey between 2010 and 2020. The relationship between Turkey's hedonic housing price index, CPI, and IPI was analyzed using ARDL boundary tests and Toda-Yamamoto causality tests with monthly data from the 2010-2020 period. Although the findings do not confirm the existence of a long-term relationship between the housing price index, CPI, and IPI, a negative short-term interaction was observed. Empirical results obtained through the application of the Autoregressive Distributed Lag (ARDL) model and boundary tests provide evidence that increases in the CPI or IPI negatively impact Turkey's hedonic housing price index in the short term.

Kalkay and Tunalı (2024) examined the relationship between unconventional monetary policies implemented by the Central Bank of the Republic of Turkey (CBRT) after the 2008 financial crisis and selected macroeconomic indicators. The Bayesian Vector Autoregression (B-VAR) method, which allows for the assumption of prior conditions, was used in the model. The model estimation covered one-month, three-month, and six-month datasets from the period 2010:1-2021:12. The findings indicate that, following 2014, there is no strong evidence of a significant impact between the unconventional monetary policy tools implemented by the CBRT and macroeconomic indicators.

Telatar et all (2003) investigated whether the term structure contained useful information regarding future inflation for Turkey during the 1990-2000 period, a time characterized by high inflation and large budget deficits. The study analyzed the relationship between interest rate types and inflation fluctuations using a Markov-switching heteroskedastic model and a time-varying parameter model. These models provide a reasonable explanation of the relationship between variables during periods of economic turbulence and help to better understand economic dynamics. During this period, frequent changes occurred in monetary and fiscal policies due to political instability, leading to shifts in stabilization regimes. This situation made models that account for regime changes and endogenous parameter variations particularly appealing for analyzing the Turkish economy. The study emphasizes the

importance of such modeling approaches in examining Turkey's economic dynamics and forecasting future inflation expectations.

Kayhan et all (2013) analyzed the behavior of the CBRT between 2002 and 2011 by constructing a forward-looking monetary policy reaction function incorporating individuals' expectations within the framework of the model developed by King (1996). The study examined the presence of asymmetric behavior in the CBRT's policy decisions using the Markov-Switching Vector Autoregression (MS-VAR) model. The analysis, conducted using monthly data from the 2002-2011 period when the inflation-targeting strategy was implemented, indicates that the Turkish economy experienced continuous expansion phases throughout this time frame.

Tüzün ve Kahyaoğlu (2015) The study aims to analyze the impact of the macroprudential monetary policy implemented by the Central Bank of the Republic of Turkey (CBRT) to ensure financial stability without deviating from its price stability objective, using monthly data from the 2008-2013 period. This analysis is conducted using the Bayesian VAR (B-VAR) method. The findings support the CBRT's decision to select capital mobility and credit expansion as indicators for maintaining financial stability without compromising price stability. Accordingly, financial stability policies should be implemented in a stable manner while considering price stability.

Albayrak ve Abdioğlu (2015) In this study, the validity of the Taylor rule was tested using monthly data for the period 2002-2014. According to which variable the CBRT shaped its monetary policy in the 2002-2007 and 2008-2014 periods, Taylor (1993) model and its extended versions were examined. According to the results, it is seen that the Central Bank targets price stability and directs interest rates steadily. The fact that the interest smoothing parameter is significant shows that financial stability is also taken into account. In the post-crisis period of 2008-2014, there is no significant difference between the forward and backward Taylor rule.

İlhan (2022) This study examines the impact of loans in different regimes on monetary policy reactions in Turkey. For this purpose, the credit gap and the extended Taylor rule were estimated using the Markov regime switching model with data for the 2006-2019 period. According to empirical findings, it defines two regimes: low and high interest rates. While the low interest rate regime reflects the priority of growth, the high interest rate regime has a significant impact on inflation and the interest rate of the loan. This shows that despite macroprudential tools after the global financial crisis, credit conditions are effective in tightening monetary policy in Turkey.

Yağcıbaşı ve Yıldırım (2019) In the study, alternative specifications of monetary policy rules were examined in the inflation targeting regime in Turkey, using monthly data for the period 2003-2017. The original Taylor rule and the exchange rate-augmented Taylor rule were estimated with Markov regime switching models. According to the findings, it shows that the Turkish economy operates in high and low interest rate regimes, and 2009 was the most prominent year of this transition. In a high interest rate regime, the response to inflation is greater and the response to the output gap is more aggressive, which means that, according to the Central Bank, output gap stability is more important than inflation stability.

Kılınç ve Tunc (2019) In this study, the asymmetric effects of monetary policy on economic activity in Turkey for the period 2006-2017 were examined using GDP and industrial production data. Markov Switching Model was applied. According to the findings, monetary policy shocks show stronger effects in contraction periods and weaker effects in expansion periods. Additionally, when credit cycles are used, asymmetric effects become more pronounced.

METHOD AND DATASET

In this study, a time series analysis is performed using monthly data covering the period 2012:07-2024:11. R programme was used in this analysis. The consumer price index, industrial production index and interest rate applied to consumer loans variables used in the analysis are taken from the Electronic Data Distribution System (EDDS).

Table 2: Variables

Variables	Symbol	Unit	Source
Consumer Prices Index	cpi	%	EVDS
Industrial Production Index	ipi	%	EVDS
Consumer Loan Interest Rate	int	%	EVDS
Weighted Average Funding Cost	wafc	LCU	EVDS
Transfers to Households	thl	LCU	EVDS
Financial Services Confidence Index	fsci	%	EVDS

In this study, as a first step, the graphs of the series are analysed to test the presence of seasonal effects. After determining that there are seasonal effects in the graphs, the variables are seasonally adjusted using the TRAMO-SEATS method. In another step of the analysis, ADF and PP unit root tests were used to test the stationarity of the series. After determining the stationarity of the series, the regression model was estimated. In another step, Markov regime switching model was constructed and potential regime changes of the series were analysed. Bayesian VAR (BVAR) model was established in order to model the uncertainties of the parameters and to make flexible and more reliable forecasts. In addition, the Classical VAR model was also established to provide a more comprehensive analysis.

Johansen cointegration analysis was performed to examine the relationships between the variables. In addition, Granger causality test was used to test the relationship between the model parameters. Impulse-Response analysis was applied as the last stage of the analysis.

The model used to make these analyses is constructed as follows:

$$cpi_t = \beta_0 + \beta_1 ipi_t + \beta_2 int_t + \beta_3 wafc_t + \beta_4 thl_t + \beta_5 fsci_t + \mu_t \quad (1)$$

In the model, inflation rate is selected as the dependent variable and industrial production index, interest rate applied to consumer loans, weighted average cost of funding, transfer expenditures to households and financial services confidence index are selected as independent variables. In order to solve the autocorrelation problem, one lagged version of the error terms is added to the model.

Table 3: Descriptive Statistics

	CPI	THL	FSCİ	INT	IPI	WAFC
Mean	6.135	12.981	162.163	24.177	85.288	15.451
S.d	0.722	1.0604	13.112	15.868	14.860	11.660
Minimum	5.319	11.745	119.850	10.274	60.365	4.647
Maximum	7.885	15.321	188.078	80.845	109.893	53.068
Median	5.968	12.719	165.097	17.152	83.993	11.602

TRAMO-SEATS method is used to seasonally adjust the inflation rate, industrial production index, interest rate on consumer loans, weighted average cost of funding, transfer expenditures to households and financial services confidence index variables. The graphs of the variables are as follows:

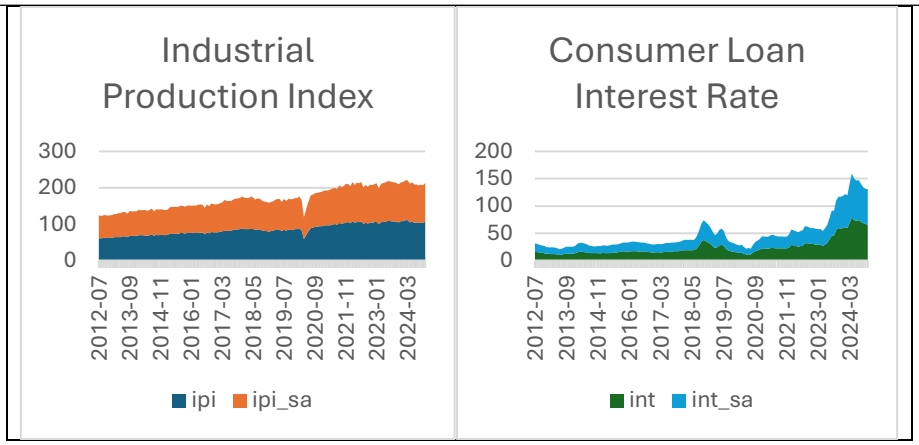

Figure 2: Seasonally Adjusted Industrial Production Index and Consumer Loan Interest Rate

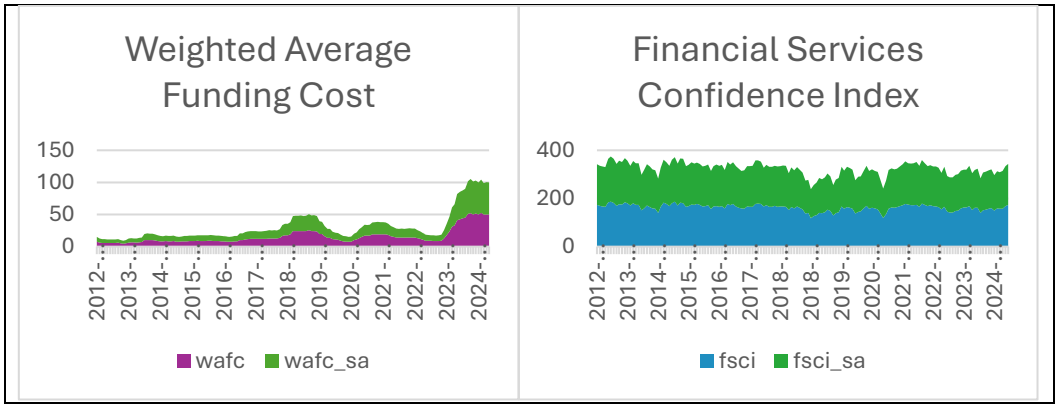

Figure 3: Seasonally Adjusted Weighted Average Funding Cost and Financial Services Confidence Index

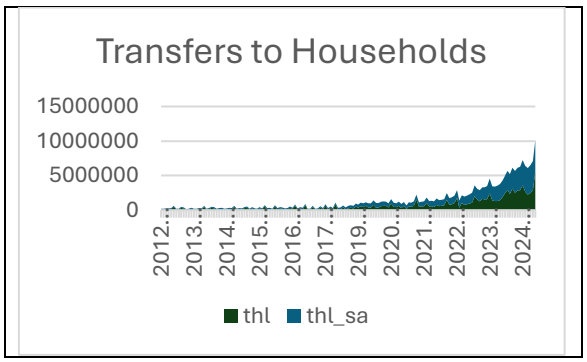

Figure 4: Seasonally Adjusted Transfers to Households

The first step in a time series analysis is to test the stationarity of the variables. In this study, the stationarity levels of the variables were determined by Augmented Dickey Fuller and Philips and Perron unit root tests.

Augmented Dickey-Fuller Unit Root Test

The Augmented Dickey-Fuller (ADF) unit root test lays the groundwork for other unit root tests. The ADF test by Dickey and Fuller is based on the assumption that error terms are independent and constant variance (Dickey and Fuller, 1979). Unit-rooted series are those containing an AR component. This test is based on the estimation of the first order autoregressive process equation. In other words, since the

AR(1) model has one characteristic root, the ADF test takes into account the presence of a single unit root. AR(1) model,

$$Y_t = \rho Y_{t-1} + \varepsilon_t \qquad (2)$$

It is expressed as. In order to perform unit root test in the established equation, firstly, Y_{t-1} is subtracted from both sides of the equation and the first order equation

$$\Delta Y_t = (\rho - 1) Y_{t-1} + \varepsilon_t \qquad (3)$$

It transforms into. When expressed here as $\rho - 1 = \delta$,

$$\Delta Y_t = \delta Y_{t-1} + \varepsilon_t \qquad (4)$$

The final form of the equation will be as follows. To test for the existence of a unit root, the hypothesis $\delta = 0\ [or\ (\rho - 1) = 0]$ is tested. Three different models are used to calculate the unit root test:

$$\Delta Y_t = \delta Y_{t-1} + \varepsilon_t \qquad (5)$$

$$\Delta Y_t = \mu + \delta Y_{t-1} + \varepsilon_t \qquad (6)$$

$$\Delta Y_t = \mu + \beta T + \delta Y_{t-1} + \varepsilon_t \qquad (7)$$

Equation number five is the model without constant and trend, equation number six is the model with constant and equation number seven is the model with both constant and trend. For all equations above, $\varepsilon_t \sim IID(0, \sigma^2)$ is assumed. The test statistic for these models is calculated as follows:

$$\tau_\delta = \frac{\hat{\delta}}{s_{\hat{\delta}}} \qquad (8)$$

The main alternative hypotheses for the test statistic will be as follows:

$$H_0: \delta = 0\ ise\ the\ series\ is\ stationary$$

$$H_1: \delta < 0\ ise\ the\ series\ is\ not\ stationary$$

Equation number six is used in this study. In other words, the unit root test model with constant is used (Güriş, Çağlayan Akay, & Güriş, 2020, p. 170).

Phillips-Perron Unit Root Test

Another unit root test used in the time series analysis conducted in this study is the nonparametric unit root test developed by Phillips and Perron (1988). According to the Dickey-Fuller unit root test, error terms are assumed to be independent and have constant variance. Phillips and Perron (1988) argued that the DF test method would give erroneous results. They added a correction factor to the Dickey-Fuller test statistics. In this context, a new method was introduced by adding a correction factor to the DF test statistic when autocorrelation is detected in the error terms.

When we express this definition mathematically,

$$(Y_t - \mu) = \alpha(Y_{t-1} - \mu) + \varepsilon_t,\ t = 1,2,3,\dots,n \qquad (9)$$

Based on the AR(1) model,

$$Y_t = \alpha_0 + \alpha Y_{t-1} + \varepsilon_t,\ t = 1,2,3,\dots,n \qquad (10)$$

It can be expressed as. In the above equation, error terms are shown as ε_t. The error terms, $\Psi(B)e_t = 1 + \beta_1 B + \beta_2 B^2 + \cdots + \beta_q B^q$, can be expressed as $\varepsilon_t = \Psi(B)e_t$. A correction factor is added to the Dickey-Fuller test statistic. The error terms from the regression of Y_t on Y_{t-1} are $\hat{\varepsilon}_t$. When we define the autocovariances of the residual squares by $\hat{\gamma}(h)$,

$$\lambda^2 = \hat{\gamma}(0) + 2\sum_{j=1}^{q}\left[1 - j\Big/(q+1)\right]\hat{\gamma}(j) \quad (11)$$

The new equation takes the form of equation number eleven. The mean squared error from the regression of Y_t on Y_{t-1} is as follows:

$$S_n^2 = \frac{1}{n-2}\sum_{t=1}^{n}\hat{\varepsilon}_t^2 \quad (12)$$

Phillips-Perron unit root test statistics,

$$Z_\mu = \left[\sqrt{\frac{\hat{\gamma}(0)}{\hat{\lambda}^2}}\right]\hat{\tau}_u - \left\{\frac{1}{2}\left(\hat{\lambda}^2 - \hat{\gamma}(0)\right)(ns_{\hat{\alpha}}/S_n)\right\}\hat{\lambda}^{-1} \quad (13)$$

It is expressed as. In this study, the stationarity degrees of the series were determined by using the constant model (Akdi, 2012, p. 261-280).

Markov Regime Switching Model

Unlike linear time series, Markov regime switching model has a non-linear structure that shows different behaviours in different periods. This model assumes that a time series can have different regimes. This assumption is determined by the Markov chain. The process consisting of at least two regimes is combined with a variable that acts as a regime variable, a dummy variable. In this context, it offers the opportunity to evaluate periods with different characteristics (Evci, 2016, p.69). Markov regime switching models have been discussed by Hamilton (1989, 1990, 1994, 1996), Kim and Nelson (1998), Krolzing (1997, 1998, 2000, 2001) in order to analyse business cycles. Hamilton (1989) constructed a two-regime Markov regime switching model as follows:

$$Y_t = \begin{cases} \phi_{1,0} + \phi_{1,1}Y_{t-1} + \cdots + \phi_{1,p}Y_{t-p} + \varepsilon_t, \ eğer \ (s_t = 1) \\ \phi_{2,0} + \phi_{2,1}Y_{t-1} + \cdots + \phi_{2,p}Y_{t-p} + \varepsilon_t, \ eğer \ (s_t = 2) \end{cases} \quad (16)$$

$$Y_t = \phi_{0,st} + \phi_{1,st}Y_{t-1} + \cdots + \phi_{p,st}Y_{t-p} + \varepsilon_t \ (17)$$

It is constructed as follows. In the model, $\phi_{1,j}$ and $\phi_{2,j}$ are the autoregressive lag parameters for each regime, s_t is the value of each regime, p is the autoregressive degree of the model, $\varepsilon_{it} \sim IID(0, \sigma^2)$ and $\sigma_i^2 < \infty$ (Mohd and Zaidi, 2006: 57; Fallahi and Rodriguez, 2007: 5), (Kayhan, S., Bayat, T., & Koçyiğit, A. (2013)).

Regimes depend on their previous values and transformation probabilities. A regime change is characterised by an unobservable state variable. This state variable is modelled as a Markov chain with different regimes:

$$P(S_t = j | S_{t-1} = i) = p_{ij}, i, j = 1,2, \ldots, k \quad (18)$$

In the eighteen numbered equation above, p_{ij} is the transition probability from regime i to regime j. The transition probability matrix P containing transition probabilities:

$$P = \begin{bmatrix} P_{11} & \cdots & P_{1k} \\ \vdots & \ddots & \vdots \\ P_{k1} & \cdots & P_{kk} \end{bmatrix}, ve \ \sum_{j=1}^{k} p_{ij} = 1 \quad (19)$$

The conditional probability density function of Y_t in each regime is expressed as follows:

$$f(Y_t | S_t = j) = \frac{1}{\sqrt{2\pi\sigma_j^2}}\exp\left(-\frac{(Y_t - X_t\beta_i)^2}{2\sigma_j^2}\right) \quad (20)$$

In the Markov regime-switching model, this equation allows both the dynamics of the series in different regimes and the transition probabilities between regimes to be modelled together.

Bayesian Vector Autoregression (B-VAR)

Litterman (1986) played an important role in the development of the Bayesian VAR model in order to solve the over-fitting and over-parameterisation problems of the classical VAR model, to make the forecasts more reliable and to reduce the uncertainties compared to the classical VAR model. The Bayesian VAR (BVAR) model adjusts parametric uncertainties with prior distributions such as Minnesota and calculates posterior distributions to fit the generated data set. The Minnesota prior allows for more consistent forecasts by preventing excessive bias of the forecasts.

The classical VAR model equation makes data-driven forecasts. Larger data sets are required compared to the Bayesian VAR model. Looking at the components of the classical VAR model:

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + \varepsilon_t,\ \varepsilon_t \sim N(0, \Sigma) \quad (21)$$

In this equation, $Y_t$represents the vector of variables, $A_i$represents the parameter matrices to be estimated and ε_t represents the error terms.

Looking at the components of the Bayesian VAR model:

$$p(\Theta|Y) \propto p(Y|\Theta)p(\Theta) \quad (22)$$

In the Bayesian approach for $\Theta = \{A_1, A_2, \ldots A_p, \Sigma\}$ the prior $p(\Theta)$ and the data-driven probability $p(Y|\Theta)$ denote the posterior probability $p(\Theta|Y)$ of the parameters.

The posterior distribution is calculated as follows:

$$p(\Theta|Y) = \frac{p(Y|\Theta)p(\Theta)}{\int p(Y|\Theta)p(\Theta)} \quad (23)$$

Bayesian VAR (BVAR) model is used in this study. The two models are compared by constructing a classical VAR model.

RESULTS

Unit Root Tests

In order to determine the stationarity of the variables used in the study, Augmented Dickey-Fuller, Phillips-Perron unit root test and Zivot-Andrews unit root test with structural breaks were used to test the stationarity of inflation rate, industrial production index, transfers to households, interest rate applied to consumer loans, weighted average cost of funding and financial services confidence index. According to the findings, it is concluded that the variables used in the model are not stationary at the level values of the series. In this context, the series were made stationary by taking the first difference. In other words, it is concluded that the series are first order integrated I (1). The results obtained are as follows:

Table 4: ADF and PP Unit Root Test

	ADF				PP			
	t- statistic	**%1**	**%5**	**%10**	**t- statistic**	**%1**	**%5**	**%10**
Cpi	-3.57	-3.46	-2.88	-2.57	-3.03	-3.47	-2.88	-2.57
İpi	-14.03	-3.46	-2.88	-2.57	-14.70	-3.46	-2.88	-2.57
İnt	-7.35	-3.46	-2.88	-2.57	-7.25	-3.46	-2.88	-2.57
Wafc	-6.68	-3.46	-2.88	-2.57	-6.87	-3.46	-2.88	-2.57
Thl	-11.63	-3.46	-2.88	-2.57	-11.71	-3.46	-2.88	-2.57
Fsci	-14.55	-3.46	-2.88	-2.57	-15.40	-3.46	-2.88	-2.57

Note: All variables are stationary at 1%, 5%, 10% significance levels

Markov Regime Switching Model

After determining the stationarity degrees of the variables, three different Markov regime switching models were constructed. For the Markov regime switching model, a model with two regimes (Regime 1 and Regime 2) was preferred. The results of the analyses are shown in the table below.

Table 5: Markov Regime Switching Model Test Results

Variables	**Model A**		**Model B**		**Model C**	
	Regime 1					
THL	0.06***(0.000)		0.0005***(0.000)		0.0004***(0.000)	
Fsci	0.4831(0.3831)		-0.3440*(0.03082)		-	
Wafc	-8.6662 ***(0.000)		-11.8201***(0.000)		-9.8309***(0.000)	
Int	9.2151***(0.000)		10.8558***(0.000)		9.7748***(0.000)	
Ipı	-0.4002(0.6577)		-		2.0617***(0.000)	
C	33.9811(0.6727)		157.0814***(0.000)		-26.2999(0.3446)	
	Regime 2					
THL	0.06***(0.000)		0.0005***(0.000)		990.3796***(0.000)	
Fsci	0.2039 (0.188073)		3.6362***(0.000)		-	
Wafc	-1.8538**(0.004459)		-3.9650**(0.007938)		15.7086***(0.000)	
Int	0.9741(0.114635)		8.5776***(0.000)		-6.7937*(0.0373)	
Ipi	3.8145***(0.000)		-		-5.1604***(0.000)	
C	-149.1142***(0.000)		-502.5410***(0.000)		-12389.6***(0.000)	
	Transition Probabilities Matrix					
	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
Regime 1	0.94734	0.02310	0.93800	0.07248	0.94954	0.06870
Regime 2	0.05265	0.9768	0.0619	0.9275	0.0504	0.931

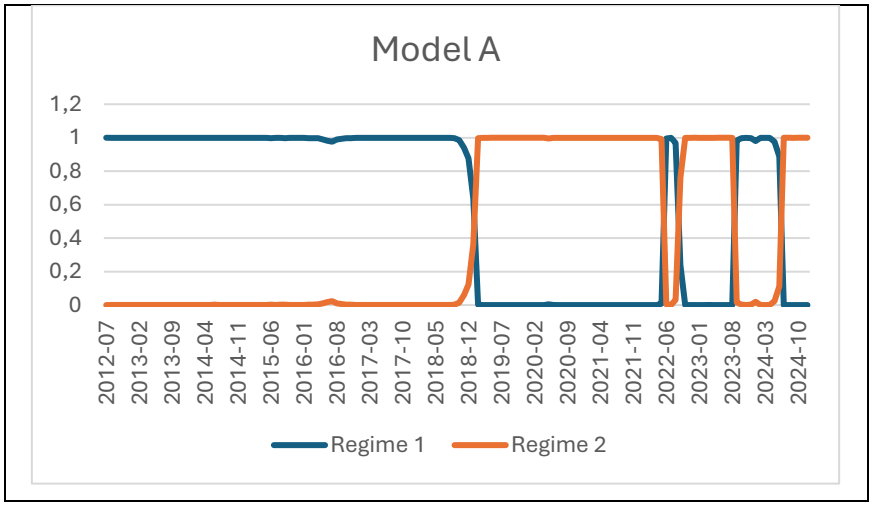

Figure 5: Model A

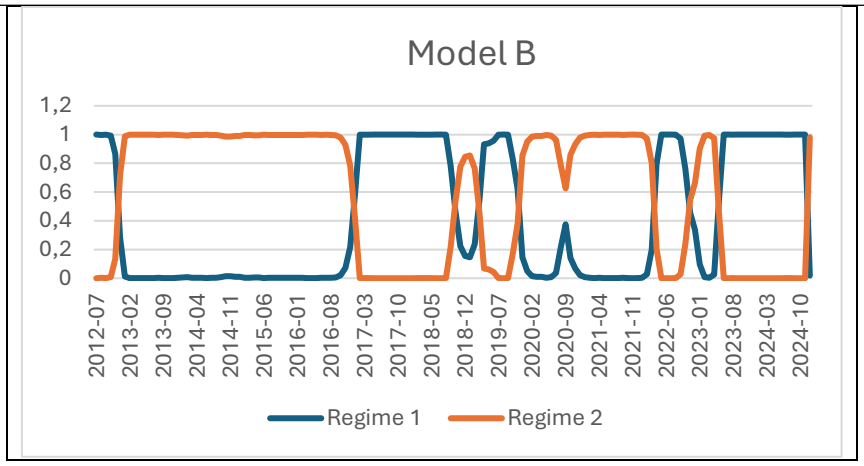

Figure 6: Model B

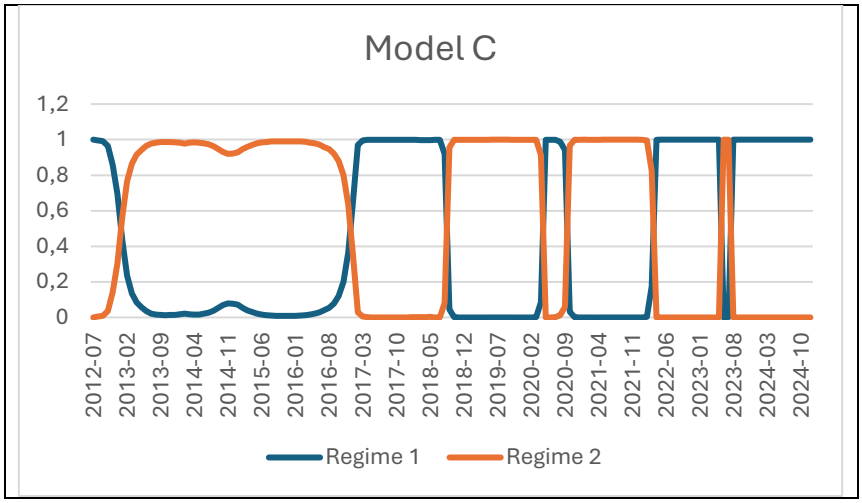

Figure 7: Model C

In this study, Markov regime switching models with two regimes, namely Model A, Model B and Model C, were established. The findings of the Markov regime switching models are presented in Table 8. Regime 1 represents the low inflation period and Regime 2 represents the high inflation period. When the results for Regime 1 in Table 8 are analysed, it is seen that Regime 1 refers to growth periods with generally lower inflation rates. In this regime, there is a positive interaction of transfer expenditures to households on inflation. This indicates that transfers increase consumption demand and exert an upward pressure on the general price level. Similarly, industrial production exerts a limited pressure on inflation in this regime. In this context, the increase in production accompanies the increase in demand and supports the price level to remain under control. The positive impact of consumer loan interest rates suggests that increased access to credit stimulates spending and may increase demand-side inflationary pressures. On the other hand, the negative effect of the weighted average cost of funding implies that the increase in financing costs restrains the increase in the price level by limiting credit utilisation. This regime covers a period of controlled inflation and economic growth.

Regime 2 represents the contractionary period observed in high inflationary environments. Looking at the results of Regime 2 in Table 8, the positive effect of transfer expenditures to households on inflation is more evident. Increased expenditures of households may lead to a rapid increase in price levels by strengthening demand-side pressure. The positive effect of the financial services confidence index contributes to inflationary dynamics by supporting economic recovery. Another parameter, the positive effect of the weighted average cost of funding on the inflation rate, indicates that the implementation of liquidity enhancing policies may further push inflation upwards. The negative effect of consumer loan

interest rates indicates that high interest rates aim to restrain price increases by reducing demand. The negative effect of industrial production, on the other hand, indicates that prices may increase uncontrollably due to the decline in production during contraction periods, rising interest rates on raw materials and equipment and supply-side problems in production. This regime refers to a period of intense demand and cost-driven inflationary pressures.

Bayesian Vector Autoregression (B-VAR)

Bayesian Vector Autoregressive Model (BVAR) model was established as the last stage of the analysis. After determining the optimal lag length suitable for the model, impulse-response graphs were created. In order to control the complexity of the model and the overfitting problem, the lambda value was set as 0.44179. The iteration value, known as the warm-up phase of the model, was accepted 4772 out of 10000 iterations. The accepted traction rate was determined as 94.4%.

Table 6: Determination of Optimal Lag Length

Criterion	1	2	3	4	5	6
AIC	**8.21998**	8.32592	8.37845	8.28179	8.27406	8.16773
HQ	**8.52449**	8.93494	9.29199	9.49983	9.79661	9.99480
SC	**8.96934**	9.82465	10.6265	11.2792	12.0208	12.6639
FPE	**3715.5**	4139.56	4388.237	4029.670	4075.801	3772.698

Note: According to AIC, HQ, SC and FPE critical values, the appropriate lag length is 1.

In this study, Bayesian Vector Autoregression (BVAR) and Vector Autoregression (VAR) models are used to examine the differences between the two models. Impulse-Response analysis results are presented in Table 13 and Table 14. When the impulse response graphs of the Bayesian VAR model are analysed, it is observed that the inflation rate's response to its own shock creates a high positive effect for a short period, while it is dampened towards the last periods. The inflation rate shows a resilient structure against exogenous effects. A one-unit shock in the transfers to households variable has a positive effect on the inflation rate until the third period. After the third period, there is a decline until the fifth period. When the last periods are approached, it is observed that the line is dampened. When the effect of financial services confidence index on inflation is analysed, it is observed that it has a positive effect until the fourth period and the curve approaches zero towards the last periods. A shock in the weighted average funding cost has a positive effect on the inflation rate. A one-unit shock in the interest rate applied to consumer loans and the industrial production index reveals that it has an increasing effect on inflation. A one-unit shock in the inflation rate has a strong negative effect on the transfers to households variable in the first period. Later on, this effect is observed to be a rapid decline in the THL variable after the shock. In the short run, a shock in the inflation rate has a negative effect on the THL variable. A one-unit shock in the financial services confidence index is found to have a strong negative effect on transfers to households in the short run. An increase in the financial services confidence index puts pressure on household expenditures. A shock in the weighted average cost of funding has a strong negative impact on transfers to households. In the first periods, transfers to households tend to decrease significantly. An increase in the weighted average cost of funding depresses transfers to households in the short run. A one-unit shock in the industrial production index has a positive effect on transfers to households. A shock in the inflation rate has a negative effect on the financial services confidence index in the initial periods. It is determined that transfer expenditures to households have a negative interaction on the financial services confidence index in the initial periods. It is observed that the curve is dampened towards the last periods. It is concluded that a one unit shock in the weighted average funding cost has a negative effect on the financial services

confidence index. A one-unit shock in consumer loans has a sharp negative trend on the financial services confidence index.

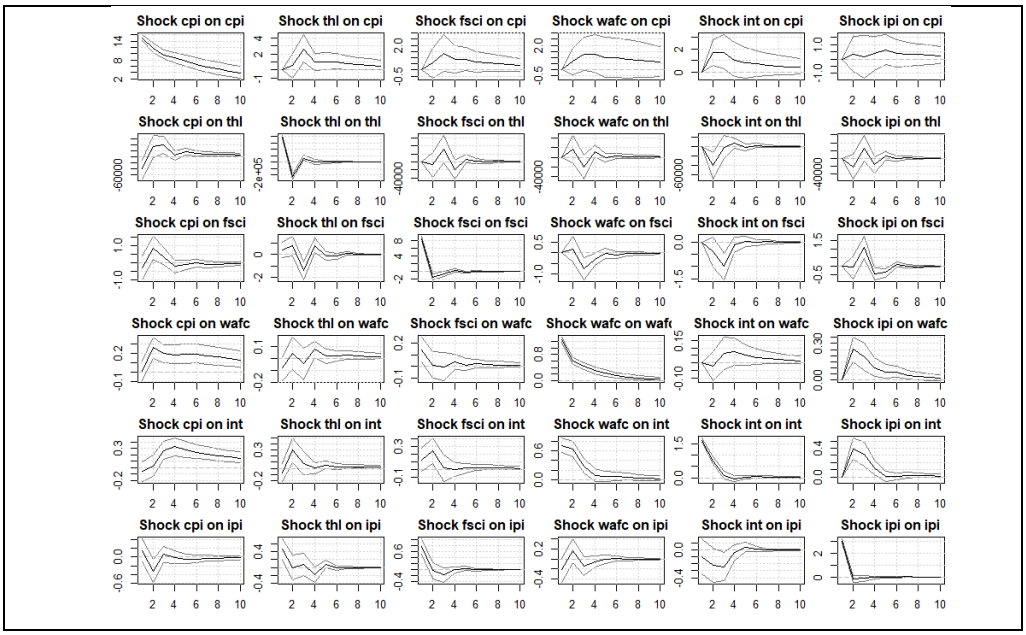

Figure 8: BVAR Impulse-Response Graphs

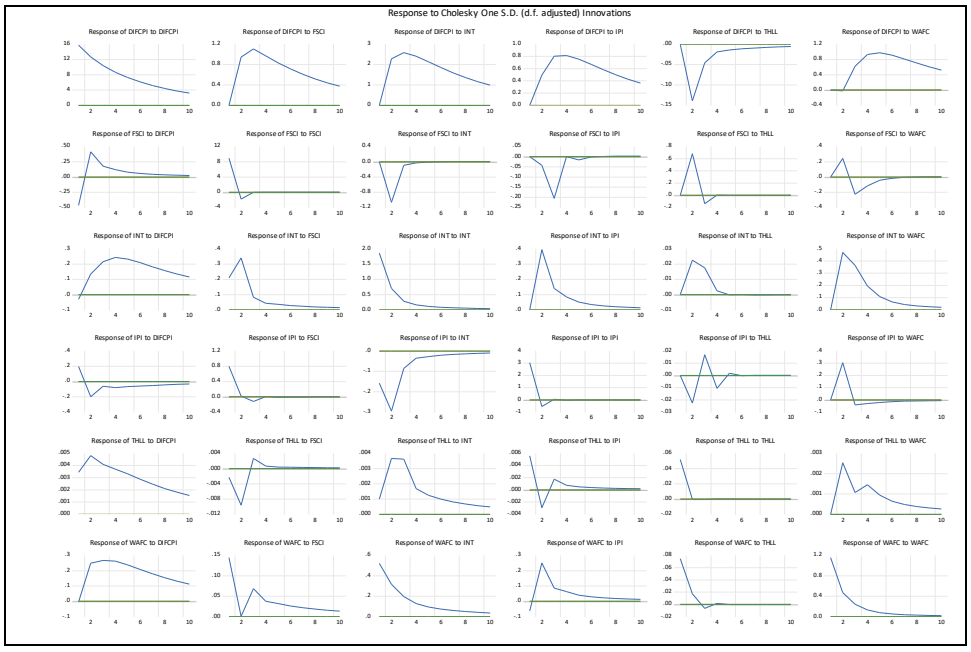

Figure 9: Classical VAR Model

CONCLUSION

This study aims to analyse the effects of transfer expenditures to households, industrial production index, weighted average cost of funding, average interest rate applied to consumer loans and financial services confidence index variables on the inflation rate and the relationships between the variables.

Markov regime switching model is used to analyse the regime transitions between variables. Regime 1 covers the period when the inflation rate is low. According to the results of regime 1, it is concluded that transfers to households increase the consumption demand of households. In this context, it is determined that it has an increasing effect on the inflation rate. Increases in the industrial production index are found

to have a suppressive effect on the inflation rate. In other words, increases in industrial production also increase aggregate demand. Increases in demand support a controlled increase in prices. Increases in interest rates applied to consumer loans are found to have a positive relationship on transfers to households. According to the results, the increase in interest rates has an incentive effect on households. This increases demand-side inflationary pressures. The negative impact of the weighted average cost of funding on inflation limits the consumer credit utilised by households. This situation suppresses the inflation rate by controlling price increases and reducing consumption demand. Regime 1 represents a period of more balanced economic activity.

Regime 2 represents the contractionary effects of periods of high inflation. According to the results of Regime 2, when the effect of the weighted average cost of funding on inflation is analysed, an increase in the weighted average cost of funding decreases the inflation rate. This increases loan rates. With increasing loan rates, banks tend to be more selective in lending. In this context, it reduces consumers' willingness to borrow and tends to reduce inflation. Decreases in the weighted average funding cost, on the other hand, increase the borrowing status of consumers and increase consumer demand. Economic activity increases with low interest rates. As a result, inflationary pressures may increase. The positive effect of transfers to households on inflation increases consumer demand. With the increase in the interest rate applied to consumer loans, households and enterprises face high costs. This situation is reflected in price increases and cost inflation is realised. When the positive effect of the industrial production index on inflation is analysed, the positive increase in the industrial production index indicates the growth status of a country, as the inflation variable is a cyclical variable. Regime 2 covers the period of intense demand and cost-driven inflationary pressures.

According to the results of the impulse-response analysis, the inflation rate shows a structure that is resistant to exogenous shocks. This shows that the inflation rate will increase steadily according to a certain level. Increases in the interest rate applied to consumer loans and the industrial production index increase inflation. This situation leads to demand and cost pressures. Weighted average funding cost increases inflation. It is also found to have a negative impact on transfers to households. According to these results, the increase in borrowing cost reduces the consumption demand of households.

When the interrelationships among the variables of inflation rate, interest rate applied to consumer loans, industrial production index, weighted average cost of funding, financial services confidence index and transfers to households are analysed, it is seen that stronger monetary policy arrangements are required for a developing country like Turkey. In addition, it is very important to conduct fiscal policy together with monetary policy. Transfers to households should be focussed on the right groups. The negative effect of financial services confidence index increases uncertainties. This situation should be eliminated by strengthening the supervision mechanisms and a more secure and transparent financial system should be created.

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Measuring Public Procurement Efficiency: The Case of EU Countries (1048)

Songül Bulut[1 *], **İhsan Alp**[1]

[1] Gazi University, Science Faculty, Statistics, Ankara, Türkiye

*Corresponding author e-mail: songul.bulut@dmo.gov.tr

Abstract

In this study, the DEA model was selected to evaluate the public procurement efficiency of EU member states and Turkey; then the Tobit model was used to analyze the effects of the factors affecting the efficiency of public procurement. The aim of the article is to measure the public procurement efficiency of countries with SBM-DEA analysis and to determine the variable/variables that have the greatest effect on efficiency with Tobit regression analysis. Efficiency scores were found for 25 EU countries and Turkey in the study. Accordingly, 13 countries were found to be efficient with a score of 1.0. Although the average public procurement efficiency score was found to be 0.62, half of the countries received a full efficiency score. When examined from a regional perspective; public procurement efficiency varies significantly. While the highest scores were obtained in the southern and northern regions, respectively, the scores of central European countries were found to be low.

Among the variables whose effects on public procurement efficiency were investigated; the number of tenders concluded with procurement, variety of products procured, framework agreement, joint procurement, central procurement and number of tenders made with electronic auction; the number of concluded tenders had a positive effect on efficiency, while the variables of joint procurement, framework agreement and product variety had a negative effect on efficiency. Tenders made with central procurement and electronic auction were not found to have any effect on efficiency.

Keywords: *Slack-based DEA, Tobit regression, public procurement, efficiency*

INTRODUCTION

Public procurement, an industrial policy tool, has shares ranging from 12-20% of GNP in countries that have made progress in areas such as production, employment, exports, development, and the environment. Public procurement has been defined as a "value" that goes beyond financial savings to include broader policy objectives such as sustainability, green digital transformation, and support for SMEs by using technological developments such as digital transformation, artificial intelligence, and blockchain; and thus has become a policy tool that aims to provide benefits by maximizing value for money (UN Environment, 2017).

Public procurement not only provides public services to citizens and businesses, but also contributes to the efficient and effective use of resources from an economic perspective. Beyond the economic aspect of value for money, public procurement is also used as a strategic tool to achieve policy objectives such as mitigating climate change, promoting innovation and social inclusion (OECD, 2019). The OECD Public Procurement Recommendations Report highlights the need to assess the effectiveness of the public procurement system at all levels of government, from individual procurement to the system as a whole, and to improve performance wherever possible (OECD, 2015).

An efficient and effective public procurement system can be defined as a good management tool that meets the following criteria:

• Promote effective and efficient procurement practices and systems to ensure timely delivery of public services

• Seek to achieve value for money based on life cycle cost

• Increase competition among suppliers through its policies and practices

• Adopt high standards of equity to ensure fairness and transparency in its practices and prevent corruption

• Ensure accountability through, among others, good record keeping, auditing and transparent review procedures

• Ensure uniformity of procurement by legal entities through the use of regulations and standard documentation in all public procurement

• Ensure that public procurement complies with the provisions of the country's public procurement law and the constitution

• Increase the quality and quantity of projects undertaken by the government

The above issues are elements of effective public procurement systems and are also at the heart of good governance (Tian et al, 2020).

Sustainable Public Procurement is about spending public funds on products/services/projects that support sustainable development. As sustainability issues become vital to the development agenda of nations, it is important to shift the focus of public procurement systems from immediate economic advantages to sustainable public procurement systems that will deliver long-term benefits (Adjei, 2010).

Sustainable Public Procurement is defined as a process by which Public Institutions meet the needs of goods, services and businesses in a way that provides value for money throughout the life cycle, while minimizing environmental damage, providing benefits not only to the organization but also to society and the economy. It is one of the seven themes promoted under the Marrakech Task Force (Tian et al, 2020)

The other six themes are:

- Sustainable Products (hosted by the UK);
- Sustainable Lifestyles (hosted by Sweden);
- Sustainable Tourism (hosted by France);
- Sustainable Buildings and Construction (hosted by Finland);
- Education for Sustainable Consumption (hosted by Italy);
- Cooperation with Africa (hosted by Germany).

Effective public procurement means that procurements are made in a way that covers all sustainability parameters and that the necessary measures are taken for this purpose. In this study, the necessary parameters for countries to make their expenditures effective, efficient and sustainable were investigated (OECD, 2023). In the study where the effectiveness of public procurement was analyzed and the variables affecting this effectiveness were investigated, the effectiveness of public procurement in 24 EU countries and Turkey was measured with SBM-DEA analysis and the parameters contributing to the effectiveness were found with Tobit regression analysis.

MATERIAL AND METHODS

Material

This study consists of a two-stage analysis. In the first stage, the public procurement efficiency scores of the countries with DMU were measured with the slack-based data envelopment analysis; in the second stage, the efficiency scores were taken as the dependent variable and the factors affecting the efficiency were found with Tobit regression. The analyses were made with the R programming language.

Methods

The Collection of the Data

The dataset used in this study was compiled from the https://opentender.eu/pt/download site, taking into account the procurements made by the countries by year. The study includes public procurement data for 24 EU countries and Turkey for the year 2023. Poland, Slovenia and Romania were excluded from the analysis because their data could not be accessed.

Statistical Analysis

In order to conduct an SBM-DEA analysis, inputs and outputs must be defined. In this context, it is important to define the factors affecting the efficiency of public procurement. In order to meet the needs of public administrations, it is essential to ensure competition among suppliers and to obtain the most economically advantageous tender price in order to prevent waste by using public resources efficiently. Public procurement is the process by which public institutions purchase goods, services and construction work and is a key activity in the economy (Kelly, 1997). EU countries have changed the way public institutions spend a large part of the €2 trillion or so spent on public procurement each year in Europe, corresponding to around 14% of EU GDP (European Commission, 2020). Therefore, improvements in the efficiency of public procurement, significant savings or an increase in the quality and quantity of procurement will contribute positively to the economy. Therefore, public procurement warrants public and academic interest (UNIDO, 2017).

For the efficiency analysis, the number of tenders, customers and approximate cost were taken as input variables in the study; the number of tenders resulting in procurement and the number of suppliers were taken as desired outputs; the number of cancelled tenders and the contract price were taken as undesirable outputs. This study aimed to measure the efficiency of public procurement, that is, the production of efficient public procurement output with efficient input variables. After the efficiency analysis, the environmental factors contributing to the efficiency were determined and how they positively/negatively affected the efficiency was found. A comparison was made between countries and it was found which countries carried out the most effective public procurements regionally.

Table 1. Input and Output Variables

Input Variables		
	I1	Number of tenders
	I2	Number of customers
	I3	Approximate cost
Output variables		
	O1	Number of tenders resulting in purchase
	O2	Number of suppliers
	O3 (undesired)	Number of cancelled tenders
	O4 (undesired)	Contract price

Table 2. Descriptive statistics of input, output and explanatory variables

Input variables	Min	Max	Mean	Standart deviation
Number of tender	291	137.907	25.131	40.024
number of customer	54	33.111	3.987	6.789
Approximate cost	4.270.000	287.089.415.453	33.188.514.504	75.540.535.277
desired output variables				
Number of bdder	153	62.249	12.710	18.900
Number of tenders resulting in purchases	243	111.491	22.145	34.390
Undesired output variables				
Number of cancelled tenders	10	26.934	1.874	5.149
contract price	3.550.141	225.729.932.119	28.184.013.788	60.808.670.159
explanatory(independent) variables				
CPV	58	4.445	975	984
Joint procurement	0	1.266	118	250
central procurement	3	11.901	620	2.275
electronic auction	0	516	59	119
Framework agrement	0	442.978	18.230	84.988

RESULTS

In DEA analysis, analysis is performed for input minimization or output maximization purposes and efficiency score is obtained. However, in this study, unlike the classical method, the Slack- Based Data Envelopment Analysis was used, which measures efficiency without telling the model the purpose of minimization or maximization with the input and output variables at hand; that is, without directing the model to the data. With TOBIT regression, which of the input and output variables used in efficiency measurement contributes to what extent to the efficiency was analyzed.

In the study, the efficiency scores of the decision-making units were found with the BCC and SBM-DEA models, and then the efficiency scores obtained were taken as independent variables and the environmental variables affecting the efficiency were examined. Table 2 includes descriptive statistics of the input and output variables.

Table 3 shows the efficiency measurements of 25 countries using SBM-DEA and BCC methods. According to the findings, 11 countries were found to be fully efficient with an efficiency score of 1.0; 14 countries were found to be inefficient.

Table 3. Efficiency Scores

DMU	SBM-DEA	BCC	Avarage Efficiency
Austria	0.91	0.98	1.0
Belgium	1.0	1.0	0.55
Bulgaria	0.12	0.24	0.13
Cyprus	1.0	1.0	1.0
Germany	1.0	1.0	0.48
Spain	1.0	1.0	1.0

Finland	0.48	0.65	0.19
France	1.0	1.0	0.19
Greece	1.0	1.0	1.0
Croatia	0.21	0.42	0.24
Hungary	0.69	0.76	0.80
Ireland	0.18	0.39	0.37
Lithuania	1.0	1.0	1.0
Luxembourg	0.65	0.98	1.0
Latvia	0.59	0.84	0.45
Malta	1.0	1.0	1.0
Netherland	0.60	0.71	0.39
Portugal	1.0	1.0	1.0
Sweden	1.0	1.0	1.0
Slovenia	0.46	0.58	0.60
Slovak Repubic	0.34	1.0	1.0
Estonia	0.32	0.59	0.34
Italy	1.0	1.0	1.0
Denmark	0.32	0.54	0.53
Turkiye	0.30	0.50	1.0
Avarage Efficiency	**0.68**	**0.80**	**0.74**

Table 4. SBM- DEA Effectiveness Score Distribution

Efficiency score	**DMU**	**Distribution**
1.0	11	44 %
0.50 and above	5	20 %
0.50 and below	9	36 %
Total	**25**	**100%**

According to Table 4; 11 countries were found to be fully effective by receiving an efficiency score of 1.0. While five country received a score of 0.50, 9 countries received a score of 0.50 and below.

The efficiency scores obtained in the SBM-DEA analysis were taken as dependent variables and the appropriate input-output variables were taken as independent variables and the analysis was conducted. In

other words, the effect of environmental factors on efficiency was measured with this analysis. The results are shown in Table 8.

Due to the nature of Tobit regression, the dependent variable will be censored and the effect of independent variables on the efficiency score will be investigated. Accordingly, in the first model, the efficiency scores were taken directly, in the second model, the average efficiency score value of 0.68 was taken as the upper value and finally, the threshold value of 0.39 found with threshold regression was taken as the minimum value and the analysis was performed. Analysis results according to the models;

Table 5. Tobit Analysis Results – Model 1

	Model 1				
Coefficients	**Estimate**	**Std.Error**	**z value**	**Pr (> |z|)**	
Intercept 1	1,10E+03	3,36E+02	3.274	0.001059	**
Intercept 2	-1,46E+03	1,89E+02	-7.741	9.89e-15	***
Number of concluded tenders	3,16E-01	1,10E-01	2.873	0.004063	**
cpv code	-1,21E+00	3,48E-01	-3.463	0.000534	***
number of tenders	-2,34E-01	1,13E-01	-2.067	0.038709	*
centralized purchasing	6,47E-02	8,02E-02	0.807	0.419563	
Number of cancelled tenders	4,74E-01	1,27E-01	3.730	0.000191	***
electronic deduction	-2,78E+00	1,70E+00	-1.636	0.101873	
joint purchasing	-1,84E+00	4,34E-01	-4.237	2.26e-05	***
framework agrements	-1,00E-03	6,42E-02	-0.016	0.987525	
sustainyes	4,33E+02	2,20E+02	1.963	0.049652	*

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Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1
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Table 6. Tobit Analysis Results – Model 2

	Model 2				
Coefficients	**Estimate**	**Std.Error**	**z value**	**Pr (> |z|)**	
Intercept 1	3,19E+02	1,10E+02	2.910	0.00362	**
Intercept 2	-1,96E+03	1,38E+02	-14.202	2E-16	***
Number of concluded tenders	6,03E-02	3,01E-02	2.004	0.04509	*
cpv code	-8,68E-02	6,10E-02	-1.423	0.15487	
number of tenders	-5,39E-02	2,77E-02	-1.946	0.05171	.
centralized purchasing	1,38E-03	3,74E-02	0.037	0.97059	
Number of cancelled tenders	9,02E-02	4,27E-02	2.114	0.03455	*
electronic deduction	-7,20E-01	5,40E-01	-1.334	0.18215	
joint purchasing	-1,13E-01	1,48E-01	-0.761	0.44693	
framework agrements	-1,46E-02	6,97E-03	-2093	0.03631	*
sustainyes	2,03E+02	1,00E+02	2.028	0.04258	*

Table 7. Tobit Analysis Results – Model 3

	Model 3				
Coefficients	**Estimate**	**Std.Error**	**z value**	**Pr (> |z|)**	
Intercept 1	9,87E+02	2,98E+02	3.313	0.000922	***
Intercept 2	-1,52E+03	1,80E+02	-8.477	2E-16	***
Number of concluded tenders	2,52E-01	1,00E-01	2.524	0.011593	*
cpv code	-8,37E-01	3,02E-01	-2.773	0.005547	**
number of tenders	-1,90E-01	1,03E-01	-1.846	0.064899	.
centralized purchasing	5,63E-03	7,48E-02	0.075	0.940002	
Number of cancelled tenders	3,52E-01	1,16E-01	3.041	0.002361	**
electronic deduction	-1,99E+00	1,60E+00	-1.242	0.214258	
joint purchasing	-1,39E+00	3,78E-01	-3.679	0.000234	***
framework agrements	-1,59E-02	5,99E-02	-0.265	0.790899	
sustainyes	3,62E+02	2,04E+02	1.779	0.075312	.

Table 8. Tobit Analysis Results

İndependent variables	**Model 1**	**Model 2**	**Model 3**
	Normal efficiency score	Upper value=mean: 0.63	Min value = threshold:0.39
Number of concluded tenders	**	*	*
cpv code	***		**
centralized purchasing			
electronic deduction			
joint purchasing	**		***
framework agrements	*	*	
tender number	*	.	.
sustainyes	*	*	.
Number of cancelled tenders	***	*	**
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1			

Among the variables whose effects on public procurement efficiency were investigated; the number of tenders resulting in procurement, CPV, framework agreement, joint procurement, the number of cancelled tenders, the number of tenders made, whether the countries have a sustainability roadmap, the number of tenders made with central procurement and electronic auction; the number of tenders made with electronic auction and the number of tenders made with central procurement were not found to be significant. In other words, there is no statistically significant effect on the effectiveness. Other variables were found to be significant. Accordingly,

The number of concluded tenders is found to be significant in all three models and has a positive effect on efficiency. A 1 br increase in the number of concluded tenders increases efficiency by 0.05, 0.04 and 0.04, respectively.

The variable of the number of tenders made was found to be significant in all 3 models and has a negative effect on efficiency. A 1 br increase in the number of tenders made reduces efficiency by 0.04, 0.03 and 0.034, respectively.

The variable of the number of cancelled tenders was found to be significant in all 3 models and has a negative effect on efficiency. A 1 br increase in the number of tenders decreases efficiency by 0.08, 0.06 and 0.064, respectively.

The CPV code variable was found to be significant in models 1 and 3 and has a negative effect on efficiency. A 1 br increase in the number of CPV codes reduces efficiency by 0.06 and 0.15, respectively.

The sustainyes variable, which indicates the countries that have determined a sustainable roadmap, is found to be significant in all 3 models and has a positive effect on efficiency. The efficiency score in countries that have determined a sustainable roadmap for public procurement is 1.59 times higher in model 1, 0.74 times higher in model 2 and 1.3 times higher in model 3 than in countries that have not.

The variable of the number of joint procurement tenders was found to be significant in the 1st and 3rd models and has a negative effect on efficiency. A 1 br increase in the number of joint procurement tenders reduces efficiency by 0.09 and 0.06, respectively.

The variable of the number of tenders made with the framework agreement was found to be significant only in the 2nd model and has a negative effect on efficiency. A 1 br increase in the number of tenders made with the framework agreement decreases efficiency by 0.009.

DISCUSSION AND CONCLUSION

The effective, efficient and sustainable public procurement, which has a share of 12-20% in the GNP of countries and is an industrial policy tool of the state, will provide a leverage effect on the national economy and make a significant contribution to the development of countries. In this context, it is imperative to ensure maximum public benefit and social benefit by using the country's resources effectively and blending the tender processes with artificial intelligence tools such as the internet of things, robotics, natural language processing, and chatbots that have become a part of our lives with digital transformation.

The study found efficiency scores for 24 EU countries and Turkey. Accordingly, 11 countries were found to be efficient with a score of 1.0. The average efficiency obtained as a result of the analysis was found to be 68%. When the efficiency scores were examined, it was seen that 11 countries had an efficiency score of 1.0, meaning they were 100% efficient. It was seen that five of the 14 countries that were not found efficient had a score above 50%, while 9 countries had a score below 50%. From a regional perspective; public procurement efficiency varies significantly. While the highest scores were obtained in the southern and northern regions, respectively; the scores of central European countries were found to be low. As a result of the efficiency analysis, it was found which country had excess input/deficit output.

Again, it can be easily seen that the importance that should be given to variables that have positive and negative contributions to the efficiency score is also revealed by the tobit analysis. As a result of the analysis, it was seen that the number of concluded tenders and the ownership of the sustainability roadmap variables positively affected the efficiency, while the variables of product diversity, joint

purchasing, framework agreement, the number of tenders made and the number of cancelled tenders decreased the efficiency.

According to the result found by Tobit regression, the fact that the tenders are concluded with purchases, that is, the number of cancelled tenders is low, affects the efficiency positively. As a result, the work and transactions carried out materially and morally throughout the tender process may cause problems such as loss of time, increase in the price of the product during this process, or the institution in need suffering due to the lack of product. Again, as in every field, it has been seen that the implementation of sustainability elements in public procurement makes a positive contribution to the effectiveness. In fact, with the digitalizing world, sustainability has also become an indispensable element.

Sustainable public procurement aims to integrate these considerations into the procurement process to promote sustainable development and create long-term value. Public authorities can help encourage businesses to adopt more sustainable practices by using their purchasing power to increase demand for sustainable products and services. Sustainable public procurement can also help support local economic development by supporting small and medium-sized enterprises that produce sustainable goods and services. To effectively implement sustainable public procurement, governments and public institutions need to have a clear strategy, policy framework and monitoring system. It is also recommended for public governance to engage in dialogue with stakeholders, including suppliers, civil society organisations and local communities, to ensure that procurement processes are aligned with sustainability goals and contribute to sustainable development.

It is recommended that the parameters that negatively affect the efficiency score be investigated and improvement efforts be made in these parameters. For example; bulk purchases are made in joint purchasing, central purchasing and framework agreement tenders, and it is essential to benefit from the economy of scale that these bulk purchases will provide to the maximum extent.

The contribution of the study to the literature is that it is the first study in which slacker-based data envelopment analysis and tobit regression analysis are performed with public procurement data. Here, the variables affecting public procurement are determined after a laborious process and then the data is obtained, the analysis is designed and the results are found. In subsequent studies, economic, environmental and social factors can be taken into account for sustainable public procurement and the analysis can be repeated. In addition, more countries can be included in the analysis, these countries can be separated with some statistical analyses or descriptive statistical values and their efficiency scores can be evaluated separately. As stated in the OECD Public Procurement Recommendations Report (2019), the need to ensure performance improvements by evaluating the effectiveness of the public procurement system at all levels of government, from individual purchases to the entire system, is emphasized. In this context, the study is expected to be a pioneer in this field and also shed light on future studies.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

Songül Bulut; methodology, literature review, writing, research

İhsan Alp: methodology, supervisor, evaluation

Data-Driven and Statistically-Informed Classification of Global MPP Voltage under Partial Shading Conditions (1069)

Hatice Gül Sezgin-Ugranlı[1*]

[1]Izmir Bakircay University, Deparment of Electrical-Electronics Engineering, 35665, İzmir, Türkiye

*Corresponding author e-mail: haticegul.ugranli@bakircay.edu.tr

Abstract

This study proposes a classification-based approach to identify the global maximum power point voltage of photovoltaic modules under partial shading conditions using artificial neural networks. A large number of unique irradiance scenarios are systematically generated to emulate realistic shading patterns at the submodule level, with each scenario represented by a 12-dimensional input vector. These irradiance vectors are mapped to their corresponding global maximum power point voltages using a detailed physics-based simulation of a photovoltaic module composed of three bypass diode-protected substrings. Due to the non-unique mapping between irradiance patterns and voltage values, direct regression approaches are found to be challenging. Hence, the voltage range is discretized into three categorical bands, and the problem is reformulated as a multi-class classification task. A three-layer feedforward neural network architecture is selected after extensive experimentation involving alternative topologies and optimization methods. The network is trained using a second-order adaptive algorithm, and the output layer employe a softmax activation to provide probabilistic class predictions. An independent test set is used to validate the model's generalization capability. Performance evaluation through error histograms, receiver operating characteristic curves, and confusion matrices confirm that the model accurately distinguishes maximum power voltage classes with over 95% accuracy on test dataset. The results indicate that data-driven classification is a robust and scalable solution for intelligent maximum power tracking strategies in photovoltaic systems under complex shading scenarios.

Keywords: *Photovoltaic Systems, Partial Shading, Artificial Neural Networks, MPP Voltage Classification, Bypass Diodes.*

INTRODUCTION

Partial shading conditions (PSCs) represent a significant challenge in the operation of photovoltaic (PV) systems, leading to nonuniform irradiance distribution across the solar array and consequently resulting in substantial power losses (Belhachat and Larbes, 2017). These losses are not only due to the reduction in the overall incident solar energy but also arise from the complex electrical behavior induced by mismatched operating points of individual PV modules or cells. Under such conditions, the current-voltage (I–V) and power-voltage (P–V) characteristics of PV systems exhibit multiple local maxima (Rehman et al., 2023), thereby complicating the task of maximum power point tracking (MPPT). To mitigate these adverse effects and improve system performance, advanced modeling and optimization techniques have been increasingly investigated (Lee et al., 2025; Eltamaly, 2023).

From a statistical perspective, the analysis of shading patterns and their probabilistic impact on PV output has enabled the development of robust models that capture temporal and spatial irradiance variability (Gupta et al., 2024). Monte Carlo simulations, stochastic irradiance profiles, and probability density function modeling are commonly used to estimate expected energy yields and identify high-risk

shading configurations (Kim and Baek, 2025; Jones and Erickson, 2013; Wang and Sheu, 2015). These methods provide insight into the likelihood of severe power mismatches and inform the design of system components such as bypass diodes, array layouts, and inverter configurations to minimize mismatch losses (Singh et al., 2023).

In recent years, artificial neural networks (ANNs) have emerged as powerful tools for both predictive modeling and real-time control of PV systems under PSCs. ANNs can learn complex, nonlinear mappings between high-dimensional input features—such as irradiance levels on submodule scale, temperature variations, and shading indicators—and corresponding power outputs (Wadehra et al., 2024; Bisht and Sikander, 2024; Kumaraswamy and Naik, 2024). Once trained, ANNs can be used for fast and accurate prediction of the global maximum power point (GMPP), enabling more responsive and efficient MPPT algorithms compared to conventional techniques. Moreover, hybrid models that integrate ANN-based forecasting with statistical filtering methods (e.g., Kalman or particle filters) have demonstrated enhanced robustness against sensor noise and measurement uncertainty (Kouser et al., 2023).

Additionally, classification-based approaches have gained traction in the development of intelligent MPPT strategies. By framing the MPPT problem as a classification task—wherein irradiance patterns or electrical signatures are mapped to a set of predefined shading scenarios—real-time decision-making can be improved. Classifiers such as support vector machines (SVM), k-nearest neighbors (k-NN), and deep learning architectures have been trained to identify the presence and severity of partial shading events (Moreira et al., 2022; Liao, 2010; Zabia et al., 2022; Yang et al., 2025), thereby enabling adaptive switching between different MPPT techniques (e.g., Perturb and Observe, Incremental Conductance, Particle Swarm Optimization). This strategy facilitates the dynamic adjustment of algorithm parameters based on the real-time classification of operating conditions.

In summary, reducing power losses under partial shading in PV systems demands a multidisciplinary approach combining probabilistic modeling, data-driven prediction, and intelligent classification. The integration of statistical tools with ANN-based and classification frameworks offers a promising pathway to develop resilient MPPT mechanisms that ensure higher energy yields, better inverter utilization, and increased reliability under real-world conditions.

In this context, this study presents a data-driven classification framework to identify the global MPP voltage in PV modules exposed to partial shading. A comprehensive irradiance dataset is generated and used to simulate MPP behavior under a wide range of shading scenarios. The continuous voltage output is discretized into categorical bands, and a multi-layer artificial neural network is trained to classify these voltage levels based on submodule irradiance patterns. The proposed approach is evaluated through extensive testing and performance metrics, demonstrating its effectiveness and robustness in capturing nonlinear irradiance-voltage relationships.

Photovoltaıc Module Under Unıform And Non-Unıform Irradıance Condıtıons

The electrical behavior of PV modules is highly dependent on the distribution of solar irradiance across the module surface. Figures 1 and 2 illustrate the typical electrical response of a PV module under uniform and non-uniform irradiance conditions, respectively. A PV module incorporating three bypass diodes is considered in this study to illustrate the effects of shading by using MATLAB (MathWorks, 2025).

Figure 1 shows a PV module operating under uniform irradiance where all cells receive 1000 W/m^2. In this scenario, all solar cells receive equal irradiance, leading to a smooth I–V and P–V characteristic. The current remains relatively constant across a wide voltage range, while the power curve exhibits a

single, well-defined global MPP. This behavior is typical of unshaded, uniformly illuminated PV modules and is often used as the baseline for performance analysis.

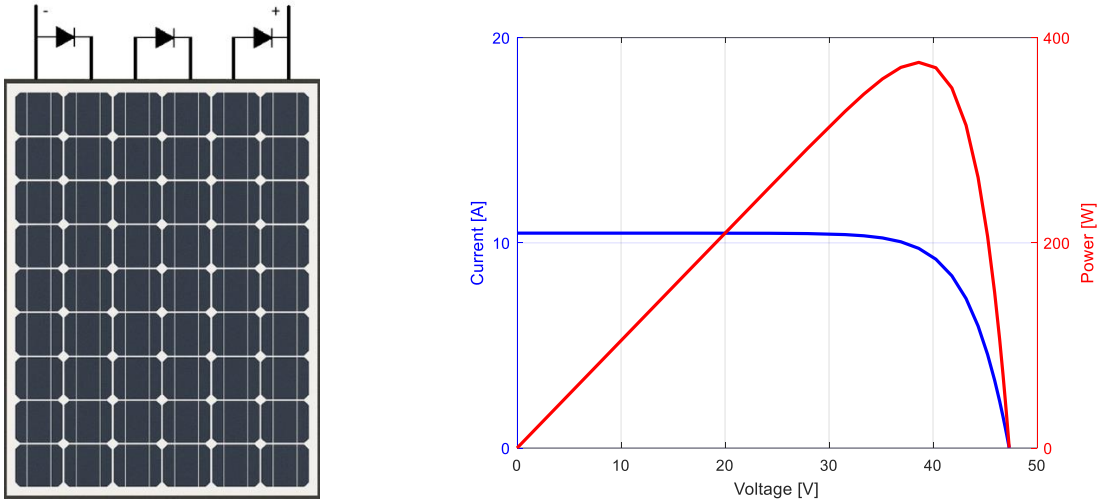


Figure 1. PV module under uniform irradiance condition

In contrast, Figure 2 illustrates a PV module under partial shading conditions, where non-uniform irradiance causes different strings of solar cells to generate unequal current levels. This disparity triggers the activation of bypass diodes, which are integrated across cell substrings to protect shaded cells from excessive reverse bias and thermal stress. As a consequence, the overall I–V and P–V characteristics exhibit multiple steps and local peaks. This multi-modal power profile results in multiple local MPPs, making MPPT more challenging for power electronics systems.

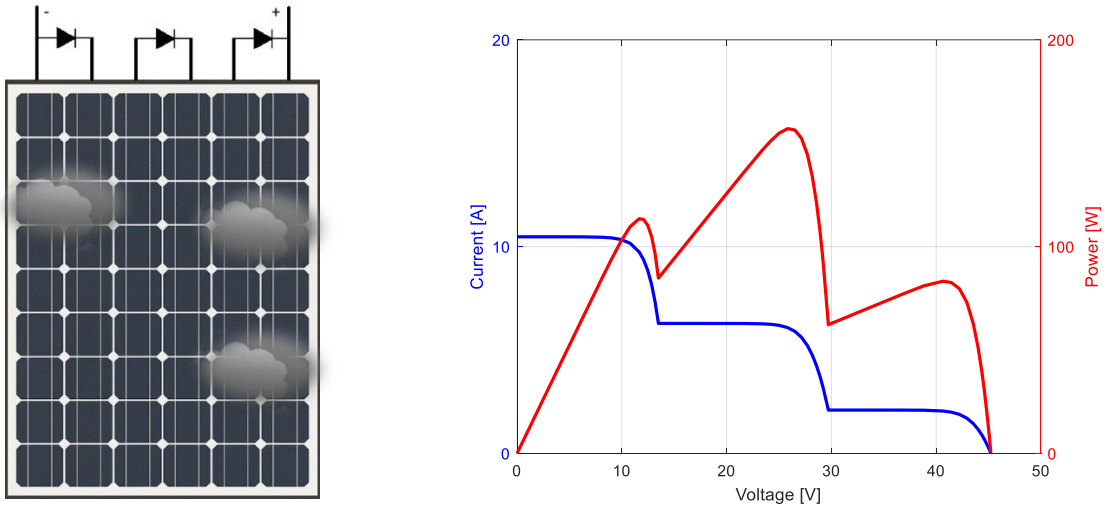


Figure 2. PV module under non-uniform irradiance condition

The modeling framework in this study considers a PV module composed of several series-connected cell groups, each protected by a bypass diode. Under shading conditions, the bypass diodes selectively conduct to isolate shaded substrings, effectively altering the equivalent circuit of the module. This behavior is critical in accurately simulating PV performance under real-world conditions, where shading is dynamic and often unpredictable.

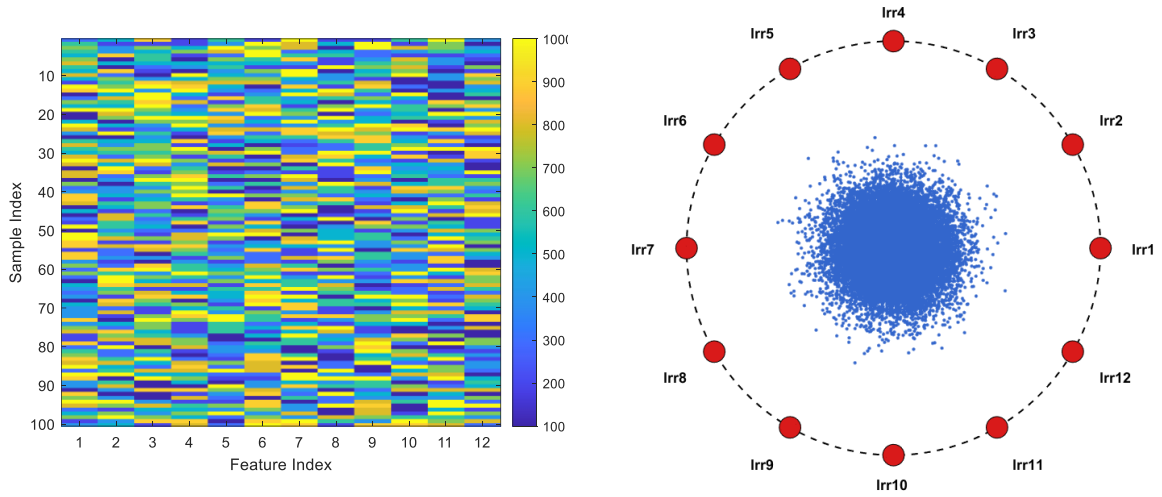
The location of the global MPP is not fixed but varies significantly depending on the irradiance distribution across the PV module. Under partial shading conditions, the dynamic behavior of bypass diode activation leads to shifting and possible fragmentation of the power curve, with global and local maxima appearing at different voltage levels. Therefore, accurately identifying and classifying the

global MPP become essential to prevent substantial energy losses during MPPT operation. To address this challenge, an ANN structure is employed in this study due to their capacity to learn nonlinear relationships and adapt to varying input patterns derived from irradiance scenarios. For this purpose, the PS-M72-405 monocrystalline PV module is used in this study. This module consists of 72 cells arranged in a 6 × 12 matrix, divided into three substrings, each protected by a bypass diode. The module delivers a rated power output of 405 W under standard test conditions (STC), with a nominal open-circuit voltage (Voc) of approximately 50.32 V and a short-circuit current (Isc) of 10.35 A.

Generatıon And Statıstıcal Mappıng Of Irradıance Scenarıos

In this study, a synthetic irradiance dataset is developed to emulate a wide range of partial shading conditions that may occur across a PV module. The irradiance values are defined at the substring level, with each substring representing a group of six solar cells. Given that the module includes 72 cells, this configuration results in 12 distinct irradiance input features for each simulation scenario.

To capture the variability and complexity of shading phenomena, 30,000 unique irradiance scenarios are randomly generated. For each of the 12 inputs, irradiance values are independently drawn from a uniform distribution ranging between 100 W/m² and 1000 W/m². This wide dynamic range is representative of various environmental conditions.



(a) **(b)**

Figure 3. (a) Heatmap of 100 samples, **(b)** Inter-feature balance and sample mapping for 30000 samples

Figure 3a presents a heatmap visualization of a randomly selected subset of 100 samples, where each row represents a unique irradiance scenario, and columns correspond to the 12 irradiance inputs (Irr1–Irr12). The distribution of colors confirms a uniform spread across the entire irradiance range, indicating that the sampling process does not introduce systemic bias toward any particular irradiance band. Such balance is crucial for training robust machine learning models capable of generalizing across unseen irradiance configurations.

Figure 3b illustrates a RadViz-based visualization showing the inter-feature distribution and sample dispersion over the 12-dimensional irradiance space for all 30,000 samples. The dense blue cluster near the center implies a strong central tendency in the generated samples, while the uniform radial distribution confirms that all features are statistically balanced in terms of value representation. This balance ensures that no individual irradiance feature disproportionately influences the model, which is vital for unbiased learning in data-driven algorithms such as ANNs.

All irradiance scenarios are then fed into a Simscape-based PV simulation environment, where a physics-based equivalent circuit model of the PS-M72-405 module is used. The model includes nonlinear diode behavior, temperature-dependent characteristics, and bypass diode switching logic. The cell temperature is fixed at 45°C, reflecting moderately heated operating conditions under partial shading. For each irradiance scenario, the corresponding global MPP voltage is determined using an exhaustive voltage sweep and recorded for downstream analysis and classification.

Classıfıcatıon Of Global Mpp Voltage

In this study, a multi-layer neural network model is developed to classify the global MPP voltage of PV modules exposed to diverse irradiance conditions. Predicting MPP voltage as a continuous output is found to be challenging due to the non-unique input-output mapping: substantially different irradiance profiles can often result in similar MPP voltages. Furthermore, the highly nonlinear and irregular nature of the MPP surface under partial shading complicates direct functional approximation. To address this, the voltage output space is discretized into three categorical bands (10-13 V, 22-29 V, and 33-45 V) by analyzing the statistical distribution and natural clustering of the simulation-derived MPP voltages. The emergence of these voltage bands is closely related to the number of bypass diodes used in the PV module. Each bypass diode allows a portion of the module to be bypassed under shading conditions, resulting in characteristic voltage drops. In modules with three bypass diodes, up to three distinct MPP voltage levels typically arise, corresponding to one, two, or all three module sections contributing to power generation. This structural behavior naturally leads to the observed clustering in the output voltage values. This transformation of the problem into a classification task significantly enhances learning stability and improves generalization. The final layer of the network produces class scores, which are then passed through a softmax activation function to yield normalized probabilities for each class, enabling a probabilistic interpretation of the model's predictions.

To train the network, a total of 30,000 uniquely generated irradiance scenarios are synthesized as mentioned before, each represented as a 12-dimensional input vector corresponding to the irradiance received by segments of a PV module partitioned into three bypass diode regions. These input vectors are systematically constructed by combining structured sampling with controlled randomness, ensuring both representational diversity and coverage of edge cases. For each scenario, the corresponding global MPP voltage is computed via detailed simulation using a physics-based model that captures the nonlinear behavior of solar cells under variable shading conditions.

An independent test set composed of 6,000 additional irradiance scenarios is similarly generated, ensuring that the evaluation is conducted on unseen data with a comparable distribution. This separation of training and test data enables robust assessment of the model’s generalization capability across a wide range of irradiance patterns.

The selected neural network architecture consists of three hidden layers with 40, 32, and 32 neurons, respectively as shown in Figure 4. This configuration is chosen following extensive experimentation involving various combinations of hidden layer numbers, neuron numbers, and training algorithms. Multiple architectures, ranging from shallow to deeper networks, are evaluated in terms of training stability, generalization accuracy, and computational efficiency. Similarly, several optimization methods—including gradient descent variants and quasi-Newton approaches—are tested to assess their performance in handling the complex input-output relationships under noisy and nonlinear irradiance conditions.

Each hidden unit in the final design utilizes a sigmoidal activation function, enabling the network to model highly nonlinear dependencies between irradiance distributions and global MPP voltage behavior. The network is ultimately trained using a second-order optimization algorithm with adaptive

step size control, which demonstrated superior convergence behavior and robustness across varying initial conditions and data subsets.

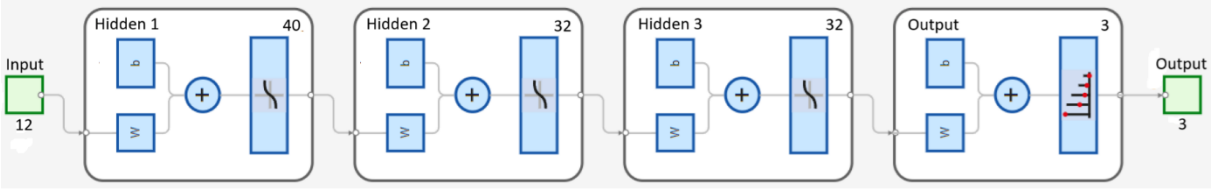
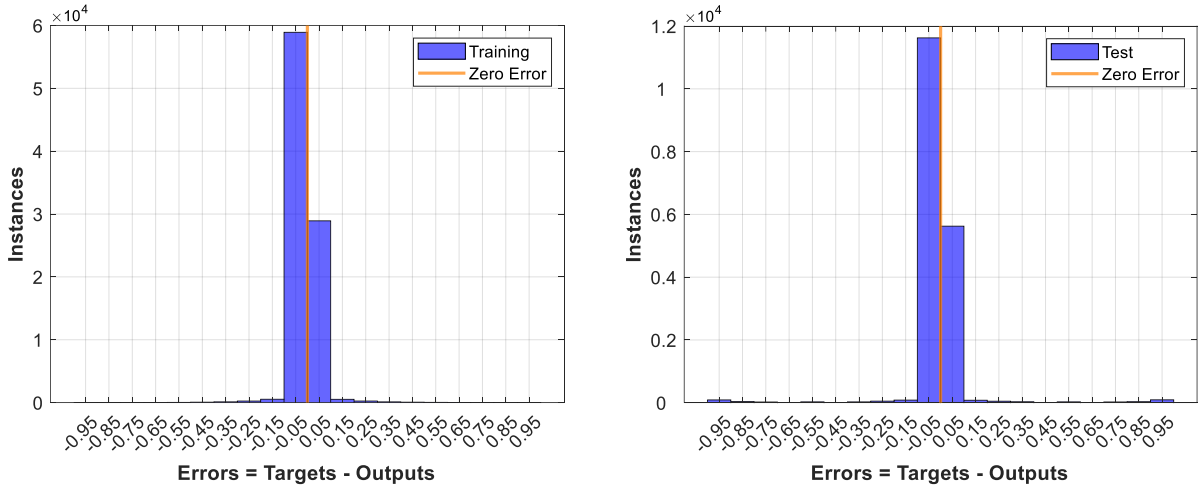


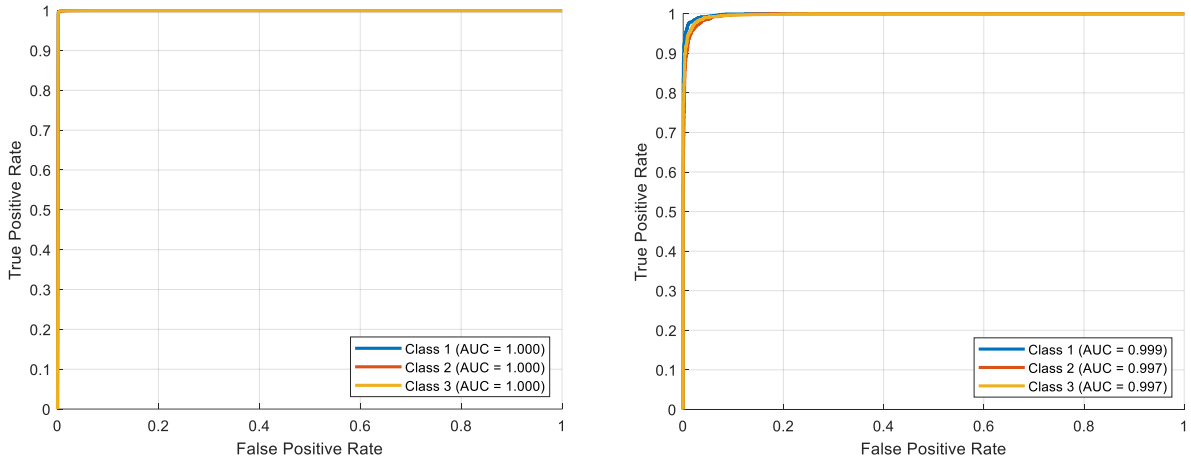
Figure 4. Structure of the multi-layer neural network for classification

After completing the training and testing phases, model evaluation based on error histograms shown in Figure 5 demonstrates that both training and test predictions are tightly centered around zero error, indicating minimal deviation from ground truth class labels.



(a) **(b)**

Figure 5. Error histogram for (a) training and (b) test dataset

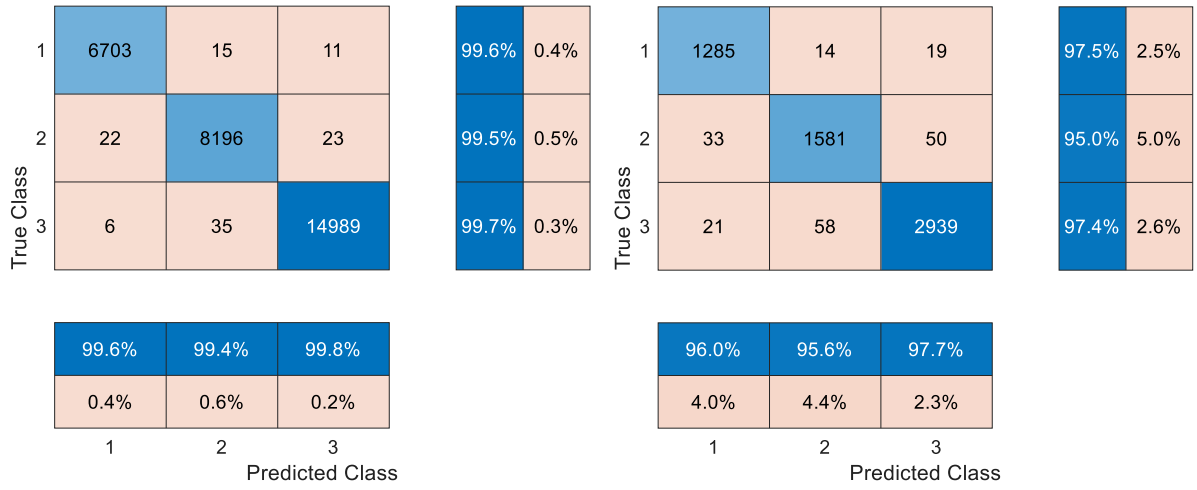


(a) **(b)**

Figure 6. Receiver operating characteristic for (a) training and (b) test dataset

Further validation is provided via receiver operating characteristic (ROC) analysis in Figure 6, where the area under the curve (AUC) exceeds 0.98 for all classes in the test set and reaches perfect separability (AUC = 1.00) in the training set. This indicates strong discriminative capability and generalization across different input scenarios.

The confusion matrices shown in Figure 7 reinforce this conclusion, with class-wise accuracies exceeding 95% on the test set and 99% on the training set. The relatively high diagonal dominance in both matrices confirms that the network has successfully learned to distinguish voltage classes despite the underlying complexity and input overlap.



(a) **(b)**

Figure 7. Confusion matrix for **(a)** training and **(b)** test dataset

By reframing the problem as a classification task and leveraging systematic scenario generation and simulation, this work provides a robust and scalable approach for global MPP voltage classification in partially shaded PV systems. The results suggest that data-driven classification is an effective alternative to regression.

CONCLUSION

This study demonstrates that the classification of global MPP voltage under partial shading conditions can be successfully achieved using a neural network trained on synthetically generated irradiance scenarios. Framing the task as a classification problem enabled the model to effectively learn the complex patterns between irradiance inputs and MPP voltage categories, facilitating more robust and interpretable predictions. The discretization of the voltage space into three representative bands is well-justified by the presence of three bypass diodes, which cause distinct voltage steps under shading events. The trained neural network, consisting of a carefully optimized three-layer architecture, is able to generalize across a wide range of irradiance conditions with high accuracy. Evaluation metrics such as error distribution, ROC analysis, and confusion matrices confirm the model's predictive performance and robustness. When compared to traditional MPPT strategies that often struggle under multi-modal voltage profiles, the proposed classification-based approach offers a promising alternative that is both interpretable and computationally efficient.

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Conflict of Interest

The authors have declared that there is no conflict of interest

Improving Production with Artificial Neural Networks and Integration into ERP Systems: An Approach within the Scope of Industry 4.0 (1083)

Gizem Sara Onay[1*], Mehmet Çakmakçı[2]

[1]Dokuz Eylül University, Institute of Science, İzmir / Turkey
[2]Dokuz Eylül University, Faculty of Engineering, İzmir / Turkey

*Corresponding author e-mail: gizemsara@hotmail.com

Abstract

This study aims to digitize production planning by utilizing prediction models based on production data and reducing human intervention to increase efficiency. Production data obtained from a real manufacturing system through the Manufacturing Execution System (MES) interface was analyzed using an artificial neural network (ANN) algorithm, and future production quantities were predicted. By integrating the production forecast results into the Enterprise Resource Planning (ERP) system, it was aimed to automatically direct the production processes. Thus, production decisions can be automatically made by the system based on past data. As a result of the implementation, dynamic and data-driven decision-making processes in production management were facilitated through the forecast outputs integrated into the ERP system. This prediction-based approach is more flexible compared to traditional production planning methods and enables quicker responses from the production system. Consequently, this study presents an innovative approach that contributes to digital transformation within the scope of Industry 4.0 and serves as an example for decision support systems in production management. With this study, the development of predictive systems that operate with real-time data flow is aimed for the future.

Keywords: *Artificial Neural Networks, Production Planning, ERP, Industry 4.0, MES, Forecasting*

INTRODUCTION

The concept of Industry 4.0 encompasses digitalization, automation, and data-driven decision support infrastructures in production systems. This study enables businesses to utilize their resources more efficiently, adopt a flexible production structure, and respond more quickly to customer expectations. However, many manufacturing facilities still operate with manual data processing procedures and traditional planning methods. This leads to several operational challenges, including excess inventory, material shortages, shipment errors, and production losses.

In this study, the losses caused by the improper planning of SMD (Surface Mount Device) type materials in the production line of a company manufacturing television mainboards were examined. Since material requests are reported to the production site as a total quantity, the need on a reel basis is not fully reflected, and an excessive number of reels are sent to production. This leads to both stock inflation and layout inefficiency on the production line.

To prevent this problem, production data was collected from the past and analyzed using an artificial neural network model; future production quantities were predicted. These predictions were integrated into the ERP system, and stock control and supply planning were automated, thereby eliminating manual errors.

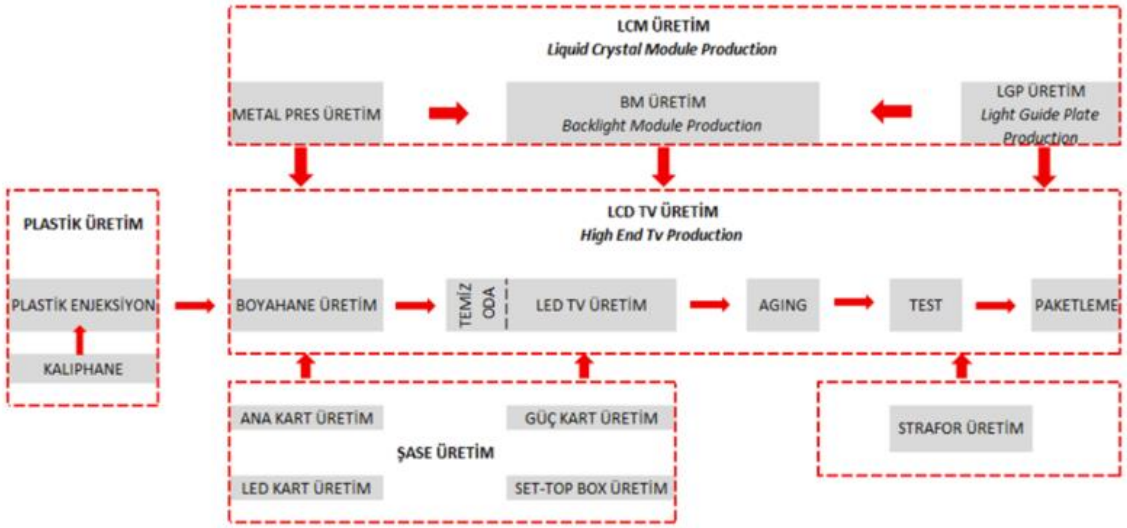

Figure 1. TV mainboard production

MATERIAL AND METHODS

Material

The mainboard production line of an electronics manufacturing facility operating in the Aegean Region was selected as the application area. The SMD materials (resistors, capacitors, etc.) used on this line are supplied in reels and automatically placed by the machines on the production line. Since the required number of reels is manually calculated based on the production quantity, over- or under-supply frequently occurs.

Theoretical Framework

In the past decade, numerous publications have been made on the concepts of Industry 4.0, MES, and ERP. The diversity of methods found in the literature stems from various approaches ranging from forecasting algorithms to system integration.

In the literature review conducted, 32 studies were examined. Some of these studies focused solely on production data forecasting and did not include ERP integration. For example, El Madany et al. (2022) proposed a hybrid time series model for supply forecasting; however, ERP integration was not included. Similarly, the ARIMA-based forecasting model developed by IFS Applications was used for demand prediction but did not support real-time MES integration.

The unique aspect of this study is the combination of ANN-based production forecasting with ERP-MES system integration. While MES systems monitor the production process, ERP systems handle corporate planning. The coordinated operation of these two systems will enhance production efficiency and the quality of decision-making.

Methods

Material tracking problems were analyzed using a fishbone diagram under the categories of material, method, human, and process:

- **Material**: Small-sized components → confusion and loss
- **Method**: Total quantity → excess reels → stock inflation
- **Human**: Inconsistency between the system and the production floor
- **Process**: Too many feeding points → irregular distribution

Consequently, this structure results in unnecessary inventory and production disruptions.

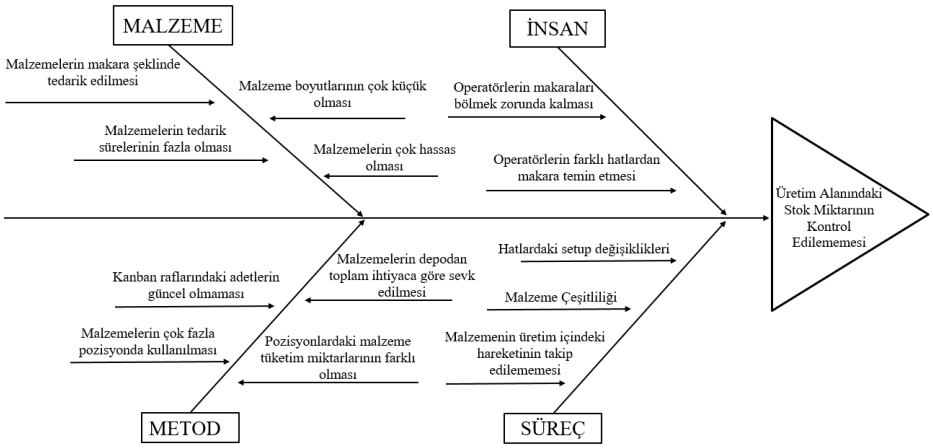

Figure 2. Fishbone Diagram

Data Collection Process

Production data, material usage records, and order information were collected through the factory's Manufacturing Execution System (MES). A Pareto analysis was applied to identify critical materials. According to the analysis, 24 out of 72 materials accounted for 80% of the total inventory. These 24 materials were selected as critical input data for the model.

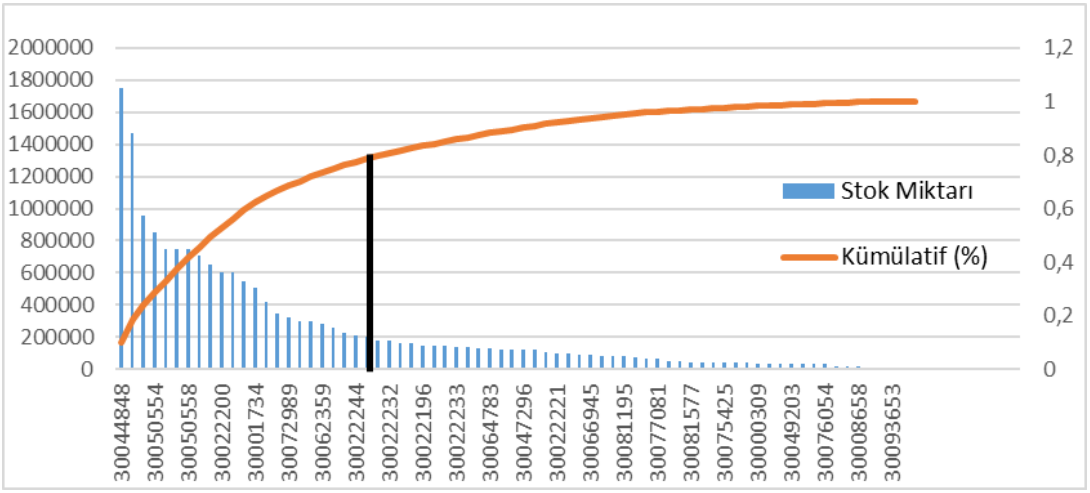

Figure 3. Distribution of the 24 most used materials according to the Pareto analysis

Artificial Neural Network Model

The modeling process was carried out in the MATLAB environment. A multilayer feedforward artificial neural network model was chosen, and historical production quantities were used as input variables. The output was the predicted production quantity for the next 10 days. 70% of the data was used for training and 30% for testing. The performance of the model was evaluated based on MSE (Mean Squared Error) and R^2 criteria.

ERP Integration

The forecasting results were transferred to the ERP system, which was developed on a PostgreSQL basis, via a REST API. The production, inventory, and order modules within the ERP system read this

data to provide real-time suggestions to managers. The system provides the purchasing team with future production forecasts based on historical data.

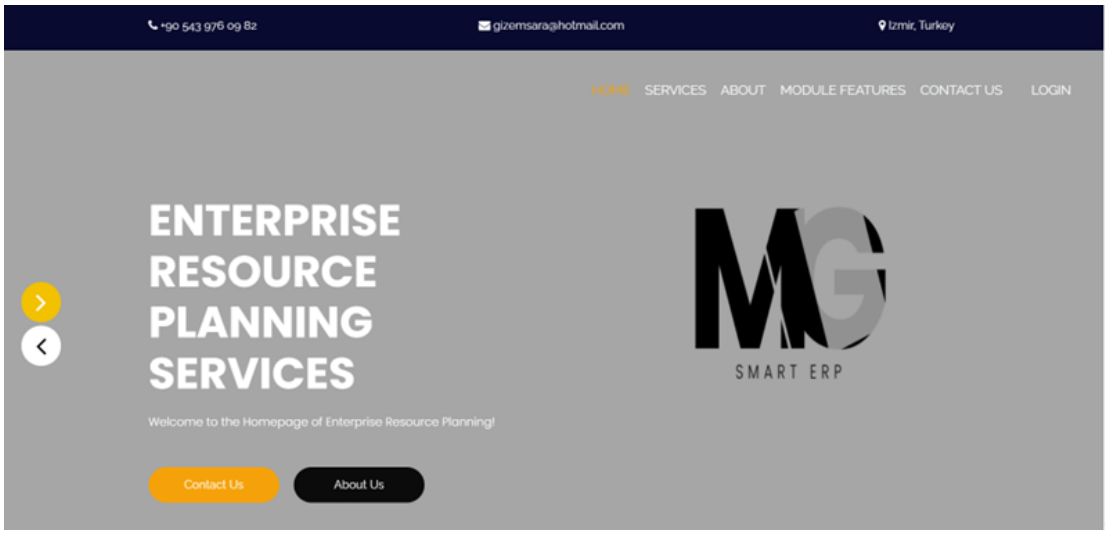

Figure 4. ERP Login Screen

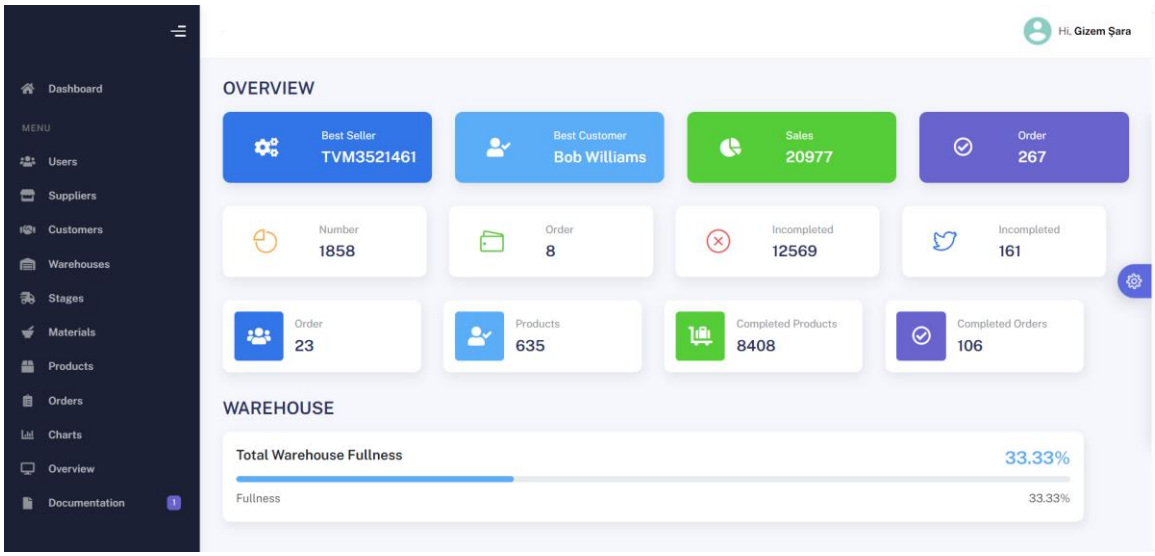

Figure 5. ERP Screen

RESULTS

This study aimed to address the problems encountered in the most critical areas of production processes such as production planning and inventory management through artificial intelligence and digital system integration. The artificial neural network model developed based on field observations and system data made highly accurate production forecasts using historical production data; these forecasts were integrated with the PostgreSQL-based ERP system, contributing to the digitalization of decision support processes.

As a result of the analyses, it was found that a large portion of production inventory consisted of a small number of critical materials, and it was understood that planning processes should be optimized around these materials. In this context, Pareto analysis and fishbone diagrams were effective both in identifying the problem and in generating solution strategies.

A high correlation coefficient was obtained for the artificial neural network model. This indicates that the model has a strong forecasting capacity and can make reliable predictions regarding production quantities. Through integration with the ERP system, these predictions were systematically transferred to the production, procurement, and inventory modules; thus, manual planning errors, time losses, and waste were significantly reduced.

Moreover, the developed system's ability to provide real-time data flow between ERP and MES not only enhances planning but also improves operational efficiency. This situation can provide transparency in the production process and allow decision-makers to intervene quickly in the field. Problems such as material surplus, misdirection, and operator confusion on the production line will be significantly reduced through this system.

This study also demonstrated that:

- Artificial intelligence models integrated into the production process provide not only efficiency but also strategic flexibility.
- The harmonious operation of ERP and MES systems is a cornerstone in the journey of digital transformation.
- With data-driven forecasting, businesses can optimize their resources not only based on the past but also for the future.

As a result, the proposed method of this study is a low-cost, sustainable digitalization example that can be easily applied in small and medium-sized manufacturing enterprises. The method is not merely a forecasting algorithm but also a digital transformation strategy that shapes decision-making processes.

DISCUSSION AND CONCLUSION

The implementation of production forecasting using the artificial neural network model developed in this study, and the integration of the forecast results into the ERP system, provided accuracy, speed, and efficiency in production planning. The high R^2 value of the model represents a significant achievement compared to similar studies. For instance, El Madany et al. (2022) used time series analysis methods for supply chain forecasting but did not integrate model accuracy with ERP systems. From this perspective, the proposed model is innovative not only in terms of forecasting performance but also in system integration.

Studies based on deep learning models, such as those by Bengio (2009), also emphasize that neural networks perform excellently in nonlinear and complex systems. In this study, the dynamic structure of the production system was successfully modeled using an artificial neural network. However, the number of data points and the sampling period used for training the system were limited. This may affect the model’s generalization capacity.

The findings supported by Pareto analysis and fishbone diagrams enabled the basic problems on the production floor to be identified within a broader framework. It was understood that the core problems encountered in material planning stem not only from technical issues but also from organizational and procedural deficiencies. In this regard, the socio-technical integration requirement frequently emphasized in literature supports the findings of this study.

In addition, the integration of predictions into the ERP system provided a significant advantage to managers in real-time decision-making processes. As emphasized by Ghadge et al. (2022) in the context of the green supply chain, digitalization positively affects not only production but also sustainability and resource efficiency.

In conclusion, this study offers a practical example of digital transformation through AI-assisted planning and ERP integration, while also contributing original insights to fill existing gaps in literature.

Future Work and Practical Implications

This study demonstrates the applicability of real-time data-driven forecasting systems integrated with ERP and MES in small and medium-sized enterprises (SMEs) in the manufacturing sector. Expanding the model to different production lines or multi-product systems may further optimize resource planning. In addition, a fully integrated MES-ERP infrastructure can be extended to include other functional modules such as supply chain management, maintenance planning, and workforce allocation, thereby evolving into a holistic digital transformation system.

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Conflict of Interest

The authors declare that there is no conflict of interest regarding this study.

Author Contributions

Gizem Şara Onay:

Conceptual framework of the study, data collection, execution of analyses, development of the artificial neural network model, ERP integration, and manuscript writing.

Mehmet Çakmakçı:

Academic supervision, methodological guidance, evaluation of the modeling process, interpretation of results, and revision of the manuscript.

A Hybrid Tree-Based Regression Model Combining Evolutionary and Simulated Annealing Techniques (1085)

Deniz Efendioğlu[1*]

[1]Ankara Yıldırım Beyazıt University, Ankara, Türkiye

*Corresponding author e-mail: defendioglu@aybu.edu.tr

Abstract

This study proposes a novel hybrid approach for regression tree optimization by combining the strengths of evolutionary algorithms and simulated annealing. The primary goal is to enhance model accuracy while maintaining interpretability, especially in complex regression tasks. The proposed method first employs an evolutionary tree algorithm (EVTREE) to explore a diverse set of tree structures through mutation and selection. Subsequently, the resulting trees are refined using a Simulated Annealing Tree (SATREE) algorithm, which enables escape from local optima and further cost reduction. Both algorithms consider not only prediction error but also tree complexity, balancing model accuracy and simplicity via a regularized cost function. The hybrid framework was tested on the California Housing dataset, where it achieved lower mean squared error (MSE) than either EVTREE or SATREE alone. Moreover, the tree structures produced remained interpretable and compact. This integration of global and local search strategies offers a promising direction for symbolic regression and interpretable machine learning. The proposed framework is modular, parallelizable, and can be adapted for broader model selection tasks.

Keywords: *Hybrid regression tree, evolutionary algorithm, simulated annealing, model interpretability, symbolic learning*

INTRODUCTION

Decision trees are among the most commonly preferred models in machine learning due to their high interpretability. However, these methods often rely on local optimization strategies, which carry the risk of getting stuck in local minima rather than reaching globally optimal solutions. This limitation can constrain model performance, especially in complex problem domains. To overcome this limitation, heuristic optimization algorithms come into play. These algorithms aim to explore wide solution spaces and optimize the structure of decision trees more effectively. In this study, regression trees constructed using two different heuristic methods are compared, and the potential of a hybrid approach is discussed:

Hybrid Use: Why and How?

In this study, we propose a novel hybrid regression tree approach that combines Simulated Annealing (SA) and Evolutionary Algorithms (EA) to address the dual objectives of improving predictive accuracy and reducing model complexity. Traditional regression tree algorithms often rely on greedy heuristics, which can lead to suboptimal solutions and overfitting. Metaheuristic approaches have shown promise in overcoming these limitations by exploring the solution space more broadly and avoiding local minima. Building on this foundation, our hybrid model integrates two complementary optimization strategies: SATREE and EVTREE.

SATREE, based on the principles of simulated annealing, conducts a guided local search through a cooling process that allows occasional acceptance of inferior solutions. This mechanism encourages exploration in the early phases and gradually focuses on exploitation, enabling the algorithm to rapidly

converge to structurally compact and computationally efficient trees. In contrast, EVTREE employs evolutionary principles by maintaining a population of candidate trees that evolve over successive generations using genetic operators such as mutation, crossover, pruning, and splitting. This population-based exploration allows EVTREE to search a wider solution space and discover low-error tree structures through stochastic variation and selection.

The integration of SATREE and EVTREE offers several synergistic benefits. First, the hybrid model achieves higher predictive accuracy while maintaining a simpler tree structure, as SATREE's tendency to generate compact initial solutions complements EVTREE's capacity for refinement. Second, the method benefits from effective initialization, where promising trees generated by SATREE form the initial population for EVTREE, thereby providing a strong foundation for further exploration. Third, the hybrid approach contributes to better control over overfitting by balancing global search and local exploitation, ensuring that the resulting models generalize well to unseen data. Finally, the overall search time is reduced, as SATREE accelerates convergence in the early phase and narrows the search space for EVTREE, making the evolutionary process more efficient.

These advantages demonstrate the potential of combining SA and EA in a unified regression tree learning framework. Similar hybridizations have been noted in prior research. For instance, Barros et al. (2011) and Grubinger et al. (2014) highlight the effectiveness of evolutionary strategies in discovering optimal tree structures, while Alvarez et al. (2004) emphasize the value of combining local and global search techniques. The proposed hybrid approach thus builds upon and extends this line of work, contributing a flexible and robust method for regression tree induction that is especially suitable for complex modeling tasks requiring both accuracy and interpretability.

Method Development: Simulated Annealing-Based Decision Tree (SATREE)

Problem Definition and Objective Function

Like any regression tree, SATREE takes an input dataset X with corresponding target values Y, recursively partitions the input space into regions (leaf nodes), and assigns a constant output (typically the mean $\bar{y}_l$) to each leaf. The goal is to find a tree structure that minimizes the following cost function:

$$\text{Cost}(\text{Tree}) = \sum_{\text{leaves } l} \sum_{i \in l} (y_i - \bar{y}_l)^2 + \lambda \cdot \text{Complexity}(\text{Tree})$$

- First term: Measures the prediction error using Mean Squared Error (MSE) across all leaf nodes.
- Second term: Imposes a penalty based on tree complexity, which may include the number of splits, tree depth, or the number of nodes.
- λ: A regularization coefficient that balances model fit and structural simplicity, helping to avoid overfitting.

Tree Representation

SATREE represents each regression tree as a complete solution within the optimization search space. This representation includes a binary tree structure in which each internal node is defined by a split variable (feature index) and a corresponding threshold value, determining how the data is partitioned. Each terminal (leaf) node contains a prediction value, typically the mean of the target values within that region. To enable efficient modification and evaluation during the optimization process, SATREE also maintains a split list that encodes the hierarchical structure of the tree. This structured representation allows SATREE to flexibly explore and refine tree configurations during the simulated annealing search.

Neighbor Solution Generation

The core of Simulated Annealing lies in its ability to explore neighboring solutions to progressively improve model performance while avoiding local minima. In SATREE, the neighbor function achieves this by generating structurally similar trees through small, controlled modifications to the current solution. These modifications include changing the splitting variable or threshold at an internal node to explore alternative partitions of the data, introducing a new split in place of a leaf node to increase the tree's complexity, removing a split to merge two child leaves into a single node for simplification, and slightly adjusting the constant prediction values at leaf nodes to fine-tune outputs without altering the overall structure. Through these operations, SATREE balances local refinement with broader structural exploration, supporting both model accuracy and complexity control during the optimization process.

Simulated Annealing Procedure

The Simulated Annealing algorithm in SATREE follows a structured procedure aimed at exploring the solution space while gradually reducing the likelihood of accepting worse solutions. The process begins with the initialization phase, where a random initial tree is generated to serve as the starting solution, and an initial temperature is set to control the probability of accepting worse solutions. During the iterative phase, a neighboring tree is generated by applying small modifications to the current tree. The cost difference is then calculated. If the new tree has a lower cost, it is accepted unconditionally. However, if the cost is higher, the new solution is accepted with a probability where T is the current temperature. This probabilistic acceptance allows the algorithm to escape local minima in the early stages of the search. After each iteration, the current solution is updated if the neighbor is accepted, and the temperature is reduced according to a predefined cooling schedule—commonly exponential decay. The algorithm continues this process until a stopping criterion is met, such as reaching a maximum number of iterations, the temperature dropping below a minimum threshold, or no improvement in solution quality over a fixed number of iterations. This framework enables SATREE to effectively balance exploration and exploitation during tree structure optimization.

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Stopping Criteria

The optimization process halts when one or more of the following stopping criteria are satisfied. First, the algorithm terminates if a predefined maximum number of iterations has been reached, ensuring computational feasibility. Second, the process stops if the temperature parameter drops below a minimum threshold, indicating that the search has sufficiently cooled and is unlikely to accept worse solutions. Third, the optimization halts if no improvement in the cost function is observed over a fixed number of consecutive iterations, suggesting convergence to a stable or optimal solution. These criteria collectively ensure a balance between thorough search and computational efficiency.

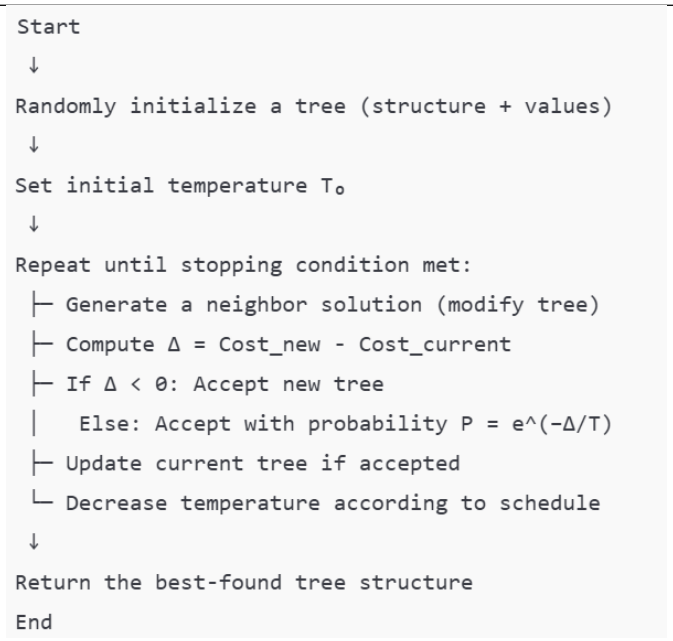

Figure 1. Algorithm of Simulated Annealing

Regression Tree Comparisons

Classical Methods: rpart, ctree, ctree2

- rpart is particularly suited for quick prototyping or when the goal is to produce easily interpretable decision rules. Its simplicity, however, comes with drawbacks such as a high risk of overfitting and limited flexibility with categorical variables. It is best used in small to medium-sized datasets and when model transparency is prioritized over predictive performance.
- ctree and ctree2, both part of the conditional inference framework, enhance statistical rigor by using hypothesis testing (p-values) for splits. While ctree is more conservative and overfitting-resistant, ctree2 relaxes these constraints to allow more flexible trees. They are ideal for applications where statistical validity of splits is important, such as in psychometrics or medical decision-making.

Ensemble and Soft Aggregation Approaches: nodeHarvest

- nodeHarvest offers an ensemble-like output without sacrificing interpretability, by aggregating predictions across multiple nodes. It smooths the decision boundary, which can be beneficial in noisy datasets. Its main limitation is the computational cost and limited performance on small datasets. It's a strong choice when a low-variance, smoothed model is desired, especially as an interpretable alternative to Random Forests.

Optimization-Focused Methods: partDSA, evtree, and SATREE

- partDSA is powerful for capturing complex, nonlinear interactions by optimizing decisions at each split. However, its computational demands make it more suitable for large datasets where modeling complexity justifies the cost.
- evtree, which uses evolutionary algorithms, is designed for global optimization of the tree structure. It excels in finding high-accuracy models, particularly in small to medium-sized datasets, but may be slow and sensitive to parameter tuning.
- The proposed satree leverages Simulated Annealing to explore the solution space more flexibly and heuristically. It aims to approximate global optima with fewer resources than evtree, especially in settings with multimodal loss surfaces. While still under development, SATREE

demonstrates potential in balancing solution quality and computational efficiency, making it particularly suitable for research contexts and for comparisons with global optimization techniques like EVTREE.

Table 1. Methods Comparison

Method	Advantages	Disadvantages	Suitable Use Cases / Notes
rpart	- Fast and easy to implement - Produces practical decision rules	- Risk of overfitting - Limited with categorical vars	- Small to medium datasets - Ideal for simple rules
ctree	- Makes statistically significant splits based on p-values	- Can be overly conservative	- When hypothesis testing is desired - Overfitting-resistant
ctree2	- As meaningful as ctree but allows more splits	- May result in more complex trees	- For more flexible structures - When better fit is needed
nodeHarvest	- Softens over-branching - Aggregates node outputs - Interpretable	- Training can be slow - Limited with small data	- For low-variance, smooth models - Ensemble alternative
partDSA	- Optimizes at every split - Sensitive to nonlinear relationships	- High computational cost	- For complex and interactive data - Advanced decisions
evtree	- Performs global optimization (evolutionary algorithm) - Searches for global optima	- May be slow - Requires parameter tuning	- High accuracy with small–medium datasets
satree *(algorithm created)*	- Searches for better splits using Simulated Annealing - Approaches global optima heuristically	- Still under development - Parameter sensitivity	- Suitable for multimodal loss functions - For comparison with evolutionary methods

The table illustrates a spectrum of regression tree algorithms, ranging from classical methods like rpart to more advanced optimization-based approaches like evtree and the proposed satree. Each method reflects trade-offs between simplicity, accuracy, interpretability, and computational efficiency.

While classical methods like rpart and ctree remain relevant for their simplicity and speed, more advanced methods such as evtree and satree provide improved modeling capabilities at the cost of higher computational complexity. SATREE, in particular, introduces a promising heuristic global optimization strategy through Simulated Annealing, making it a valuable addition for tasks where structural optimization and interpretability must be balanced. Its development marks a step forward in the growing field of metaheuristic-based regression tree learning.

Table 2. Goal/Scenario of Methods

Scenario / Goal	Recommended Method(s)
Fast and easy prototyping	rpart, ctree
Meaningful splits + interpretability	ctree, ctree2
Soft/ensemble-style output	nodeHarvest
Complex structures, interactions	partDSA, evtree, satree
Targeting global optimization	evtree, satree
Small dataset	ctree, rpart
Large dataset	ctree2, partDSA, nodeHarvest

Lıterature Revıew

Decision trees offer significant advantages in terms of interpretability and the explainability of decision rules. However, the limitations of classical algorithms have prompted the development of more effective optimization methods for these structures. In this context, the literature on optimizing decision trees with heuristic and evolutionary algorithms can be examined under three main headings: classical decision trees, heuristic approaches (particularly Simulated Annealing), evolutionary algorithms, and finally hybrid models.

Classical Decision Trees

Among the most common classical decision tree algorithms are CART (Breiman, 1986), C4.5, and ID3. These methods generally use greedy approaches, selecting the best local split at each step based only on the current data. However, this strategy carries the risk of getting stuck in local minima rather than achieving a global optimum.

Optimization of Decision Trees with Heuristic Methods Simulated Annealing

Simulated Annealing (Kirkpatrick et al., 1983) offers a more flexible optimization strategy compared to classical methods, particularly due to its ability to avoid local minima. By accepting higher-energy solutions with some probability, it performs a broader search over the solution space. Notable studies applying SA to decision trees include:

- Bucy and Diesposti (1993): The SA algorithm was used for structural design of decision trees to improve classification performance. Its random search capability helped avoid local minima where traditional algorithms get stuck.
- SACS Algorithm (1994): The Simulated Annealing Classifier System integrated SA with the Minimum Description Length (MDL) principle, achieving successful results on both noisy and clean datasets.
- MIT Research (2023): Studies within the Optimal Classification Trees (OCT) framework demonstrated that SA is effective in producing optimal decision trees for classification, prescription, and survival analysis problems.

Optimization of Decision Trees with Evolutionary Algorithms

Genetic algorithms and evolutionary strategies aim to optimize decision trees through evolutionary processes. These methods use population-based search strategies with crossover and mutation operators to explore a wide solution space. Key works in this area include:

- Evolutionary Design of Decision Trees (2021): These studies developed evolutionary approaches as alternatives to traditional methods, producing interpretable and robust models especially for medical applications.
- Evolutionary Decision Trees Survey (2024): A comprehensive recent survey focused on the increasing use of evolutionary algorithms in contrast to the top-down "divide and conquer" approach.
- Matrix-Based Application (2023): A new matrix encoding method was proposed to optimize evolutionary decision trees more efficiently, accelerating prediction through numerical computations.
- EvTree (Grubinger et al., 2014): The EvTree algorithm optimizes decision trees using evolutionary operators, theoretically enabling solutions closer to the global optimum.

Hybrid Approaches

Combining SA and EA in complex optimization problems, the combined use of heuristic methods like Simulated Annealing (SA) and Evolutionary Algorithms (EA) holds the potential to harness the advantages of both. Noteworthy studies on these hybrid approaches include:

- Predict-Then-Optimize Framework (2023): This framework combined the predictive power of decision trees with the optimization capabilities of evolutionary algorithms to develop more effective and balanced models.
- Interpretable Reinforcement Learning (2023): Evolutionary optimization was applied to improve interpretable decision trees, enhancing transparency in reinforcement learning models.
- Tsang et al. (2020): It was shown that hybrid optimization structures combining SA and EA yield effective results especially in high-dimensional and multimodal search spaces.

Literature Summary and Comparative Evaluation

The innovative aspect of this study is the systematic comparison of SA (SATREE) and EA (EVTREE) based decision tree models in terms of structure, performance, and interpretability. Additionally, the differences between SA's individual-based optimization strategy and EA's population-based approach are analyzed in detail.

Table 3. Strength & Weakness of Methods

Approach	Strengths	Weaknesses
Simulated Annealing (SA)	Avoids local minima, produces simpler models, interpretable	Single-individual search, can be time-consuming
Evolutionary Algorithms (EA)	Population-based search over wide solution space, successful in complex datasets	Complexity, risk of reduced interpretability
Hybrid Approaches	Combines advantages of both methods, balances accuracy and interpretability	Implementation and parameter tuning can be complex

METHODOLOGY

Dataset: California Housing

The California Housing dataset is designed to predict housing prices in the state of California and is widely used in regression problems. Based on the 1990 US Census data, this dataset is available built-in within the scikit-learn library.

The dataset includes a range of socioeconomic and geographic features that describe housing conditions in California neighborhoods. Key variables include median household income (MedInc), average house age (HouseAge), and room characteristics such as the average number of rooms (AveRooms) and bedrooms (AveBedrms). It also accounts for population density (Population), average occupancy per household (AveOccup), and location data through Latitude and Longitude. These features collectively provide a comprehensive view of the factors influencing housing values. (MedHouseVal) represents the median house value in the neighborhood, measured in $100,000 units. This is the primary dependent variable used for regression modeling tasks in the California Housing dataset. It reflects the socioeconomic condition of the area and is influenced by various features such as location, population, and housing characteristics.

Table 4. Features and Descriptions of Independent & Dependent Variables

Feature (Input Variable)	Description
MedInc	Median household income in the neighborhood (in $10,000)
HouseAge	Average age of houses (years)
AveRooms	Average total number of rooms
AveBedrms	Average number of bedrooms
Population	Total population in the neighborhood
AveOccup	Average household size
Latitude	Latitude information
Longitude	Longitude information
Target Variable	**Description**
MedHouseVal	Median house value in the neighborhood (in $100,000)

Applications

The SATREE (Simulated Annealing-Based Decision Tree) algorithm starts with a randomly initialized decision tree. In each iteration, a small neighboring tree is generated by applying slight structural changes, such as altering a branch or modifying a split condition. The acceptance of the new solution is based on the Metropolis criterion, which allows for accepting worse solutions with a certain probability depending on the difference in energy (i.e., cost). This probability decreases over time through a cooling schedule as the temperature is gradually lowered. SATREE is particularly suitable for escaping local optima and offers a high potential for discovering compact, low-cost tree structures by maintaining a controlled level of randomness during search.

In contrast, the EVTREE (Evolutionary Decision Tree) algorithm uses a population-based approach. It begins with a set of randomly generated trees and evolves over several generations. Each generation involves the application of evolutionary operators such as mutatemajor, pmutateminor, psplit, pprune, and pcrossover. These operators introduce variation into the population and allow the exploration of different tree structures. At each iteration, the best-performing trees are selected based on an objective function, and the population is updated accordingly. EVTREE is effective in maintaining population diversity and supports progressive improvements by combining selection, crossover, and mutation mechanisms.

Both SATREE and EVTREE aim to minimize a common objective function that balances prediction accuracy and model complexity. The cost function is defined as: Cost = MSE + $\lambda \times$ Node Complexity

Here, MSE refers to the mean squared error of the model's predictions, and λ is a regularization parameter that penalizes trees with excessive node complexity. This formulation discourages overfitting by promoting simpler, interpretable trees without sacrificing predictive performance.

A hybrid approach can leverage the strengths of both SATREE and EVTREE. This integration is designed to enhance the balance between exploration and exploitation in the search for optimal tree structures. There are several strategies to implement this hybrid model.

The first strategy is to start with SATREE and improve with EVTREE. In this method, a compact and efficient tree is initially generated using SATREE and then introduced into the EVTREE population as

an elite individual. This boosts the initial quality of the population and reduces the time needed for convergence.

The second strategy involves using SATREE as a local search method within EVTREE. At the end of each generation in the evolutionary process, the best-performing individuals undergo further optimization through SATREE. This allows fine-tuning of promising solutions and offers a flexible improvement mechanism beyond traditional evolutionary operations.

The third strategy is parallel use through either ensemble learning or model selection. Trees generated by both SATREE and EVTREE can be combined to form an ensemble model, enhancing robustness and predictive power. Alternatively, the model with the lowest composite cost (MSE + complexity) can be selected as the final output. This parallel structure ensures flexibility and improved generalization performance across different datasets.

How to Hybridize?

1. Start with SATREE, Improve with EVTREE
    - SATREE produces a compact and low-cost initial tree.
    - This tree is included as an elite individual in the EVTREE population.
    - Initial quality is increased; search time is shortened.
2. Local Search with SATREE inside EVTREE (Hybrid Evolutionary-SA)
    - At the end of each generation in EVTREE, the best individual(s) are subjected to local optimization using SATREE.
    - Thus, a more flexible improvement process is provided, not only relying on crossover and mutation.
3. Parallel Use (Ensemble or Model Selection)
    - Trees generated by SATREE and EVTREE can be used together within an ensemble model.
    - Alternatively, the model with the lowest MSE + complexity cost can be selected.

Contributions of the Hybrid Approach and Potential Challenges

The hybridization of SATREE and EVTREE brings several notable advantages by combining their complementary strengths. One of the key contributions is the increased initial quality of the evolutionary process. By injecting a well-optimized initial tree from SATREE into the EVTREE population, the search begins with a stronger foundation, leading to faster convergence and more effective exploration.Another major benefit is achieving a better balance between prediction error and model complexity. While EVTREE is capable of discovering highly accurate trees, SATREE's preference for simpler structures helps maintain interpretability and reduces overfitting. This synergy supports the development of models that are both powerful and generalizable. Furthermore, the stochastic nature of simulated annealing enhances the ability to escape local optima. This becomes particularly valuable in search spaces where evolutionary operators alone may struggle to overcome suboptimal regions. By allowing occasional acceptance of worse solutions, SATREE injects necessary randomness that expands the search frontier. The hybrid approach also contributes to reducing overfitting, as SATREE typically favors smaller, less complex trees. This helps in preventing the model from memorizing noise or over-adapting to training data. Additionally, the integration fosters greater diversity in the search process, with EVTREE's population-based exploration complemented by SATREE's temperature-controlled, probabilistic transitions.

Table 5. Contribution of Hybrid Approach

Contribution	Explanation
Increased initial quality	EVTREE starts with strong individuals, accelerating the evolutionary process.
Error–complexity balance	EVTREE's accuracy and SATREE's simplicity are optimized together.
Escape from local optima	SA's randomness helps overcome regions where EVTREE might get stuck.
Reduction of overfitting	Simpler structures reduce the risk of overfitting.
Diversity in the search process	EVTREE's population diversity combined with SATREE's temperature-based strategy.

Despite these advantages, there are some challenges to consider. A key difficulty lies in parameter management, as each algorithm requires careful tuning (e.g., cooling schedules in SATREE, mutation rates in EVTREE). Coordinating these parameters to ensure synergy without destabilizing the process can be complex. Another issue is the computational time cost, especially when local search operations using SATREE are applied frequently during evolutionary cycles. Although performance improves, this may significantly increase overall runtime. Lastly, the implementation complexity of a hybrid algorithm is non-trivial. Managing two optimization frameworks within a single system demands a more sophisticated codebase, potentially increasing development and maintenance effort.

Table 6. Challenge for Hybrid Approach

Challenge	Explanation
Parameter management	Optimizing parameters of two algorithms can be complex.
Time cost	Especially hybrid local search processes can increase total runtime.
Implementation complexity	Code structure may become more complex.

RESULTS

In this study, three different decision tree-based approaches were compared: SATREE (Simulated Annealing Tree), EVTREE (Evolutionary Tree), and a Hybrid approach combining both techniques. Each method was evaluated in terms of predictive performance (measured by Final Test Mean Squared Error) and model complexity (measured by the number of nodes in the resulting tree).

The SATREE algorithm demonstrated the best predictive accuracy, with a final test MSE of 0.7662. Its decision tree consisted of 11 nodes, indicating a moderately complex structure. SATREE's success stems from its ability to explore the solution space through localized stochastic changes and gradually refine tree structures. By using the Metropolis acceptance criterion, SATREE allows the algorithm to escape poor local optima during early stages while converging to a more optimal structure over time. This flexibility enables SATREE to produce relatively compact yet high-performing trees, particularly effective in balancing generalization and precision. However, its performance may vary depending on the initial configuration and parameter tuning, and convergence time can fluctuate based on the temperature decay schedule.

The EVTREE algorithm, in contrast, generated the simplest model, consisting of only 3 nodes, which is advantageous for interpretability and speed in application. Nevertheless, this came at the cost of accuracy; the final MSE reached 0.9401, the highest among the three. EVTREE relies on a population-based optimization strategy using evolutionary operators such as mutation, crossover, pruning, and splitting. While this allows for greater diversity in the search process, the algorithm can sometimes

prioritize structural simplicity too aggressively, leading to underfitting—especially when not adequately tuned or when diversity isn't sufficiently maintained. Moreover, due to its population-based nature and repeated evaluations over generations, EVTREE tends to be computationally more expensive, particularly for large datasets.

Table 7. Results of Each Approach

Method	*Final Test MSE*	*Node Count*	*Highlights*
SATREE	0,7662	11	Lowest MSE, most accurate predictions; good balance between complexity and performance; sensitive to initialization.
EVTREE	0,9401	3	Simplest model, highly interpretable; promotes diversity but may underfit; high computational cost.
Hybrid	0,7981	9	Balances accuracy and simplicity; compact and efficient; combines fast local search with global diversity; faster in practice.

The hybrid method, which integrates the strengths of both SATREE and EVTREE, produced a balanced outcome. Its final test MSE was 0.7981, which, while slightly higher than SATREE, still represents a substantially better performance than EVTREE. The hybrid tree had 9 nodes, making it structurally more compact than SATREE but more detailed than EVTREE. This approach begins by generating a promising initial solution using SATREE and then incorporates that solution into EVTREE's population. Alternatively, it can apply SATREE's local optimization process to the best-performing individuals in each generation of EVTREE. This combination enhances search efficiency, improves convergence speed, and retains model quality while controlling complexity. Additionally, the hybrid method was observed to have faster convergence than the standalone methods, making it more practical for real-time or large-scale applications. Its architecture benefits from the exploratory nature of evolutionary algorithms and the fine-tuning capability of simulated annealing, creating a well-rounded optimization process.

Table 8. Leaf Node Details

Model	*Final Test MSE*	*Number of Nodes*	*Used Variables*	*Leaf Node Details*
SATREE	0.7662	11	MedInc, AveRooms, AveOccup	- MedInc ≤ 3.0596 ∧ AveRooms ≤ 3.9189 → 1.7103- MedInc ≤ 3.0596 ∧ AveRooms > 3.9189 → 1.2260- MedInc ≤ 3.0596 ∧ AveOccup ≤ 2.3474 → 2.7704- MedInc ≤ 3.0596 ∧ AveOccup > 2.3474 → 1.8411- MedInc ∈ (3.0596, 5.3698] → 1.7794- MedInc > 5.3698 → 3.5038
EVTREE	0.9401	3	MedInc	- MedInc ≤ 5.0102 → 1.7280- MedInc > 5.0102 → 3.2975
Hybrid	0.7981	9	MedInc	- MedInc ≤ 3.0652 ∧ MedInc ≤ 4.7732 → 1.7026- MedInc ≤ 3.0652 ∧ MedInc > 4.7732 → 3.1799- MedInc ∈ (3.0652, 4.6111] → 2.0247- MedInc ∈ (4.6111, 6.3663] → 2.6814- MedInc > 6.3663 → 4.0331

Overall, the results show that while SATREE excels in accuracy, and EVTREE provides simplicity and interpretability, the hybrid approach achieves an optimal trade-off, delivering solid predictive performance with reduced structural complexity and runtime cost.

Contributions and Future Work

This study provides a comprehensive comparison of three decision tree optimization strategies—SATREE, EVTREE, and a hybrid approach—focusing on their performance in terms of prediction accuracy, tree complexity, and computational efficiency. Each method offers unique strengths and weaknesses, and the combination of these methods introduces a powerful framework for constructing regression trees.

The SATREE (Simulated Annealing Tree) algorithm demonstrated its strength in achieving locally optimized solutions with relatively low complexity. Due to its controlled exploration mechanism, it is particularly effective for applications where simpler, interpretable models are preferred and computational resources are limited. Its fast convergence in local regions makes it a practical choice for tasks that require rapid feedback or streamlined models. However, its sensitivity to the initial solution remains a limitation; suboptimal starting points can hinder performance and cause convergence to less favorable tree structures.

The EVTREE (Evolutionary Tree) method, on the other hand, explores the solution space more thoroughly by maintaining a diverse population and applying various evolutionary operators. This makes it more robust in escaping local optima and exploring complex solution regions, thus enhancing the likelihood of reaching global optima. While EVTREE has the potential to produce highly adaptive models, its computational intensity and longer runtime—especially on larger datasets—can limit its practical applicability in real-time or resource-constrained settings.

The hybrid approach was developed to harness the strengths of both methods: the search diversity and global reach of EVTREE, and the local optimization and model simplicity of SATREE. The results showed that the hybrid method was able to achieve a near-optimal trade-off between prediction accuracy and structural simplicity, maintaining a relatively low error with fewer nodes compared to standalone EVTREE and with faster runtime than either method individually. Additionally, the hybrid model introduces greater stability and consistency in performance across multiple runs, as it combines global search with local refinement. Nevertheless, it introduces additional structural and tuning complexity, as it requires careful coordination between the parameters of both algorithms to ensure synergy.

These findings provide strong evidence that a hybrid optimization framework can substantially enhance the performance of regression trees, making it a promising approach for real-world predictive modeling where both accuracy and interpretability are essential.

Building on the insights from this research, several directions are proposed to further improve the robustness and applicability of the hybrid decision tree model:

- Integration of categorical variables: Current implementations primarily handle continuous data. Future developments should incorporate methods to natively process and split on categorical features, increasing the model's flexibility and domain relevance.
- Extension to time series data: Adaptations of the hybrid model can be developed to accommodate temporal dependencies, potentially involving dynamic tree structures or memory-aware nodes that evolve over time.
- Enhanced interpretability: As tree models gain complexity, tools like SHAP (SHapley Additive exPlanations), feature importance rankings, and partial dependence plots can be

integrated to provide deeper insights into the decision process and to support transparent model deployment.

- Dynamic parameter adaptation: Instead of manually tuning parameters like temperature decay or mutation rates, future versions can incorporate adaptive control mechanisms that adjust these values during training based on performance metrics.

Overall, this study lays a solid foundation for improving regression tree models through hybrid optimization. By blending global and local search paradigms, the proposed framework enhances both model quality and operational practicality. Future research will refine these methods further, aiming for scalable, interpretable, and domain-adaptable solutions in machine learning and decision support systems.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

Deniz Efendioğlu developed the algorithm.

Effective Feature Selection in High-Dimensional Datasets: A Binary Artificial Locust Swarm Optimization Approach (1095)

Buğra Kaan Tiryaki[1*]

[1]Karadeniz Technical University, Faculty of Science, Computer Science, Türkiye

*Corresponding author e-mail: btiryaki@ktu.edu.tr

Abstract

Working with high-dimensional data in machine learning can negatively affect the performance of the model. Especially in classification problems, the presence of unnecessary or irrelevant features both reduces the accuracy of the model and increases the computational cost. Therefore, feature selection is a critical step to identify important and meaningful features and eliminate the rest. In recent years, nature-inspired algorithms based on artificial intelligence have provided effective tools to solve this problem. In this context, Binary ALSO, the binary version of the Artificial Locust Swarm Optimization (ALSO) algorithm, is a powerful optimization approach adaptable to feature selection problems.

The Binary ALSO algorithm continuously updates candidate solutions by mimicking the natural behavior of swarms of locusts. In this algorithm, each locust represents a solution vector, i.e. the selected features. The vectors consist of 0's and 1's, and these values indicate whether the corresponding feature has been selected or not. The algorithm works iteratively; in each iteration, the individuals in the population are guided towards the best solution, but at the same time a certain amount of randomization (mutation) is maintained. In this way, the solution space is scanned efficiently. The fitness function is determined by balancing the number of features with the accuracy obtained from a classifier trained using selected features.

The Binary ALSO algorithm increases the accuracy of the model by effectively selecting features, especially in high-dimensional datasets, while eliminating unnecessary data. In applications, it has been observed that this algorithm achieves higher or comparable classification success with fewer features compared to classical methods. This reduces both model complexity and training time. Binary ALSO is an innovative and applicable approach that reflects the synergy between artificial intelligence and optimization.

Keywords: *Feature Selection, Binary Optimization, Locust Swarm, Metaheuristics, Machine Learning*

INTRODUCTION

The performance of machine learning models directly depends not only on the algorithms used, but also on the quality and size of the data. In particular, high-dimensional datasets can negatively affect the performance of the model in supervised learning problems such as classification. As the dimensional complexity increases, the learning process of the model becomes more difficult, the risk of overlearning increases and the computational cost increases. Therefore, feature selection is recognized as a critical preprocessing step in systems dealing with high-dimensional data.

In general, feature selection methods fall into three main groups: filter methods, wrapper methods and embedded methods. Filter methods rely on statistical criteria independent of the classifier, while wrapper methods integrate the classifier directly into the evaluation process. Embedded methods integrate feature selection into the learning process of the model. Although each approach has different advantages and

disadvantages, wrapper-based methods can produce more effective results on complex and high-dimensional datasets (Chandrashekar and Sahin, 2014).

However, wrapper-based methods have high computational cost. To overcome this challenge, nature-inspired meta-heuristic algorithms have been widely used in recent years. Methods such as Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Simulated Annealing (SA) and Ant Colony Optimization (ACO) aim to obtain optimal or near-optimal feature subsets by efficiently scanning the search space. These algorithms are well suited to feature selection problems since they have a high probability of reaching the global optimum and can work independently of the problem structure (Xue et al., 2016).

In particular, recently developed algorithms such as Artificial Locust Swarm Optimization (ALSO) offer a more balanced exploration-exploitation strategy inspired by the principles of collective intelligence and movement in nature. The applicability of these algorithms in binary form makes them ideal for problems such as feature selection, where the decision is "selected/not selected" (1/0). Meta-heuristic algorithms adapted to binary form determine whether each feature should be included in the model by encoding the solution vectors as 0 and 1.

In conclusion, developments in literature show that nature-inspired meta-heuristic algorithms offer more effective and flexible solutions than traditional feature selection methods, especially in high-dimensional and complex data structures. In this context, algorithms such as Binary ALSO make significant contributions in terms of both improving classification performance and preserving the simplicity of the model. başarımını artırma hem de modelin yalınlığını koruma açısından önemli katkılar sağlamaktadır.

MATERIAL AND METHODS

Material

The datasets used in this study were obtained from publicly available and widely used machine learning repositories of different sizes and domains. In particular, high-dimensional datasets encountered in classification problems were selected and the performance of the Binary Artificial Locust Swarm Optimization (Binary ALSO) algorithm was evaluated on these datasets. The two main datasets used are as follows: Wisconsin Breast Cancer Diagnostic Dataset (WBCD) (Frank, 2010) and Leukemia Gene Expression Dataset (CuMiDa) (Grisci, 2021). These datasets were chosen to measure the performance of the algorithm on both low and high dimensional data. The characteristics of the dataset used are given in Table 1.

Table 1. Characteristics of the datasets used

	Wisconsin Breast Cancer Diagnostic Dataset (WBCD)	**Leukemia gene expression Dataset: CuMiDa**
Objective	Classify tumors	Classify types of leukemia
Number of Observations	569	64
Number of Attributes	30 (numeric / continuous values)	22.284 (numeric, each one a gene expression)
Target	2 class: 0 = malignant,1 = benign	5 class: AML, Bone_Marrow, PB, PBSC_CD34, Bone_Marrow_CD34

Categorical Data	None (all numerical)	None (all numerical)
Missing Data	None	None

Methods

The Collection of the Data

Data were obtained from the UCI Machine Learning Repository and Kaggle platforms. The data sets were taken directly into the analysis process without any pre-processing steps. Especially since the attributes are numeric and do not contain missing data, they were directly suitable for the meta-heuristic algorithm. Each dataset was given as input to the Binary ALSO algorithm to be used in solving the classification problem.

Statistical Analysis

The basic method used in this study is the Binary Artificial Locust Swarm Optimization (Binary ALSO) algorithm. Each locust is represented by a binary solution vector (consisting of 0s and 1s). Each 1 in this vector indicates that the feature is included in the model, while a 0 indicates that it is excluded. The algorithm iteratively updates the population, gravitates towards the best solution and incorporates a certain amount of randomization (mutation). In this way, it is aimed to reach the global optimum in the search space.

The algorithm works iteratively, directing each individual to the best individual of the swarm and at the same time introducing a certain amount of randomness (mutation). It aims to achieve more effective results by maintaining a balance of both exploration and exploitation in the search space.

The Binary ALSO algorithm has been adapted to work with binary vectors consisting of 0 and 1 values instead of continuous valued position vectors. The following approaches were adopted in this transformation:

- *Position Representation*

  Solution vectors in binary space ($x \in \{0,1\}^D$) are used instead of position vectors in continuous space ($x \in R^D$).

- *Position Update Mechanism*

  The position update formula in the continuous ALSO algorithm is modified as it is not suitable for the binary problem structure. Therefore, for each updated velocity value, a probability value in the range 0-1 is obtained using the sigmoid function:

$$S(v_j) = \frac{1}{1 + e^{-v_j}}$$

According to this value, selection is made for each feature according to the following rule:

$$x_j = \begin{cases} 1 & if\ rand < S(v_j) \\ 0 & otherwise \end{cases}$$

Here, v_j is the updated position value, rand is a randomly generated number in the interval [0,1] and $S(v_j)$ is the sigmoid output (probability of selection). With these adaptations, the continuous-valued optimization algorithm can be successfully applied to binary decision problems such as feature selection.

For each solution (bitstring), a fitness function is used to measure how good this solution is. The fitness function used in feature selection balances two criteria. These are the number of features selected (aimed at reducing) and the classification accuracy (aimed at increasing).

$$Fitness = Accuracy \times \left[\alpha + (1-\alpha)\left(1 - \frac{|S|}{D}\right)\right]$$

Here, $|S|$ is the number of features selected, D is the total number of available features and $\alpha \in [0,1]$ is used to balance the two objectives. The proposed fitness function directly maximizes accuracy, while at the same time encouraging the selection of a small number of features and achieving a weighted balance between accuracy and number of features.

Classification accuracy was evaluated using a logistic regression classifier on the dataset. The following metrics were used as performance measures: Accuracy, Precision and F1-Score.

In order to increase the reliability of the analyses, 5-fold cross-validation was applied on each data set. This approach allowed the classification performance to be evaluated free from random effects. Furthermore, the performance of the developed Binary ALSO algorithm is analyzed in comparison with two different meta-heuristics widely used in the literature, Binary Genetic Algorithm (GA) (Sivanandam and Deepa, 2008) and Binary Particle Swarm Optimization (PSO) (Kennedy and Eberhart, 1997). This comparison provided a comprehensive benchmark to evaluate the effectiveness of the algorithms on datasets of different sizes.

RESULTS

In this study, the performance of the Binary Artificial Locust Swarm Optimization (Binary ALSO) algorithm for feature selection is evaluated on both low and high dimensional datasets. The datasets used include two different classification problems with different feature sizes: breast cancer diagnosis data (low dimensional) and leukemia gene expression data (high dimensional). The results are analyzed with statistical measures such as accuracy, precision and F1-score, and the results are compared with Binary Genetic Algorithm (GA) and Binary Particle Swarm Optimization (PSO) algorithms widely used in the literature. The values obtained with the full feature set were used as a baseline. Comparative results for the breast cancer (WBCD) dataset are given in Table 1.

Table 1. Comparative results of the breast cancer (WBCD) dataset

		Binary		
		Genetic Algorithm	**Particle Swarm Optimization**	**Artificial Locust Swarm Optimization**
No. Sel. Feat.[a]	30	10	7	3
Feature No	All	1,2,3,7,9,14,17,20,21,25	2,7,11,20,24,25,28	2,9,23
Accuracy	0.9806	0.9719	0.9754	0.9467
Precision	0.9782	0.9699	0.9679	0.9417
F1-Score	0.9847	0.9763	0.9822	0.9590

[a] Number of Selected Features

As seen in Table 1, the Binary ALSO algorithm achieved acceptable classification performance despite working with only 3 attributes. This offers a significant advantage, especially in models where interpretability is important. The GA and PSO algorithms also produced strong results with fewer attributes, but in terms of the number of attributes, Binary ALSO managed to obtain a leaner model.

The results of the selection among the first 1000 features in the leukemia gene expression dataset are given in Table 2. Since all features are used, the running time of the algorithms is quite long. For this reason, although there are more features in this dataset, the first 1000 were selected.

Table 2. Leukemia gene expression dataset comparative results (selection from top 1000 features)

		Binary		
		Genetic Algorithm	**Particle Swarm Optimization**	**Artificial Locust Swarm Optimization**
No. Sel. Feat.[a]	1000	471	464	476
Accuracy	0.9512	0.9512	0.9512	0.9833
Precision	0.9667	0.9667	0.9667	0.9867
F1-Score	0.9642	0.9642	0.9642	0.9875

[a] Number of Selected Features

Table 2 shows that the Binary ALSO algorithm performs much better than competing methods on high-dimensional gene expression data. With higher accuracy and F1-score, it both improves the classification performance and reduces the number of features by almost half compared to the reference model.

The number of common features selected between binary algorithms for the leukemia gene expression dataset is given in Table 3.

Table 3. The number of common features selected between binary algorithms (selection from top 1000 features)

Binary Algorithms	Number of common features
GA-PSO	221
GA-ALSO	237
PSO-ALSO	216

Table 3 shows that there is a certain consistency in attribute selection across algorithms. This suggests that some attributes play a decisive role in classification performance and that different algorithms make similar choices.

DISCUSSION AND CONCLUSION

This paper proposes the Binary Artificial Locust Swarm Optimization (Binary ALSO) algorithm to solve the feature selection problem in high-dimensional datasets and experimentally evaluates the effectiveness of this method on two different datasets. The results show that the algorithm performs competitively and successfully on both low-dimensional (WBCD) and high-dimensional (Leukemia) datasets.

In particular, the Binary ALSO algorithm is remarkable in that it provides acceptable classification accuracy with a minimum number of attributes. The 94.67% accuracy with only three attributes on the breast cancer dataset demonstrates the algorithm's capacity to produce lean models. On the other hand, the fact that it works with 98.33% accuracy and 98.75% F1-score on the high-dimensional leukemia dataset shows that the algorithm can produce effective results even in high-dimensional complex data structures.

Comparisons with Binary Genetic Algorithm (GA) and Binary Particle Swarm Optimization (PSO) methods commonly used in the literature show that Binary ALSO provides similar or higher classification performance. In addition, the high level of commonality in the selected features between the algorithms indicates that the selected features are statistically consistent and have high information value.

The findings of the study directly contribute to the need to build low complexity but high-performance models, especially in application areas such as explainable artificial intelligence, biomedical

classification systems, gene expression analysis and decision support systems. In addition, successfully balancing high accuracy with a low number of features also offers practical benefits in terms of reducing computational load.

In future work, it is proposed to integrate the Binary ALSO algorithm with different classifiers (e.g. SVM, KNN, RF), extend it for multi-class problems and combine it with other dimensions such as sample selection to create hybrid models.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

The entire study was conducted by a single author. All stages of the research—including the conception of the idea, literature review, development of the algorithm, execution of simulations, data analysis, interpretation of the results, and writing of the manuscript—were carried out solely by the author.

A Balanced Fuzzy Random Sampling Approach for Fair Workload Assignment in Shift Scheduling (1096)

Buğra Kaan Tiryaki[1*]

[1]Karadeniz Technical University, Faculty of Science, Computer Science, Türkiye

*Corresponding author e-mail: btiryaki@ktu.edu.tr

Abstract

Shift planning is of vital importance, especially in sectors that require uninterrupted service such as health, safety and production. In this process, fair distribution of workload to employees is a factor that directly affects both employee satisfaction and organizational efficiency. However, most of the time, work packages are assigned to employees randomly or at the discretion of the manager, which causes some employees to constantly face heavier workloads. Such imbalances may lead to loss of motivation and damage to the sense of justice among individuals working with fixed salaries.

In this study, an assignment algorithm based on balanced fuzzy sampling is proposed to ensure a balanced distribution of workload to individuals working with fixed salaries. Unlike classical random sampling methods, this approach dynamically updates the selection probabilities by considering the past workload of each employee. While the selection probability of individuals with higher-than-average workload is reduced in subsequent assignments, the selection probability of employees with less workload is increased. Thus, a certain degree of randomness is maintained, and the workload balance is achieved in the long run. The developed model was applied to the case of fair assignment of night shifts to nurses according to different difficulty levels and satisfactory results were obtained.

The proposed balanced fuzzy sampling approach shows that randomness and fairness can be achieved simultaneously in workforce planning. This method is applicable not only in the field of health, but also in many sectors from production to logistics, in the distribution of tasks of personnel working with fixed salaries. Since fair workload distribution is a critical factor in increasing employee satisfaction and productivity, the model presented in this study can be considered an effective tool in organizational resource planning.

Keywords: *Fuzzy Random Sampling, Employee Scheduling, Nonprobability Sampling, Workload Assignment*

INTRODUCTION

Today, in sectors such as health, security, production and logistics where shift work is common, managing the workforce in a balanced manner is of great importance in terms of both employee satisfaction and organizational efficiency. Especially for individuals working with fixed salaries, fair distribution of workload is considered as a fundamental element in establishing organizational justice. However, in current planning processes, task assignments are often made randomly or based on the subjective evaluations of managers, which results in some employees constantly facing heavier workloads. The constant uneven distribution can lead to negative consequences such as dissatisfaction, burnout and demotivation among employees (Ernst et al., 2004; Sahraeian, 2024).

In literature, task assignment and shift planning problems are generally addressed with scheduling algorithms, mathematical programming, heuristics and multi-criteria decision making (MCDM)

approaches (Kletzander & Musliu, 2020). Most of these methods work on the basis of deterministic rules and generally do not consider the past workloads of employees. However, in order to fairly distribute the workload of employees working with fixed salaries, it is necessary to consider past load data and develop dynamic and adaptive solutions.

The "Balanced Fuzzy Sampling" approach, developed to address this need, differs from classical random sampling methods by incorporating the past workload of each employee and dynamically updating the selection probabilities accordingly. This method, which has been previously described as a "Fuzzy Replacement" based sampling technique (Kesemen et al., 2021), offers a flexible model that can consider both randomness and fairness at the same time, especially in task allocation.

The main objective of this study is to develop a fair workload allocation mechanism that considers past workloads and task difficulty levels for individuals working on a fixed salary. The algorithm developed in this context aims to provide a balanced distribution over time by updating the job assignment probabilities based on both the difficulty coefficients of the tasks and the past workloads of the employees. The proposed model is tested on night shift assignments for nurses and simulation results show that the workload imbalance is largely eliminated.

This approach is not only limited to the health sector but can be applied in many different fields from production to logistics, from security to public services. It can be considered as an effective decision support tool for organizations that want to balance fairness and efficiency in their corporate planning processes.

MATERIAL AND METHODS

Material

In this study, a balanced fuzzy sampling-based algorithm is developed and tested through simulation to ensure fair task assignments in workforce planning. Nurses with fixed salaries are selected as the application group, with the aim of distributing night shifts fairly among them. Each task has a different difficulty level, ranging from 1 (lightest: outpatient clinic) to 5 (most difficult: intensive care). Difficulty levels were determined based on the physical and mental burden of the tasks.

The data used in the simulation were artificially generated and the workload of the nurses in the previous weeks was randomized. The difficulties of the shifts to be assigned were randomized and the total workload of each nurse was updated with the new tasks. Simulations were performed with Python programming language.

Methods

Unlike classical random sampling methods, the proposed algorithm is based on a balanced fuzzy sampling model that considers the past workload of each employee and updates the selection probabilities accordingly. The basic operation of the model is as follows.

First, all employees are given equal probability of selection. Then, a random number is generated for each task and an employee is selected by finding the equivalent of this value in the cumulative probability distribution. The selected employee is assigned the relevant task and the difficulty coefficient associated with this task is added to the workload.

After each election, the normalized workload ratio of every employee is computed, and the selection probabilities are updated accordingly. This adjustment reduces the likelihood of employees with a high workload being selected in subsequent elections, while increasing the probability for those with a lower workload. Consequently, a degree of randomness is preserved, ensuring a balanced distribution of tasks over the long term.

Algorithm of Balanced Fuzzy Sampling

The algorithm used in this study is based on the Balanced Fuzzy Sampling approach to achieve balanced workload distribution. It dynamically updates the selection probabilities for each task assignment by considering the employees past workloads. The basic steps of the algorithm are outlined below:

Algorithm 1. Algorithm of the *Balanced Fuzzy Sampling*

Step 1. Defining Initial Probabilities:

All employees are defined to have equal initial probabilities of being selected:

$$p_i = \frac{1}{N}$$

Here, N represents the number of employees.

Step 2. Defining the Workload Vector:

The system is initialized by entering the total workload vector of the employees for the previous weeks. If the initial load is not specified, all employees are initialized with zero load.

Step 3. Shift Assignments:

One worker for each shift is randomly selected according to the available probability distribution. In this selection, unlike classical random sampling, past workload imbalance is considered.

Step 4. Updating Probabilities:

After the selection, the workload vector is updated and the normalized workload ratio (r) of each employee is calculated:

$$r_i = \frac{y_i}{\bar{y}}$$

Here, y_i represents the total workload of the employee, and $\bar{y}$ denotes the average workload.

Step 5. Weight Adjustment with Balancing Factor:

$$p_i = p_{0i} \,.\, e^{-r_i^{\frac{1-\alpha}{\alpha}}}$$

Here, $\alpha \in (0,1)$ is the balance factor. Smaller values of α allow for a more sensitive response to workload differences. p_0 represents the initial probability distribution.

Step 6. Use of Difficulty Levels:

Each task is assigned a difficulty score (ω_j). These scores range from 1 for outpatient clinic tasks to 5 for intensive care tasks. When task j is assigned to employee i, the employee's total workload y_i is increased by the corresponding task difficulty coefficient:

$$y_i \leftarrow y_i + w_j$$

Thus, more demanding tasks are reflected in the system as a higher workload.

RESULTS

The findings of this study show that the proposed balanced fuzzy sampling algorithm can distribute the workload both randomly and fairly. Simulations were conducted over three basic scenarios:

Scenario I - Task Assignment in the Same Work Department

A total of 30-night shifts were distributed among 5 nurses working in the same department (e.g., general ward). The simulation results are presented in Table 1, while the initial and total workloads are shown in Table 2.

Table 1. Distribution of a total of 30-night shifts among 5 nurses working in the same department

Simulation Results		
1. Shift: Nurse C	11. Shift: Nurse E	21. Shift: Nurse D
2. Shift: Nurse B	12. Shift: Nurse D	22. Shift: Nurse A
3. Shift: Nurse D	13. Shift: Nurse A	23. Shift: Nurse C
4. Shift: Nurse D	14. Shift: Nurse D	24. Shift: Nurse B
5. Shift: Nurse D	15. Shift: Nurse B	25. Shift: Nurse D
6. Shift: Nurse B	16. Shift: Nurse E	26. Shift: Nurse E
7. Shift: Nurse B	17. Shift: Nurse E	27. Shift: Nurse B
8. Shift: Nurse D	18. Shift: Nurse B	28. Shift: Nurse A
9. Shift: Nurse A	19. Shift: Nurse E	29. Shift: Nurse C
10. Shift: Nurse D	20. Shift: Nurse A	30. Shift: Nurse D

Table 2. Number of initial and total workloads

Nurse	Previous Workload	Total Workload (New + Previous)
A	20	25
B	18	25
C	22	25
D	15	25
E	19	24

Table 1 and Table 2 show that the total workload of all nurses is approximately equalized. This reveals that the algorithm achieves balance among workers by taking into account past loads.

Scenario II - Sections with Different Difficulty Levels

The shifts were planned in 5 different departments ranging from outpatient clinic (challenge 1) to intensive care (challenge 5). A total of 30-night shifts were distributed among 5 nurses working in different departments. Initial and total workloads are given in Table 3, simulation results are given in Table 4 and the distribution by departments with difficulty scores are given in Table 5.

Table 3. Number of initial and total workloads

Nurse	Previous Workload	Total Workload (New + Previous)
A	20	36
B	18	38
C	22	38
D	15	36
E	19	37

Table 4. Distribution of a total of 30-night shifts among 5 nurses working in different departments

Simulation Results		
1. Shift: Nurse A (SU)	11. Shift: Nurse E (ED)	21. Shift: Nurse D (SU)
2. Shift: Nurse E (OC)	12. Shift: Nurse E (SU)	22. Shift: Nurse E (SU)
3. Shift: Nurse B (OC)	13. Shift: Nurse D (IM)	23. Shift: Nurse A (ED)
4. Shift: Nurse D (ED)	14. Shift: Nurse C (IM)	24. Shift: Nurse C (SU)
5. Shift: Nurse B (SU)	15. Shift: Nurse C (IM)	25. Shift: Nurse D (IC)

6. Shift: Nurse D (OC)	16. Shift: Nurse D (OC)	26. Shift: Nurse B (IC)
7. Shift: Nurse D (IC)	17. Shift: Nurse B (IC)	27. Shift: Nurse C (SU)
8. Shift: Nurse B (IC)	18. Shift: Nurse C (SU)	28. Shift: Nurse E (IM)
9. Shift: Nurse A (ED)	19. Shift: Nurse A (ED)	29. Shift: Nurse E (IC)
10. Shift: Nurse C (IM)	20. Shift: Nurse D (OC)	30. Shift: Nurse A (OC)

*The hospital departments are abbreviated as follows: Outpatient Clinic (OC), Internal Medicine (IM), Surgery (SU), Emergency Department (ED), and Intensive Care (IC).

According to Table 3 and Table 4, the distribution of nurses according to their previous workloads showed that nurses who previously worked in difficult tasks tended to be selected for lighter tasks in new assignments.

The difficulty scores of hospital departments and the total number of assignments are given in Table 5.

Table 5. Difficulty scores of hospital departments and total number of assignments

Deparments	Difficulty Levels	Total Number of Assignments
Outpatient Clinic	1	6
Internal Medicine	2	5
Surgery	3	8
Emergency Department	4	5
Intensive Care	5	6

Table 5 shows that the total workload of all nurses was approximately equalized.

Scenario III – Monthly Workload Monitoring

The total workload for 10 nurses over one month, two months, and three months have been calculated. The initial and total workloads are presented in Table 6.

Table 6. Number of initial and total workloads by month for 10 nurses

Nurse	Previous Workload	Month 1	Month 2	Month 3
H1	20	26	35	43
H2	9	23	36	43
H3	22	31	38	44
H4	15	26	36	46
H5	19	29	36	44
H6	19	29	34	44
H7	10	26	36	44
H8	22	29	36	42
H9	18	27	33	46
H10	20	27	35	42

The system balances the workload over time by assigning fewer tasks to heavily loaded employees and more tasks to those with lighter workloads. These results demonstrate the algorithm's effectiveness in achieving workload equality in the long term.

The number of shifts assigned by the algorithm according to hospital departments is presented in Table 7.

Table 7. Number of shift assignments by hospital departments per month

Months	Outpatient Clinic	Emergency Department	Surgery	Internal Medicine	Intensive Care	Total
1	5	8	7	3	7	30

2	12	10	17	9	12	60
3	19	17	24	15	15	90

Table 7 shows that the number of appointments of hospital departments varies on a monthly basis.

DISCUSSION AND CONCLUSION

In this study, a balanced fuzzy sampling-based algorithm developed to ensure fair task assignments among employees with fixed salaries is introduced and tested under different scenarios. Unlike classical random assignment methods, the proposed model dynamically updates each employee's task selection probability by considering their past workload data, thereby enabling both short-term randomness and long-term equitable distribution. In this respect, the model offers a holistic approach to workforce planning that considers not only mathematical fairness but also psychological and organizational balance.

The simulations show that the proposed algorithm successfully balances the workload over time. Especially in night shift scheduling for nurses working in the same or different departments, it was observed that employees with high workload in the previous weeks were directed to less difficult tasks in subsequent assignments, while employees with low workload were made more active in the system. This contributes to strengthening the perception of justice within the organization and creates a mechanism that can increase employee satisfaction.

The findings indicate that the proposed algorithm can be successfully applied not only in the healthcare sector but also in other industries involving shift workers, such as manufacturing, logistics, and security. If integrated into user-friendly software, the model has the potential to become an effective decision support tool for human resources departments. Future studies are recommended to extend the difficulty scores to encompass not only physical but also psychological burdens and to validate the model using large-scale corporate datasets.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

The entire study was conducted by a single author. All stages of the research—including the conception of the idea, literature review, development of the algorithm, execution of simulations, data analysis, interpretation of the results, and writing of the manuscript—were carried out solely by the author.

Selective Classification with Bayesian Neural Networks: A Posterior-Based Uncertainty Approach (1098)

Fatih Sağlam[1*]

[1]Ondokuz Mayıs University, Faculty of Science, Department of Statistics, Türkiye

*Corresponding author e-mail: fatih.saglam@omu.edu.tr

Abstract

This study investigates how classification performance can be improved through selective learning by leveraging the posterior distributions provided by Bayesian Neural Networks (BNNs). Selective learning is a strategy in which the model abstains from making predictions in uncertain situations and only produces outputs when it is sufficiently confident. The ability of Bayesian models to quantify predictive uncertainty via posterior distributions makes them naturally suited for this approach. In this work, a BNN was trained on a two-dimensional, binary classification dataset. For each test instance, 1000 posterior samples of the positive class probability were drawn. Using the 2.5% and 97.5% quantiles of these samples, credible intervals were constructed to determine whether the classification threshold fell within or outside the posterior interval. Predictions were made only on the "confident" instances, where the threshold lay outside the interval, while "risky" instances were abstained from. Selective performance metrics such as accuracy, F1 score, MCC, balanced accuracy, and Youden's index were computed across 50 different confidence levels, represented by varying α values in the range [0, 1]. The results demonstrate that abstaining from predictions on high-uncertainty samples can significantly improve model performance on the remaining confident subset. Furthermore, contour plots of posterior means and uncertainty regions were generated to visually interpret the model's decision behavior. Importantly, this approach offers a principled framework that can be integrated into predictive software systems to ensure that predictions are issued only when the model is sufficiently certain. This level of uncertainty-aware decision-making is difficult to achieve using conventional machine learning algorithms, highlighting the added value of Bayesian methods in real-world applications requiring reliability and interpretability.

Keywords: *Bayesian Neural Networks, Selective Classification, Uncertainty Estimation, Posterior Sampling, Credible Intervals*

INTRODUCTION

Machine learning (ML) is a subfield of artificial intelligence that enables computer systems to learn from experience and improve their performance without being explicitly programmed (Mitchell, 1997). These methods, particularly in prediction problems such as classification and regression, can achieve high accuracy rates. However, classical machine learning models generally do not provide information about the uncertainty of their decisions, which poses a significant problem in domains where incorrect predictions may have serious consequences (e.g., medicine, finance, forensic informatics) (Gal, 2016).

Uncertainty estimation reveals the situations in which a model is not confident, allowing automatic decisions to be deferred to human experts in uncertain regions. This approach is referred to as selective classification or risk-based prediction, and it establishes a cooperative mechanism between the model and the human expert (Geifman & El-Yaniv, 2017). The model only makes predictions when it is sufficiently confident; otherwise, it abstains.

Classical (frequentist) machine learning approaches typically focus solely on epistemic uncertainty—uncertainty due to limited knowledge of the training data. However, aleatoric uncertainty—noise inherent in the data—is often overlooked by these approaches. Bayesian machine learning methods model both types of uncertainty, providing a more comprehensive estimation framework (Kendall & Gal, 2017). As a result, model outputs are not merely point estimates but full distributions, allowing for statistical assessments such as confidence intervals.

Selective classification has recently become a prominent area of research to enhance the reliability of deep learning models. Geifman and El-Yaniv (2017) proposed selective prediction strategies for deep neural networks, offering a framework in which the model decides whether to predict or abstain based on its confidence level. In this study, uncertainty was estimated using softmax outputs, and selective accuracy was analyzed via confidence-ranked curves. The same authors later introduced a new architecture, "SelectiveNet," optimized specifically for selective classification tasks (Geifman & El-Yaniv, 2019). Meanwhile, studies using Bayesian approaches for uncertainty estimation identified risky regions around decision boundaries via posterior distributions (Gal & Ghahramani, 2016; Kendall & Gal, 2017). These studies have shown that deep Bayesian models, which account for both epistemic and aleatoric uncertainty, perform more effectively in uncertainty-sensitive applications such as selective learning.

Bayesian Neural Networks (BNNs) provide a framework for uncertainty by performing posterior sampling using stochastic techniques like dropout (Gal, 2016). Within this framework, selective classification can be achieved by evaluating the model's predictive intervals and making predictions only when the credible interval is sufficiently narrow. Lakshminarayanan et al. (2017) demonstrated that even non-Bayesian methods such as ensemble deep networks can estimate predictive uncertainty, though Bayesian methods—capable of sampling from true posteriors—are considered more reliable, especially in critical decision support systems (Ovadia et al., 2019).

In terms of applications, selective classification is increasingly adopted in high-risk areas such as medical diagnosis (Caruana et al., 2015), autonomous driving (Michelmore et al., 2018), cybersecurity (Thulasidasan et al., 2019), and credit risk prediction (Khosravi et al., 2011). In such domains, basing decisions only on confident predictions significantly enhances the reliability of systems.

In this study, a selective learning approach is developed using Bayesian Neural Network classifiers (BNNET). For the modeling process, the Banana dataset—sourced from the KEEL data repository (Alcalá-Fdez et al., 2011)—was chosen due to its two-dimensional structure and class overlap. In the first phase, BNNET models trained with different hyperparameter settings were compared, and the model with the highest area under the curve (AUC) was selected. This selection prioritized the model's capacity for making reliable decisions rather than just predictive accuracy. The decision threshold of the selected model was determined based on ROC analysis, aiming to maximize potential performance. Using the obtained threshold, posterior predictive distributions for the positive class were represented with 1000 samples. From these distributions, 95% credible intervals were constructed, and uncertainty was evaluated for each test instance. If the decision threshold fell outside the interval, the model was deemed confident in its decision; otherwise, it indicated uncertainty.

Within this framework, selective BNNET models were created to make predictions only for "confident" instances, and their performance was analyzed using various metrics (accuracy, F1 score, MCC, balanced accuracy, Youden index). The findings suggest that avoiding uncertain instances can significantly enhance classification performance on the remaining confident cases.

MATERIAL AND METHOD

This study aims to model uncertainty in classification problems using Bayesian Neural Networks (BNNET) and to develop selective learning strategies based on these uncertainties. While traditional machine learning algorithms produce only point estimates, Bayesian models can provide both predictions and associated uncertainty levels. This capability enables the development of a reliability-based classification approach in which the model makes predictions only for instances with a sufficient confidence level.

At the core of the study lies the sampling from the posterior distributions obtained from neural networks trained with a Bayesian approach. Through these samples, prediction uncertainties are estimated and used to guide selective decision-making processes. Accordingly, the methodological framework consists of three main components:(i) training the model using Bayesian Neural Networks and performing posterior sampling, (ii) evaluating prediction uncertainties through quantile-based analysis, and (iii) developing selective learning strategies that make predictions only for instances with adequate confidence levels.

Bayesian Neural Networks

Bayesian Neural Networks (BNNs) aim to explicitly represent uncertainty in a model's decisions by modeling the deterministic parameters (weights and biases) of traditional neural networks as probabilistic variables (MacKay, 1992). In a conventional neural network, the prediction function is expressed as:

$$\hat{y} = f(\mathbf{x}, \mathbf{w}) = \phi_L \circ \phi_{L-1} \circ \cdots \circ \phi_1(\mathbf{x})$$

Here, $\mathbf{x}$ is the input vector, $\mathbf{w}$ is the set of parameters (weights and biases), and $\phi_l(\cdot)$ represents the composition of linear and activation operations in each layer. In the classical approach, $\mathbf{w}$ has a fixed value, whereas in the Bayesian approach, it is represented by a probability distribution: $\mathbf{w} \sim p(\mathbf{w})$.

The goal in the Bayesian framework is to obtain the posterior distribution of the parameters conditioned on the data:

$$p(w \mid D) = \frac{p(\mathcal{D} \mid \boldsymbol{w})\, p(\boldsymbol{w})}{p(\mathcal{D})}$$

Here, $p(\mathcal{D} \mid \boldsymbol{w})$ is the likelihood, $p(\boldsymbol{w})$ is the prior distribution (typically assumed as $\mathcal{N}(0, \sigma^2 I)$), and $p(\mathcal{D})$ is the marginal likelihood or evidence, which is usually intractable.

Since the posterior cannot be computed analytically, approximate inference methods are employed. The most common approach is *variational inference*, where a parametric distribution, $q(\boldsymbol{w}; \theta)$ is defined and optimized to approximate the true posterior. The objective is to minimize the Kullback-Leibler (KL) divergence:

$$\mathrm{KL}\big(q(\boldsymbol{w}; \theta) \parallel p(\boldsymbol{w} \mid \mathcal{D})\big) = E_q[\log q(\boldsymbol{w}) - \log p(\boldsymbol{w} \mid \mathcal{D})]$$

This divergence is indirectly minimized by maximizing the variational lower bound (ELBO):

$$\mathcal{L}(\theta) = E_{q(\boldsymbol{w};\theta)}[\log p(\mathcal{D} \mid \boldsymbol{w})] - \mathrm{KL}\big(q(\boldsymbol{w}; \theta) \parallel p(\boldsymbol{w})\big)$$

This expression accounts for both model fit (log-likelihood) and prior conformity (KL divergence). During training, the learned parameters include not just the mean (μ_i) of the weights but also their variance σ_i^2, so that each weight w_i is modeled as $w_i \sim \mathcal{N}(\mu_i, \sigma_i^2)$.

Using this setup, predictions are made by sampling $\boldsymbol{w} \sim q(\boldsymbol{w}; \theta)$ during feedforward inference. For an input $\boldsymbol{x}$, the outputs from T posterior weight samples $\{\hat{y}^{(t)}\}_{t=1}^{T}$ are used to compute predictive mean, $\bar{y} = \frac{1}{T}\sum_{t=1}^{T} \hat{y}^{(t)}$,and predictive variance, $\mathrm{Var}(\hat{y}) = \frac{1}{T}\sum_{t=1}^{T}\left(\hat{y}^{(t)} - \bar{y}\right)^2$.

In BNNs, several uncertainty estimation metrics are used for classification tasks based on the analysis of class probability distributions derived from posterior samples. One of the most widely used metrics. Based on the average prediction $\overline{\boldsymbol{p}} = \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{p}^{(t)}$ entropy is computed as:

$$H[\overline{\boldsymbol{p}}] = -\sum_{k=1}^{K} \bar{p}_k \log \bar{p}_k$$

This reflects the total (epistemic + aleatoric) uncertainty. A high value indicates that class probabilities are close to each other, suggesting low confidence (Gal & Ghahramani, 2016).

A more direct measure of confidence, representing the dominance of the most frequently predicted class. It is computed as:

$$\mathrm{VR} = 1 - \frac{1}{T}\max_{k}\sum_{t=1}^{T} I\left[\arg\max \boldsymbol{p}^{(t)} = k\right]$$

Here, I is the indicator function. If the same class is predicted in all samples, VR = 0, indicating high confidence (Geifman & El-Yaniv, 2017).

Another uncertainty measure, Mutual Information is used specifically to isolate epistemic uncertainty. It is computed as the difference between the predictive entropy and the mean entropy of the posterior samples:

$$\mathrm{MI} = H[\overline{\boldsymbol{p}}] - \frac{1}{T}\sum_{t=1}^{T} H\left[\boldsymbol{p}^{(t)}\right]$$

A large difference indicates uncertainty due to lack of knowledge about model parameters (Houlsby et al., 2011; Kendall & Gal, 2017).

Margin measures the difference between the top two average class probabilities using $\mathrm{Margin} = \bar{p}_{(1)} - \bar{p}_{(2)}$. A small margin implies the prediction is near the decision boundary and thus uncertain. A large margin indicates higher confidence (Settles, 2012). And, one of the simplest metrics, confidence score, is calculated as $\mathrm{Confidence} = \max_{k} \bar{p}_k$. A value close to 1 indicates high confidence, while a value far from 1 suggests high uncertainty. Some studies also use $1 - \max_{k} \bar{p}_k$ directly as the uncertainty score (Lakshminarayanan et al., 2017). Each of these metrics captures different aspects of uncertainty and can be used complementarily in confidence-based decision-making scenarios such as selective classification.

Selective Learning

Selective learning is a classification approach that allows a model to make predictions only for instances in which it has a sufficient level of confidence. In this method, the model abstains from prediction when uncertainty is high, thereby reducing the risk of error. As a result, performance metrics such as accuracy can be improved at the expense of coverage (El-Yaniv & Wiener, 2010).

A selective classifier makes a prediction only if the confidence function $g(x)$ exceeds a specified threshold τ:

$$f_s(x) = \begin{cases} f(x) & \text{If } g(x) \geq \tau \\ \text{abstain} & \text{otherwise} \end{cases}$$

The confidence function $g(x)$ can be derived from various uncertainty measures such as maximum class probability, margin, entropy, or posterior confidence intervals. In this study, the selection decision is based on the posterior confidence interval obtained from model samples.

Proposed Selective Learning Approach

In this study, the selective classification decision is based on the posterior distribution of the positive class probability predicted by the Bayesian neural network. For each test instance, T posterior samples are drawn from the model, and these samples are used to obtain the probability distribution for the positive class. From this distribution, the 95% credible interval (CI) is computed as:

$$\text{CI}_{95\%} = [q_{2.5\%}, q_{97.5\%}]$$

Here, $q_{2.5\%}$ and $q_{97.5\%}$ are the 2.5th and 97.5th percentiles of the posterior distribution, respectively. The selection decision is made by checking whether this confidence interval contains the predefined decision threshold t. kapsayıp kapsamadığına göre verilir. If $t \in \text{CI}_{95\%}$, the model is considered uncertain about its prediction, and no prediction is made for that instance. If $t \notin \text{CI}_{95\%}$, the model is considered sufficiently confident, and a prediction is made.

This method enables a direct assessment of posterior uncertainty and grounds the selective prediction decision in statistical confidence. In this way, predictions are made only for instances that fall outside the decision boundary—i.e., those associated with high confidence. The strategy aims to improve performance metrics such as accuracy, F1 score, and MCC, even at the cost of reduced coverage.

Application

In this section, the proposed selective learning approach is evaluated using Bayesian Neural Networks. The implementation process is addressed under three main headings. First, information about the dataset used is provided. Then, the architecture of the Bayesian model, the training procedure, and the decision mechanism for selective prediction are explained under the application design heading. Finally, the coverage and performance metrics obtained at different confidence levels are reported under the findings section.

Dataset

The dataset used in this study is the banana dataset, a commonly used benchmark available in the KEEL data repository (Alcalá-Fdez et al., 2011). The dataset contains $p = 2$ numerical features and two class labels. The entire dataset was randomly split into 80% training and 20% test sets, resulting in 2,112 observations in the training set and 528 in the test set. To introduce class imbalance, the sampling was adjusted such that the ratio of negative to positive class instances is $\frac{n^{\text{neg}}}{n^{\text{pos}}} = 9$.

Experimental Design

In this study, a Bayesian Neural Network (BNN) is employed as the classification model. The model consists of one input layer, two hidden layers (each with 16 neurons and ReLU activation), and an output layer with two units. All layers are Bayesian linear layers, where both the mean and variance parameters are learned for each weight. As a result, the model contains a total of 328 learnable parameters.

The loss function includes the standard cross-entropy (CE) term along with a Kullback-Leibler (KL) divergence term that penalizes the divergence between the prior and posterior distributions:

$$\mathcal{L} = \text{CE} + \lambda \cdot \text{KL}, \quad \lambda = 0.001$$

Here, CE refers to the cross-entropy loss. The model is trained for 300 epochs using the Adam optimization algorithm. After training, the proposed selective learning approach is applied, and the prediction decision for each test instance is made based on the posterior confidence interval.

The model's performance is evaluated in terms of coverage levels corresponding to different α thresholds. In addition to accuracy (ACC), performance metrics sensitive to class imbalance—namely balanced accuracy (BACC), F1 score, Matthew's correlation coefficient (MCC), and Youden's index (YI)—are also used.

Findings

The trained Bayesian Neural Network model was evaluated on the test data, and the resulting ROC curve is presented in Figure 1. The Area Under the Curve (AUC) was calculated as 0.9397. Based on the points obtained from the ROC curve, the optimal prediction threshold was determined to be $= 0.07$. This threshold was selected by identifying the point where the true positive rate (TPR) and true negative rate (TNR) were closest to each other, taking class imbalance into account. At this threshold, the TPR was 0.887 and the TNR was 0.854.

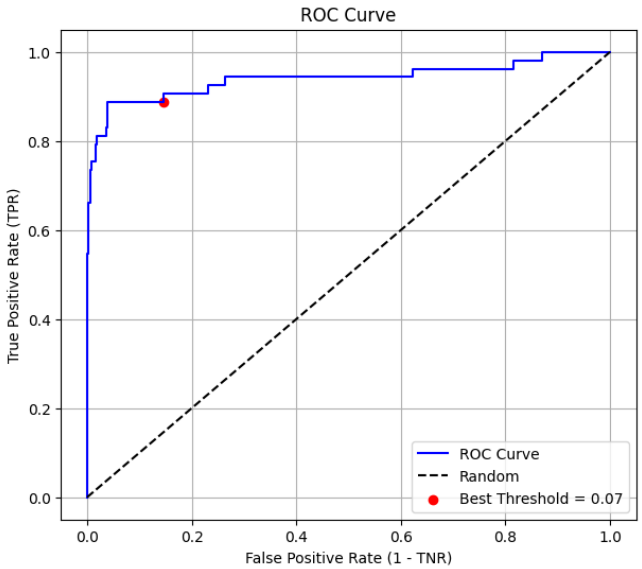

Figure 1. ROC Curve of the Trained Model

Using the prediction threshold $= 0.07$, the model's performance metrics under full coverage were calculated as follows: ACC = 0.858, BACC = 0.871, MCC = 0.530, F1 = 0.556, and YI = 0.741. As part of the selective classification approach, selective predictions were made using the confidence interval corresponding to a significance level of $= 0.05$. Figures 2 and 3 illustrate examples of confident and uncertain predictions for the positive and negative classes, respectively. Uncertainty assessment was based on whether the predicted probability fell within the 95% confidence interval (i.e., the 2.5%–97.5% quantile range). Predictions falling within this interval were considered uncertain and thus rejected; otherwise, a prediction was made.

As a result of selective prediction with $\alpha = 0.05$ applied to the entire test set, a coverage of 80.49% was achieved. Within this coverage, model performance metrics improved significantly, yielding ACC = 0.911, BACC = 0.923, MCC = 0.691, F1 = 0.712, and YI = 0.848.

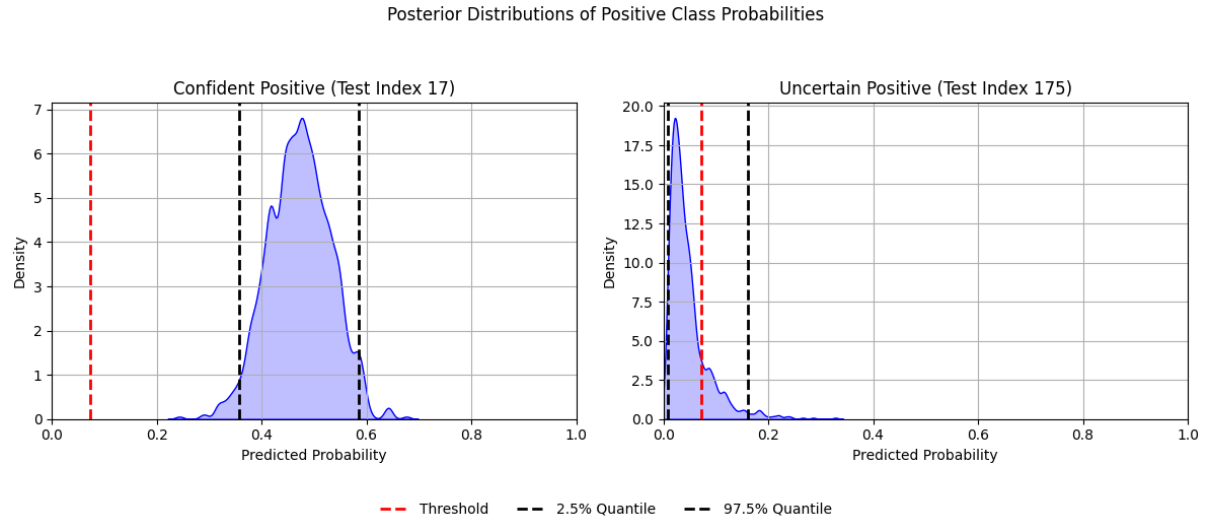

Figure 2. Examples of Confident and Uncertain Predictions for the Positive Class

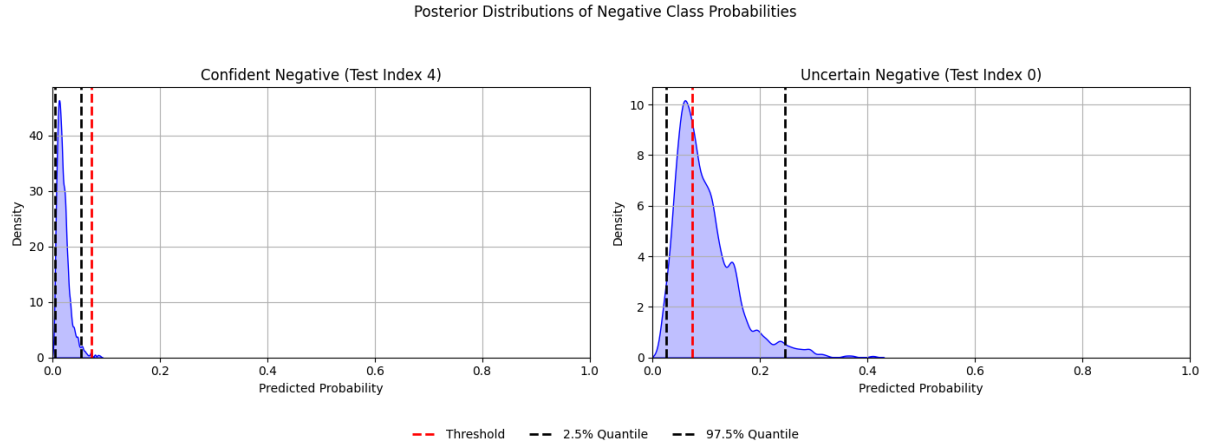

Figure 3. Examples of Confident and Uncertain Predictions for the Negative Class

Figure 4 illustrates the spatial distribution of the model's positive class probabilities using a contour plot. Yellow regions represent areas with a high probability of the positive class, while blue regions indicate areas dominated by the negative class. Black dots denote uncertain regions where the model abstains from prediction under the selective classification framework. The red dashed line represents the decision boundary corresponding to the threshold $t = 0.07$. Upon examination, it is evident that the abstention regions coincide with areas where the classes overlap—regions with a high risk of misclassification. This contributes to a reduction in erroneous predictions.

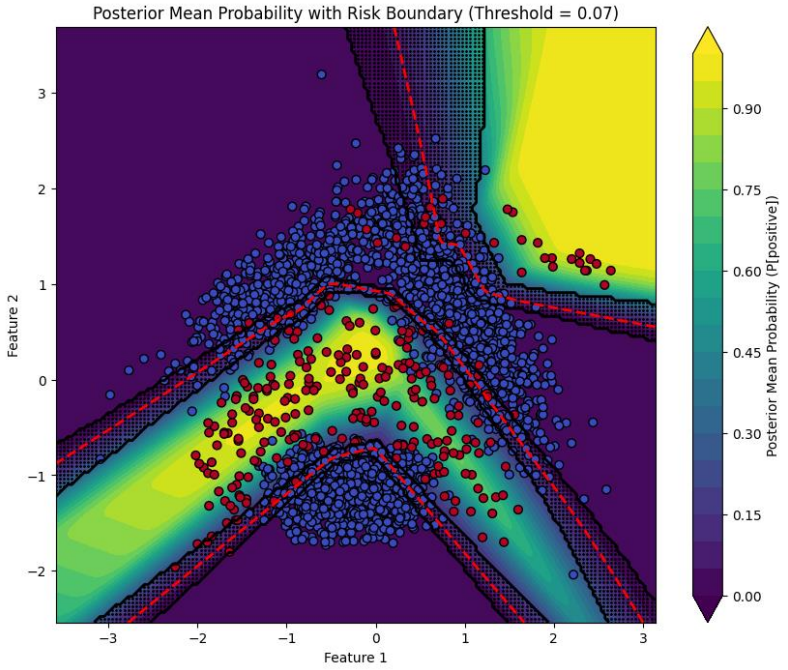

Figure 4. Contour Plot of Positive Class Probabilities with Selective Prediction Regions

Coverage and performance values obtained at different α levels are presented in Table 1. Here, $\alpha = 1$, represents the classical classification scenario in which the model makes predictions for all instances, while $\alpha = 0$ corresponds to the strictest selective setting, where predictions are made only for instances whose decision threshold falls entirely outside the posterior samples. At $\alpha = 0$, the model made predictions on only 66.10% of the data subset; however, this low coverage yielded the highest performance metrics. In this case, the model abstained from predicting in 33.90% of the instances.

Figure 5 presents a line chart showing the variation of different performance metrics with respect to the percentage of rejected instances. The findings reveal that the proposed selective learning approach systematically and consistently improves accuracy, F1 score, MCC, and other metrics as coverage decreases.

Table 1. Model performance results at different α levels

α	CI	Rejection rate	Coverage	ACC	BACC	MCC	F1	YI
1	0.00%	0.00%	100.00%	0.858	0.871	0.556	0.538	0.742
0.99	1.00%	0.38%	99.62%	0.861	0.873	0.563	0.544	0.745
0.95	5.00%	0.76%	99.24%	0.865	0.874	0.570	0.551	0.749
0.9	10.00%	1.89%	98.11%	0.875	0.880	0.591	0.571	0.760
0.75	25.00%	3.60%	96.40%	0.886	0.894	0.618	0.601	0.788
0.5	50.00%	6.44%	93.56%	0.891	0.896	0.635	0.615	0.793
0.25	75.00%	11.17%	88.83%	0.893	0.914	0.653	0.638	0.828
0.1	90.00%	15.91%	84.09%	0.905	0.921	0.691	0.673	0.841
0.05	95.00%	19.51%	80.49%	0.911	0.923	0.712	0.691	0.847
0.01	99.00%	26.89%	73.11%	0.922	0.930	0.758	0.733	0.860
0	100.00%	33.90%	66.10%	0.931	0.934	0.789	0.763	0.868

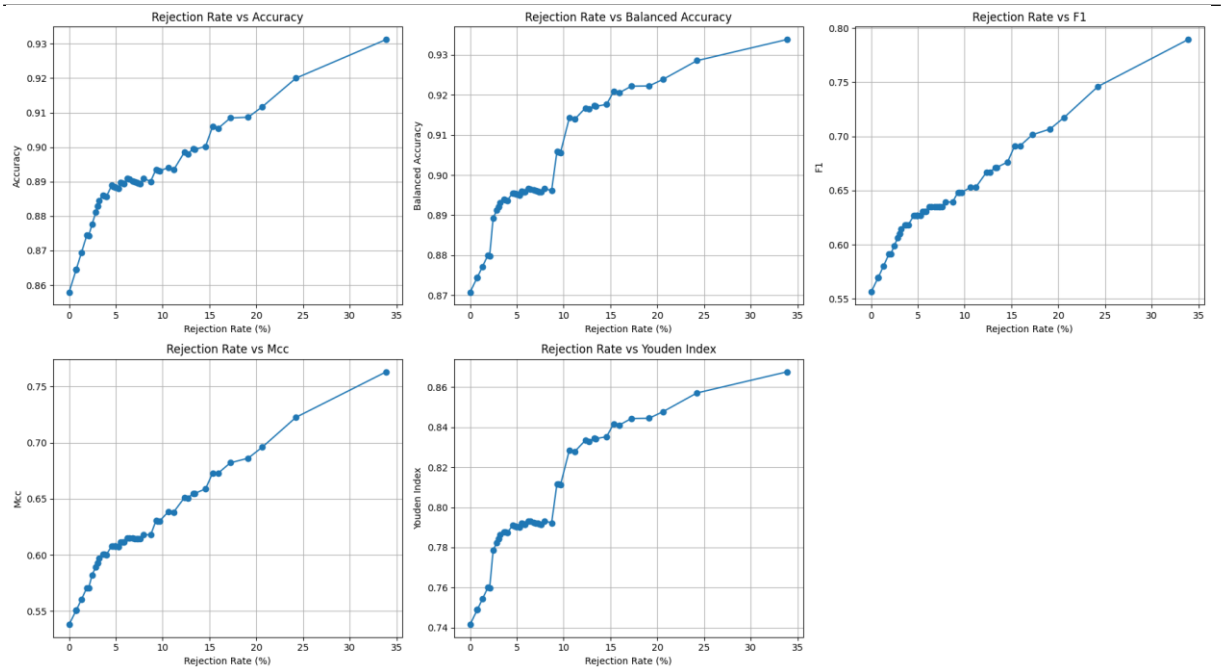

Figure 5. Line Graph Showing the Relationship Between Performance Metrics and Rejection Rates

CONCLUSION AND DISCUSSION

In this study, a selective learning approach was proposed for classification problems, allowing predictions to be made only for instances where the model is sufficiently confident. Using Bayesian Neural Networks (BNNs), prediction decisions were based on 95% posterior credible intervals derived from the predictive distributions. Predictions were abstained in cases deemed uncertain.

In the experimental evaluation conducted on the banana dataset from the KEEL repository, the model's accuracy and its ability to handle uncertainty were thoroughly assessed. Through ROC analysis, the optimal threshold was determined as $t = 0.07$, at which point the model achieved ACC = 0.858, BACC = 0.871, MCC = 0.530, F1 = 0.556, and YI = 0.741. However, when selective prediction was applied—i.e., predictions were made only for instances whose posterior confidence intervals did not contain the threshold—the coverage rate decreased to 80.49%. In return, significantly improved performance metrics were obtained: ACC = 0.911, BACC = 0.923, MCC = 0.691, F1 = 0.712, and YI = 0.848.

These results demonstrate that Bayesian models offer significant advantages over classical neural networks—not only in representing uncertainty but also in integrating that information into the decision-making process. Particularly in problems involving class imbalance, the selective classification strategy helps prevent misclassifications by eliminating the necessity for the model to make predictions on all instances in pursuit of high accuracy.

As illustrated in Figures 4, the model's tendency to abstain from prediction in uncertain regions is concentrated in areas where class overlap occurs. This provides both visual and statistical evidence of how accurately Bayesian uncertainty estimates can represent decision boundaries. Moreover, analyses conducted with different α levels reveal that a meaningful and predictable trade-off can be established between coverage and performance metrics.

In conclusion, this study shows that Bayesian Neural Networks are effective not only in terms of prediction accuracy but also in terms of prediction confidence. Selective learning strategies can offer substantial value, especially in applications where reliability is paramount—such as healthcare, finance, and autonomous systems. Future work may benefit from testing the proposed approach on multi-class problems, more complex real-world datasets, and alternative Bayesian architectures.

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Following Major Damages in Insurance: Threshold-Based Warning and Maximum Damage Distribution (1106)

Fahreddin Kalkan[1*], İsmail Kınacı[1]

[1]Selçuk University, Faculty of Science, Department of Actuarial Sciences, Türkiye

*Corresponding author e-mail: fahreddin.kalkan@selcuk.edu.tr

Abstract

Claims are among the most critical concepts in the insurance industry, directly affecting both operational risk and financial stability. Insurers must manage not only the frequency of claims but also their severity. While the overall distribution of claims is important, special attention must be paid to claims that exceed a certain high threshold, as these large claims can significantly strain an insurer's payment capacity and pose a threat to solvency. In practice, a series of such extreme claims within a short time frame, either within a single policy or across a portfolio, can severely impact the insurer's reserves and trigger the need for emergency risk management measures. To mitigate such risks, insurance companies may establish internal warning or monitoring systems based on the occurrence of high-severity claims. In this study, we consider a monitoring mechanism in which a warning is triggered when the number of consecutive major claims reaches a predetermined number k. The main objective is to investigate the statistical behavior of the maximum claim observed among those that exceed the threshold c up to the warning point. The distribution of this maximum excess claim is theoretically derived, and its properties are analyzed using graphical tools. By understanding the distributional characteristics of such extreme events, insurers can better estimate potential losses and improve risk mitigation strategies. This research contributes to the literature on extreme value theory in insurance and provides practical insights for actuarial modeling and operational risk assessment.

Keywords: *Maximum Claim, Order k Distribution, Claim Severity.*

INTRODUCTION

This study focuses on a stochastic model designed to monitor the progression of claims in an insurance portfolio. In particular, the model assumes that an alert mechanism is activated whenever k successive claim amounts exceed a specified critical level c. This claim process for $k = 3$ is shown in Figure 1. Damages, especially major damages, are quite important in the insurance sector. The successive occurrence of major damages will negatively affect the company's solvency. In this study, damages exceeding a certain c value are considered major damages and it is thought that a warning is created for the company in case of major damages k times in a row. The main purpose here is to obtain the distribution of the largest damage encountered until the warning occurs. This distribution will of course depend on the number of damages that occur until the warning. Damages greater than c are considered as "success" and are indicated by 1, and those smaller than c are considered as "failure" and are indicated by 0. Then, the number of damages that occur until the warning can be expressed as the number of trials performed until k successes are achieved in independent Bernulli trials with a probability of success p. The distribution of the number of trials in this way has been studied by many authors Yalcin and Eryilmaz (2014), Aki and Hirano (1994), Makri and Philippou (2005), Aki and Hirano (1989), Mohanty (1994).

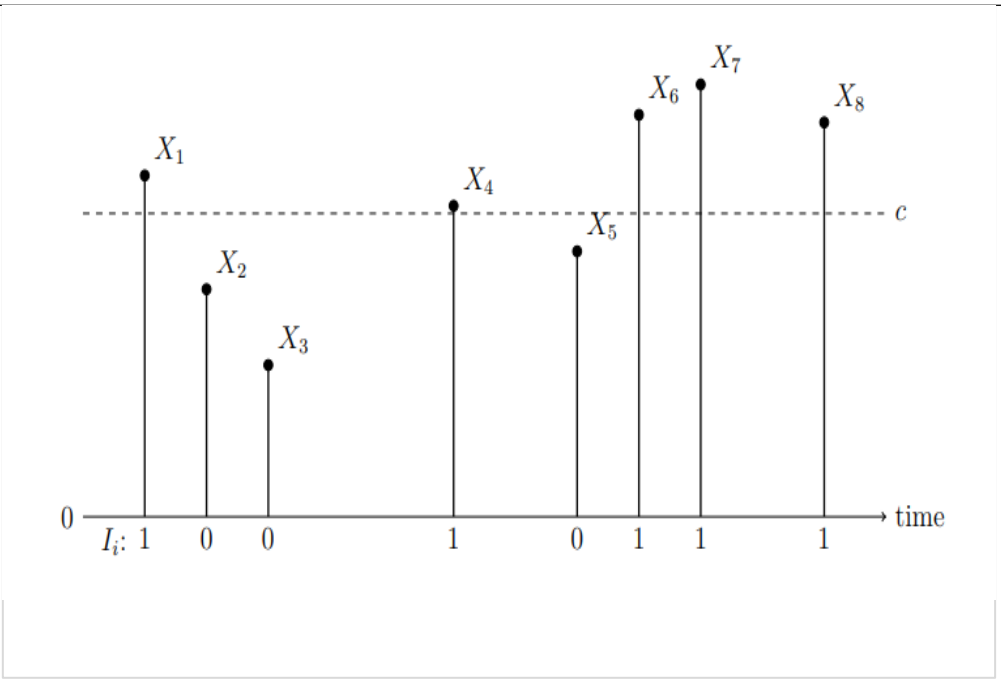

Figure 1. The warning process for $k = 3$

Let X_1, X_2 , ... denote the independent claim amounts distributed as X and $X \sim Exp(\theta)$ (with rate θ). Then the *pdf* and *cdf* of X can be expressed as

$$f_X(x) = \theta e^{-\theta x}, x > 0 \tag{1}$$

$$F_X(x) = 1 - e^{-\theta x}, x > 0, \tag{2}$$

respectively. The success probability in Bernoulli trials is

$$p = P(X > c) = 1 - F_X(c) = e^{-\theta c}. \tag{3}$$

Let N denote the number of trials until the first consecutive k successes obtained. Then the pmf and pgf of N are

$$P(N = n) = \sum_{i=1}^{n-k} p^{n-i} q^i A(i, n-i-k), n \geq k, \tag{4}$$

$$\phi(t) = \frac{p^k (1-pt) t^k}{1 - t + q p^k t^{k+1}} \tag{5}$$

where $q = 1 - p$ and

$$A(i, n-i-k) = \sum_{j=0}^{n-i-k} (-1)^j \binom{n-k-i}{j} \binom{n-k(j+1)-1}{n-k-i-1} \tag{6}$$

Yalcin and Eryilmaz (2014). The distribution of N is called as the geometric distribution of order k and it is denoted by $N \sim G_k(p)$.

Maximum Claim

This section examines the distribution of the maximum claim observed up to the warning point, defined as the moment when the claim exceeds the threshold c for k consecutive times. This random variable is denoted by Y and it can be expressed as

$$Y = max\{X_1, X_2, \dots, X_N\} \mid \{N^* \geq k\} = max\{X_1^*, X_2^*, \dots, X_{N^*}^*\} \quad (7)$$

where X_i^* denotes the claim exceeding c and N^*denotes the number of successes until the first consecutive k successes obtained. It is clear that the random variable X^*can be expressed as

$$X^* = X \mid \{X > c\} \quad (8)$$

and has a left-truncated exponential distribution. Accordingly, the pdf and cdf of the random variable X^* will be in the form of

$$f_X^*(x) = \frac{f_X(x)}{1 - F_X(c)} = \theta e^{-\theta(x-c)}, x > c \quad (9)$$

and

$$F_X^*(x) = \frac{F_X(x) - F_X(c)}{1 - F_X(c)} = 1 - e^{-\theta(x-c)}, x > c. \quad (10)$$

It is clearly seen in Equation 7 that the distribution of the maximum damage Y depends on the distribution of N^*. For this reason, the distribution of the random variable N^* before the distribution of the random variable Y is mentioned here. Aki and Hirano (1989) obtained the distribution of random variable N^* as the shifted geometric distribution of order $k-1$ so that its support begins with k and it is denoted by $N^* \sim G_{k-1}(p, k)$. Then the pmf and pgf of N^* are

$$P(N^* = n) = \sum_{i=1}^{n-k-1} p^{n-i-1} q^i A(i, n-i-k), n \geq k \quad (11)$$

$$\phi^*(t) = \frac{p^{k-1}(1-pt)t^k}{1 - t + qp^{k-1}t^k} \quad (13)$$

Finaly, the pdf of the maximum claim observed up to the warning point, Y, is obtained as

$$f_Y(y) = P(Y = y) = \sum_{n=k}^{\infty} n f_X^*(y) \left(F_X^*(y)\right)^{n-1} P(N^* = n) \quad (14)$$

$$= \theta e^{-\theta(y-c)} \sum_{n=k}^{\infty} n \left(1 - e^{-\theta(y-c)}\right)^{n-1} \sum_{i=1}^{n-k-1} p^{n-i-1} q^i A(i, n-i-k), y > c. \quad (15)$$

Despite the complexity of the probability density function presented in Equation 13, the required computations can be carried out efficiently with a short code implementation. Figure 2 presents the graph of the probability density function f_Y of the random variable Y for various values of θ, given that $k = 4$.

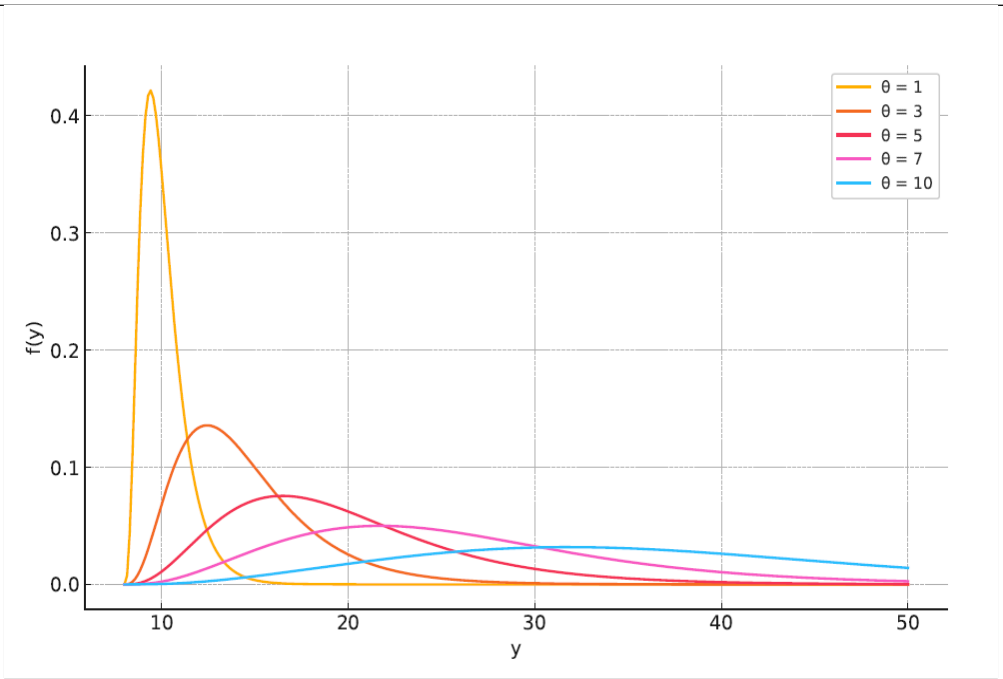

Figure 2. Plot of $f_Y(y)$ for $k = 4$

The density function $f_Y(y)$ is unimodal, exhibiting a single peak shortly after the threshold c. The distribution is right-skewed, indicating that moderate excess claims are more probable, while extremely large claims are rare. This shape aligns with typical insurance loss behavior, where the risk is concentrated around a specific damage level.

DISCUSSION AND CONCLUSION

In this study, we examined a stochastic monitoring framework designed to trigger a warning when major claims defined as those exceeding a high threshold c occur consecutively k times. Recognizing the critical importance of such extreme claim sequences in insurance risk management, we focused on the statistical characterization of the maximum claim observed up to the warning point.

We derived the exact probability distribution of the maximum excess claim and demonstrated that, although the resulting density function is analytically intricate, it can be computed efficiently with basic numerical tools. This makes the model both theoretically robust and practically implementable. The results offer valuable insights for insurers seeking to design early warning systems and assess capital adequacy in the face of clustered large losses. Understanding the behavior of the maximum claim prior to a critical risk event enhances the insurer's ability to respond proactively, allocate reserves appropriately, and refine underwriting or reinsurance strategies.

Future work may extend the model by considering dependent claims, time-varying thresholds, or alternative claim severity distributions, thereby further enhancing its relevance to real-world insurance applications.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

All authors contributed equally to all stages of the study.

Adaptive Smart Antenna MIMO Techniques for Subchannel Optimization in Next-Generation Wireless Networks (Study Case) (1107)

Ardit Dervishi[1*], Algenti Lala[1], Olimpjon Shurdi[1]

[1]Polytechnic University of Tirana, Faculty of Information & Technology, Electronic & Telecommunication Engineering, Albania

*Corresponding author e-mail: ardit.dervishi@fti.edu.al

Abstract

The ever-growing demand for high-performance wireless communication has intensified research efforts in optimizing wireless systems, particularly in environments with stringent energy and bandwidth constraints. Wireless Sensor Networks (WSNs) and adaptive smart antenna MIMO systems both face critical challenges related to energy efficiency and data transmission performance. In WSNs, limited battery resources require efficient energy management strategies to prolong network longevity, while in MIMO-based wireless communication, subchannel optimization is crucial for maximizing spectral efficiency and minimizing power consumption. This research will introduce an integrated solution that combines adaptive spatial diversity with advanced data compression. Techniques such as ALDC and FELACS are evaluated within a MIMO-OFDM context using MATLAB-based simulations. The results from the study aims to minimize transmission overhead in MIMO-based wireless networks. ALDC employs Huffman coding with adaptive encoding tables to enhance energy savings, while FELACS utilizes Golomb-Rice coding, which is particularly effective in reducing variations in dataset patterns. In our study will use different modulation types: QPSK, 16-QAM using OFDM mapping, where from output will check quantity bit savings from ALDC and FELACS in realistic payloads. These techniques are adapted to improve MIMO subchannel transmission efficiency by reducing redundant data transmission and optimizing channel coding strategies for real-time communication. This paper gives contribution to developing intelligent, low-power wireless systems applicable in IoT infrastructures and mobile vehicular communication.

Keywords: *Huffman Coding, subchannel, FELACS, MIMO, WSN*

INTRODUCTION

Nowadays, the 21st century society is characterized by a global demand for wireless communications. The main scientific research challenges in this field involve the development of broadband wireless systems for multimedia services in heterogeneous and propagation-challenging environments, where the limited availability of radio frequency spectrum represents a significant obstacle. The most promising approach to address these challenges is the use of multiple antennas, combined with advanced multicarrier signal modulation techniques such as OFDM (Orthogonal Frequency Division Multiplexing) or MC-CDMA (Multi-Carrier Code Division Multiple Access). Recent studies have shown that wireless systems utilizing Multiple Input – Multiple Output (MIMO) technology can achieve high data transmission rates essentially in direct proportion to the number of antennas—without increasing transmission bandwidth or power. As a result, there is a growing global research interest in MIMO systems.

To address these shortcomings, Multiple-Input Multiple-Output (MIMO) technology has emerged as a transformative solution. MIMO systems (Björnson, 2014, December) utilize multiple antennas at both the transmitting and receiving ends, allowing the simultaneous transmission of multiple data streams. This technique enhances throughput, improves coverage, and significantly increases spectral efficiency (A. Goldsmith). When combined with Orthogonal Frequency Division Multiplexing (OFDM) a modulation strategy that divides the spectrum into narrowband subcarriers MIMO systems become more robust to frequency-selective fading and inter-symbol interference, making MIMO-OFDM (Anouar, 2024) the foundation for advanced standards such as LTE, Wi- A critical enabler of MIMO performance is the use of smart antenna systems, which enhance communication quality by dynamically controlling the radiation pattern of antennas. Smart antennas typically operate in two configurations: switched-beam systems, which select from a predefined set of directional beams based on user location, and adaptive beamforming systems, which continuously adjust beam direction in real time based on channel feedback (Shivapanchakshari). Although adaptive beamforming achieves higher performance, it also introduces increased processing complexity and energy consumption (A. Paulraj).

To further improve resilience in multipath-rich environments (Borges, 2021), spatial diversity techniques are employed within MIMO architectures. Methods such as transmit diversity (e.g., the Alamouti scheme) and receive diversity (e.g., maximal ratio combining) allow the system to transmit redundant signal replicas across multiple paths or antennas. This improves signal robustness against fading and interference, ensuring more reliable communication (Z. Wu, Jun. 2018,). These techniques are particularly valuable in mission-critical scenarios such as vehicular ad hoc networks (VANETs) and industrial IoT, where consistent and low-latency communication is essential (Q. Nadeem, 2017,).

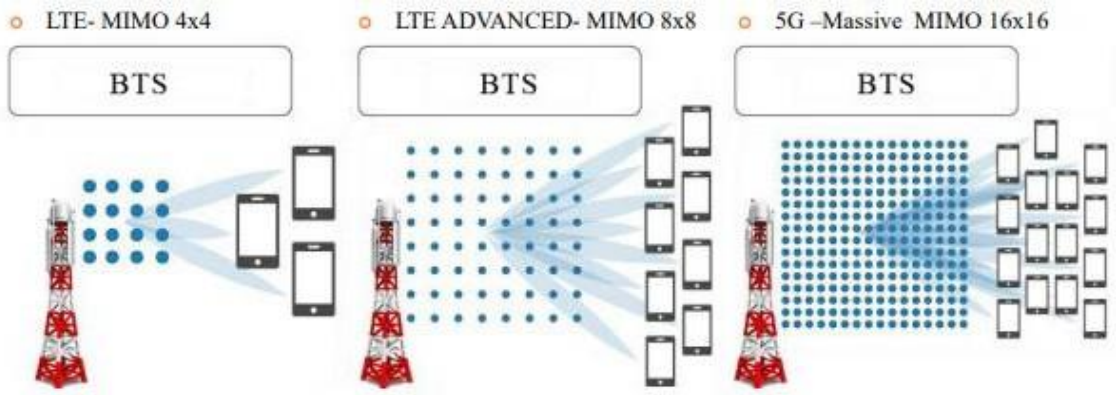

Figure 1. Relationship between MIMO complexity vs. spectral efficiency and users

ADAPTIVE TECHNIQUES

Adaptive Lossless Data Compress Algorithm

Adaptive compression techniques dynamically modify their parameters according to variations in the input data source. In this paper[22], a proposal of the Adaptive Lossless Data Compression (ALDC) scheme, which employs two distinct adaptive lossless entropy compression (ALEC) coding methods for efficient, lossless data compression. These coding methods termed the 2-Huffman Table ALEC and the 3-Huffman Table ALEC were initially developed and described in our earlier research focused on enhancing compression efficiency specifically for wireless sensor networks.

Both ALEC methods utilize adaptive Huffman coding strategies, with the 2-Huffman Table ALEC adaptively selecting between two Huffman tables, and the 3-Huffman Table ALEC adaptively using three Huffman tables, based on real-time data characteristics. These Huffman tables were meticulously constructed through extensive experimentation involving diverse real-world datasets collected from wireless sensor nodes, each exhibiting varying degrees of data correlation. Tables 1, 2, and 3 detail the specific Huffman tables developed for these adaptive coding methods, optimized to handle distinct data correlation scenarios encountered in practical wireless sensor network deployments.

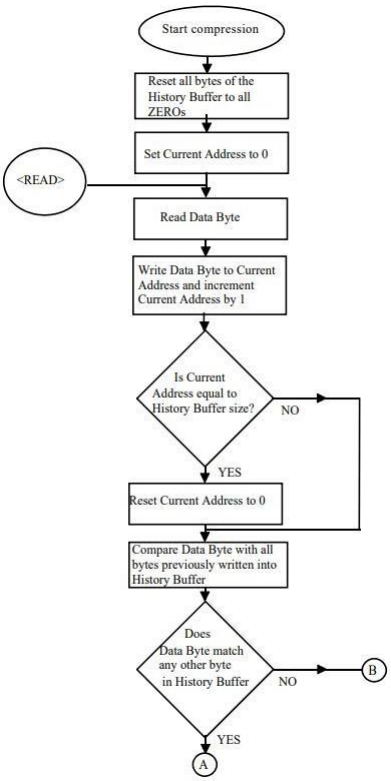

Figure 2. Flowchart scheme of ALDC algorithm

Pseudocode Steps:

1. *Map input data symbols (e.g., bytes or quantized data) to binary.*
2. *Track symbol frequency dynamically (sliding window or update after each block).*
3. *Build/update Huffman tree from frequencies.*
4. *Encode binary data using updated Huffman table.*
5. *Send Huffman table (or updates) as header for decoding*

//Adaptive is the function that determines the boundary for the decision regions.

//Compute F, the total of the absolute value of residues.

//K denotes the residue length. Call Residual() and calculate K.

//Determine the F region. IF F < 3K

//Set code ID for using two Huffman Tables A, B ID → '0'. ELSE IF 3K < F <= 12K

//Set ID for using three Huffman tables A, B and C ID → '1'. ENDIF RETURN ID.

The adaptive function helps in structuring the sum of absolute values, residues length by generating a code option identifier.

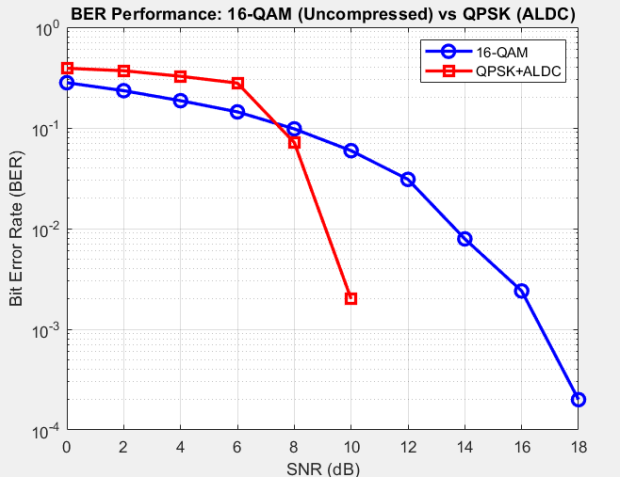

Figure 4. Difference in performance and BER analysis between 16-QAM and QPSK modulation using ALDC

The comparative BER analysis demonstrates that while uncompressed 16-QAM offers slightly lower bit error rates at low SNR levels, the integration of ALDC compression with QPSK modulation quickly surpasses its performance as channel conditions improve. The QPSK+ALDC approach exhibits a sharp decline in BER at moderate-to-high SNR values, achieving near error-free transmission with far fewer transmitted bits. This result highlights the dual benefit of enhanced spectral and energy efficiency alongside robust data integrity in favorable channel conditions. Although the compressed transmission system is somewhat more susceptible to errors in low SNR scenarios due to the cascading impact of bit errors during decompression, its superior performance at higher SNR makes it an attractive solution for bandwidth- and energy-constrained wireless sensor network applications, where minimizing data volume without sacrificing reliability is essential.

Fast Efficient Lossless Adaptive Compression Scheme

The Fast and Efficient Lossless Adaptive Compression Scheme (FELACS) is a block-based, entropy coding algorithm tailored for wireless sensor networks (WSNs), built upon the Golomb–Rice coding method. The scheme assumes that sensor samples fall within the interval [0, $2N-1$] where NNN represents the resolution of the node's analog-to-digital converter (ADC).

To make FELACS suitable for low-power and resource-constrained WSN environments, several optimizations were introduced over traditional Golomb–Rice coding:

Limited Code Set: Based on empirical analysis of sensor data residue distributions, FELACS restricts its coding options to eight Golomb–Rice codes, where $k \in \{0, 1, 2, \ldots, 7\}$This limitation reduces overhead by encoding the selected k-value using just a 3-bit identifier, while still effectively compressing data with source entropy between 1.5 and 9.5 bits.

Block-Based Encoding for Resilience: Each data block is compressed (Al-Jobouri, 2015) independently to improve robustness against packet loss due to interference or congestion. The first sample in every block serves as a reference and is sent uncompressed using N-bit binary representation. The remaining samples are encoded using the selected k-value and appended to the packet. This ensures that even in the presence of packet drops, each block can be individually decoded.

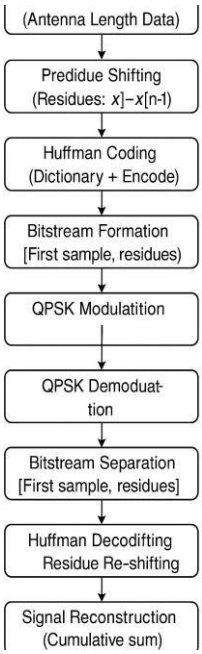

Figure 5. *QPSK modulation block scheme using ALDC algorithm*

Efficient Code Signaling: To inform the decoder of the encoding parameters, a 3-bit binary code is used to signal the chosen Golomb–Rice code. For instance, if k=2 is used, the identifier "010" is appended to the block for decoding reference.

Lightweight k-Value Selection: (Ketshabetswe, 2024) Instead of evaluating all possible code options per block as in traditional Rice encoding, FELACS uses a simplified approach by summing the residuals (excluding the reference sample) within a block. This sum helps estimate the optimal k- value with minimal computational cost, requiring at most seven bit-shift operations per block.

These enhancements collectively make FELACS a fast, lightweight, and energy-efficient lossless compression technique, well-suited for real-time applications in WSNs.

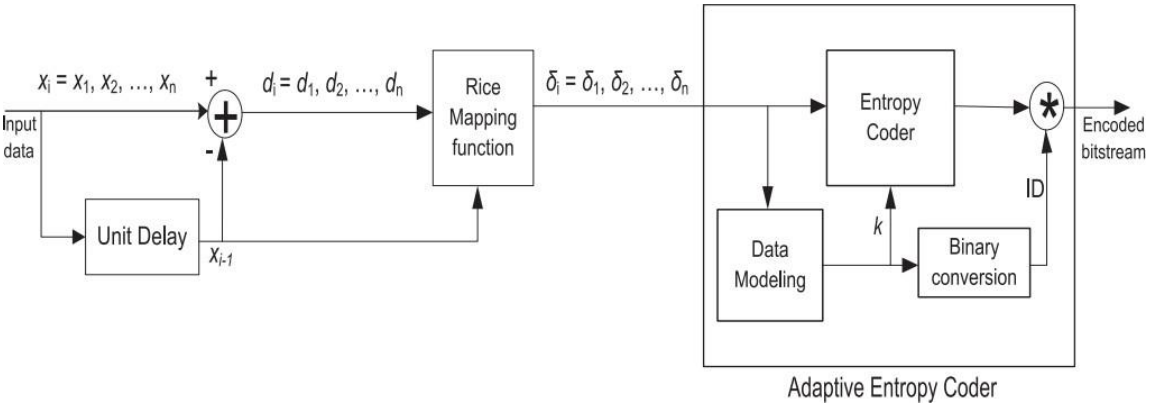

Figure 6. Flowchart scheme of FELACS algorithm

In this paper (T. Chen, 2017,) a deep learning based CNN framework for video compression is proposed where scalar quantization and Huffman coding are the techniques part of the framework.

SYSTEM MODEL

We examine a multiuser uplink MIMO communication system consisting of a base station (BS) equipped with M receive antennas and N single-antenna devices (users), where $N \geq M$. In any given time slot, a maximum of M users can be scheduled to transmit their status updates simultaneously. The BS is assumed to apply a zero-forcing (Qamar, 2017) (ZF) detection technique due to its relatively low computational complexity and suitability in multiuser decoding scenarios (Z. Wang and G. B. Giannakis, May 2000.).

Time is discretized into equal-length slots indexed by t =1, 2, ..., T where T defines the total simulation horizon. The generation of status update packets at each user follows a stochastic model. Specifically, the arrival of status packets at device i is modeled as a Bernoulli process with arrival probability λi and arrivals are assumed to be independent across users and time slots.

Each device maintains a buffer that stores only the most recent status update. When a new update arrives, it overwrites the existing packet. Once an update is successfully transmitted and acknowledged by the BS, the buffer is cleared. Let $It \in \{0, 1\}$ denote the buffer occupancy indicator for device i at slot t, where It = 1 implies a valid status update is present.

The BS manages user scheduling centrally. At the start of each slot t, the BS determines a subset $Kt \subseteq \{1, 2, \ldots, N\}$ of users to be scheduled for transmission. A binary scheduling variable $at \in \{0, 1\}$ indicates whether device i is scheduled in slot t. If at=1, the device is granted a transmission opportunity, conditional on having a valid update in its buffer (i.e., It = 1). The set of active devices with both scheduling and data availability is defined as $Ka \subseteq Kt$.

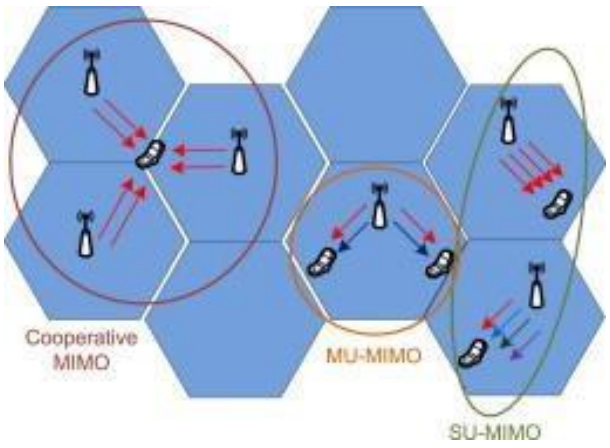

Figure 7. Differences in typology between MU-MIMO, SU-MIMO and Cooperative MIMO regarding to complexity, performance and overall cell data coverage

We assume that M orthogonal pilot sequences are available for uplink training (Ali, 2017). Each active user in Ka is assigned one unique pilot symbol, which is transmitted as a preamble. These pilots allow the BS to estimate the channel coefficients and detect which users are actively transmitting (Zeng, 2021). This aligns with uplink scheduling protocols in IEEE 802.11ax and similar WiFi standards (] L. You, Apr. 2019) (Kolo, 2012). The wireless channel is modeled as quasi-static Rayleigh fading, with coefficients constant within each time slot and independently changing between slots. Assuming uniform distance L from all users to the BS, and a path loss exponent τ, the average channel gain is given as Ω=L $-\tau$. The noise power at the receiver is σ2, and the transmit power per user is P. The resulting SNR is $\gamma = \boldsymbol{P}$. The probability of successful decoding for any user in the active set Ka, given $K = Ka\ \boldsymbol{\sigma 2}\ t\ a$ concurrent transmissions, is modeled by: $p(K) = \sum M - Ka\ 1\ (\sigma 2\ \gamma) m \exp(-\sigma 2\ \gamma)$, for

K >0, where γ is the SNR threshold required for a m=0 $m!$ $P\boldsymbol{\Omega}$ th $P\boldsymbol{\Omega}$ th a th reliable detection. For idle time slots Ka>0, we define p(0)=0 for consistency.

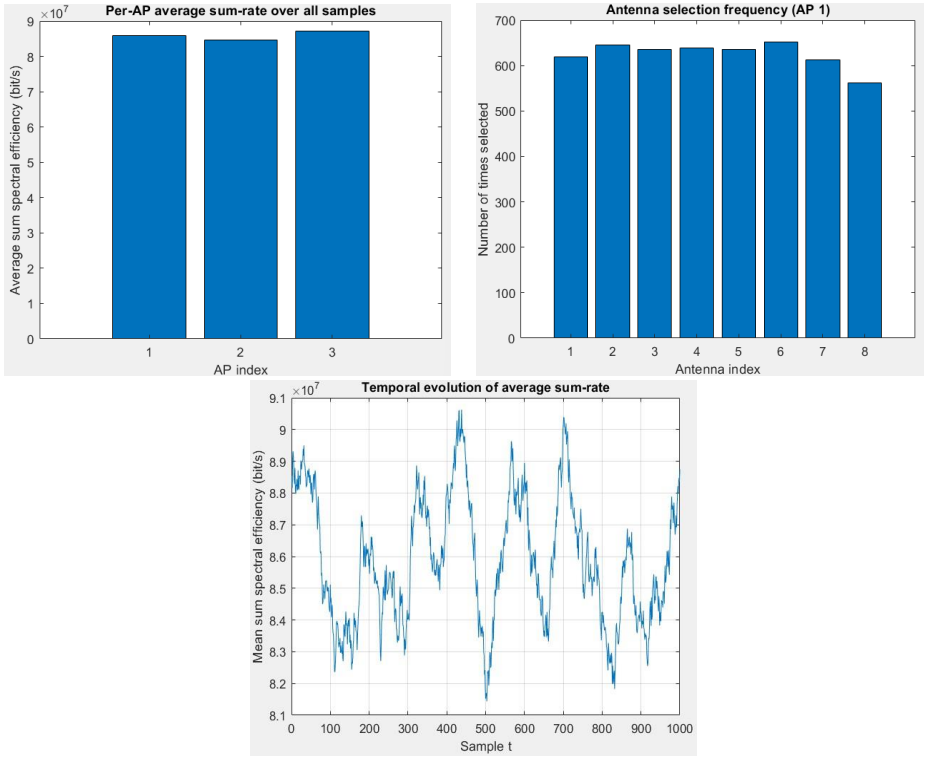

Figure 8. *Data output after plotting sum-rate per AP, plot distribution for selected antenna and plot evolution of mean sum-rate (left-right & up-down)*

There are several processes that undergo in generating the model system. Estimation of the channel during τc samples of time-frequency, where TDM occurs with its adoption for better estimation and data transmission. The system uses a block fading model, assuming static and flat channels over a coherence block. A TDD (Time Division Duplexing) protocol is applied, dividing each block into pilot, uplink, and downlink phases where each user is assigned a unique orthogonal pilot sequence for channel estimation (Khan, 2024). The received pilot signal at the Access Point (AP) is used to compute the LMMSE estimate of the user channels. Channel estimation error is modeled statistically, accounting for noise and path loss.

In every coherence block, only a subset (M out of N) antennas is activated per AP where the selected antenna subset determines the effective user-AP channel and influences system performance. The system considers cell-free MIMO, where multiple APs cooperatively serve all users over the same time-frequency resources. Spectral efficiency is expressed as a lower-bound due to imperfect channel knowledge (Björnson E. S., (2016, November)). SINR (Signal-to-Interference-plus-Noise Ratio) is derived by modeling estimation error as additional noise. SINR formulation includes contributions as: channel estimates, interference, estimation error covariance, additive noise (Gupta, 2004). Our goal is to maximize the total spectral efficiency by choosing the optimal antenna subset (combinatorial decision), designing corresponding beamforming (precoding) vectors.

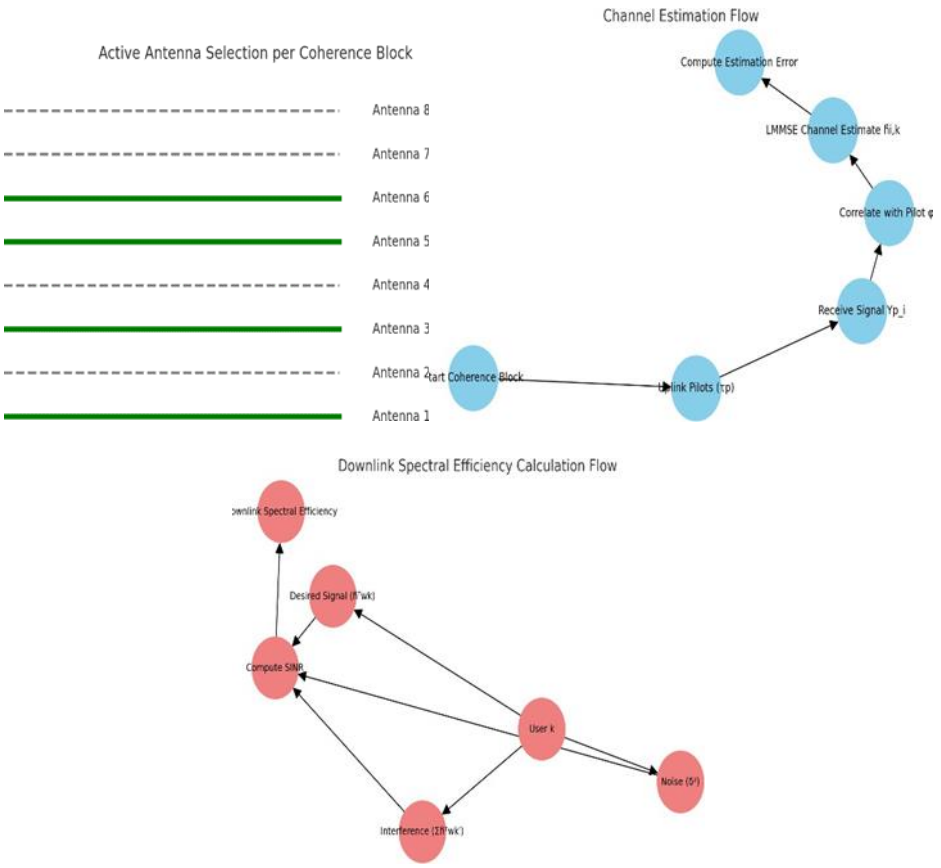

Figure 9. CEF (process from coherence block initiation to the LMMSE channel estimation), AAS (subset of antenna selected per coherence block) and SEC (computation graph of downlink SINR)

CONCLUSION

This study successfully demonstrates the integration and effectiveness of advanced lossless data compression algorithms, particularly ALDC and FELACS, in MIMO-OFDM wireless communication systems. The proposed solution significantly reduces transmission overhead by adapting data compression strategies dynamically according to signal characteristics. This adaptive approach not only enhances data transfer rates but also substantially improves energy efficiency, leading to prolonged network lifetime and optimized resource utilization in environments with limited bandwidth, such as Wireless Sensor Networks (WSNs) and IoT infrastructures. Furthermore, the inclusion of machine learning techniques like convolutional neural networks (CNNs) offers potential for context-aware optimization, extending the applicability of the proposed methods to more complex data types, including image, audio, and video (Alzubaidi, 2021). The combined benefits of reduced data volume, improved reliability, and lower bit error rates position the proposed system as a robust and efficient communication framework suitable for next-generation wireless networks, vehicular communications, and IoT-enabled applications.

Future Work

Future research will expand current data compression methods to handle diverse and complex data types such as images, audio, and video, which play critical roles in emerging wireless applications. Machine learning techniques, including convolutional neural networks (CNNs), will be incorporated to achieve adaptive, context-sensitive optimization of compression parameters in real-time. Hybrid algorithms integrating the advantages of ALDC, FELACS, and other leading compression methods will be

developed to create robust, scalable, and energy-efficient solutions suitable for dynamic wireless environments. Additionally, extensive testing in realistic wireless scenarios, addressing challenges such as fading effects and mobility, will be conducted to verify and refine these solutions. This comprehensive approach aims to facilitate the creation of intelligent, power-efficient communication protocols for next-generation IoT systems, vehicular networks, and critical real-time applications.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

An Analysis of Schooling Performance Using Multi-Criteria Decision-Making Methods: The Case of G7 Countries (1112)

Mulla Veli Ablay[1]

[1]Osmaniye Korkut Ata University, Rectorate, UZEM, Türkiye

*Corresponding author e-mail: veliablay@gmail.com

Abstract

The schooling rate is a key indicator that reflects individuals' level of participation in the education system, and it is closely associated not only with educational policies but also with economic, social, and technological factors. In this study, the Entropy and TOPSIS techniques, which are among the multi-criteria decision-making methods, were employed to comprehensively evaluate the dynamics affecting the schooling rate in G7 countries during the period from 2017 to 2021. In determining the level of schooling, factors such as economic capacity (measured by GDP), gender equality (represented by the female labor force participation rate), the level of public investment (indicated by public expenditure on education), and access to information (reflected by internet usage rates as a proxy for digital accessibility) were taken into consideration. Through these indicators, the study aims to evaluate the schooling rate not merely as an educational outcome, but as a reflection of the multidimensional development process. In the study, the weights of the criteria were determined using the Entropy method, while the performance ranking of the countries was conducted through the TOPSIS method. According to the analysis based on 2021 data, GDP was identified as the most significant criterion, while the female labor force participation rate was the least significant. In terms of performance ranking, the United States ranked first, whereas Canada ranked last. It was observed that there were no changes in the criterion weights or the performance rankings of the countries over the past five years. The multi-dimensional analysis conducted using the Entropy-TOPSIS method is expected to contribute to the objective identification of areas for improvement for policymakers

Keywords: *Schooling Rate, G7 Countries, MCDM*

INTRODUCTION

Education is regarded as one of the fundamental pillars not only for the personal development of individuals but also for the social, cultural, and economic advancement of societies (UNESCO, 2020). In modern knowledge societies, the quality and accessibility of education systems play a critical role in enabling countries to achieve their sustainable development goals (Hanushek & Woessmann, 2015). In this context, the school enrollment rate is one of the key indicators used to measure individuals' participation in education and serves as a commonly utilized quantitative criterion for assessing the effectiveness of a country's educational policies (Barro & Lee, 2013).

Education policies vary across countries worldwide, influenced by factors such as the level of economic development, socio-cultural structure, public investment, and technological infrastructure, all of which play a significant role in shaping these policies (OECD, 2020). Within this framework, the G7 group of developed countries (the United States, the United Kingdom, Canada, France, Germany, Italy, and Japan) represents a significant portion of the global economy and holds a leading position in areas such as quality of education, equal opportunity, and educational accessibility.

When considering countries' school enrollment rates, it becomes inevitable that many criteria influencing educational performance are interrelatedThe multi-criteria decision-making (MCDM) method is preferred for the simultaneous evaluation of multiple criteria and alternatives and for conducting effective analysis (Wang & Lee, 2009). The MCDM method simultaneously calculates the relative weighting of multiple criteria and ranks the performance of decision alternatives.

In this study, the variables influencing school enrollment rates in G7 countries were analyzed using Entropy-TOPSIS techniques, which are among the multi-criteria decision-making methods. In the study, the variables used to analyze school enrollment rates included GDP, government expenditure on education, female labor force participation rate, and the proportion of individuals using the internetThrough a comprehensive analysis using these variables, the strengths and weaknesses of countries in the field of education, as influenced by the factors affecting school enrollment rates, were identified. Moreover, the findings aim to serve as a reference for researchers and policymakers.

Literature Review

The investigation of school enrollment rates has become an important research topic at both national and international levels. In studies conducted on this subject, the methods applied have become as important as the variables used. The MCDM method has increasingly been employed effectively in studies on school enrollment rates. MCDM is effectively used in weighting different decision criteria relative to each other and in ranking alternatives according to their level of importance.

In their study, Büyüközkan and Öztürkcan (2010) developed a new model by combining the DEMATEL and ANP multi-criteria decision-making methods to determine the most suitable project alternative through an empirical study in logistics companies. The study identified that the most effective project alternative was cost, followed by the improvement of business processes. In his study, Dağdeviren (2010) used the ANP and TOPSIS techniques to evaluate an effective personnel selection process. Five candidates who passed the preliminary selection process were evaluated in the study. The application and results were considered acceptable by the company. In his study, Lorcu (2015) used the MDS technique to analyze the achievement of educational goals by EU member and candidate countries in 2020. The study found that the two most effective countries in achieving educational goals were Denmark and Finland. In his 2017 study, Masça used 2015 data and applied the TOPSIS technique to analyze the economic performance of EU countries. According to the performance ranking based on the TOPSIS method in the study, Sweden was identified as the top-ranked country, while Greece was the lowest-ranked.

In his 2022 study, Yaman Yılmaz analyzed the educational performance of OECD countries using 2019 data with the SPSS software. In the study, similar countries were evaluated within a group based on cluster analysis, and these countries were divided into three clusters according to their shared characteristics. The study identified countries with similar and distinct characteristics in terms of educational performance.

In their 2024 study, Pınar and Erdoğan analyzed the economic performance of G7 countries by examining the impact of inflation rate, current account balance, GDP growth rate, and unemployment rate on GDP for the years 2018–2022, using the CRITIC and TOPSIS methods. In the study, the CRITIC technique was used to determine that macroeconomic performance is the most influential criterion of economic growth. Using the TOPSIS method, the performance ranking of the countries was conducted, with Germany identified as the most successful country. It was found that the performance rankings of the other countries varied over the years.

MATERIAL AND METHODS

The study data were compiled from The World Bank dataset (The World Bank, 2025) and consist of criteria affecting the school enrollment rates of G7 countries. This section of the study will present the application steps and mathematical formulations of the Entropy-TOPSIS techniques used in the analysis.

Entropy Method

The entropy technique is one of the multi-criteria decision-making (MCDM) methods used to determine the relative importance weights of the criteria in the decision matrix during the decision-making process. It was originally developed by Rudolph Clausius in 1865 to measure uncertainty in a system, and later adapted to information theory by Claude E. Shannon in 1948 (Zhang et al., 2011). The application steps of the entropy method are as follows (Ömürbek et al., 2017):

Step 1: Decision Matrix

$$A_{ij}=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \quad (1)$$

Step 2: Normalize Decision Matrix

$$P_{ij}=\frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a_{ij}}};A_j \quad (2)$$

Step 3: ENTROPY Value and Eij Matrix

$$E_j=\left(\frac{-1}{\ln(m)}\right)\sum_{i=1}^{m}[P_{ij}\ln(P_{ij})];A_j \quad (3)$$

Step 4: Determining the Degrees of Differentiation, Dij Values

$$D_j=1\text{-}E_j;A_j \quad (4)$$

Step 5: Establishing the Entropy Criterion Weight Values

$$w_j=\frac{d_j}{\sqrt{\sum_{j=1}^{n} d_j}};A_j \quad (5)$$

TOPSIS Method

The TOPSIS method, one of the multi-criteria decision-making techniques, ranks alternatives by identifying their ideal and non-ideal solutions. The widely used TOPSIS method was first developed by Hwang and Yoon in 1981 and later enhanced by Chen and Hwang in 1992. The application steps of the method are as follows (Wang et al., 2016):

Step 1: Construction of the Decision Matrix

The decision matrix is provided by Equation 6.

$$A_{ij}=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \quad (6)$$

Step 2: Obtaining the Normalized Matrix

The matrix was normalized using the decision matrix and Equation 7.

$$r_{ij}=\frac{x_{ij}}{\sqrt{\sum_{k=1}^{m} x_{kj}^2}},\ i=1,2,\ldots,m;\ j=1,2,\ldots,n \quad (7)$$

The values obtained from Equation 7 are substituted into Equation 8 to obtain the R_{ij} matrix.

$$R_{ij} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & & r_{2n} \\ \vdots & & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} \quad (8)$$

Step 3: Obtaining the Weighted Normalized Matrix

The weighted normalized matrix was obtained using Equation 9.

$$V_{ij} = w_j r_{ij}, \quad \text{i=1,2,…,m; j=1,2,…,n} \quad (9)$$

Step 4: Obtaining the Ideal and Negative Ideal Solution Values

The ideal solution (A^+) and the negative ideal solution (A^-) were obtained from the weighted normalized matrix derived by Equation 9, using Equation 10.

$$A^+ = \{v_1^+, \dots, v_n^+\}$$

$$A^- = \{v_1^-, \dots, v_n^-\} \quad (10)$$

Step 5: Obtaining the Distance Values from the Ideal and Negative Ideal Points

The values obtained from Equations (9) and (10) are substituted into Equation 11 to calculate the ideal separation (D^+) and negative ideal separation (D^-) measures.

$$D_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^+)^2}$$

$$D_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2} \quad (11)$$

Step 6: Calculating the Relative Closeness to the Ideal Solution

To calculate the school enrollment rate performance values of G7 countries, the relative closeness to the ideal solution is calculated using Equation 12, with the separation measures obtained in Equation 11.

$$C_i = \frac{D_i^-}{D_i^- + D_i^+} \quad (12)$$

RESULTS

In this section, the criterion weights will first be determined using the Entropy technique to calculate the school enrollment rate of G7 countries, and then the performance ranking of the alternatives will be carried out using the TOPSIS method.

Calculation of Criterion Weights Using the Entropy Technique

The criteria for the G7 countries have been analyzed using the Entropy-TOPSIS model. The codes and optimization of the criteria used are provided in Table 1.

Table 1: Criteria

Indicator	Kod	Optimizasyon
GDP per capita	K1	Max
Labor force participation rate, female	K2	Max
Government expenditure on education	K3	Max
Individuals using the Internet	K4	Max

To obtain the importance weights of the G7 countries' performance, the decision matrix, which is the first step of the Entropy technique, has been constructed as shown in Table 2.

Table 2: Decision Matrix

Country Names	K1	K2	K3	K4
Canada	2,01E+12	47,0485	4,7486	93,9000
France	2,97E+12	48,4182	5,4304	86,1000
Germany	4,35E+12	46,4180	5,4549	91,4000
Italy	2,18E+12	42,4568	4,2183	74,9000
Japan	5,03E+12	44,3722	3,3356	82,9000
United Kingdom	3,14E+12	47,9771	5,9046	96,2000
United States	2,37E+13	45,1872	5,4283	91,3000

Source: The World Bank, 2025

The normalized decision matrix values are provided in Table 3. These values were obtained from the decision matrix using Equation 2, taking into account the direction of the criteria.

Table 3: Normalized Decision Matrix

Country Names	K1	K2	K3	K4
Canada	0,0463	0,1462	0,1376	0,1523
France	0,0684	0,1504	0,1573	0,1396
Germany	0,1003	0,1442	0,1580	0,1482
Italy	0,0503	0,1319	0,1222	0,1215
Japan	0,1161	0,1379	0,0966	0,1344
United Kingdom	0,0725	0,1491	0,1710	0,1560
United States	0,5461	0,1404	0,1572	0,1480

Table 4 presents the entropy values, which were obtained using the normalized decision matrix through Equation 3.

Table 4: Ej Values

K1	K2	K3	K4
0,7592	0,9995	0,9923	0,9984

Table 5 presents the degree of diversification. These values were calculated using the entropy values through Equation 4.

Table 5: Dij Values

K1	K2	K3	K4
0,2408	0,0005	0,0077	0,0016

Table 6 presents the criterion weights. These values were obtained by dividing the diversification degree of each criterion by the total diversification degree, as expressed in Equation 5.

Table 6: Criterion Weights

K1	K2	K3	K4
0,9612	0,0019	0,0306	0,0063

An examination of Table 6 reveals that the criterion with the highest weight is GDP per capita, while the criterion with the lowest weight is the labor force participation rate, female. The ranking of the criteria in Table 1 in terms of their weights is presented in Table 6 as K1> K3> K4> K2.

Measuring Performance Using the TOPSIS Method

The decision matrix, which constitutes the first step of the TOPSIS method for evaluating the performance of the G7 countries' schooling components, was constructed in Table 2. Using the data in Table 2, the normalized decision matrix was obtained through Equation 7 and is presented in Table 7.

Table 7: Normalized Decision Matrix

Country Names	K1	K2	K3	K4
Canada	0,0798	0,3864	0,3589	0,4016
France	0,1180	0,3976	0,4104	0,3683
Germany	0,1729	0,3812	0,4123	0,3910
Italy	0,0866	0,3487	0,3188	0,3204
Japan	0,2002	0,3644	0,2521	0,3546
United Kingdom	0,1250	0,3940	0,4463	0,4115
United States	0,9416	0,3711	0,4103	0,3905

The weighted normalized matrix obtained using Equation 8 is presented in Table 8.

Table 8: Weighted Normalized Decision Matrix

Country Names	K1	K2	K3	K4	$\mathbf{Q_i^{(1)}}$
Canada	0,0767	0,0007	0,0110	0,0025	0,0767
France	0,1134	0,0007	0,0126	0,0023	0,1134
Germany	0,1662	0,0007	0,0126	0,0025	0,1662
Italy	0,0833	0,0007	0,0098	0,0020	0,0833
Japan	0,1924	0,0007	0,0077	0,0022	0,1924
United Kingdom	0,1201	0,0007	0,0137	0,0026	0,1201
United States	0,9051	0,0007	0,0126	0,0025	0,9051

The A^+ ideal decision point was calculated using Equation 10 and obtained as shown in Table 9.

Table 9: Ideal Decision Point

K1	K2	K3	K4
0,9051	0,0007	0,0137	0,0026

The distance to the D^+ ideal solution was calculated using Equation 11 and is shown in Table 10.

Table 10: Distance to the D^+ Ideal Solution

$\boldsymbol{D_1^+}$	$\boldsymbol{D_2^+}$	$\boldsymbol{D_3^+}$	$\boldsymbol{D_4^+}$	$\boldsymbol{D_5^+}$	$\boldsymbol{D_6^+}$	$\boldsymbol{D_7^+}$
0,8284	0,7917	0,7389	0,8218	0,7127	0,7850	0,0011

The A^- negative decision point was calculated using Equation 10 and obtained as shown in Table 11.

Table 11: Negative Decision Point

K1	K2	K3	K4
0,0767	0,0007	0,0077	0,0020

The distance to the D^- negative ideal solution was calculated using Equation 11 and is shown in Table 12.

Table 12: Distance to the D^- Negative Ideal Solution

$\boldsymbol{D_1^-}$	$\boldsymbol{D_2^-}$	$\boldsymbol{D_3^-}$	$\boldsymbol{D_4^-}$	$\boldsymbol{D_5^-}$	$\boldsymbol{D_6^-}$	$\boldsymbol{D_7^-}$
0,0033	0,0370	0,0896	0,0069	0,1157	0,0438	0,8284

Equation 12 was used to calculate the relative closeness to the ideal solution, and the resulting performance scores and rankings are presented in Table 13.

Table 13: Countries' Performance Scores (2021)

Country Names	C* Performans puanları	Rank
Canada	0,0040	7
France	0,0446	5

Germany	0,1081	3
Italy	0,0083	6
Japan	0,1397	2
United Kingdom	0,0529	4
United States	0,9987	1

Table 14 presents the performance rankings of the countries over the years. An examination of Table 14 reveals that there have been no changes in the countries' performance rankings.

Table 14: Countries' C Performance Rankings Over the Years

Country Names	2017	2018	2019	2020	2021
Canada	7	7	7	7	7
France	5	5	5	5	5
Germany	3	3	3	3	3
Italy	6	6	6	6	6
Japan	2	2	2	2	2
United Kingdom	4	4	4	4	4
United States	1	1	1	1	1

DISCUSSION AND CONCLUSION

The entropy method used in the study provides objectivity to MCDM analyses by determining weights independently of subjective judgments, while the TOPSIS method offers robust comparative analyses through the ranking of decision alternatives based on the ideal solution. The combined use of these methods contributes to a more reliable and comparable assessment of the performance of education systems. In this context, the schooling performances of the G7 countries were analyzed using the Entropy-TOPSIS method, providing a comparative perspective on the educational achievements of these countries.

In the study, when calculations were performed using the Entropy technique for the criteria of GDP, female labor force participation, government expenditures on education, and the proportion of individuals using the internet, it was observed that the criterion with the highest weight was GDP per capita, while the criterion with the lowest weight was the female labor force participation rate. When country performances were ranked using the WASPAS technique, the United States ranked first, while Canada ranked last.

This study provides significant findings by analyzing the key factors affecting schooling rates in G7 countries using the Entropy-TOPSIS method. In future studies, it would be beneficial to expand the scope of the analysis. In addition to the criteria used in the study, other factors affecting the schooling rate can also be included in the model.

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Enhanced Rank-Based Correlation Estimation Using Smoothed Wilcoxon Rank Scores (1039)

Feridun Taşdan[1*], Rukiye Dağalp[2]

[1]Department of Mathematics Western Illinous University, Macomb, IL 61455
[2]Department of Statistics Ankara University, Ankara, Turkey.

*Corresponding author e-mail: f-tasdan@wiu.edu

Abstract

This article proposes an improved version of the Spearman rank correla- tion based on using Wilcoxon rank score function. A smoothed empiri- cal cumulative distribution function (ecdf) computes the smoothed ranks and replaces the regular ranks in the Wilcoxon rank score function. The smoothed Wilcoxon rank scores are then used for estimation of the Spear- man's correlation. The proposed approach is similar to the Spearman's rho (ρ) estimator which uses ranks of the random samples of X and Y but the proposed method improves Spearman's approach such as handling ties and gaining higher efficiency under monotone associations. A Wald type hypothesis test has been proposed for the new estimator rsa and the asymptotic properties are shown.

Keywords: *Spearman's Rank Correlation, Smoothed Empirical Distribu- tion Function, Wilcoxon Scores.*

INTRODUCTION

The rank based estimation and testing procedures for the correlation coefficient ρ are commonly used in many data analysis problems. Especially Spearman (Spearman (1904)) and Pearson's correlation (Pearson (1920) coefficients are the leading methods of estimation the correlations between two dependent random variables. Pearson's traditional approach works good if the relationship is linear and the underlying joint distribution is normal. On the other hand, Spearman's approach is based on the ranks of the random samples and does not depend on the normality assumption or any other underlying distribution. Spearman's approach works good if the relationship is linear or monotonic (Santos et al.(2011) and Reshef et al. (2011)). Therefore, Spearman has some superior properties over the Pearson's correlation if the underlying conditions are not ideal such as having a monotonic association instead of a linear. The figures below depict the cases of linear, nonlinear monotonic and non-monotonic nonlinear relationships.

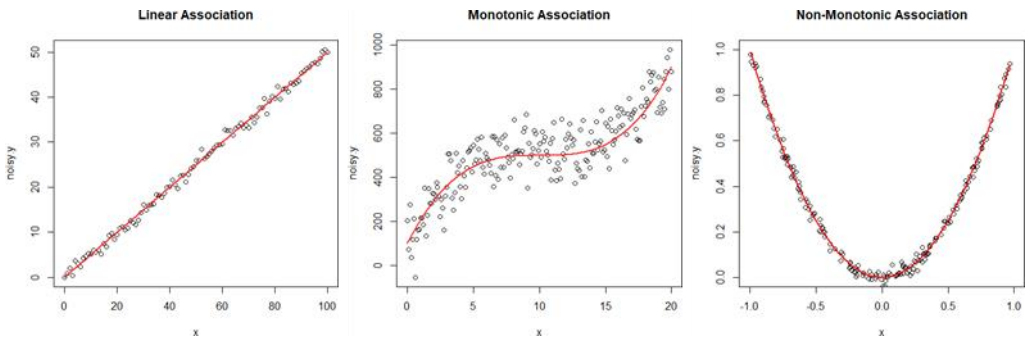


Figure 1: Examples of linear, monotone and non-monotone relationships

In Section 2 traditional correlation estimations are reviewed. In Section 3 score based estimation of the correlation coefficient ρ is discussed. Moreover, the relationship between the Spearman's rank based

correlation estimation and the Wilcoxon score based correlation estimations are shown. In Section 4 it is shown that a non-traditional approach of smoothed ranks are used in the Wilcoxon's score function to estimate the correlation coefficient. In addition, Wald type hypothesis testing option has been proposed. In Section 5 Monte Carlo simulation study has been performed to show small and large sample properties under several bivariate distributions. Final conclusion about the proposed estimator and the results of the Monte Carlo simulations that study are presented in Section 6.

Pearson and Spearman's Correlation Estimations

The measure of association between random variables X and Y is defined as ρ which is called the population correlation coefficient. Pearson's ρ definition which is given by (Pearson (1920)) can be written as

$$\rho = \frac{E[(X - \mu_x)(Y - \mu_y)]}{\sigma_x \sigma_y} = \frac{Cov(X, Y)}{\sigma_x \sigma_y}. \qquad (1)$$

where —1 ≤ ρ ≤ +1 and Cov(X, Y) = E[XY] — E[X]E[Y], which is defined as covariance between X and Y . It should be noted that if ρ = 0 (independence of X and Y) implies Cov(X, Y) = 0 but the opposite is not true.

To find the method of moment estimator for Pearson's ρ, let (X1, Y1), (X2, Y2), ..., (Xn, Yn) be a random sample from a bivariate continuous cdf of F (X, Y) and let r be the estimated sample correlation coefficient. The method of moment estimate of Cov(X, Y) = E[(X — μx)(Y — μy)] can be defined as Co^v(X, Y) = 1 Σn (Xi —X)(Yi — Y). The σx can be estimated with σ^x = q 1 Σn (Xi — X)2 and σ^y = q 1 Σn (Yi — Y)2. Thus, the Pearson's method of moment correlation coefficient estimate can be written as

$$r_p = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2}\sqrt{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}}. \qquad (2)$$

The result above can be further simplified to the version below,

$$r_p = \frac{\sum_{i=1}^{n} X_i Y_i - \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right)\left(\sum_{i=1}^{n} Y_i\right)}{\sqrt{\sum_{i=1}^{n} X_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right)^2}\sqrt{\sum_{i=1}^{n} X_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right)^2}} \qquad (3)$$

A nonparametric (rank based) alternative of Pearson's correlation coefficient is proposed by Spearman (Spearman (1904)). Spearman's ρ estimator uses the ranks of random samples of X and Y instead of the original observations. Let R(Xi) be the rank of ith observation Xi for i = 1, ..., n. Similarly, Yi is replaced by R(Yi). By substituting the ranks R(Xi) and R(Yi) in the Pearson's correlation coefficient formula in Eq(2), we obtain the Spearman's correlation coefficient estimate below

$$r_s = \frac{\sum_{i=1}^{n}(R(X_i) - \overline{R(X_i)})(R(Y_i) - \overline{R(Y_i)})}{\sqrt{\sum_{i=1}^{n}(R(X_i) - \overline{R(X_i)})^2}\sqrt{\sum_{i=1}^{n}(R(Y_i) - \overline{R(Y_i)})^2}} \qquad (4)$$

We should note that the ranks $R(X_i)$ (also $R(Y_i)$) are uniformly distributed for the integers on $i = 1, \ldots, n$. Also, sum of the ranks can be written as $\sum_{i=1}^{n} R(X_i) = \sum_{i=1}^{n} R(Y_i) = n(n+1)/2$ for $i = 1, \ldots, n$. Thus, $\overline{R(X_i)} = \overline{R(Y_i)} = \frac{n+1}{2}$. Moreover, it can be shown that $E[R(X_i)] = (n+1)/2$ and $nV[R(X_i)] = \sum_{i=1}^{n} (R(X_i) - \overline{R(X_i)})^2 = \frac{n(n^2-1)}{12}$. So, we can use these results to obtain much simpler version of r_s as described below;

$$r_s = \frac{\sum_{i=1}^{n} (R(X_i) - \frac{n+1}{2})(R(Y_i) - \frac{n+1}{2})}{\frac{n(n^2-1)}{12}} \tag{5}$$

Yet, there is another version of the Spearman's correlation coefficient has been developed in the literature or used in teachings of the correlation coefficients. So, let $D_i = R(X_i) - R(Y_i)$ for $i = 1, \ldots, n$. It is also true that

$$D_i = [R(X_i) - \overline{R(X_i)}] - [R(Y_i) - \overline{R(Y_i)}] \tag{6}$$

since $\overline{R(X_i)} = \overline{R(Y_i)} = \frac{n+1}{2}$ Thus, it can be written that

$$\sum_{i=1}^{n} D_i^2 = \sum_{i=1}^{n} [R(X_i) - \overline{R(X_i)}] - [R(Y_i) - \overline{R(Y_i)}]^2 \tag{7}$$

$$= \sum_{i=1}^{n} (R(X_i) - \overline{R(X_i)})^2 + \sum_{i=1}^{n} (R(Y_i) - \overline{R(Y_i)})^2$$

$$- 2 \sum_{i=1}^{n} (R(X_i) - \overline{R(X_i)})(R(Y_i) - \overline{R(Y_i)})$$

$$\sum_{i=1}^{n} D_i^2 = \frac{n(n-1)}{12} + \frac{n(n-1)}{12} - 2 \sum_{i=1}^{n} (R(X_i) - \overline{R(X_i)})(R(Y_i) - \overline{R(Y_i)})$$

If the last expression on the right of the equation is isolated, we can obtain

$$\sum_{i=1}^{n} (R(X_i) - \overline{R(X_i)})(R(Y_i) - \overline{R(Y_i)}) = \frac{n(n^2-1)}{12} - \frac{\sum_{i=1}^{n} D_i^2}{2} \tag{8}$$

So, the numerator in Eq (4) is replaced with the above result, and the denominator terms (square root of variances) are replaced with $\sum_{i=1}^{n} (R(X_i) - \overline{R(X_i)})^2 = \frac{n(n^2-1)}{12}$ then a further simplification gives us yet another definition of r_s which can algebraically be reduced to

$$r_s = 1 - \frac{6 \sum_{i=1}^{n} D_i^2}{n(n^2-1)} \tag{9}$$

where Di = R(Xi) — R(Yi). If some ties exist in the samples, a small estimation error might occur in the last formula but a few ties can be tolerated without a significant difference. The average method can be used to break the ties if necessary. All three versions of rs produce the same result but each has somewhat different computational difficulties.

Spearman's Rank Correlation Based on General Score Functions

Spearman's r_s estimator of the correlation parameter ρ can be computed with using a general score function $\phi(u)$ where the score function must be nondecreasing function defined on the interval (0, 1) such that $\int_0^1 \phi^2(u)du < \infty$. Hettmansperberger and McKean (2011) defines the correlation coefficient estimator based on a general score function $\phi(u)$ as

$$r_a = \frac{1}{s_a^2} \sum_{i=1} a(R(X_i))a(R(Y_i)). \quad (10)$$

where $a(i) = \phi(i/n + 1)$ and $s_a^2 = \sum_{i=1}^{n} a^2(i)$ for $i = 1, ..., n$. The score functions $\phi(u)$ can be standardized so that it satisfies $\int_0^1 \phi(u)du = 0$ and $\int_0^1 \phi^2(u)du = 1$. Moreover, $E(r_a)$ and $Var(r_a)$ can be derived with the following theorem.

Theorem 1. Let (X1, Y1), ..., (Xn, Yn) be a random sample from a bivariate con- tinuous distribution function F (X, Y). Under the null hypothesis of H0 : ρ = 0 (F (X, Y) = Fx(X)Fy(Y) or independence of X and Y), the test statistics ra sat- isfies the following properties:

a) E[ra] = 0

b) V ar[ra] = 1/(n — 1)

Proof. Since it is assumed that the scores are generated by $a(i) = \phi(i/n + 1)$ for $i = 1, ..., n$ and $a(1) \leq a(2) \leq ... \leq a(n)$ based on the assumptions that the score function $\phi(u)$ is nondecreasing. The score function is also centered so that $\int_0^1 \phi(u)du = 0$ and $\int_0^1 \phi^2(u)du = 1$. By using the Reimann-Sum approximation on the integral, it can be shown that $\int_0^1 \phi(u)du \doteq \sum_{i=1}^{n} \phi(i/(n + 1)\frac{1}{n} = \sum_{i=1}^{n} a(i)\frac{1}{n} \approx 0$. Also, $\int_0^1 \phi^2(u)du \doteq \sum_{i=1}^{n} \phi^2(i/(n+1))\frac{1}{n} = \sum_{i=1}^{n} a^2(i)\frac{1}{n} = s_a^2 \approx 1$ can be approximated in a similar fashion.

It is known that the ranks, R(Xi)(also R(Yi)), is uniformly distributed for the integers on i = 1, ..., n. Thus, P [(R(Xi) = k] = 1/n, where k is the rank of ith observation. So, each rank of X_i is equally likely distributed with a probability of $1/n$. Therefore, from the expected value of $a(R(X_i))$ it can be found that $E[a(R(X_i))] = \sum_{n} a(i)\frac{1}{n} = 0$ since $\sum_{n} a(R(X_i)) = 0$. Similarly, $E[a(R(Y_i))] = \sum_{n} a(i)\frac{1}{n} = 0$. When above results are substituted into the expression of r_a as defined in Eq 10, it shows $E[r_a] = 0$.

To show part b, the variance definition of V [ra] = E[(ra)2] — (E[ra])2 can be used for the estimator ra. In the first part of the proof (part-a), it has been shown that E[ra] = 0. Thus, the variance expression can be simplified to V [ra] = E[(ra)2].

Assuming under $H_0 : \rho = 0$,

$$E[(r_a)^2] = E\{[\frac{1}{s_a^2}\sum_{i=1}^{n} a(R(X_i))a(R(Y_i))]^2\}$$
$$= \frac{1}{s_a^4}\sum_{i=1}^{n} E[a(R(X_i))a(R(Y_i))]\sum_{j=1}^{n} E[a(R(X_j))a(R(Y_j))]$$
$$= \frac{1}{s_a^4}\sum_{i=1}^{n}\sum_{j=1}^{n} E[a(R(X_i))a(R(Y_i))]E[a(R(X_j))a(R(Y_j))]$$
$$= \frac{1}{s_a^4}\sum_{i=1}^{n}\sum_{j=1}^{n} E[a(R(X^i))a(R(X^j))]E[a(R(Y^i))a(R(Y^j))]$$

To find the result of the expectations in the last expression, there are two cases ($i = j$ and $i \neq j$) that must be considered separately. For $i = j$,

$$E[a(R(X_i))a(R(X_j))] = E[a^2(X_i)] = \sum_{i=1}^{n} a^2(X_i)\frac{1}{n} = \frac{1}{n}s_a^2.$$

Similarly, it can be found that

$$E[a(R(Y_i))a(R(Y_i))] = \frac{1}{n}s_a^2.$$

For $i \neq j$, it can be written

$$E[a(R(X_i))a(R(X_j))] = \sum_{i=1}^{n}\sum_{j=1}^{n} a(R(X_i))a(R(X_j))\frac{1}{n(n-1)} = \frac{1}{n(n-1)}s_a^2.$$

The last result can be applied to $E[a(R(Y_i))a(R(Y_j))] = \frac{1}{n(n-1)}s_a^2.$

So, by substituning both cases of $i = j$ and $i \neq j$,

$$E[(r_a)^2] = \frac{1}{s_a^4}\sum_{i=1}^{n}\sum_{j=1}^{n} E[a(R(X_i))a(R(X_j))]E[a(R(Y_i))a(R(Y_j))]$$
$$\frac{1}{s_a^4}\{\sum_{i=j=1}^{n} E[a^2(X_i)]E[a^2(Y_i)] + \sum_{i\neq j} E[a(R(X_i))a(R(X_j))]E[a(R(Y_i))a(R(Y_j))]\}$$
$$\frac{1}{s_a^4}\{\sum_{i=j=1}^{n}\frac{1}{n}s_a^2\frac{1}{n}s_a^2 + \sum_{i\neq j}\sum\frac{1}{n(n-1)}s_a^2\frac{1}{n(n-1)}s_a^2\}$$
$$= \frac{1}{s_a^4}\frac{ns_a^4}{n^2} + n(n-1)\frac{s_a^4}{n^2(n-1)^2}$$
$$= \frac{1}{s_a^4}\frac{s_a^4}{n} + \frac{s_a^4}{n(n-1)}$$
$$= \frac{1}{s_a^4}\frac{(n-1)s_a^4}{n(n-1)} + \frac{s_a^4}{n(n-1)}$$
$$= \frac{1}{s_a^4}\frac{s_a^4}{n-1}$$
$$E[(r_a)^2] = \frac{1}{n-1}$$

Thus, using the results of $E[r_a] = 0$ and $E[(r_a)^2] = \frac{1}{n-1}$ as shown above, we prove that $V[r_a] = \frac{1}{n-1}$ The finite sample distribution of r_a is not easy to find but it can be claimed that (by the central limit theorem) $z_a = \frac{r_a}{\sqrt{1/(n-1)}} = r_a\sqrt{n-1} \rightarrow N(0, 1)$ as n goes to ∞. For an asymptotic (large sample) α level test, it can be defined that reject $H_0 : \rho = 0$ if $|z_a| > z_{\alpha/2}$.

For example, $\phi(u)$ can be taken as $\phi(u) = \sqrt{12}(u - 1/2)$ which is called the Wilcoxon's linear score function. It can be shown (with the following theorem) that Spearmen's rank correlation (r_s) is equal to to the r_a which is defined in Eq(10) if Wilcoxon's linear score function is used to estimate the correlation coefficient of r_a.

Theorem 2. Let (X1, Y1), ..., (Xn, Yn) be a random sample from a bivariate con- tinuous distribution function F (X, Y). Also let ϕ(u) = √12(u — 1/2) which is called the Wilcoxon's linear score function. Under the null hypothesis of H0 : ρ = 0 (F (X, Y) = Fx(X)Fy(Y) or independence of X and Y), the test statistics based on Wilcoxon's rank score ra is equal to the Spearman's rs with the same variance and expected value as defined in Theorem 1.

Proof. First, recall that $a(i) = \phi[i/(n + 1)] = \sqrt{12}(i/(n + 1) - 1/2)$ with using Wilcoxon's score function. Also, s_a^2 is defined as

$$
\begin{aligned}
s_a^2 &= \sum_{i=1}^{} a^2(i) \\
&= \sum_{i=1} \sqrt{12}[i/(n+1) - 1/2]^2 \\
&= \frac{12}{(n+1)^2}\left[\sum_{i=1}^{n} i^2 - (n+1)\sum_{i=1}^{n} i + \frac{n(n+1)^2}{4}\right] \\
&= \frac{n(n-1)}{n+1}
\end{aligned}
$$

So, we also let $a(R(X_i)) = \sqrt{12}[R(X_i)/(n + 1) - 1/2]$, and similarly, $a(R(Y_i)) = \sqrt{12}[R(Y_i)/(n + 1) - 1/2]$. We plug in these results into the r_a which is defined in Eq (10). Then, we can find

$$
r_a = \frac{\sum_{i=1}^{n}[(R(X_i) - (n + 1)/2)(R(Y_i) - (n + 1)/2)]}{n(n^2 - 1)/12}. \qquad (11)
$$

which is also defined as rs in Eq (5). So, the result of the Theorem 2 proves that Spearman's rs can also be estimated using a score-based methods such as Wilcoxon's score function.

Yet there is another nonparametric estimator available to estimate ρ, which is called Kendall's τ estimator (Kendall, 1938). It still uses the ranks of the observations but it counts the concordant and discordant pairs of the observations in the pairs of random samples. An estimator of Kendall's τ can be derived

$$r = \frac{(n_c - n_d)}{(n(n-1))/2} \qquad (12)$$

where nc shows the number of concordant pair and nd shows the number of discordant pairs.

If we compare Pearson against Kendall's and Spearman's estimators, Pear- son's correlation coefficient measures how well the linear relationship between two variables are but in reality there could be other ways that two variables can be correlated such as monotonic relationship which does not show relation- ship with a straight line. Monotonic relationships between two variables can show exponential or logistics distributions such as while one variable increases, the other increases or decreases consistently. Spearman and Kendall's correlation coefficients are basically a non-parametric ways to investigate the monotonic rela- tionships between two random variables with a continuous bivariate distribution function of F (X, Y).

The Spearman correlation test assumes only that your data are a random sample. Spearman's correlation is calculated on the ranks of the observations and will therefore work with any type of data that can be ranked, including ordinal, interval, or ratio data. Because it based on ranks, it is less sensitive to outliers than the Pearson correlation test, and it is sometimes used to evaluate a correlation when outliers are present.

Proposed Smooted Ranks Based Correlation Estimator

First, recall that the general score function is defined as $a(i) = \phi[(i/n + 1)] = \sqrt{12}(i/(n + 1) - 1/2)$ based on Wilcoxon's linear score function. Then us- ing ranks, $a(R(X_i)) = \sqrt{12}\,[R(X_i)/(n+1) - 1/2]$, and similarly, $a(R(Y_i)) = \sqrt{12}\,[R(Y)/(n+1) - 1/2]$. As it was shown by Theorem 1 and Theorem 2, r is equal to the Spearman's r_s as defined in Eq(9). The proposed method re- places $R(X_i)$ and $R(Y_i)$ with a smoothed ranks and call them $\hat{R}(X_i)$ and $\hat{R}(Y_j)$, respectively.

In order to generate a rank set of a random sample of X1, X2, .., Xn, the empirical cdf of the random sample is defined as

$$F_n(x) = \frac{\#\{X_i \le x\}}{n} = \frac{1}{n}\sum_{j=1} \mathbf{I}(X_j \le x). \qquad (13)$$

where **I** is called the indicator function. This is a stepwise discrete function at each Xi. Then, the ranks of a random sample can be expressed as

$$R(X_i) = nF_n(X_i) = \sum_{j=1} \mathbf{I}(X_i \le X_i) \qquad (14)$$

where R(Xi) is the rank of the ith observation. In a similar fashion,

$$R(Y_i) = nF_n(Y_i) = \sum_{j=1} \mathbf{I}(Y_i \le Y_i) \qquad (15)$$

We now consider a smooth approximation to the indicator function by replac- ing I with a continuous distribution function H(x/h), where the scale parameter h is called the bandwidth or smoothing parameter in kernel smoothing applica- tions and h approaches zero as the sample size increases. The distribution func- tion H(x) is chosen to be a continuous, nondecreasing, and also bounded function that generates smoothed ranks as described below. The resulting smoothed em- pirical function can be written as

$$F_s(t) = \frac{1}{n} \sum_{i=1} H\left(\frac{t - X_i}{h}\right). \qquad (16)$$

By using this result, we can now define the smoothed ranks

$$\widehat{R(X_i)} = nF_s(X_i) = \sum_{i=1} H\left(\frac{X_i - X_i}{h}\right) \qquad (17)$$

where the notation $\widehat{R(X_i)}$ indicates the smoothed rank of the observation X_i. It is important to mention that the purpose of smoothing is not about estimating a density function but to use H(x) function as a smooth approximation of the indicator function I. Therefore, the smoothed ranks can be generated from the

$F_s(t)$ function. Moreover, it should be noted that when $x_i > X_j$, $H\left(\frac{x_i - X_j}{h}\right) \to 1$ when h approaches zero as n gets large. Similarly, when $x_i < X_j$, $H\left(\frac{x_i - X_j}{h}\right) \to 0$

when h approaches zero as n gets large.

Also, it should be noted that

$$H_n(X_{(i)}) = \frac{i - 1}{n} + \frac{X_{(i)} - V_i}{n(V_{i+1} - V_i)} \qquad (18)$$

where X(i) is the ith order observation and Vi is the midpoints between Xi and Xi+1. Also, V1 = X1 — (V2 — X1) and Vn+1 = Xn + (Zn — Vn).

As a result of the smoothing and replacing the indicator function I with H(x/h), we now obtain the smoothed ranks using smoothed distribution function of Fs(Xi) and write them as

$$\widehat{R(X_i)} = nF_s(X_i) \tag{19}$$

and

$$\widehat{R(Y_i)} = nF_s(Y_i) \tag{20}$$

Then using these smoothed ranks, we rewrite the smoothed score function $a(\widehat{R(X_i)}) = \sqrt{12}\left[\widehat{R(X_i)}/(n+1) - 1/2\right]$, and similarly, $a(\widehat{R(Y_i)}) = \sqrt{12}\left[\widehat{R(Y_i)}/(n+1) - 1/2\right]$.

We can plug in the new scores into r_a as defined in Eq (10) to derive the new version of Spearman's correlation estimator based on the smoothed Wilcoxon score function,

$$r_{sa} = \frac{1}{s_a^2}\sum_{i=1}^{n} a(\widehat{R(X_i)})a(\widehat{R(Y_i)}). \tag{21}$$

where $s_a^2 = \sum_{i=1}^{n} a^2(i) = \frac{n(n-1)}{n+1}$ with using Wilcoxon's score function. A further simplification of the above results gives

$$r_{sa} = \frac{\sum_{i=1}^{n}[(\widehat{R(X_i)} - (n+1)/2)(\widehat{R(Y_i)} - (n+1)/2)]}{n(n^2-1)/12}. \tag{22}$$

Moreover, we can replace $R(X_i)/(n+1) = \frac{n}{n+1}F_n(X_i)$ by $H_n(X_{(i)})$ and $R(Y_i)/(n+1) = \frac{n}{n+1}F_n(Y_i)$ by $H_n(X_{(i)})$ and $H_n(Y_{(i)})$, respectively. Then,

$$a(\widehat{R(X_i)}) = \sqrt{12}\left[H_h(X)_i - 1/2\right] \tag{23}$$

Similarly,

$$a(\widehat{R(Y_i)}) = \sqrt{12}\left[H_h(Y)_i - 1/2\right] \tag{24}$$

Then we can plug in these result into r_a and found the new version of the Spearman correlation coefficient estimator based on kernel function

$$r_{sa} = \frac{\sum_{i=1}^{n}[(H_n(X_{(i)}) - (n+1)/2)(H_n(Y_{(i)}) - (n+1)/2)]}{n(n^2-1)/12}. \tag{25}$$

Bandwidth Selection

It is well known that the selection of bandwidth h is more important than the shape of the kernel function H(x) as stated by Sheather (2004) and Silverman (1986). As a result, an appropriate selection of a bandwidth h must be con- sidered in order to obtain a smoothed approximation of the indicator function.

There are many bandwidth h options available in the smoothing applications but the bandwidths suggested by Silverman (1986), Sheather and Jones (1991), and Bowman (1984) are options to be considered because of their common us- age, high performance and also availability in R software. The bandwidth sug- gested by Silverman (1986) is also called the ”rule of thumb” bandwidth approach in the literature. It has a numerical formula that equals to h = 0.9σˆn—0.20, where σˆ= min{s, IQR/1.349}. Bowman (1984) suggests a least squares cross-validation approach and it is also called an unbiased cross-validation smoother in the literature. Finally, Sheather-Jones's plug-in bandwidth approach is also commonly used and considered to be a good performer as suggested by Simonoff (1996). The last two options have no closed form formulas. Tasdan and Yeniay (2014) discussed these three bandwidth options for the smoothed version of the Kolmogorov-Smirnov statistic, and has shown all three bandwidth options per- formed similarly in estimating the shift parameter for location problems. Moreover, we also considered the smoothing parameter h = σˆn—0.26 as suggested by Heller (2007). It also satisfies nh → ∞ and nh4 → 0 conditions in order to have an optimal rate of convergence. The σˆ is the estimated pooled standard deviation from the data. For robustness purposes, we consider using MAD (Me- dian Absolute Deviation) to estimate σ instead of a regular standard deviation approach. Comparative study of these four different bandwidth parameters will be discussed in detail later in Section 5.

Monte Carlo Simulation Study

In this section, Monte Carlo Simulation has been performed and the Mean Square Error (MSE) of the ρ parameter for each estimator r is computed. The MSE is formulated as MSE(r) = (r — ρ)2 + V (r) which is found by average of bias2 and adding the variance of correlation estimator(r). First, two random samples of size n = 50 are generated from a bivariate normal distribution with parameters μ1 = 2, μ2 = 4, σ1 = 1, σ2 = 1 and with the correlation parameter ρ which is set to the sequence of ρ = {0, 0.02, 0.04, ..., 1}. For each parameter value of the ρ, Person's rp, Spearman's rs, Kendall's rk and the proposed smoothed ranked based rsa estimator are used for estimating the MSE values. The Figure 2 shows the result of the bivariate normal distribution which is defined below.

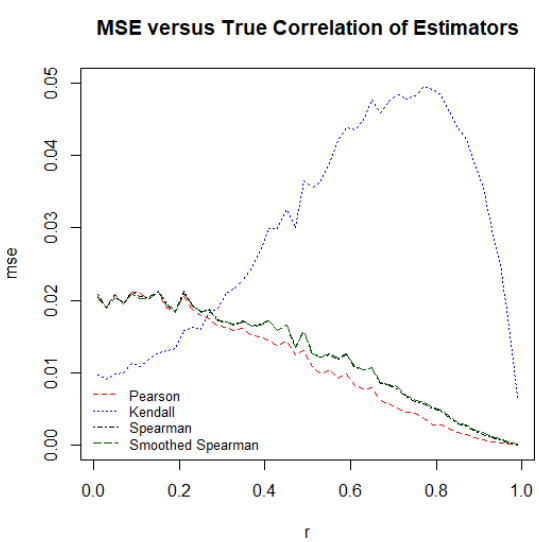
$$f_{XY}(x, y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left(x^2 - 2\rho xy + y^2\right)\right\}$$

where $-1 \leq \rho \leq 1$.

The Figure 3 is generated from the bivariate exponential distribution, also known as Farlie-Gumbel-Morgenstern distribution, is given by

$$F(x, y) = F_1(x)F_2(y)[1 + \rho(1 - F_1(x))(1 - F_2(y))] \qquad (26)$$

for x ≥ 0 and y ≥ 0 and the marginal distribution functions F1 and F2 are exponential with scale parameters θ1 and θ2 and correlation parameter ρ where—1 ≤ ρ ≤ 1. It is important to point out that the marginal distributions of X and Y are exponential with parameters α and β.



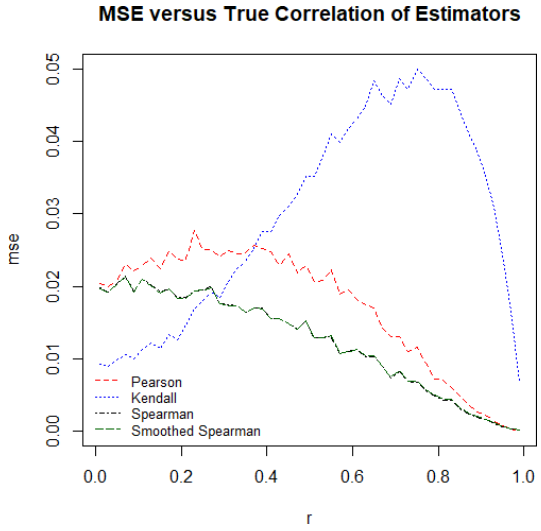


Figure 2: Estimation of MSEs of ρ using Bivariate Normal Distribution

Figure 3: Estimation of MSEs of ρ using Bivariate Exponential Distribution

As it can be seen from the Figure 2 under the assumption that X and Y ran- dom samples are coming from a bivariate normal distribution, Kendall's rk gives lower MSEs when about $\rho < 0.30$ but it worsens when ρ approaches 1. On the other hand, Pearson's rp performs the best overall, gives minimun MSEs when ρ approaches 1. Moreover, the proposed smoothed rank and Spearman estimates MSE estimates exactly overlaps. Therefore, if we compare the all estimators, Pearson's rp performs the best under the bivariate normality whereas Spearman and the smoothed ranked estimates perform similarly and looks very stable. On the other hand, Kendall's rk is the worst performer as shown by the Figure 2. As the Figure 3 shows that if we assume X and Y are coming from a bivariate exponential distribution, then proposed smoothed rank and Spearman's correla- tion estimates performs the best overall. Pearson's MSE values are higher than Spearman and the smoothed rank based estimates under bivariate exponential case. This is expected since Pearson known to be a good estimator under nor- mality or linear relationships. Kendall's rk shows a fluctuating MSEs as shown previously under the bivariate normal case.

In the second part of the simulation, the relative efficiencies of the smoothed ranked rsa with respected to the other estimators are compared based on the ratios of the estimated MSEs. The results are presented in the Table 1. The most obvious result is that the proposed smoothed rank's rsa estimator performs very similar to the Spearmans's rs method since the relative efficiencies are very close to 1 which is expected since we saw that similarity on the Figure 2. Pearson shows superiority over the other estimators if the underlying distribution is bivariate normal since the relative efficiency results as ρ goes to 1. Between smoothed and Kendall's estimator, Smoothed rank based estimator has increasing relative efficiencies as ρ increases.

Table 1: Relative efficienct rates of the ρ estimators basen on MSEs under bi- variate normal

Correlation(ρ)	Pears-Smoot	Kendal-Smoot	Spear-Smoot	Pears-Spear
0	0.9931	0.4690	1.0044	0.9888
0.25	0.9758	0.9021	1.0053	0.9707
0.50	0.8644	2.4973	1.0074	0.8581
0.75	0.6583	7.6484	1.0092	0.6523
0.95	0.3530	38.7826	0.9981	0.3537

Above simulation is repeated under the bivariate exponential distribution con- dition and results are presented in Table 2.

As we see Table 2, Smoothed rank's rsa and Spearman's rs performs equally as ρ approaches 1. Both are better than the Pearson's estimator especially when ρ is around 0.50—0.75. The proposed smoothed rank based estimator performs better than the Kendall since relative efficiencies goes as high as 40 as ρ approaches 1.

Table 2: Relative efficienct rates of the ρ estimators basen on MSEs under bi- variate exponential

Correlation(ρ)	Pears-Smoot	Kendal-Smoot	Spear-Smoot	Pears-Spear
0	1.0064	0.4669	1.0024	1.0040
0.25	1.3173	0.9166	1.0057	1.3098
0.50	1.6154	2.5556	1.0065	1.6050
0.75	1.7028	7.8280	1.0131	1.6807
0.95	1.1578	40.9222	1.0479	1.1049

CONCLUSION

The Spearman's rank correlation coefficient (ρ) is a nonparametric measure that assesses the strength and direction of association between two ranked variables. Tradionality, *p* is calculated 1- 6 – d^2 / a(n-1) where a_i is the difference between the ranks of corresponding variables x and y, and n is the number of observations. However, this method can be sensitive to the exact rank assignments, particularly when there are ties or the data is noisy.

To address these issues, kernel smoothing of ranks can be employed in the Wilcoxon's linear score function which would enhance the robustness and sen- sitivity of Spearman's correlation under several conditions as mentioned below.

Kernel smoothing transforms the ranks $R_x(i)$ and $R_y(i)$ into smoothed ranks $RS_x(i)$ and $RS_y(i)$ using the formula $RS_x(i) = \sum_{j=1}^{n} K\left(\frac{R_x(i) - R_x(j)}{h}\right) R_x(j)$ and

similarly for $RS_y(i)$, where K is the kernel function and h is the bandwidth parameter. This smoothing mitigates the impact of rank ties and small rank perturbations.

The advantages of using smoothed ranks in Spearman's correlation are man- ifold. First, they provide a more nuanced understanding of the relationship be- tween variables, particularly in the presence of noisy or tied data. Second, the approach is more robust to outliers and non-linear relationships. Third, it allows for a more flexible interpretation of ranks, accommodating the specific character- istics and distributions of the data. Finally, the proposed method can yield more accurate and meaningful insights into the underlying monotonic relationships, especially in complex datasets where traditional methods might falter.

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